Adjoint Current-Based Approaches to Brachytherapy Optimization

Jeremy A. Roberts

Department of Engineering Physics
University of Wisconsin–Madison

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Outline

Introduction

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- Focus here is on *permanent* sources

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- Near-optimal results...but could they be better?
- Here, two methods incorporating further aspects of the “adjoint” theory explored

The Boltzmann Equation

A time-independent treatment of particle interactions suffices, done via

$$[\hat{\Omega} \cdot \vec{\nabla} + \Sigma(\vec{r}, E)]\psi(\vec{r}, \hat{\Omega}, E) =$$
$$q_{ex}(\vec{r}, \hat{\Omega}, E) + \int dE' \int d\Omega' \Sigma_s(\vec{r}, E' \to E, \hat{\Omega}' \cdot \hat{\Omega})\psi(\vec{r}, \hat{\Omega}', E') \quad (1)$$

where $\vec{r}$ denotes position, $\hat{\Omega}$ denotes the direction of travel, and E is the energy of a particle. [3]

Transport Phase-Space

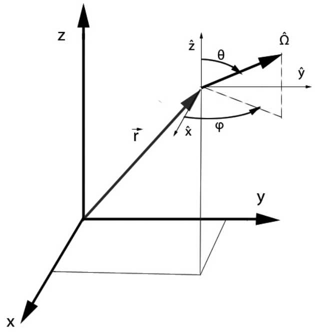

Figure: The position and direction variables describing a particle.

Forward Flux

The solution of the Boltzmann equation is the *angular flux*, $\psi(\vec{r}, \hat{\Omega}, E)$. From this, the *scalar flux* is defined

$$\phi(\vec{r}, E) = \int_{4\pi} d\Omega \psi(\vec{r}, \hat{\Omega}, E). \tag{2}$$

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- $\phi(\vec{r}, E)$ describes the particle population at a given point and at a given energy $\rightarrow$ *direction of motion not considered*
- Reaction rates, e.g. dose rates, calculated via

$$R(\vec{r}) = \int_0^\infty \phi(\vec{r}, E) DF(E) dE \tag{3}$$

where $DF(E)$ are flux-to-dose conversion factors.

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- Let a source q^+ be the "pseudo particles" representing a desired *detector response*
- Set those particles in motion, but apply "reverse physics"...
- ... the adjoint equation:

$$[-\hat{\Omega} \cdot \vec{\nabla} + \Sigma(\vec{r}, E)]\psi^+(\vec{r}, \hat{\Omega}, E) =$$
$$q^+(\vec{r}, \hat{\Omega}, E) + \int dE' \int d\Omega' \Sigma_s(\vec{r}, E \rightarrow E', \hat{\Omega} \cdot \hat{\Omega}')\psi^+(\vec{r}, \hat{\Omega}', E') \tag{4}$$

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- Adjoint flux $\approx$ *relative importance some component of a forward source has on detector response*

Current Vector

- The *forward current vector* is defined

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- Current represents *net flow* of particles at $\vec{r}$, a *vector quantity*

The Adjoint Function

- For a region of interest (ROI) with adjoint flux $\phi^+_{ROI}(\vec{r})$, the *adjoint function* is defined

$$D^+_{ROI}(\vec{r}) = C_{1S_u}\phi^+_{ROI}(\vec{r})/V_{ROI} \tag{8}$$

where C_{1S_u} yields units of Gy/S_u and S_u denotes one *source unit* (0.43 mCi for a ^{125}I seed) [4]

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- The *adjoint ratio* is defined

$$\rho(\vec{r}) = \frac{w_u D^+_u(\vec{r}) + w_r D^+_r(\vec{r}) + w_n D^+_n(\vec{r})}{D^+_t(\vec{r})} \tag{9}$$

where w_i weight tissue importance; these are set to unity. [4]

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- ...seed clustering? $\rightarrow$ *dose-update*

$$S_{dose}(j) = \frac{1}{D_p} \sum_{i=1}^{n} D(i,j) \tag{10}$$

$$\rho_d(j) = \rho(j) S_{dose}(j) \tag{11}$$

- $\rho_d(j)$ the *greedy criterion*

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- Lowest three energy groups of the FENDL-42 group photon library used

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- ICRP 21 flux-to-dose conversion factors (DF) used [2]

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- Current vector: a more complete description of the physics...
- ...may generate better insight to the physical nature of the problem $\longrightarrow$ *better results*

