MAY 2022

TEXAS
The University of Texas at Austin

ASE 366K: PROJECT 2

Plotting an orbit from orbital elements

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Some contents have been redacted from this version

The problem

Given orbital elements…

- Propagate orbit
- Plot in various coordinate systems
- Animation
- Text outputs

Process

Part 1

Functions

- **`R1`**, **`R2`**, **`R3`**: coordinate rotation matrices
- **`oe2rv`**: convert orbital elements to **r** and **v** (similar to Homework 8)
- **`f`**: represents differential equation (similar to Project 1)
- **`testCase`**: holds most repetitive tasks

Orbital Elements to Vectors

- Only for first point
- Easier to propagate orbit of <u>equally spaced times</u> numerically

Orbital Elements to Vectors

- Similar to Homework 8

Propagating Orbits

- Create derivative as anonymous function to match format
- Solve with **`ode45`**
- Results are returned as <u>row vectors</u>

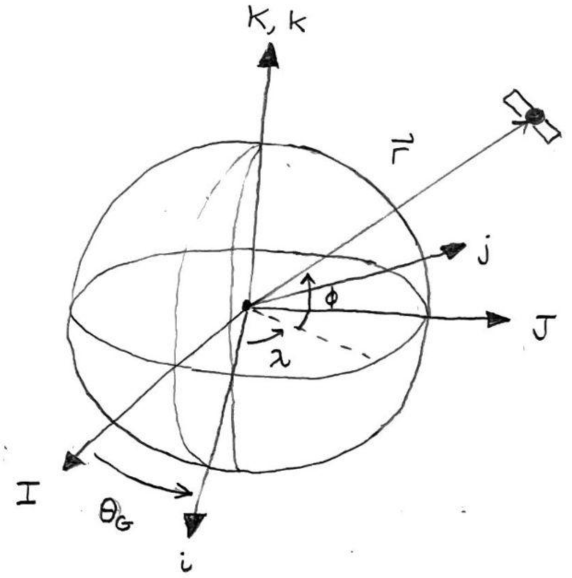
Propagating Orbits

- Similar to Project 1

Coordinate Systems

- Inertial (IJK): given
- Body-fixed (ijk)
  - Latitude/longitude
- Topocentric (sez)
  - Azimuth/elevation

Body-Fixed/Latitude/Longitude



$$\theta_G = \theta_{G0} + \frac{2\pi}{T_\oplus}\Delta t$$

$$\vec{r}_{ijk} = R_3(\theta_G)\,\vec{r}_{IJK}$$

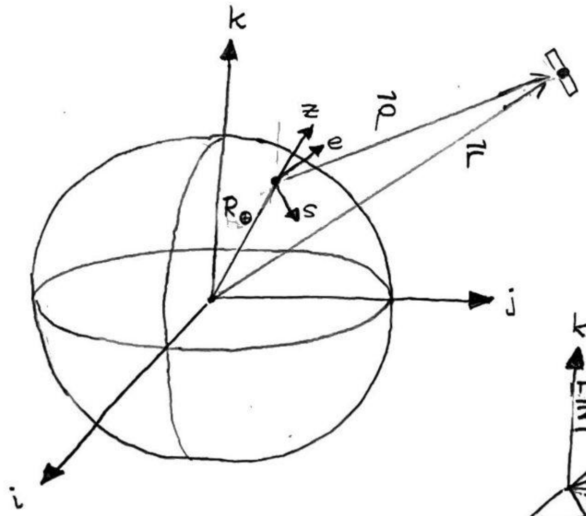
$$\phi = \arcsin\left(\frac{\vec{r}_{ijk}\cdot\hat{k}}{\|\vec{r}_{ijk}\|}\right)$$

$$\lambda = \operatorname{arctan2}\left(\vec{r}_{ijk}\cdot\hat{j},\ \vec{r}_{ijk}\cdot\hat{i}\right)$$

Body-Fixed/Latitude/Longitude

- Rotate coordinate axes to match Earth
- No ambiguity for angles
  - ϕ is restricted
  - λ uses **`atan2`**

