Supplemental Information to "Dynamic pie: A strategy for modeling trade-offs in compositional variables over time"

1 Robustness Check: Monte Carlo Experiments

The classic error correction setup is one in which both the independent variable and the dependent variable are non-stationary and have the same order of integration, usually I(1). As we point out on page 8 of the paper, it is possible for the categories of the same dynamic compositional dependent variable to have different orders of integration.[1] When this is the case, the use of an error correction framework might lead to temporal autocorrelation.[2] In particular, we were interested in the robustness of our models across the following conditions:

- one or more of the original series for categories of the dependent variable being I(0) instead of I(1)
- whether the baseline category is I(0) or I(1)
- different levels of temporal autocorrelation in the original series for categories of the dependent variable that are I(0)
- different levels of contemporaneous error correlation[3]

To test the robustness of our models to different combinations of stationary and non-stationary series in terms of the resulting residual temporal autocorrelation in our models, we performed a series of Monte Carlo experiments. For each variation in our experiments, we simulated 10,000 sets of five series from a dynamic compositional variable, y_{jt} ($j = 1,2,3,4,5$), each with 100 observations from the following data-generating process:

$$y_{jt} = \beta x_t + u_{jt} \tag{1}$$

[1] In our empirical work, we have only ever encountered mixtures of series that were I(0) and I(1). Although higher orders of integration are certainly possible and worthy of further investigation, we confine the present discussion and analyses to these cases.

[2] This is likely to be the case because we are over-differencing an I(0) series which can lead to a non-invertible moving average.

[3] One of the advantages of the SUR framework is that it takes advantage of contemporaneous error correlation to provide more efficient estimates.

where:

$$u_{jt} = \rho_{jj} u_{jt-1} + v_{jt} \tag{2}$$

indicating that the error process is temporally autocorrelated for $0 <\mid \rho_{jj} \mid< 1$ and/or

$$u_{jt} = \rho_{ij} u_{it} + v_{jt} \forall i \neq j \tag{3}$$

indicating that the error process is contemporaneously correlated unless $\rho_{ij} = 0$. We set $\beta = 2$, and the exogenous variable x_t is an I(1) unit root process.[4] The term v_{jt} is an error term that meets the usual OLS assumptions. We simulated the five y_{jt} series to be either I(1) or I(0).

We varied the simulated scenarios as follows:

1. Keeping the "baseline" category ($j = 5$) always as an I(1) series
   - varying the number of non-baseline, non-unit root series, I(0), from 0 to 4
   - series that are I(0) were set to have a range of temporal autocorrelation values, from $\rho_{jj} = 0.0, 0.2, 0.4, 0.6, 0.8$
   - contemporaneous correlation of errors across all series in a composition were set to the same value, from $\rho_{ij} = 0.0, 0.2, 0.4, 0.6$
2. Keeping the "baseline" category ($j = 5$) always as an I(0) series
   - varying the number of non-baseline, non-unit root series, I(0), from 0 to 4
   - series that are I(0) were set to have a range of temporal autocorrelation values, from $\rho_{jj} = 0.0, 0.2, 0.4, 0.6, 0.8$
   - contemporaneous correlation of errors across all series in a composition were set to the same value, from $\rho_{ij} = 0.0, 0.2, 0.4, 0.6$

[4] Note that the independent variable is not indexed by j, indicating that it is a common regressor across each y_j, as are the independent variables in our models in the main paper.

Following the strategy outlined in our paper, we then created four logged ratio variables (our compositional dependent variables), s_{jt}, and estimated error-correction models using a seemingly unrelated regression setup. We used the Harvey test for global autocorrelation (Judge et al. 1985) to examine the conditions under which autocorrelation threatens our inferences from the compositional variables.[5] The test is a modified form Lagrange-multiplier statistic given as

$$Harvey\ LM = T \sum_{j=1}^{s} \hat{\rho}_{jj}^2 \tag{4}$$

where the test statistic is asymptotically distributed $\chi^2(s)$, and in our case has four degrees of freedom ($s = 4$) since we estimate four equations in our SUR. The test examines the joint hypothesis of whether residual temporal autocorrelation exists in the system (H_0 =no autocorrelation remains in the system).

