1 ODE Integration: Simplified Stellar Structure

In this exercise we solve the equations of stellar structure by integrating them from the center to the surface of the star by using Runge-Kutta methods of second, third and fourth order. A code skeleton was provided and our task was to inspect it and fill in the missing segments.

The structure of the code is the following:

- Routine called *setgrid* is used to define a radial grid throughout the entire star.
- Routine *tovRHS* defines the equations of stellar structure (equations 1 and 2 in the worksheet) and the equation of state (equation 3 with parameters given in equation 4).
- Routine $tovRK2$, $tovRK3$ and $tovRK4$ are used for integrating equations over one step in the grid using Runge-Kutta methods of order 2, 3 and 4, respectively.
- Routine *tovintegrate* is used to integrate equations over the entire star by calling RK routines for every step in the grid.

Knowing what all these routines do, it is not difficult to understand what the entire code does. First it sets up the grid that represents a star with spherical symmetry (the only coordinate in the problem is radius) and then it solves (i.e. integrates) the equations of stellar structure one radial step at a time, until it reaches the surface.

Figure 1 shows the results obtained using RK4 for the highest resolution that I used (1000 steps). The values of $M(r)$, $\rho(r)$ and $P(r)$ are all normalized to their largest values (cental pressure and density, and final mass) indicated in the legend.

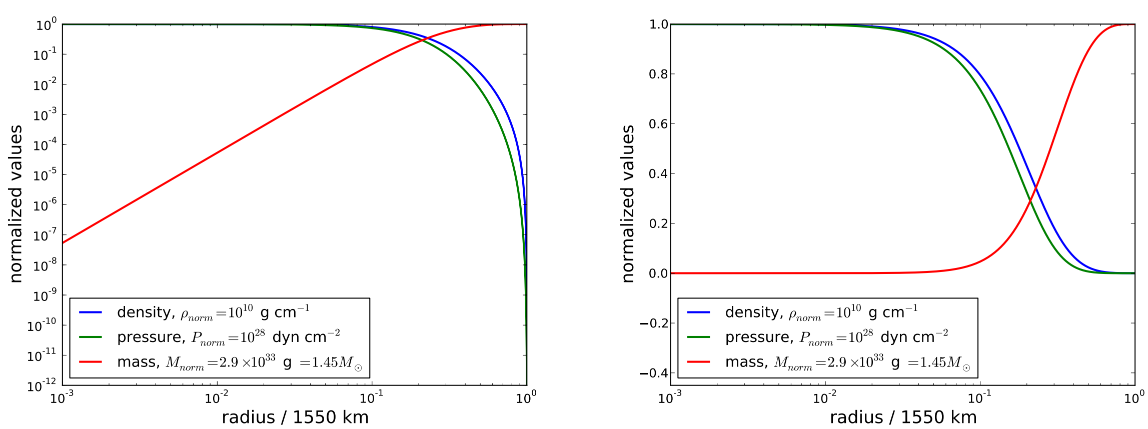

Figure 1: Mass, density and pressure profiles normalized to their largest values, indicated in the box. The radius is normalized to 1550 km. Left panel has logarithmic scales on both axes, whereas the right panel shows a linear y-axis.

I ran the code for all three RK methods for three different resolutions: 600, 800 and 1000 steps, corresponding to step sizes h_1, h_2 and h_3, respectively. To evaluate the convergence, I compute the self-convergence factor Q_s for final mass obtained using three resolutions

$$Q_s = \frac{|M_{surface}(h_3) - M_{surface}(h_2)|}{|M_{surface}(h_2) - M_{surface}(h_1)|} \tag{1}$$

and compare it to the convergence factor of order n:

$$Q_s(n) = \frac{h_3^n - h_2^n}{h_2^n - h_1^n} \; . \tag{2}$$

Convergence factors calculated using equation (2) are: $Q_s(n=2) = 0.463$, $Q_s(n=3) = 0.356$ and $Q_s(n=4) = 0.273$. Second order RK method resulted in $Q_s = 0.451$, RK3 in $Q_s = 0.355$ and RK4 in $Q_s = 0.272$. These results suggest that our ODE integration methods converge as expected.