1 Finite Difference Approximation and Convergence

1.1 In-class exercise

The task of this problem was to numerically compute the first derivative of function $f(x) = x^3 - 5x^2 + x$ using two different methods, forward and central differencing, and to compare the obtained results with the analytic derivative $f'(x) = 3x^2 - 10x + 1$, for two resolutions h_1 and $h_2 = h_1/2$.

My results are shown in Figure 1. The upper panel shows the difference between the first derivative computed using the forward difference method and the analytic result for lower (h_1, solid line) and higher resolution (h_2, dashed). Since the forward difference method introduces an error that has a linear dependence on the grid step h, it is expected that the error for $h_2 = h_1/2$ is half the error of h_1, which is exactly what the plot shows.

The lower panel shows the same, but for central differencing. The expected absolute error for this method has a quadratic dependence on the step size h. Hence the error for h_2 (dashed line) should be four times smaller than for h_1 (solid line), and my plot confirms that.

Comparing the results for the two methods at same resolution suggest that the central difference method has much smaller errors at a given resolution.

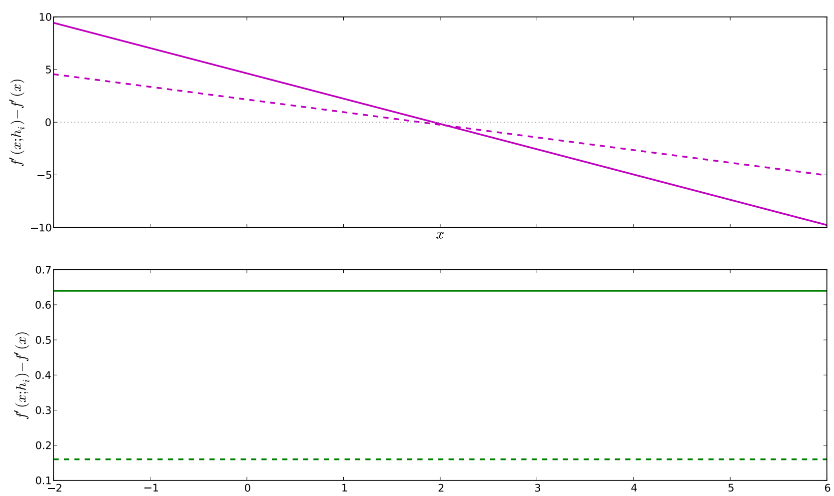

Figure 1: *Top panel:* Absolute error introduced by forward difference method at resolutions h_1 (solid) and $h_2 = h_1/2$ (dashed). *Bottom panel:* Error introduced by central difference method at resolutions h_1 (solid) and $h_2 = h_1/2$ (dashed).

1.2 Homework

In this problem, I need to derive a second-order central finite difference approximation for the second derivative of a function $f(x)$, assuming a fixed step size h. Since it is a central difference approximation, a good place to start is to write the Taylor expansions of functions $f(x+h)$ and $f(x-h)$:

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{6}f'''(x) + \mathcal{O}(h^4)$$
$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{6}f'''(x) + \mathcal{O}(h^4) \quad .$$

Now we can add these two equations to get

$$h^2 f''(x) = f(x+h) + 2f(x) + f(x-h) + \mathcal{O}(h^4) \tag{1}$$

and if we divide the equation by h^2 we end up with

$$f''(x) = \frac{f(x+h) + 2f(x) + f(x-h)}{h^2} + \mathcal{O}(h^2) \ . \tag{2}$$

2 Interpolation: Cepheid Lightcurve

The task of this problem was interpolate the given data (time and apparent magnitue for a Cepheid star) using various methods of interpolation.

2.1 In-class exercise

Figure 2 shows the results of using a single global Lagrange interpolation polynomial $p_8(x)$ of degree 8 (left panel), and piecewise linear and piecewise quadratic interpolation (right panel). Lagrange interpolation seems very good in the central region, however near the end points it shows strong oscillations known as Runge's phenomenon. Piecewise interpolations do not have this problem and they seem to be reasonably good interpolation schemes, quadratic of course better than linear.