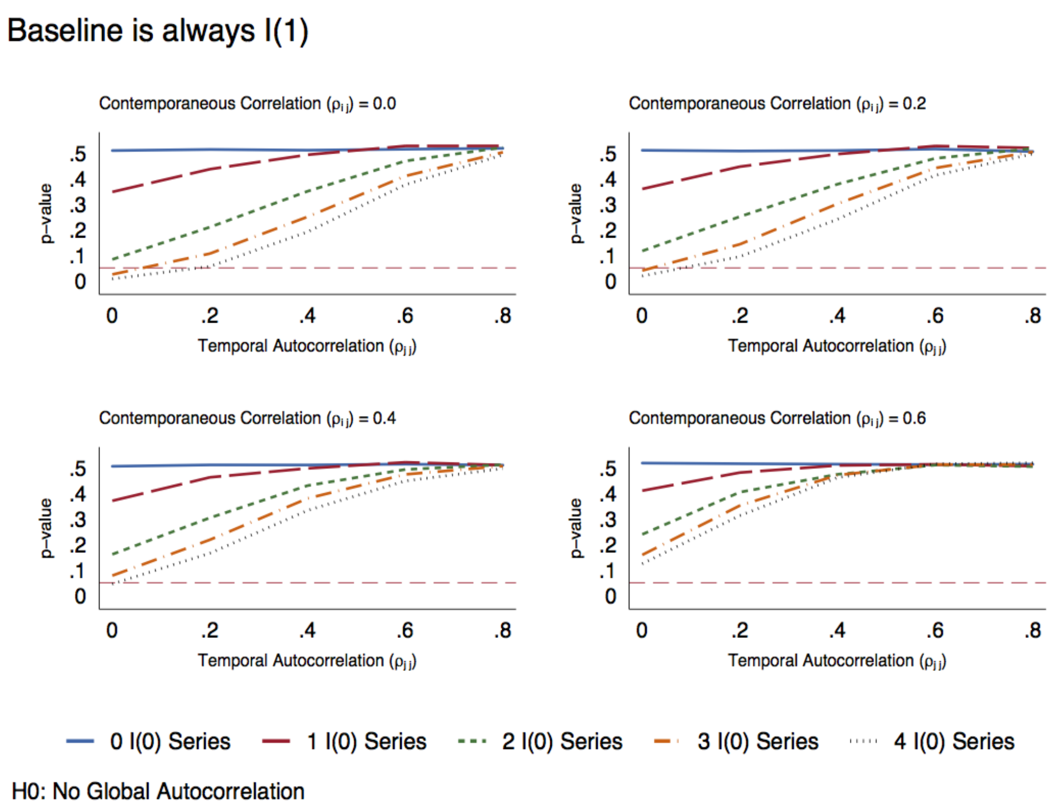

Figure S 1: Rejection Rates From Harvey LM Test for No Global Autocorrelation, Baseline is I(1)

[5] This test is implemented in Stata with the program **lmanlsur** by Shehata (2012).

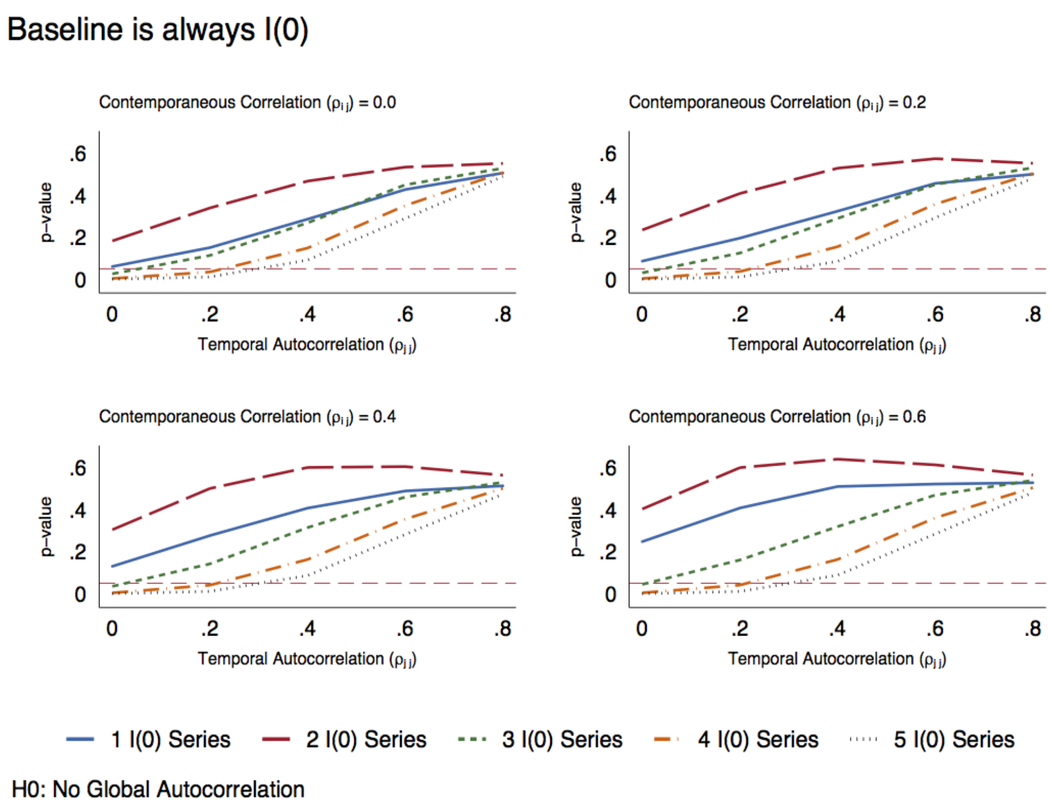

Figure S 2: Rejection Rates From Harvey LM Test for No Global Autocorrelation, Baseline is I(0)

The results from our Monte Carlo experiments are presented in Figures S 1 and S 2 which correspond to the scenarios #1 and #2 above. The plots in Figure S 1 depict the p-values for the Harvey test from simulations where the baseline is always I(1) while the plots in Figure S 2 depict the p-values for the Harvey test from simulations where the baseline is always I(0). In each figure we have four different plots reflecting varying levels of contemporaneous correlation (ρ_{ij}). Within each of the plots, each line represents the number of series that are I(0) and the horizontal axis is the level of temporal autocorrelation, ρ_{jj} for each of the I(0) series in that plot. The vertical axis is the average p-value for the simulations. P-values below the dashed horizontal line at 0.05 indicate a rejection of the null of no global autocorrelation at conventionally-accepted levels.

The four plots in Figure S 1 indicate that if the baseline category (the denominator when the logged ratios are calculated) is I(1), we are almost always *unable* to reject the null hypothesis of no global autocorrelation. Evidence of temporal autocorrelation (rejection of the null) only occurs when there are more series that are I(0) than I(1), when ρ_{jj} is very low, and when the contemporaneous correlation (ρ_{ij}) is low.

The four plots in Figure S 2 indicate that if the baseline category is I(0) there are somewhat greater threats from residual autocorrelation. As was the case in Figure S 1, all of the evidence of temporal autocorrelation occurs when there are more series that are I(0) than I(1). We can also see that the choice of a baseline category that is I(0) leads to evidence of autocorrelation at higher levels of ρ_{jj} than when the baseline category is I(1) and that the variation in contemporaneous correlation has less of an effect when the baseline category is I(0).

Our Monte Carlo experiments suggest that, as long as the number of I(1) series in a composition is greater than the number of I(0) series, temporal autocorrelation is unlikely to be a problem. They also suggest that one should choose an I(1) series as the baseline category.

