1 pp-Chain Nucleosynthesis

For this homework we needed to use a pp-chain nuclear reactions code written in Fortran 90 by Frank Timmes. Before running the code, we needed to make small changes to the code, such as specifying the initial mass fractions of ^1H (75%) and ^4He (25%), the central density (150 g cm^{-3}) and temperature (one of the following values: 10^7 K, 2×10^7 K and 3×10^7 K).

Figure 1 shows the resulting mass fractions of hydrogen and helium as functions of time, for three values of the initial temperature.

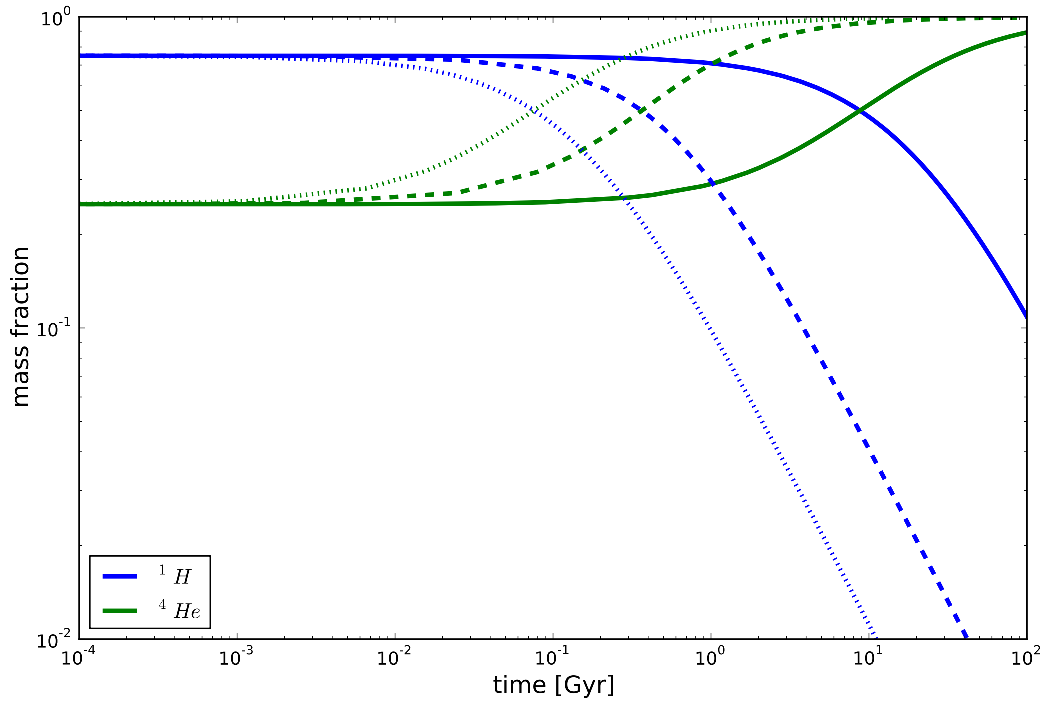

Figure 1: Mass fractions of hydrogen and helium-4 as functions of time. Solid lines represent the values obtained for $T_i = 1 \times 10^7$ K, dashes lines are for $T_i = 2 \times 10^7$ K, and dotted for $T_i = 3 \times 10^7$ K.

Assuming that the main sequence lifetime of the Sun is 10 Gyr and that it turns off of the main sequence when its hydrogen mass fraction at the center becomes ~ 0.02, we can estimate its initial central temperature from Figure 1. At $t = 10$ Gyr the line for hydrogen that would pass through the mass fraction of 0.02 is that for a temperature between $T_i = 2 \times 10^7$ K and $T_i = 3 \times 10^7$ K (i.e. between the dashed and dotted blue line).

Currently, the Sun is ~ 5 Gyr old, so if we assume the value for the initial temperature of $T_i \approx 2 \times 10^7$ K, the Sun's current mass fraction of hydrogen is already down at ~ 0.08.

The most probable pp-chain (ppI) consists of the following nuclear reactions:

$$\begin{aligned}
{}^1H +{}^1 H &\rightarrow {}^2H + e^+ + \nu_e + \gamma \\
{}^1H +{}^2 H &\rightarrow {}^3He + \gamma \\
{}^3He +{}^3 He &\rightarrow {}^4He + 2{}^1H + \gamma \ .
\end{aligned}$$

The second most probable branch (ppII) is:

$$\begin{aligned}
{}^1H +{}^1 H &\rightarrow {}^2H + e^+ + \nu_e + \gamma \\
{}^1H +{}^2 H &\rightarrow {}^3He + \gamma \\
{}^3He +{}^4 He &\rightarrow {}^7Be + \gamma \\
{}^7Be + e^- &\rightarrow {}^7Li + \nu_e + \gamma \\
{}^7Li +{}^1 H &\rightarrow {}^4He +{}^4 He + \gamma \ .
\end{aligned}$$

The least common branch of pp-chain (ppIII) is:

$$\begin{aligned}
{}^1H +{}^1 H &\rightarrow {}^2H + e^+ + \nu_e + \gamma \\
{}^1H +{}^2 H &\rightarrow {}^3He + \gamma \\
{}^3He +{}^4 He &\rightarrow {}^7Be + \gamma \\
{}^7Be +{}^1 H &\rightarrow {}^8B + \gamma \\
{}^8B &\rightarrow {}^8Be + e^+ + \nu_e + \gamma \\
{}^8Be &\rightarrow 2{}^4He + \gamma \ .
\end{aligned}$$

Nuclear reaction network for the first branch is:

$$\begin{aligned}
\frac{dn_H}{dt} &= -\frac{2}{2}n_H^2\lambda_{HH} - n_H n_D \lambda_{HD} + 2\frac{1}{2}n_3^2\lambda_{33} \\
\frac{dn_D}{dt} &= \frac{1}{2}n_H^2\lambda_{HH} - n_H n_D \lambda_{HD} \\
\frac{dn_3}{dt} &= -\frac{2}{2}n_3^2\lambda_{33} + n_H n_D \lambda_{HD} \\
\frac{dn_4}{dt} &= \frac{1}{2}n_3^2\lambda_{33} \ .
\end{aligned}$$

Subscripts 3 and 4 are used to denote ${}^3He$ and ${}^4He$, respectively and D stands for deuteron.