University of Waterloo
CS 341 — Algorithms
Winter 2009
Problem Set 3

Distributed Tuesday, January 20 2009.

Due Tuesday, January 27 2009. Hand in to assignment boxes on the 3rd floor of MC by 3 PM.

1. [10 marks]

   Recall from Lecture 4 (`http://www.student.cs.uwaterloo.ca/~cs341/lecture4.html`) that we obtained the following recurrence for the running time of our algorithm `printperm` (for $m \geq 0, n \geq 1$):

   $$T(m,n) = \begin{cases} n(T(m+1,n-1)+m+1+n), & \text{if } n > 1; \\ m+1, & \text{if } n = 1. \end{cases} \tag{1}$$

   Also recall that I claimed that the solution to this equation was

   $$T(m,n) = (m+1+n)a(n) - n! \tag{2}$$

   where the sequence a is defined by

   $$a(n) = \begin{cases} n(a(n-1)+1), & \text{if } n > 0; \\ 0, & \text{if } n = 0. \end{cases} \tag{3}$$

   Your job is to prove that (2) holds. Hint: use induction on n.

2. [10 marks]

   Prove that $a(n) = n!\left(\frac{1}{0!} + \frac{1}{1!} + \cdots + \frac{1}{(n-1)!}\right)$ for $n \geq 1$. Hint: use induction on n.

3. [10 marks] In Lecture 4, we guessed Eq. (2) by looking at the values produced by the recurrence. But there's another, more direct way to to derive Eq. (2): by analyzing the computation tree formed by all the recursive calls of the `printperm` algorithm (given in the notes to Lecture 4). Do this.

4. [5 bonus marks; extra credit only]

   Also in the Lecture 4 notes (`http://www.student.cs.uwaterloo.ca/~cs341/lecture4.html`), we observed that $a(n)$ (defined above) satisfies $a(n) = \lfloor e \cdot n! \rfloor - 1$ for $n \geq 1$. Prove this.