To see how our two examples from the main paper fit into the findings from the Monte Carlo simulations above, we performed unit root tests on each of the series in the compositions. Tables S 1 and S 2 show the results from augmented Dickey-Fuller and Phillips-Perron tests on both the

Table S 1: Unit Root Tests for UK party support series

	Aug. D-F Z(t)-stat.		P-P Z(t)-stat.	
	Un-Differenced	Differenced	Un-Differenced	Differenced
Conservatives	-2.32	-9.55*	-2.27	-9.64*
Labour	-2.21	-7.94*	-2.41	-7.95*
Lib. Dems.	-2.88*	-9.18*	-2.72	-9.47*

Note: Augmented Dickey Fuller Z(t) test statistics. * $p < .05$. Phillips-Perron Z(t) test statistics. * $p < .05$.

differenced and un-differenced series for UK party support and US budget examples. In Table S 1 for the un-differenced series, we are able to reject the null hypothesis that the series contains a unit root at the 0.05 level only for the Liberal Democrats, and only when using the Dickey-Fuller test. We do not use the Liberal Democrats as a baseline category and the other two series for this dependent variable are clearly I(1). In the US budget series, we were never able to reject the null hypothesis of a unit root for any of the un-differenced spending variables, across either the Dickey-Fuller or Phillips-Perron tests. This suggests that our model is less likely to experience problems of residual autocorrelation than what was found under certain conditions in the Monte Carlo analyses. All of the series were made stationary by first-differencing, as shown in Table S 2.

Table S 2: Unit Root Tests for the US budget series

	Aug. D-F Z(t)-stat.		P-P Z(t)-stat.	
	Un-Differenced	Differenced	Un-Differenced	Differenced
Defense	-1.71	-5.68*	-1.69	-5.71*
Income	-2.24	-8.98*	-2.18	-8.86*
Social Sec.	-1.82	-4.23*	-1.77	-4.31*
Interest	-0.42	-3.79*	-0.93	-3.83*
Other	-0.57	-5.51*	-0.69	-5.45*

Note: Augmented Dickey Fuller test statistics. * $p < .05$. Phillips-Perron Z(t) test statistics. * $p < .05$.

Finally, we performed the Harvey LM test on each of our models. As in our main paper, the baseline category for the UK was Labour, and defense spending was the baseline category for the

US. The test returned a χ^2 statistic of 0.06 for the UK data, and 4.85 for the US data.[6] Since we cannot reject the null hypothesis of no global autocorrelation for both the US and the UK, and we have investigated the conditions under which this does occur (as shown in Figure S 2), we conclude that autocorrelation poses little threat to our inferences.

To summarize, we list the recommendations of our Monte Carlo analysis in regards to the characteristics of the compositional dependent variables and the threat of remaining temporal autocorrelation:

- A situation in which all categories/series of the dependent variable exhibit I(1) characteristics is preferable to having a mixture of I(1) and I(0) series.
- If not all categories are I(1), the user should take care to make sure the baseline category is one of the I(1) categories.
- For those categories that are not I(1), higher levels of autocorrelation in the series results in a lower likelihood of remaining global autocorrelation after the estimation.
- The higher the level of contemporaneous correlation across the series, the lower the likelihood that remaining global autocorrelation poses a threat to inference, especially when the baseline category is an I(1) series.

[6] The p-value of the UK global LM test was 0.97. Running LM tests on the single equations also indicated there was no remaining autocorrelation. For the US models, the p-value of the LM test was 0.30. In addition, Harvey LM tests on the single equations in the US model fail to reject the null hypothesis of no autocorrelation for all equations, although the equation $\frac{interest}{defense}$ is borderline at a p-value of 0.051.

2 Explanation of Program "dynsim"

The modeling strategy proposed in our paper has thus far been implemented in two Stata .do files that will be available on our website along with replication data.[7] The first .do file, **dynsimpie_AJPS_US**, is a replication file of our results for the US section of the paper. Since we are dealing with a compositional dependent variable that have five categories, this program was written to handle sets of equations. After the program has been called into Stata, and the user has used the **tlogit** command available in Clarify to transform the dependent variables into compositions (Tomz, Wittenberg, and King 2003), the **dynsimpie** program can be run as the following:

dynsimpie [indepvars] **, dvs(**[depvars]**) time(**10**) shock( ) shockvar(**[shock vars]**) dummy1( ) sig( )**

where

- The names of the un-differenced, un-lagged independent variables come right after the command.
- The option **dvs( )** is required and is comprised of all the compositional dependent variables created in **tlogit**.
- The required option **time( )** is the time when the shock to the system occurs. Since the dynamic simulation is across 20 time points, we find that a shock at time between 5 and 10 works best.
- The required option **shock( )** is the value of the shock to the system that the independent variable specified in **shockvar( )** takes on. Note that the variable in **shockvar( )** was not included in the earlier set of independent variables.

[7] As we discuss below, we have several additional pieces of software under development that we will make publically available at the time of publication.

The last parts of the command are optional:

- **dummy1( ) dummy2( ) dummy3( )** allow for up to three dummy variables.
- **sig( )** allows the user to change the confidence intervals from the default setting of 95 percent.

The **dynsimpie_AJPS_UK** program is very similar to above, with only slight differences. First, since there are three categories of the dependent variable in our analysis, this program can handle two compositional trade-offs. Second, we allow for up to three shocks at the same time period through the use of **shockvar( ) shockvar2( ) shockvar3( )** that take on the value of **shock( ) shock2( ) shock3( )**. As with the US, there must always be one shock variable specified.

2.1 Software Currently in Development

As is apparent from the discussion in this section, we currently have separate Stata .do files that implement our modeling strategy depending on the number of categories of the dependent variable. We are currently in the final stages of writing a single .ado file that will work for compositional dependent variables with as many ten categories. This .ado file will also include an option for conducting out-of-sample forecasts in which uncertainty about the predicted values of the dependent variable is incorporated and thus compounded into future predicted values. Upon acceptance for publication, we will make all of our Stata code available as both .do and .ado files on the website of the corresponding author. At this time, we are developing several additional software tools. These include a stand-alone version of our current Stata code that does not rely on Clarify and an R version of our software. We plan to make these programs available on the website of the corresponding author.