Summing Adjoint Currents

- Define the horizontal (and vertical) summed tissue adjoint currents:

$$J_x^{tot}(i) = w_r J_x^r(i) + w_u J_x^u(i) + w_n J_x^n(i) + w_t J_x^t(i) \qquad (14)$$

$$J_y^{tot}(i) = w_r J_y^r(i) + w_u J_y^u(i) + w_n J_y^n(i) + w_t J_y^t(i) \qquad (15)$$

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- Also have a *forward* current for seeds, $\vec{J}^s$

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- b relates tissue and seed currents; a defines appropriate displacement
- This displacement is defined

$$i' = (x,y)' \longleftarrow (x + \vec{C}_x(i), y + \vec{C}_y(i)) = i + \vec{C}(i) \tag{18}$$

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- Relating currents:

$$b = \max ||\vec{J}^{tot}||_2 / (k \max ||\vec{J}^s||_2), \tag{19}$$

$$k = 1/\sqrt{m_1 \sqrt{m_2/m_1}}. \tag{20}$$

$$m_1 = \text{mean}((J_x^s)^2 + (J_y^s)^2) \tag{21}$$

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$$a = k / \max ||\vec{J}^{tot}||_2 \tag{23}$$

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$$||\vec{C}(i)||_2 = \sqrt{C_x(i)^2 + C_y(i)^2} \tag{24}$$

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$$||\vec{C}(i)||_2 = \sqrt{C_x(i)^2 + C_y(i)^2} \tag{24}$$

- Diminish $\vec{C}$ with each iteration:

$$\vec{C}(i) = \text{round}(\vec{C}(i)/it^c). \tag{25}$$

where "round" accounts for discretization

		$f=1$	$f=2$	$f=3$	**Total**	**% Δ**
TARGET						
$V_{100}(\%)$	*CD*	98.08	98.87	99.39	98.78	-0.26
	GY	98.42	99.23	99.47	99.04	
$V_{150}(\%)$	*CD*	79.55	81.50	85.32	82.12	-1.44
	GY	80.20	82.87	86.89	83.32	
CN	*CD*	0.51	0.49	0.45	0.48	-2.30
	GY	0.50	0.50	0.45	0.49	
DNR	*CD*	0.81	0.82	0.86	0.83	-1.03
	GY	0.81	0.83	0.87	0.84	
URETHRA						
$V_{125,ur}(\%)$	*CD*	57.16	69.29	80.42	68.96	-8.40
	GY	67.73	75.81	82.32	75.29	
$V_{360,ur}(\%)$	*CD*	0.00	0.00	0.00	0.00	0.00
	GY	0.00	0.00	0.00	0.00	
$D_{90,ur}(\text{Gy})$	*CD*	167.35	177.00	195.86	180.07	-1.81
	GY	170.95	180.90	198.33	183.39	

Table: Average evaluation parameters, by f and total.

		$f = 1$	$f = 2$	$f = 3$	**Total**	**% Δ**
RECTUM						
$V_{80,re}(\%)$	*CD*	26.53	38.70	64.86	43.36	-2.65
	GY	28.15	41.45	64.03	44.54	
$V_{90,re}(\%)$	*CD*	80.87	87.73	95.68	88.09	0.11
	GY	80.78	87.54	95.66	87.99	
$D_{90,re}(\text{Gy})$	*CD*	86.40	90.07	102.98	93.15	0.13
	GY	86.47	90.41	102.21	93.03	
NORMAL TISSUE						
$D_{90,no}(\text{Gy})$	*CD*	46.74	50.79	53.37	50.30	1.65
	GY	45.71	49.91	52.82	49.48	
QUALITATIVE						
Seeds	*CD*	19.60	10.40	7.60	12.53	0.00
	GY	19.60	10.40	7.60	12.53	

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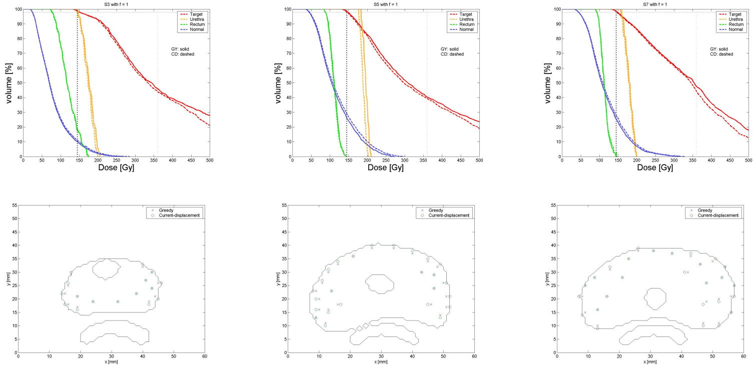

Figure: DVH’s and placement for slices 3, 5, and 7 with $f = 1$.

