Ay190 Final Project:

Dark Matter Halo

Antonija Oklopčić

1 Introduction

Dark matter halos are often used in N-body cosmological simulations. They are usually represented by a distribution of particles that corresponds to the Navarro-Frenk-White (NFW) density profile, given by

$$\rho(r) = \frac{\rho_0}{\frac{r}{r_s}\left(1 + \frac{r}{r_s}\right)^2} \ , \tag{1}$$

where the scale length r_s and the corresponding density ρ_0 are characteristics of a given halo that can be used to describe it.

The aim of this project is to generate an N-body representation of a halo of scale length $r_s = 20$ kpc and the total mass of 10^{12} $M_\odot$ within ~ 200 kpc. In addition to having the right spatial distribution, each particle in a halo must also have a velocity assigned to it.

Finally, I should demonstrate that the constructed halo is in a state close to equilibrium. I can do that by evolving the halo with an N-body code *Gadget-2* for a few dynamical times, and comparing the initial and final profiles.

2 Initial Positions

NFW profiles are spherically symmetric, so it is most appropriate to first find the desired distribution of particles in spherical coordinates and then make coordinate transformations to convert the positions into Cartesian system, used by Gadget.

For a given radial coordinate r, particles must be uniformly distributed on a sphere of that radius. To that end, I use NumPy's random module to

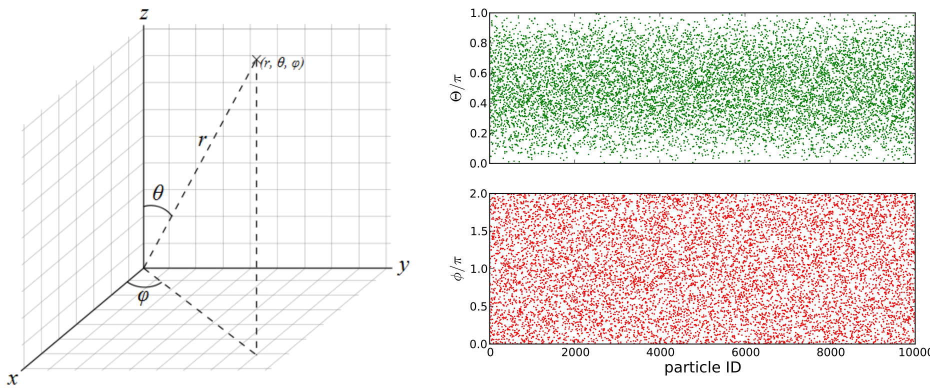

Fig. 1: *Left:* Spherical and Cartesian coordinate systems, both used in assigning initial positions of dark matter particles. *Right:* Generated distribution of angles ϕ and Θ for 10^4 particles.

assign values of the azimuth angle ϕ between 0 and 2π. Simply choosing a random value between 0 and π for the polar angle Θ would over-populate polar regions of the sphere. Hence, I introduce a new variable $u = \cos\Theta$, randomly assign to it values between 0 and 1 and then convert it back to Θ. The obtained angular distributions for a realization of a halo with 10^4 particles are shown in Figure 1.

Instead of using the NFW profile that is simply truncated to zero at the radius of 200 kpc, I use a more realistic distribution:

$$
\begin{aligned}
\rho(r) &= \frac{\rho_0}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2} && \text{for } r < 200 \text{ kpc} \\
\rho(r) &= \frac{\rho_0}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2} e^{-\frac{r}{R}} && \text{for } r > 200 \text{ kpc}
\end{aligned}
$$

where $R = 250$ kpc. I divide the entire halo into small radial bins of equal size[1]. Using the NFW formula I calculate the number of particles in each bin and then I assign a radial coordinate to each particle. The values of the radial coordinate are random numbers between the lower and upper limit of the corresponding bin.

[1] Radial bins have equal radial lengths δr, not volumes.