3 Compositional Independent Variables in the UK Party Choice Models

As mentioned in footnote 22 of the paper, in the UK models, one of our independent variables, "party best on most important issue", is a compositional variable. In the paper we show an increase to Labour as best manager of the most important issue with a corresponding decrease evenly split between the Conservatives and Liberal Democrats.[8] This "ratio-preserving counterfactual" is recommended by Adolph (2013, 108) so as not to conflate a change in our variable of interest (a one standard deviation increase in Labour as best manager of the economy) with alternative effects given by a disproportionate reallocation among the remaining alternative variables in the composition (Liberal Democrats and Conservatives).

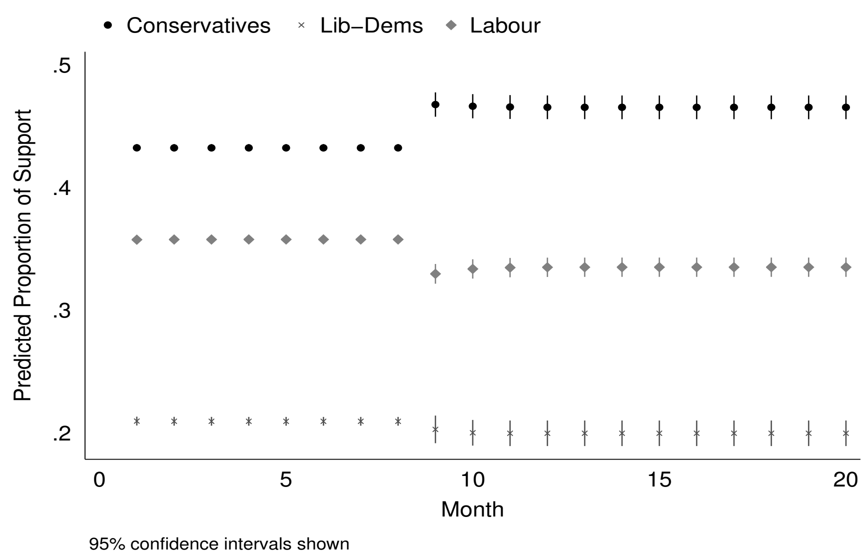

Figure S 3: 1 Standard Deviation Decrease in Labour as Best Manager of the Most Important Issue, All of Which Goes to the Conservatives

In this section we show two example figures where we move away from the ratio-preserving counterfactual and toward some other substantively interesting changes. To do so we now include

[8]This is true since by including only the variable for Labour, we are in effect comparing it to all other categories (Liberal Democrats and Conservatives) lumped together.

three independent variables measuring the percentage of respondents who think that each major party is the party best for the most important issue facing the nation, one for Labour, one for the Conservatives, and one for the Liberal Democrats. This is shown in Figure S 3, where a one standard deviation decrease in the opinion that Labour is the best party to handle the most important problem facing the nation is met with a corresponding rise in the Conservative leader evaluation at time $t = 9$. No change is made to the variable for the Liberal Democrats. From this figure, it is clear that the Conservative support takes an instantaneous, statistically significant increase at the cost of Labour. Support for the Liberal Democrats rises slightly, but this rise does not achieve statistical significance.

In Figure S 4, two-thirds of the increase caused by a one standard deviation decrease in Labour at the best party to handle the most important problem facing the nation goes the the Conservatives, and one-third to the Liberal Democrats. The results are similar to Figure S 3, except now there is a large short-run drop in support for Labour that benefits both the Liberal Democrats and the Conservatives, and now the Liberal Democrat proportion of support is statistically significantly higher than before the increase at time $t = 9$.

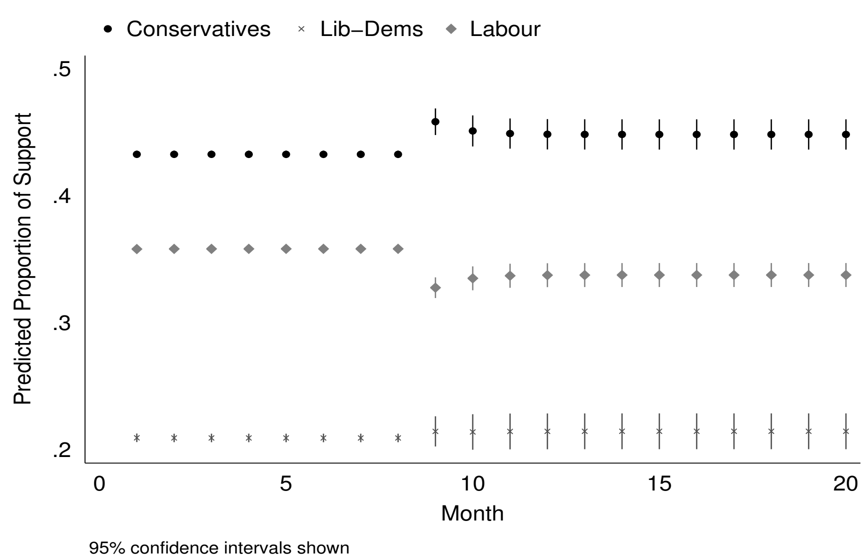

Figure S 4: 1 Standard Deviation Decrease in Labour as Best Manager of the Most Important Issue, $\frac{2}{3}$ of Which Goes to the Conservatives, and $\frac{1}{3}$ to the Liberal Democrats

4 US Budgetary Data

As footnote 28 describes, in the US budgetary section we disaggregated the budget into five categories: defense, social security, welfare (income security), interest on debt payments, and other. The categories we analyze consist of the following categories from the original data:

- National defense consists of spending on the department of defense: military, atomic energy defense activities, and other defense-related activities

- Social security consists of social security spending

- Income security (or welfare) consists of general retirement and disability insurance (excluding social security), federal employee retirement and disability, unemployment compensation, housing assistance, food and nutrition assistance, and other forms of income security

- Debt payment consists of payment of interest on the public debt, interest received by on-budget trust funds, interest received by off-budget trust funds, other interest, and other investment income

- Other consists of all other spending not accounted for by the above categories. It contains 15 general categories: international affairs; general science, space, and technology; energy; natural resources and environment; agriculture; commerce and housing credit; transportation; community and regional development; education, training, employment, and social services; health; medicare; income security; veterans benefits and services; administration of justice; and general government.