Results and Discussion

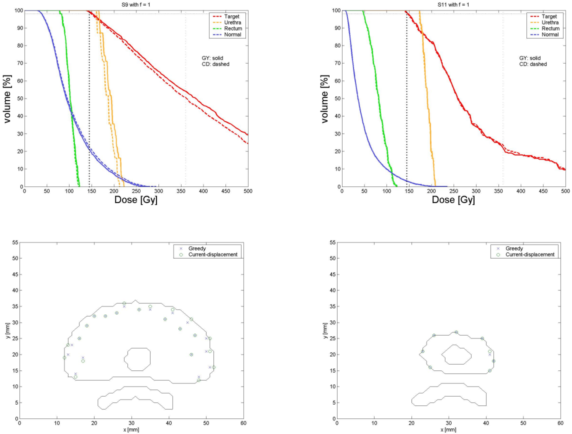

Figure: DVH's and placement for slices 9 and 11 with $f = 1$.

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- $D_{90,no}$ increased slightly by CD $\rightarrow$ slightly reduced normal tissue sparing
- *Applying CD leads to better treatment plans on average*

Redefining Current-displacement

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- a *new greedy criterion*

$$||\vec{C}(i)||_2 = \sqrt{C_x(i)^2 + C_y^2(i)} \quad (28)$$

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- Define for a tissue i the parameter:

$$rms_i = \sqrt{\mathrm{mean}\Big((J_x^i)^2 + (J_y^i)^2\Big)} \tag{29}$$

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- Target not included!

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	GY	0.00	0.00	0.00	
$D_{90,ur}(\mathrm{Gy})$	*CD*	178.86	171.95	175.41	-1.89
	GY	181.63	175.92	178.78	

Table: Average evaluation parameters for grid, no grid, and overall.

		With Grid	Without Grid	Total	% Δ
RECTUM					
$V_{80,re}$(%)	*CD*	42.69	42.41	42.55	10.85
	GY	41.97	34.80	38.38	
$V_{90,re}$(%)	*CD*	84.95	88.00	86.48	2.68
	GY	84.28	84.16	84.22	
$D_{90,re}$(Gy)	*CD*	90.73	91.73	91.23	2.10
	GY	90.27	88.44	89.35	
NORMAL TISSUE					
$D_{90,no}$(Gy)	*CD*	38.74	40.48	39.66	-14.12
	GY	44.55	47.81	46.18	
QUALITATIVE					
Seeds	*CD*	13.20	13.90	13.55	-7.19
	GY	14.20	15.00	14.60	

Table: Average evaluation parameters for grid, no grid, and overall.

Results and Discussion

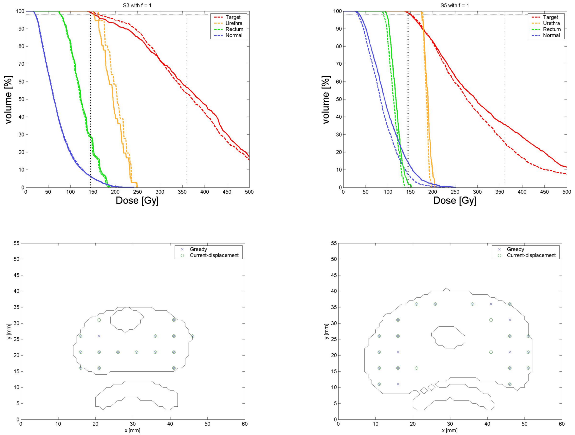

Figure: DVH’s and placement for slices 3 and 5 with $f = 1$ and grid.

