1 Integration via Newton-Cotes Formulae

The task of this problem is to compute two finite integrals ($\int_0^\pi \sin x dx$ and $\int_0^\pi x \sin x dx$) using two methods, the trapezoidal and Simpson's rule for different step size (or different number of subintervals, since the total interval remains fixed). Figure 1 shows the difference between computed values and analytical solutions

$$\int_0^\pi \sin x dx = 2$$

$$\int_0^\pi x \sin x dx = \pi$$

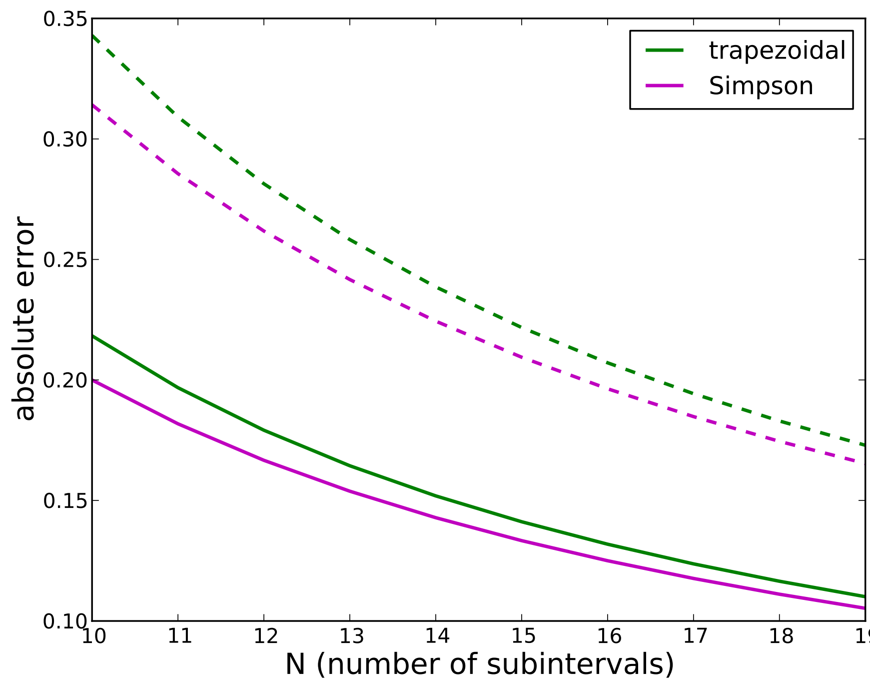

Figure 1: Difference between numerically and analytically computed integrals using two methods, trapezoidal and Simpson's rule (shown in green and magenta curves respectively) for different number of subintervals. Solid lines show the results for $\int_0^\pi \sin x dx$, and dashed for $\int_0^\pi x \sin x dx$.

2 Gaussian Quadrature

The first part of this problem required the computation of the total number density of electrons given by

$$n_{e^\pm} = \frac{8\pi (k_B T)^3}{(2\pi\hbar c)^3} \int_0^\pi \frac{x^2 dx}{e^x + 1} \ . \tag{1}$$

To solve the integral we were supposed to use Gauss-Laguerre Quadrature. We were allowed to use a scipy routine for finding roots of Laguerre polynomials. The obtained value of the integral for different number of nodes used in the computation is shown in Figure 2. Using this result and evaluating the constant in front of the integral in equation (1) gives the total number density of electrons of $\sim 2 \times 10^{35}$ cm^{-3}.

The second part of this problem required producing a spectral distribution of electrons using energy bins of $\Delta E = 5$ MeV in the energy range from $E = 0$ to $E \approx 150$ MeV. For calculating integrals we were supposed to make use of Gauss-Legendre Quadrature. We were allowed to use a scipy routine for finding roots of Legendre polynomials. My result is shown in Figure 2. The computed value is slightly lower than what I obtained in the first part of the problem (1.78 vs. 1.80) because I didn't calculate the integral over the entire range (which is infinite), but made a cut-off at $E = 150$ MeV.

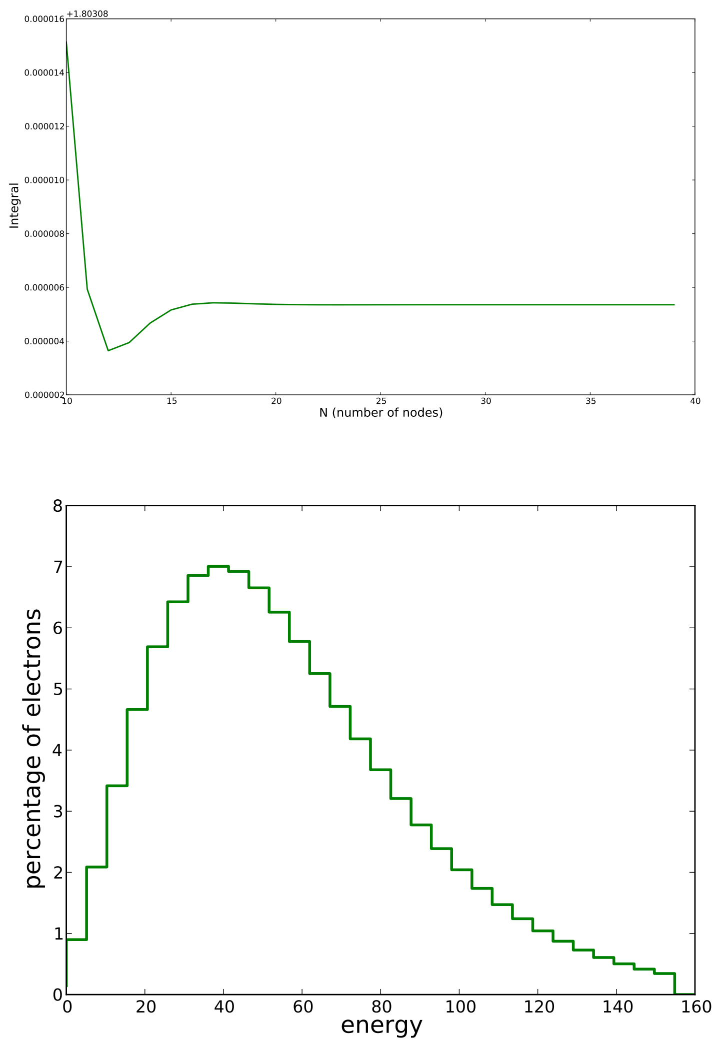

Figure 2: *Upper panel:* The calculated value of the integral for different number of nodes. *Lower panel:* Spectral distribution of electrons shows the percentage of the total number of electrons contained in each bin of width $\Delta E = 5$ MeV.