CS 365, Winter 2009
Assignment 1
Due Thursday, January 22, in class

1. [20 marks] For each of the following languages, state whether or not it is regular. If it is regular, give a finite automaton or a regular expression for it, with a brief explanation; if it is not regular, give a detailed proof.

   (a) [10 marks] $\{\, a^i b^j \mid \gcd(i,j) > 1 \,\}$

   (b) [10 marks] The language L defined as follows. For a string over $\Gamma = \{0,1\}^3$, we can view each element of Γ as a column, e.g.

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

   We define a string over Γ^* to be in L if a correct binary addition problem is formed by the elements (where the third row is the sum of the first and the second). Leading 0's are allowed in any of the numbers.
   For example, since $2 + 3 = 5$, the following string is in L:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

2. [10 marks] Problem 1.54 in Sipser, p. 91. (Showing that the language $\{\, a^i b^j c^k \mid \text{if } i = 1 \text{ then } j = k \,\}$ in non-regular, even though it satisfies the pumping lemma.)

3. [10 marks] Let T be the language over alphabet $\{0,1\}$ consisting of all strings that are the binary representation of a multiple of 3. Leading 0s are allowed; thus $T = \{\varepsilon, 0, 00, 11, 000, 011, 110, \ldots\}$.

   Let R be the language represented by the regular expression $E = \big(1(01^*0)^*1 \cup 0\big)^*$.

   (a) Show that $R \subseteq T$; that is, that every string in $L(E)$ represents a multiple of 3.

   (b) Show that $T \subseteq R$; that is, that every string that represents a multiple of 3 is in $L(E)$.

4. [10 marks] Recall that, for an automaton $(Q, \Sigma, \delta, q, F)$, the extended transition function δ^* is defined by

   - $\delta^*(q, \varepsilon) = q$ for all states q, and
   - $\delta^*(q, ya) = \delta\big(\delta^*(q,y), a\big)$ for all states q, strings y and symbols a.

   The function could have equally well been defined by "peeling off" symbols from the front of the string; that is, the function δ^* satisfies

$$\delta^*(q, ay) = \delta^*\big(\delta(q,a), y\big)$$

   for all states q, strings y and symbols a. Prove this statement, by induction.