UiO : Department of Physics
University of Oslo

Project 5: The diffusion equation

FYS3150 - Computational physics

Kristine Baluka Hein
Anders Johansson

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Abstract

Hei.

1 Mathematical theory

1.1 Forward Euler

1.1.1 Derivation and error analysis

The Forward Euler scheme is an explicit scheme based on Taylor polynomials. To find an approximation of the time derivative of u at point (x_i, t_j), a first order Taylor polynomial around x_i, t_j is used to calculate $u(x_i, t_{j+1})$ is used:

$$u_{i,j+1} = u_{i,j} + \Delta t \frac{\partial u_{i,j}}{\partial t} + \frac{1}{2}(\Delta t)^2 \frac{\partial^2 u(x_i, \tilde{t})}{\partial t^2} \implies \frac{\partial u_{i,j}}{\partial t} = \frac{u_{i,j+1} - u_{i,j}}{\Delta t} + \frac{1}{2}\Delta t \frac{\partial^2 u(x_i, \tilde{t})}{\partial t^2}$$

where $\tilde{t} \in (t_j, t_{j+1})$ and the last term is the truncation error, which is proportional to Δt.

Similarly, the three point approximation to the second derivative (derived in [1])) with its error term is used to approximate the second derivative of u at point (x_i, t_j) with respect to position:

$$\frac{\partial^2 u_{i,j}}{\partial x^2} = \frac{u_{i+1,j} + u_{i-1,j} - 2u_{i,j}}{h^2} +$$

1.1.2 Stability analysis

1.2 Backward Euler

1.2.1 Derivation and error analysis

1.2.2 Stability analysis

1.3 Crank-Nicolson

1.3.1 Derivation and error analysis

1.3.2 Stability analysis

References

[1] Anders Johansson. "Project 1". In: *FYS3150, Computational Physics* (Sept. 2016), pp. 4–6. URL: https://github.com/anjohan/Offentlig/blob/master/FYS3150/Oblig1/Johansson_Anders_FYS3150_Oblig1.pdf.