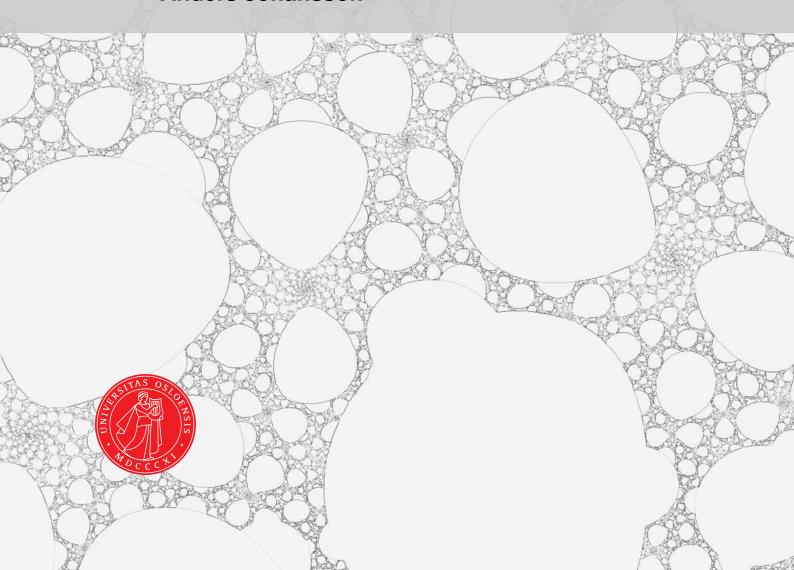


# **Project 5: The diffusion equation**

FYS3150 - Computational physics

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#### Abstract

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### 1 Mathematical theory

#### 1.1 Forward Euler

#### 1.1.1 Derivation and error analysis

The Forward Euler scheme is an explicit scheme based on Taylor polynomials. To find an approximation of the time derivative of u at point  $(x_i, t_j)$ , a first order Taylor polynomial around  $x_i, t_j$  is used to calculate  $u(x_i, t_{j+1})$  is used:

$$u_{i,j+1} = u_{i,j} + \Delta t \frac{\partial u_{i,j}}{\partial t} + \frac{1}{2} (\Delta t)^2 \frac{\partial^2 u(x_i, \tilde{t})}{\partial t^2} \implies \frac{\partial u_{i,j}}{\partial t} = \frac{u_{i,j+1} - u_{i,j}}{\Delta t} + \frac{1}{2} \Delta t \frac{\partial^2 u(x_i, \tilde{t})}{\partial t^2}$$

where  $\tilde{t} \in (t_j, t_{j+1})$  and the last term is the truncation error, which is proportional to  $\Delta t$ .

Similarly, the three point approximation to the second derivative (derived in [1])) with its error term is used to approximate the second derivative of u at point  $(x_i, t_j)$  with respect to position:

$$\frac{\partial^2 u_{i,j}}{\partial x^2} = \frac{u_{i+1,j} + u_{i-1,j} - 2u_{i,j}}{h^2} +$$

#### 1.1.2 Stability analysis

#### 1.2 Backward Euler

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- 1.2.1 Derivation and error analysis
- 1.2.2 Stability analysis
- 1.3 Crank-Nicolson
- 1.3.1 Derivation and error analysis
- 1.3.2 Stability analysis

## References

[1] Anders Johansson. "Project 1". In: FYS3150, Computational Physics (Sept. 2016), pp. 4—6. URL: https://github.com/anjohan/Offentlig/blob/master/FYS3150/Oblig1/Johansson\_Anders\_FYS3150\_Oblig1.pdf.