1 Measuring π with an MC Experiment

In this problem we have to implement an MC experiment in order to obtain the value of π. This method is based on the fact that the area of a circle of radius r is $A_{circ} = \pi r^2$, whereas the area of a square with a side $2r$ is $A_{square} = 4r^2$. Hence, the ratio of A_{circ} and A_{square} is equal to $\pi/4$.

For this *experiment* we make use of NumPy's random module to generate random numbers with a uniform distribution. For every pair of generated random numbers between -1 and 1 (x, y), we need to check if $\sqrt{x^2 + y^2}$ is less than 1. If it is, we count this pair as being inside of a circle. After many iterations, we take the ratio of the number of (x, y) pairs that are in the circle to the total number of generated pairs, we then multiply this ratio by 4 and get our estimate of π.

Figure 1 shows the error between my experimental value of π and the true value as a function of the number of generated random pairs. As described in the Lecture Notes (section III.6.3.1), for a large number of random variables N, we expect the convergence rate to be proportional to $1/\sqrt{N}$. As shown in Figure 1, my results match this prediction fairly well.

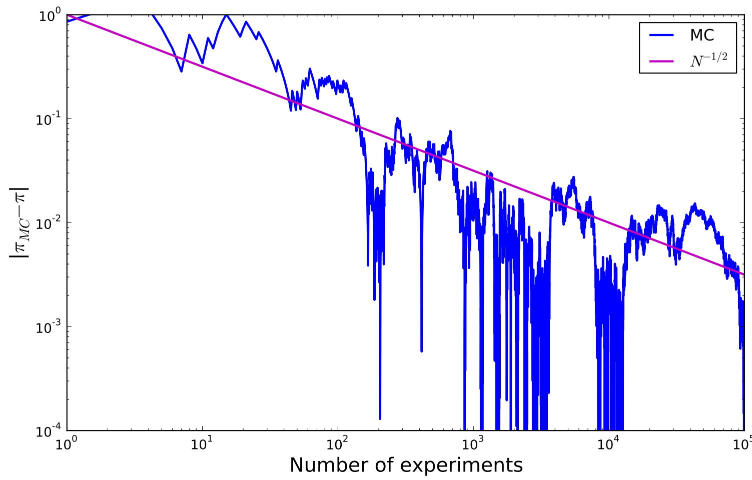

Figure 1: Error in measuring π as a function of the number of random variables used in the experiment.

2 Random Walk in 1D

In this problem we need to simulate random walk in 1D. Depending on the outcome of drawing a radnom number (using NumPy's random module), the walker takes a step either in the + or - direction. The size of each step is $\lambda = 0.01$ and we need to keep tract of the number of steps needed for the walker to come to the edge of the domain [-1, 1]. The prediction is that it should take around $N = d^2/\lambda^2 = 1/10^{-4} = 10^4$ steps.

In Figure 2 I show my results - the avarage number of steps as a function of the number of experiments. We see that the number of steps converges to the expected value of 10000.

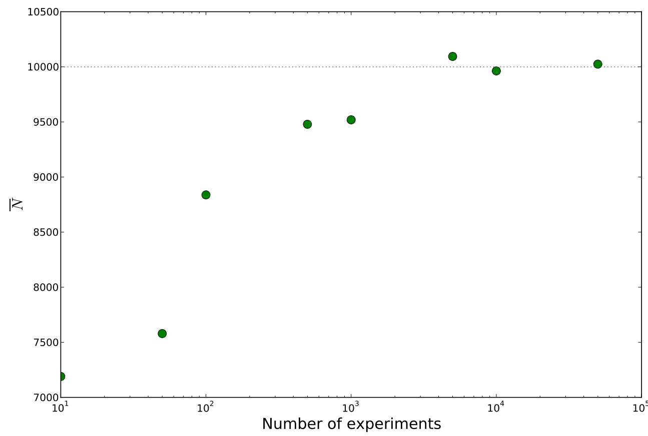

Figure 2: The average number of steps in 1D as a function of the number of experiments. The result converges to the expected value of 10000.

3 Random Walk in 2D

In the case of a 2D random walk, our particle can move in eight possible directions (left, right, up, down, diagonally up right/left, diagonally down left/right). I choose the step size ($\lambda = 0.015$) and follow the particle until it reaches the edge of a circle of radius 1. I repeat the excercise for four particles with different random seeds and plot their trajectories. The result is shown in Figure 3.

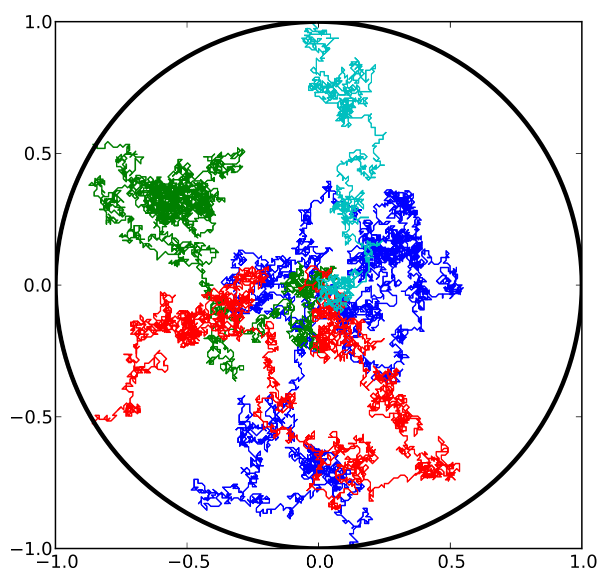

Figure 3: Random walk paths in 2D.