2.2 Homework

Figure 3 shows the results of interpolating the same data set using piecewise cubic Hermite interpolation and a scipy routine for cubic spline interpolation. Both of these interpolation schemes seem very good, better than the previous methods.

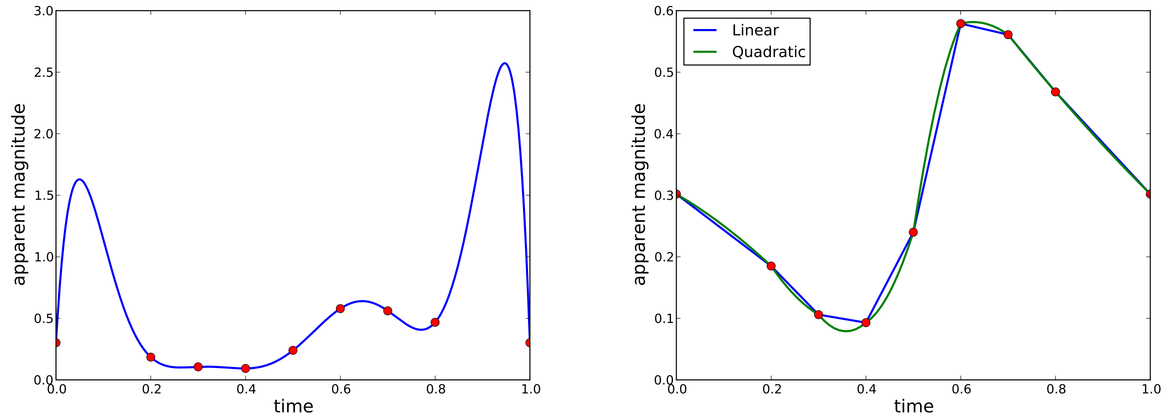

Figure 2: *Left panel:* Interpolation obtained using a Lagrange polynomial of degree 8. *Right panel:* Piecewise linear and piecewise quadratic interpolation of the same data. Data points are shown in red.

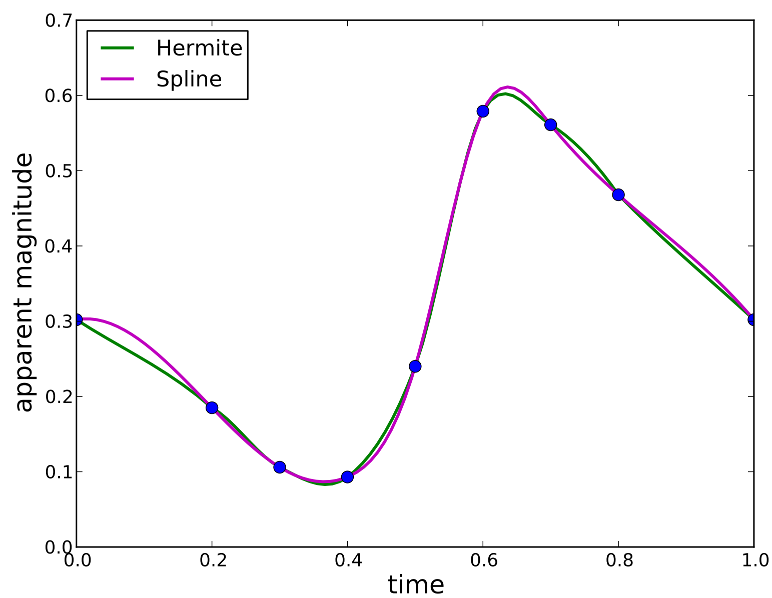

Figure 3: Interpolation obtained using piecewise cubic Hermite interpolation and cubic spline interpolation.