As mentioned in footnote 31, Table S 3 contains a more detailed listing of our theoretical expectations for the impact of an increase in each independent variable on the trade-offs between each category of dependent variables. For example, we believe a Democratic president's liberal ideology will increase welfare spending relative to defense. An expected increase in spending for the numerator relative to the denominator is represented by a +, while the opposite effect is represented by a –. Independent variables for which we have no expectations are labeled by $\approx$.

Table S 3: Theoretical Expectations for the US

Independent Variable	Theoretical Expectation	Justification
Dependent Variable Pairing: $\frac{Defense}{Welfare}$		
Democratic President	–	Guns vs. Butter: More liberal ideology leads to more welfare spending than defense
Democratic Legislature	–	Guns vs. Butter: More liberal ideology leads to more welfare spending than defense
Natural Unemployment	–	More concern on welfare spending than defense
MIDs	+	More conflict leads to more defense spending
% change in GDP	+	GDP growth leads to more defense spending relative to welfare
Population Growth	–	Increasing population leads to greater welfare demand relative to defense
Old Age Dependency Ratio	≈	
Policy Mood	–	Guns vs. Butter: More liberal ideology leads to more welfare spending than defense
Dependent Variable Pairing: $\frac{Other}{Welfare}$		
Democratic President	≈	
Democratic Legislature	≈	
Natural Unemployment	–	Greater unemployment leads to more welfare spending relative to other
MIDs	≈	
% change in GDP	+	GDP growth leads to less welfare dependency, other may be slightly higher
Population Growth	≈	
Old Age Dependency Ratio	≈	
Policy Mood	–	Both areas may benefit, but liberals likely prefer welfare spending relative to others
Dependent Variable Pairing: $\frac{Social\ Security}{Welfare}$		
Democratic President	–	More liberal ideology prefers welfare spending to Soc. Sec.
Democratic Legislature	–	More liberal ideology prefers welfare spending to Soc. Sec.
Natural Unemployment	–	Greater unemployment leads to more welfare spending relative to Soc. Sec.
MIDs	≈	
% change in GDP	+	GDP growth leads to less demand for welfare; maybe a small demand for Soc. Sec.
Population Growth	+	Increasing population leads to greater welfare demand relative to Soc. Sec.
Old Age Dependency Ratio	+	Higher dependency leads to greater social security demand,

		less welfare need
Policy Mood	$-$	More liberal mood prefers welfare spending to Soc. Sec.
Dependent Variable Pairing: $\frac{Debt\ Payment}{Welfare}$		
Democratic President	$-$	More liberal ideology prefer welfare spending over debt payments
Democratic Legislature	$-$	More liberal ideology prefer welfare spending over debt payments
Natural Unemployment	$-$	Greater unemployment drives welfare up at the cost of decreased debt payments
MIDs	$\approx$	
% change in GDP	$+$	GDP growth allows for debt repayment, less need for welfare
Population Growth	$+$	Growing population allows for payment of debt, relative to welfare
Old Age Dependency Ratio	$\approx$	
Policy Mood	$-$	More left-leaning mood prefer welfare over debt payments
Dependent Variable Pairing: $\frac{Other}{Defense}$		
Democratic President	$+$	Unclear, but liberal ideologies likely prefer other social spending relative to defense
Democratic Legislature	$+$	Unclear, but liberal ideologies likely prefer other social spending relative to defense
Natural Unemployment	$\approx$	
MIDs	$-$	More conflict leads to more defense spending relative to other
% change in GDP	$\approx$	
Population Growth	$\approx$	
Old Age Dependency Ratio	$\approx$	
Policy Mood	$+$	Unclear, but liberal mood likely prefer other social spending relative to defense
Dependent Variable Pairing: $\frac{Other}{Social\ Security}$		
Democratic President	$+$	Likely rise in both, but greater rise in other social spending relative to Soc. Sec.
Democratic Legislature	$+$	Likely rise in both, but greater rise in other social spending relative to Soc. Sec.
Natural Unemployment	$\approx$	
MIDs	$\approx$	
% change in GDP	$\approx$	
Population Growth	$\approx$	
Old Age Dependency Ratio	$-$	Greater dependency leads to greater social security demand relative to other

Policy Mood	+	Likely rise in both, but greater rise in other social spending relative to Soc. Sec.
Dependent Variable Pairing: $\frac{Other}{Debt\ Payment}$		
Democratic President	+	Greater other social spending relative to debt payment with liberal ideology
Democratic Legislature	+	Greater other social spending relative to debt payment with liberal ideology
Natural Unemployment	≈	
MIDs	≈	
% change in GDP	−	Economic growth allows greater debt repayment relative to other spending
Population Growth	+	Population growth increases demand for other social spending, less demand for debt repayment
Old Age Dependency Ratio	≈	
Policy Mood	+	Greater other social spending relative to debt payment with liberal mood
Dependent Variable Pairing: $\frac{Defense}{Social\ Security}$		
Democratic President	−	Guns vs. Butter: liberal spends more on Soc. Sec. than defense
Democratic Legislature	−	Guns vs. Butter: liberal spends more on Soc. Sec. than defense
Natural Unemployment	≈	
MIDs	+	More conflict leads to more defense relative to Soc. Sec.
% change in GDP	≈	
Population Growth	≈	
Old Age Dependency Ratio	−	Greater dependency leads to greater social security demand relative to defense
Policy Mood	−	Guns vs. Butter: liberal spends more on Soc. Sec. than defense
Dependent Variable Pairing: $\frac{Defense}{Debt\ Payments}$		
Democratic President	≈	
Democratic Legislature	≈	
Natural Unemployment	≈	
MIDs	+	More conflict leads to more defense spending, debt payment comes secondary
% change in GDP	≈	
Population Growth	≈	
Old Age Dependency Ratio	≈	
Policy Mood	≈	
Dependent Variable Pairing: $\frac{Social\ Security}{Debt\ Payments}$		
Democratic President	+	More liberal ideology leads to more Soc. Sec. spending,

		less need for debt repayments
Democratic Legislature	+	More liberal ideology leads to more Soc. Sec. spending, less need for debt repayments
Natural Unemployment	≈	
MIDs	≈	
% change in GDP	≈	
Population Growth	≈	
Old Age Dependency Ratio	+	Higher dependency leads to greater social security demand relative to debt
Policy Mood	+	More liberal mood leads to more Soc. Sec. spending, less need for debt repayments

+: spending on numerator relative to denominator. – : spending on denominator relative to numerator.
≈: no prior theoretical expectation.