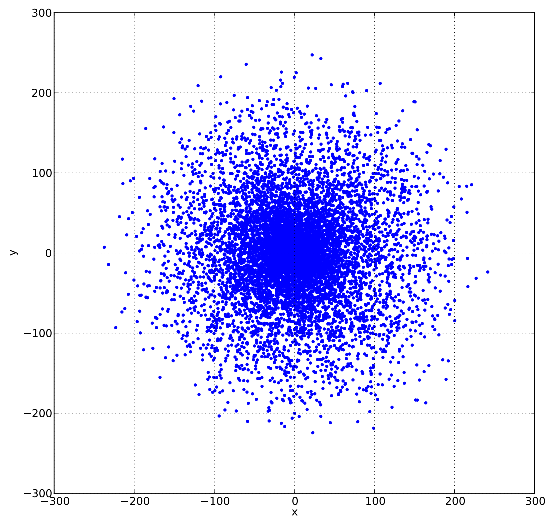

Fig. 2: Generated distribution of 10^4 particles.

After assigning spherical coordinates to all particles, I need to convert them to Cartesian coordinates x, y and z because *Gadget-2* takes the initial conditions in that format. To do that, I make use of the following coordinate transformations:

$$x = r \sin \Theta \cos \phi \quad (2)$$
$$y = r \sin \Theta \sin \phi \quad (3)$$
$$z = r \cos \Theta \ . \quad (4)$$

The resulting distribution of 10^4 particles in the xy-plane is shown in Figure 2. The density profile of that halo is shown in Figure 3, together with the analytic NFW profile.

3 Initial Velocities

Velocities of dark matter particles in a halo can be described as Gaussian random variables with isotropic dispersions ($\sigma_r = \sigma_\phi = \sigma_\Theta$). To find the right value of the velocity dispersion as a function of radius, I numerically

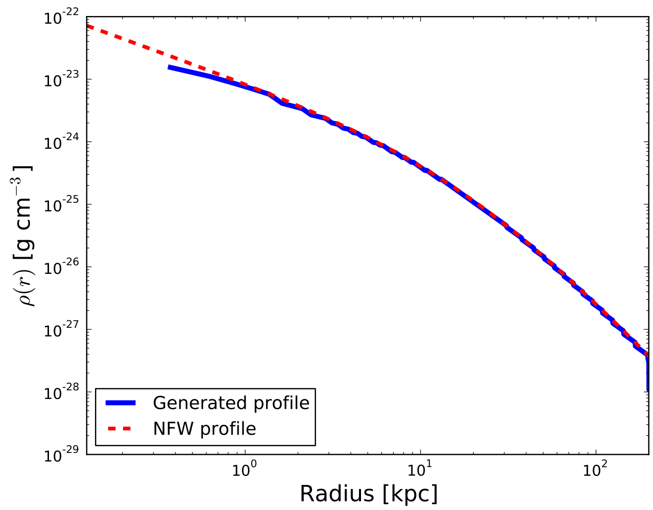

Fig. 3: Density profile for a distribution of 10^4 particles (blue solid line), shown together with the analytic NFW profile (red dashed line).

solve the Jeans equation for $\beta = 0$:

$$\frac{d(\rho\sigma_r^2)}{dr} = -\frac{GM(r)}{r^2}\rho(r) \ . \tag{5}$$

The calculated solution is shown in Figure 4 as the black solid curve. For a particle at radius r, I take the initial velocities in all three directions (v_x, v_y and v_z) to be random variables drawn from a Gaussian distribution centered on zero, with dispersion given by σ_r.

Figure 4 also shows the distribution of generated particles. I divide my sample into bins containing 100 particles each, and in this figure I show the standard deviation of x, y and z velocities (red, green and blue dots) in each bin, as a function of the mean radius of that bin. Generated distribution matches the solution of the Jeans equation fairly well.

