BRIEF NOTE - CD AS 'GREEDY CRITERION' w/ DOSE UPDATE

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1 Background

After investigating various methods to use "current-displacement" (CD) as a means to improve (via adjustment) the results of the adjoint ratio-based Greedy heuristic, it was found that such methods were not easy to implement; thus, it was believed that time was better spent looking at other possibilities. One idea is to use a CD-driven Greedy heuristic. This note details the method and gives a brief example to compare it with the adjoint ratio-based approach.

2 Current-displacement

2.1 (Review of the) Definition

We recall our definition of the general CD vector,

$$\mathbf{D}(i) = a\left(-\mathbf{J}^{tot}(i) + b\sum_{\substack{j=1\\ j\neq i}}^{n}\mathbf{J}^{s}(i,j)\right) \tag{1}$$

which gives us the displacement at a location i from both tissues and all seeds not at i. Here, the value b relates tissue currents to seed currents and a relates the total current function to a reasonable displacement value (in units of mm or 'pixels').

In its first use, **D(i)** was applied to a given initial set of seeds with coordinates specified by the adjoint-based Greedy. In essence, seeds were 'displaced' with the primary objective being that a better treatment were generated; unfortunately, this proved to be difficult and–as of yet–unrealized.

2.2 CD: a Viable Criterion?

What if instead of using CD as a 'displacement' mechanism we were to use it as the Greedy criterion? If we think back to some basic physics, the *flux*, the scalar measure for the number of particles at a position and the associated *current*, a vector measure for the net flow of particles at a position, share a relationship that bears some similarity to a function and its gradient[1]. If we were to use the point *minimum* CD as the optimal coordinate for the next seed, what exactly does this mean *physically*? Is it intuitive?

In some sense, *yes*, this is intuitive. If the *current* is like the *gradient*, then finding its minimum (here meaning its value closest to zero) tells us approximately the point at which our underlying sensitive tissue adjoint fluxes (and thus their doses) are changing slowly, possibly even reaching a minimum.

Why might this matter? It seems such slow variance would allow a maximum sparing of urethra, rectum, and normal tissue simultaneously; as of yet, the target is not included, but there may be a way to do so that makes sense for this theory.

3 Implementation

Two basic approaches to implementing CD as the Greedy criterion are presented.

3.1 Method 1: CD All the Way

The first approach was to use CD throughout the Greedy process. First, the CD map (containing both a horizontal and vertical component) was generated for the sensitive tissues according to

$$\mathbf{D}(i) \longleftarrow \mathbf{J}^{tot}(i) \tag{2}$$

where *tot*, as above, refers to the (weighted) sum[2] of tissue currents. The minimum point, e.g. point j, was found and a seed was placed. Then, the CD map was updated according to

$$\mathbf{D}(i) \longleftarrow \mathbf{D}(i) + b\mathbf{J}^{s}(i,j) \tag{3}$$

[1] And indeed this is quite true for the diffusion approximation of transport theory.

[2] This sum is not trivial; no good method of determining weights has been found yet. However, this difficulty does not detract from the basic theory.

where again, b is a seed-to-tissue current factor, and $\mathbf{J}^s(i,j)$ refers to the seed current at point i due to a seed placed at point j.

This method, as implemented, failed. Because direction is considered, when two seeds are placed in relatively near proximity (as must sometimes happen to achieve target coverage), two things happen: first, a seed's current dominates its immediate surrounding (i.e. is larger than the tissue currents) and two, between the seeds, their respective current componenents cancel (i.e. they are equal and opposite in magnitude and direction). The net effect is that a superficial minimum is produced, and seed clustering is promoted– definitely not the intent!

To account for this, a slightly different approach was taken. As the Greedy criterion, CD no longer needs direction-specific components; we are concerned primarily with magnitudes of 'push' in a given locale. If we redefine our criterion D' to be $||\mathbf{D}(i)||$, we have for the first map generated

$$D'(i) \longleftarrow ||\mathbf{J}^{tot}(i)|| \tag{4}$$

with subsequent updated defined by

$$D'(i) \longleftarrow D'(i) + ||b\mathbf{J}^s(i,j)||. \tag{5}$$

This method, while generating decent plans, did not seemingly best the adjoint-based approach, largely due to what seemed to be a lack of seed-dispersion. Thus, a second method employing the familiar tool of 'dose-update' was investigated.