The parameter estimates used to conduct our graphical simulations for the US section are shown in Table S 4. Notice that with five outcomes for our dependent variable we now have to display 10 columns of results in order to show every possible pairwise trade-off.

Table S 4: Estimated short-run and long-run effects on US Budget Spending

	$\frac{Other}{Defense}$	$\frac{Income}{Defense}$	$\frac{SocialSecurity}{Defense}$	$\frac{Interest}{Defense}$	$\frac{Other}{Income}$	$\frac{SocialSecurity}{Income}$	$\frac{Interest}{Income}$	$\frac{Other}{SocialSecurity}$	$\frac{Interest}{SocialSecurity}$	$\frac{Interest}{Other}$
Population	0.288	0.050	0.247	0.583**	0.147	0.225	0.510*	-0.060	0.254	0.300
Growth, Short	(0.178)	(0.255)	(0.131)	(0.174)	(0.200)	(0.186)	(0.229)	(0.122)	(0.146)	(0.185)
Population	0.804	-0.739	0.584	1.560**	0.756*	1.124*	2.882**	-0.129	1.178*	1.334
Growth, Long	(0.880)	(0.937)	(0.340)	(0.482)	(0.351)	(0.495)	(1.114)	(0.216)	(0.721)	(0.723)
Hostility	-0.001	-0.004	0.0002	-0.002	0.002	0.004	0.002	-0.002	-0.003	-0.001
Level, Short	(0.002)	(0.004)	(0.002)	(0.002)	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)	(0.031)
Hostility	-0.007	-0.046	-0.008	0.004	0.015	0.027	0.057	-0.004	0.011	0.014
Level, Long	(0.031)	(0.037)	(0.012)	(0.016)	(0.013)	(0.018)	(0.038)	(0.008)	(0.027)	(0.027)
National	0.049	0.188**	0.003	-0.022	-0.146**	-0.186**	-0.222**	0.040*	-0.038	-0.083**
Unemployment, Short	(0.029)	(0.043)	(0.021)	(0.031)	(0.033)	(0.031)	(0.038)	(0.021)	(0.024)	(0.031)
National	-0.069	0.189	-0.107	-0.122	-0.165**	-0.277**	-0.416*	0.076	-0.043	-0.159
Unemployment, Long	(0.159)	(0.171)	(0.064)	(0.091)	(0.063)	(0.090)	(0.199)	(0.043)	(0.133)	(0.137)
Old Age	-33.33	-14.66	-15.672	55.061*	-26.748	6.068	68.137*	-32.206*	59.495**	91.819**
Dependency, Short	(23.74)	(34.16)	(17.57)	(23.001)	(26.312)	(25.558)	(30.433)	(15.944)	(19.271)	(24.451)
Old Age	34.30*	43.862**	35.615**	36.015**	-13.546**	-3.018	-13.682	-13.396**	-5.035	0.263
Dependency, Long	(14.120)	(14.723)	(4.825)	(6.695)	(4.926)	(7.210)	(15.187)	(4.452)	(11.086)	(12.952)
Mood,	0.006	-0.010	-0.013*	-0.017*	0.015	-0.003	-0.008	0.017**	-0.004	-0.023**
Short	(0.007)	(0.011)	(0.006)	(0.007)	(0.009)	(0.008)	(0.010)	(0.005)	(0.006)	(0.008)
Mood,	-0.033	-0.016	-0.056**	-0.110**	0.009	-0.032	-0.155*	0.028*	-0.102*	-0.133**
Long	(0.047)	(0.052)	(0.019)	(0.035)	(0.019)	(0.028)	(0.075)	(0.014)	(0.051)	(0.050)
% GDP	0.952	3.636**	0.872	0.550	-2.941**	-2.589**	-3.373**	-0.453	-0.702	-0.552
Change, Short	(0.818)	(1.184)	(0.601)	(0.826)	(0.923)	(0.867)	(1.078)	(0.616)	(0.674)	(0.871)
% GDP	14.510	32.354	4.212	5.426	-12.667*	-16.428*	-33.025	-0.943	-6.135	-6.520
Change, Long	(14.722)	(19.340)	(4.920)	(6.338)	(5.244)	(7.960)	(19.655)	(3.440)	(11.672)	(11.858)
Democratic	0.128**	0.078	0.048	0.041	-0.023	-0.038	-0.036	0.071**	-0.011	-0.085*
Congress	(0.037)	(0.054)	(0.028)	(0.037)	(0.043)	(0.040)	(0.049)	(0.026)	(0.030)	(0.039)
Democratic	-0.014	0.016	-0.056	-0.086*	-0.033	-0.081	-0.098	0.061	-0.034	-0.072
President	(0.043)	(0.062)	(0.032)	(0.042)	(0.049)	(0.047)	(0.056)	(0.033)	(0.035)	(0.044)
$\hat{\alpha}$	-0.127**	-0.164*	-0.235**	-0.241**	-0.353**	-0.251**	-0.143**	-0.331**	-0.119**	-0.151**
	(0.052)	(0.065)	(0.039)	(0.071)	(0.073)	(0.068)	(0.047)	(0.120)	(0.042)	(0.042)
Constant	-0.716	-1.532	-0.923*	-0.709	1.245*	0.943	1.638*	0.378	0.579	0.904
	(0.521)	(0.811)	(0.459)	(0.842)	(0.615)	(0.555)	(0.697)	(0.630)	(0.432)	(0.608)
N	48	48	48	48	48	48	48	48	48	48
R^2	0.48	0.55	0.54	0.55	0.58	0.63	0.67	0.51	0.58	0.57

Seemingly unrelated regression model with error correction specification. * $p < .05$, ** $p < .01$. One-tailed t-tests with standard errors in parentheses. Dependent variables are logged odds ratios relative to a baseline spending category.