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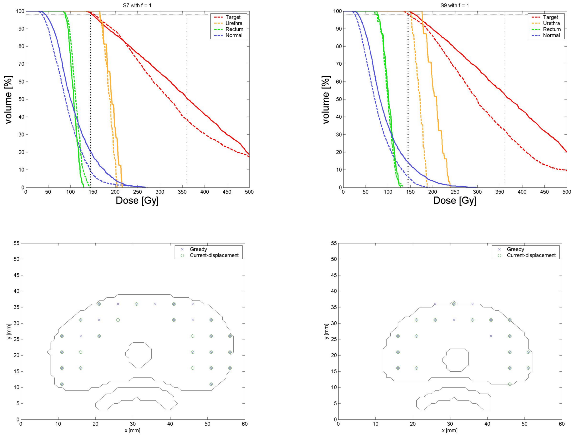

Figure: DVH's and placement for slices 7 and 9 with $f = 1$ and grid.

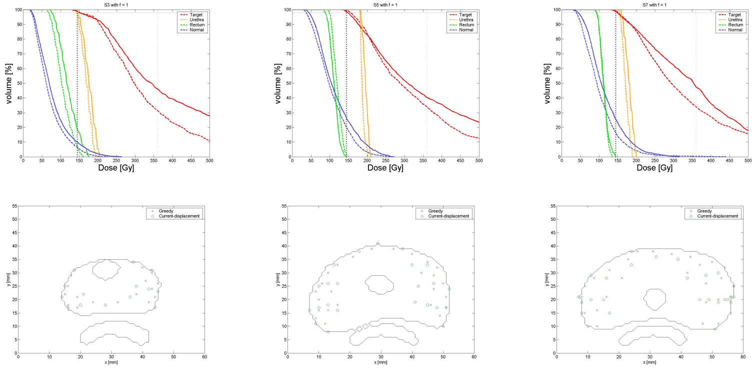

Figure: DVH's and placement for slices 3, 5, and 7 with $f = 1$ and no grid.

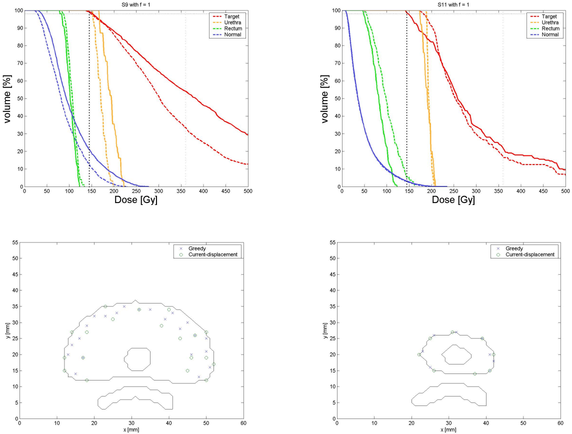

Figure: DVH's and placement for slices 9 and 11 with $f = 1$ and no grid.

Key Points

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- *CD as the greedy criterion leads to better treatment plans on average*

Concluding Remarks

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- **Questions?** *Jeremy Roberts, jaroberts2@wisc.edu*

Thanks!

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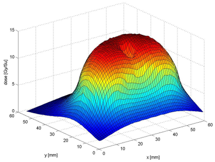
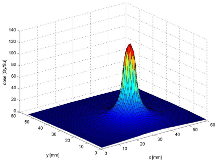
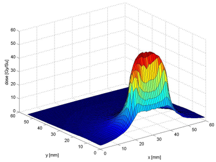
The Adjoint Function - Plots



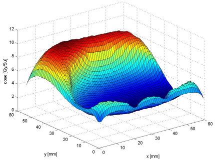
(a) Target adj. function.



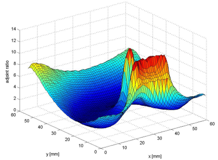
(b) Urethra adj. function.



(c) Rectum adj. function.

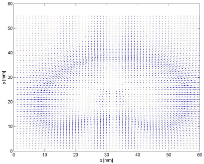


(d) Normal tissue adj.

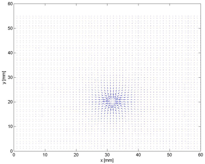


(e) Adjoint ratio.

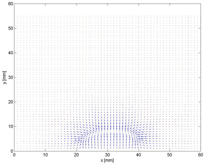
Adjoint Current - Plots



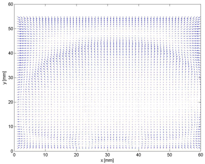
(a) Target adjoint current.



(b) Urethra adjoint current.



(c) Rectum adjoint current.



(d) Normal adjoint cur-