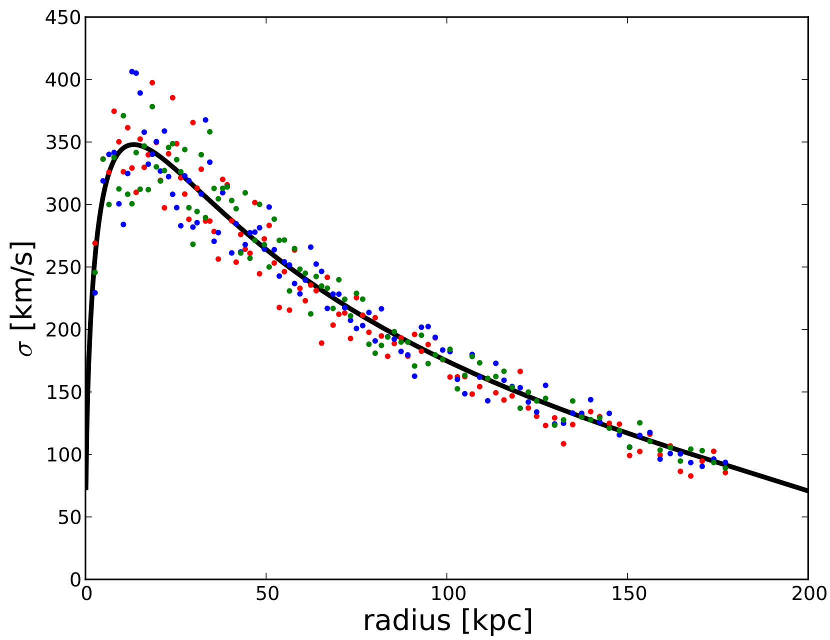

Fig. 4: Solid black curve shows the numerical solution to the Jeans equation. Red, green and blue dots represent velocity dispersions in all three coordinates of subsets of 100 generated particles as a function of the mean radius of each subset.

4 Evolving a Halo with Gadget-2

After creating a representation of a dark matter halo by setting up the initial positions and velocities of a large number of particles[2], I need to evolve it with the use of *Gadget-2* to show whether the generated halo is in an equilibrium state or not.

Figure 5 shows snapshots of the halo at different times. Sadly, my halo does not appear to be in an equilibrium state since its size is drastically increasing with time. I don't know exactly what is the reason for this, but I suspect that the initial velocities that I assigned are too large (although they do nicely match the solution of the Jeans equation, but maybe that is wrong too), or some of the Gadget's initial parameters (which are many and I don't fully understand what all of them stand for) are given wrong values. Unfortunately, I didn't have time to explore all these options to get a better result.

5 Acknowledgements

I would like to thank Andrew Benson and Allison Strom for helping me with this project. Without their help I wouldn't be able to run *Gadget* or use its output.

[2] I made two types of halos: the smaller one, with only 10^4 particles, which is more convenient for making plots and showing the position and velocity distributions, and a larger one, with 10^5 particles, which I evolved using *Gadget*.

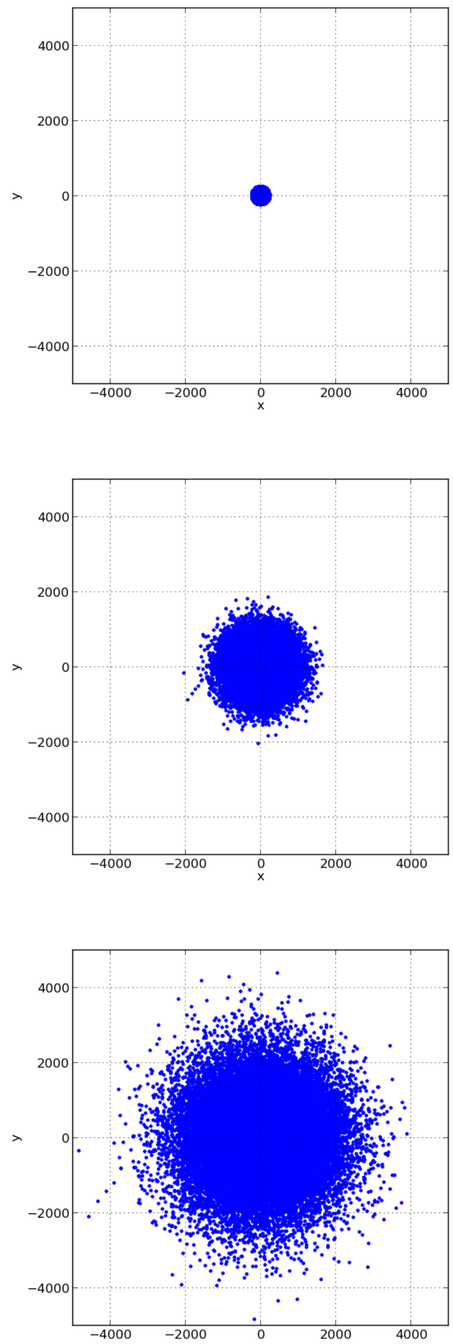

Fig. 5: Gadget snapshots showing the distribution of 10^5 particles at three different times (starting from the initial distribution in the top panel, to the final distribution at the bottom). Clearly, the halo is not in an equilibrium state because its size changes dramatically with time.