5 Comparison of Current and New Method

Table S 5: **Estimated short-run and long-run effects of valence variables on UK party support**

	$ln(\frac{\text{Conservatives}}{\text{Labour}})$		$ln(\frac{\text{Lib Dems}}{\text{Labour}})$		$ln(\frac{\text{Lib Dems}}{\text{Conservatives}})$	
Independent Variable	short	long	short	long	short	long
Labour Leader Evaluations	-.24**	-.35**	-.26**	-.38**	-.03	-.08
Conservative Leader Evaluations	.15**	-.01	-.13*	-.13	-.28**	-.16
Liberal Democratic Leader Evaluations	-.01	-.05	.46**	.28**	.48**	.30**
% Labour Party ID	.56	-3.41**	-2.21*	-1.83	-2.24*	2.16
National Retrospective Evaluations	-.07	-.18**	.08	-.01	.15	.17
% Labour best on most important issue	-1.72**	-.16	-1.42**	-1.12*	.27	-.85
constant	1.41**		.83		-1.07	
$\hat{\alpha}$	-.43**		-.75**		-.58**	
N	68		68		68	
R^2	.79		.77		.72	

Notes:
-* $p < .05$, ** $p < .01$ (two-tailed Wald χ^2 tests despite directional hypotheses)
-Dependent variable is the logged ratio of support for the top party to the bottom party.
-Estimated effects are from a seemingly unrelated regression model with error-correction specifications to model both short-run and long-run effects.
-Long-run effects are calculated by $LR = \frac{\beta_{Lj}}{-\alpha_j}$ using the delta method.

While we argue that our proposed approach for modeling compositional dynamic dependent variables is an improvement over current methods, space limitations prohibit us from comparing and contrasting the results from our models with the results from models using exiting approaches in much detail. Below, we present such comparisons for both the UK party support and US budget allocation analyses to highlight where and how our approach adds additional insight that can expand current theories. Current models are defined here as an ECM that predicts a dependent variable that is measured as one category versus all others in a composition.

Table S 6: Current Model Comparison, UK Party Support

	$(\frac{\text{Labour}}{\text{All Other}})$	
Independent Variable	short	long
Labour Leader Evaluations	.056**	.075**
Conservative Leader Evaluations	-.012	.001
Liberal Democratic Leader Evaluations	-.036**	-.022**
% Labour Party ID	.165	.625**
National Retrospective Evaluations	-.009	.027**
% Labour best on most important issue	.330**	.127
constant	-.081	
$\hat{\alpha}$	-.697**	
N	68	
R^2	.81	

Notes: - * $p < .05$, ** $p < .01$ (two-tailed t-tests)

5.1 UK Party Support Comparison

In the case of party support in the UK, Table S 5 duplicates Table 2 from the paper while Table S 6 presents findings that represent the current method used by political science scholars. We focus on support for the Labour Party as the party of the prime minister. In both tables 5 and 6, two similarities are evident. Labour leader evaluations and the percent of individuals who identify as Labour are positively related to Labour party support; these variables are positive in the Table S 6 and are negative in Table S 5 (Labour is denominator in the latter case). However, other variables provide quite different and more interesting stories about causal relationships across the two tables. While evaluations of the Conservative leader appear insignificant in Table S 6, this variable proves significant in Table S 5 such that these evaluations help the Conservative party in comparison to Labour and the Liberal Democrats but, notably, also help Labour as compared to the Liberal Democrats. Similarly, Liberal Democrat leader evaluations only appear to be negatively correlated with Labour support in Table S 6. However, our proposed approach provides additional depth to this relationship by showing that higher Liberal Democrat leader evaluations lead to higher levels of support for the Liberal Democratic party at the expense of Labour or the Conservatives *as well as* favoring Labour at the expense of the Conservatives. Outside of leader evaluations, national retrospective evaluations are largely consistent. Still, Table S 5 provides greater evidence that this variable is insignificant in both the short and long term whereas Table S 6 would lend support to linking this variable to long term party support. Here, the latter case may lead to inaccurate conclusions that do not mirror real world trade-offs. Finally, the percent who think Labour is best on the most important issue is significantly and positively related to Labour in both Table S 6 as well as Table S 5. The latter method, however, provides more extensive information; this variable is not significant for Labour support in the long term, and it has no effect on the trade-off between support for Liberal Democrats or Conservatives.

5.2 US Budget Allocation Comparison

For our second example, Tables S 7 and S 8 provide a comparison of the determinants of the percent of the budget allocated for defense. For Table S 8, we model allocations for defense as a proportion of all budget categories, since this category is often the subject of much budget research. Notably, only two variables appear to have any effect on allocations for defense in Table S 8 – old age dependency and mood. However, even a cursory glance at Table S 7 suggests that several pieces of the external environment drive multiple trade-offs related to defense spending. For example, as expected, population growth pushes funding away from defense and towards social security in both the short and long term. Similarly, higher levels of national unemployment move allocations from defense over to income security in the short run. Additionally, a Democratic congress is more likely to allocate money to both social security and the summative other category as compared to defense. While we only focus on the differences in the current strategies and our models for defense spending, it is evident from the extensiveness of the trade-offs presented in Table S 7 that we gain insights into other theoretically interesting trade-offs.