3.2 Method 2: CD-based Greedy with Dose Update

As before, we begin with the initial CD map

$$D'(i) \longleftarrow ||\mathbf{J}^{tot}(i)||$$

However, after placing the first seed (and all subsequent seeds), we update the CD map via

$$D'(i) \longleftarrow D'(i) \times \Delta(i)^k \tag{6}$$

where Δ is the current dose profile of the total region, and k is a constant.

By using dose-update, we are able to disperse seeds quite simply and effectively. This method has shown promising results that compare well and sometimes exceed those of the adjoint-based approach.

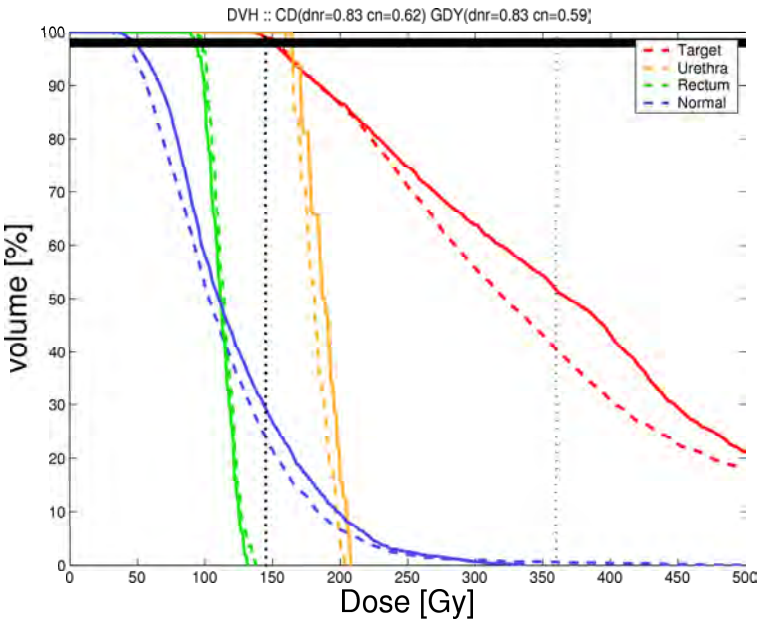

Figure 1: Patient slice number 7 with 1 U seeds. (CD is dashed) CD-25 seeds, GDY-25 seeds

4 Some (More) Graphical Results

Figures 1 and 2 represent two slices of a patient 3-d profile for which the latter CD-based Greedy method and the adjoint ratio-based method were employed to produce treatment plans. The seed strength is 1 U (about 0.4 mCi, and unlike the 2 U and 6 U plots provided in earlier discussions). Figures 3 and 4 plot the associated seed placement.