Table S 7: Estimated short-run and long-run effects on US Budget Spending

	$\frac{Other}{Defense}$	$\frac{Income}{Defense}$	$\frac{SocialSecurity}{Defense}$	$\frac{Interest}{Defense}$	$\frac{Other}{Income}$	$\frac{SocialSecurity}{Income}$	$\frac{Interest}{Income}$	$\frac{Other}{SocialSecurity}$	$\frac{Interest}{SocialSecurity}$	$\frac{Interest}{Other}$
Population	0.288	0.050	0.247	0.583**	0.147	0.225	0.510*	-0.060	0.254	0.300
Growth, Short	(0.178)	(0.255)	(0.131)	(0.174)	(0.200)	(0.186)	(0.229)	(0.122)	(0.146)	(0.185)
Population	0.804	-0.739	0.584	1.560**	0.756*	1.124*	2.882**	-0.129	1.178*	1.334
Growth, Long	(0.880)	(0.937)	(0.340)	(0.482)	(0.351)	(0.495)	(1.114)	(0.216)	(0.721)	(0.723)
Hostility	-0.001	-0.004	0.0002	-0.002	0.002	0.004	0.002	-0.002	-0.003	-0.001
Level, Short	(0.002)	(0.004)	(0.002)	(0.002)	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)	(0.031)
Hostility	-0.007	-0.046	-0.008	0.004	0.015	0.027	0.057	-0.004	0.011	0.014
Level, Long	(0.031)	(0.037)	(0.012)	(0.016)	(0.013)	(0.018)	(0.038)	(0.008)	(0.027)	(0.027)
National	0.049	0.188**	0.003	-0.022	-0.146**	-0.186**	-0.222**	0.040*	-0.038	-0.083**
Unemployment, Short	(0.029)	(0.043)	(0.021)	(0.031)	(0.033)	(0.031)	(0.038)	(0.021)	(0.024)	(0.031)
National	-0.069	0.189	-0.107	-0.122	-0.165**	-0.277**	-0.416*	0.076	-0.043	-0.159
Unemployment, Long	(0.159)	(0.171)	(0.064)	(0.091)	(0.063)	(0.090)	(0.199)	(0.043)	(0.133)	(0.137)
Old Age	-33.33	-14.66	-15.672	55.061*	-26.748	6.068	68.137*	-32.206*	59.495**	91.819**
Dependency, Short	(23.74)	(34.16)	(17.57)	(23.001)	(26.312)	(25.558)	(30.433)	(15.944)	(19.271)	(24.451)
Old Age	34.30*	43.862**	35.615**	36.015**	-13.546**	-3.018	-13.682	-13.396**	-5.035	0.263
Dependency, Long	(14.120)	(14.723)	(4.825)	(6.695)	(4.926)	(7.210)	(15.187)	(4.452)	(11.086)	(12.952)
Mood,	0.006	-0.010	-0.013*	-0.017*	0.015	-0.003	-0.008	0.017**	-0.004	-0.023**
Short	(0.007)	(0.011)	(0.006)	(0.007)	(0.009)	(0.008)	(0.010)	(0.005)	(0.006)	(0.008)
Mood,	-0.033	-0.016	-0.056**	-0.110**	0.009	-0.032	-0.155*	0.028*	-0.102*	-0.133**
Long	(0.047)	(0.052)	(0.019)	(0.035)	(0.019)	(0.028)	(0.075)	(0.014)	(0.051)	(0.050)
% GDP	0.952	3.636**	0.872	0.550	-2.941**	-2.589**	-3.373**	-0.453	-0.702	-0.552
Change, Short	(0.818)	(1.184)	(0.601)	(0.826)	(0.923)	(0.867)	(1.078)	(0.616)	(0.674)	(0.871)
% GDP	14.510	32.354	4.212	5.426	-12.667*	-16.428*	-33.025	-0.943	-6.135	-6.520
Change, Long	(14.722)	(19.340)	(4.920)	(6.338)	(5.244)	(7.960)	(19.655)	(3.440)	(11.672)	(11.858)
Democratic	0.128**	0.078	0.048	0.041	-0.023	-0.038	-0.036	0.071**	-0.011	-0.085*
Congress	(0.037)	(0.054)	(0.028)	(0.037)	(0.043)	(0.040)	(0.049)	(0.026)	(0.030)	(0.039)
Democratic	-0.014	0.016	-0.056	-0.086*	-0.033	-0.081	-0.098	0.061	-0.034	-0.072
President	(0.043)	(0.062)	(0.032)	(0.042)	(0.049)	(0.047)	(0.056)	(0.033)	(0.035)	(0.044)
$\hat{\alpha}$	-0.127**	-0.164*	-0.235**	-0.241**	-0.353**	-0.251**	-0.143**	-0.331**	-0.119**	-0.151**
	(0.052)	(0.065)	(0.039)	(0.071)	(0.073)	(0.068)	(0.047)	(0.120)	(0.042)	(0.042)
Constant	-0.716	-1.532	-0.923*	-0.709	1.245*	0.943	1.638*	0.378	0.579	0.904
	(0.521)	(0.811)	(0.459)	(0.842)	(0.615)	(0.555)	(0.697)	(0.630)	(0.432)	(0.608)
N	48	48	48	48	48	48	48	48	48	48
R^2	0.48	0.55	0.54	0.55	0.58	0.63	0.67	0.51	0.58	0.57

Table S 8: Comparison Table, US Budget Spending

	$\frac{Defense}{AllOther}$
Population	-.049
Growth, Short	(0.037)
Population	-.080
Growth, Long	(0.126))
Hostility	-.0004
Level, Short	(0.001)
Hostility	.003
Level, Long	(0.004)
National	-.011
Unemployment, Short	(0.006)
National	.010
Unemployment, Long	(0.025)
Old Age	2.204
Dependency, Short	(5.011)
Old Age	-5.949**
Dependency, Long	(1.782)
Mood,	.001
Short	(0.002)
Mood,	.010
Long	(0.007)
% GDP	-.301
Change, Short	(0.170)
% GDP	-2.458
Change, Long	(2.191)
Democratic	-.016*
Congress	(0.008)
Democratic	-.007
President	(0.009)
$\hat{\alpha}$	-.181*
	(0.079)
Constant	.162
	(0.141)
N	48
R^2	0.50

Notes: * $p < .05$, ** $p < .01$
with standard errors in parentheses.

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