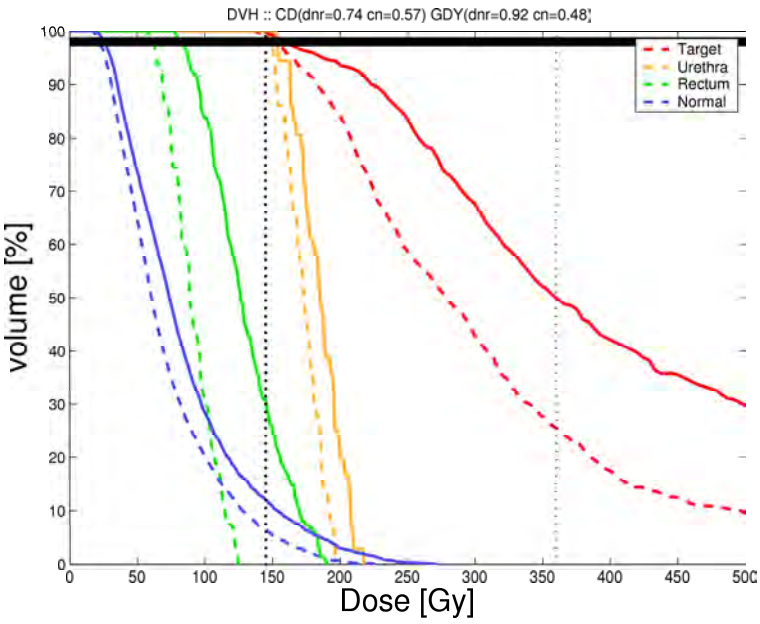

Figure 2: Patient slice number 3 with 1 U seeds. (CD is dashed) CD-12 seeds, GDY-15 seeds

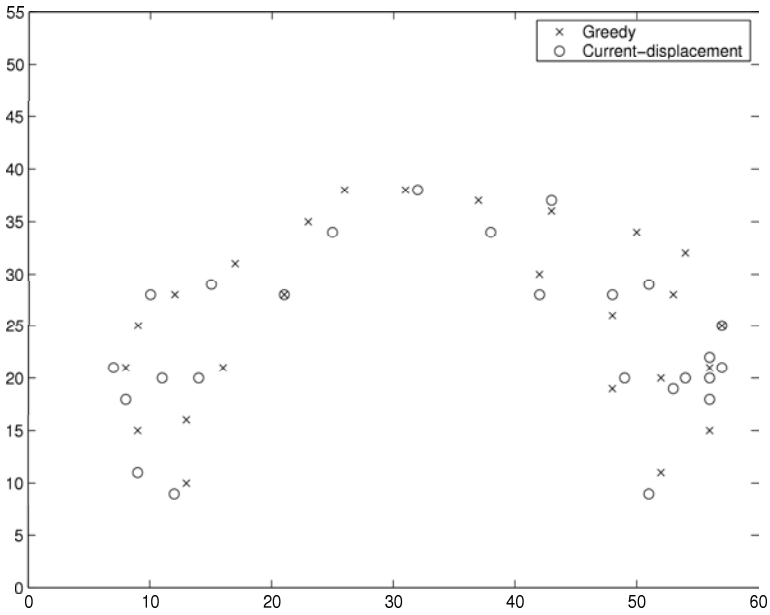

Figure 3: Patient slice number 7 seed placement

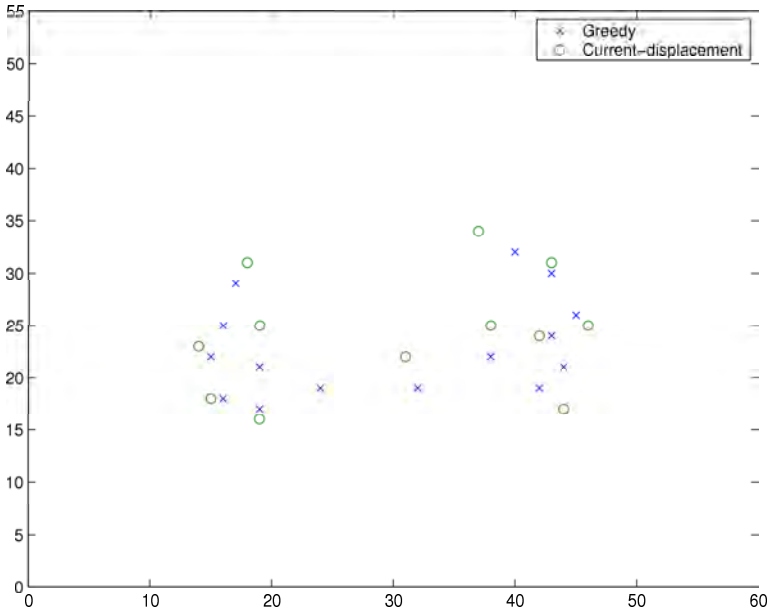

Figure 4: Patient slice number 3 seed placement