Topocentric/Azimuth/Elevation

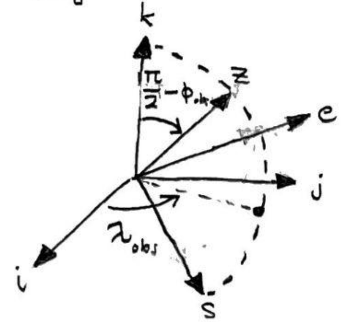


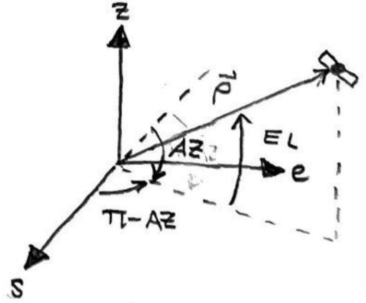
$$\vec{r}_{sez} = R_2\left(\frac{\pi}{2} - \phi_{obs}\right) R_3(\lambda_{obs}) \vec{r}_{ijk}$$

$$\vec{\rho}_{sez} = \vec{r}_{sez} - R_\oplus \hat{z}$$

$$EL = \arcsin\left(\frac{\vec{\rho}_{sez} \cdot \vec{z}}{\|\vec{\rho}_{sez}\|}\right)$$

$$AZ = \pi - \arctan2(\vec{\rho}_{sez} \cdot \hat{e}, \vec{\rho}_{sez} \cdot \hat{s})$$





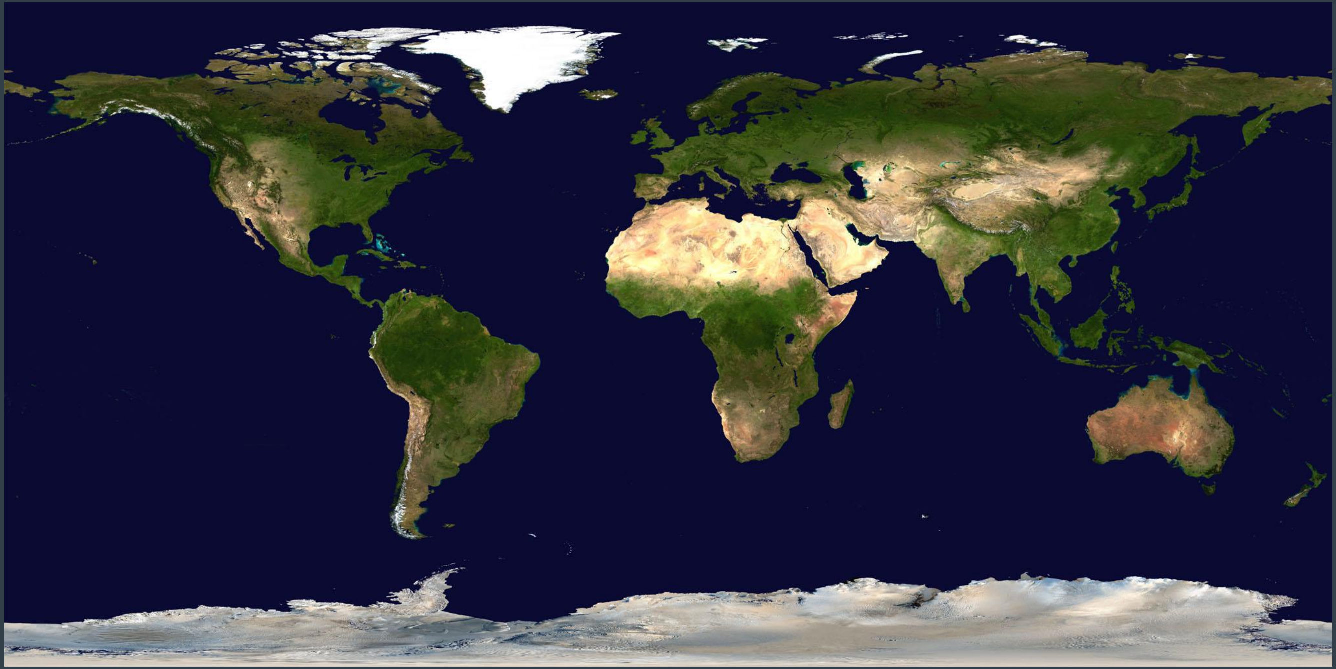
Topocentric/Azimuth/Elevation

- Rotate coordinates to match observer
- No ambiguity for angles
  - EL is restricted
  - AZ uses `atan2`

Visibility to Observer

- If EL > 0, the object is visible
  - Static: separate points into different vectors to be plotted separately
  - Animated: check EL for each point

Plotting Earth



https://earthobservatory.nasa.gov/blogs/elegantfigures/wp-content/uploads/sites/4/2011/10/land_shallow_topo_2011_8192.jpg

Plotting Earth: Sphere

- Help from fora
- Create spherical surface
- Load image
- Flip image

Plotting Earth: Flat

- Help from fora
- Create spherical surface
- Load image
- Flip image

Animation

- Always plots black dots beforehand
  - Keeps plots stationary during animation
- If animating
  - Plot dots one-by-one
  - Pause 0.05 seconds
- Else
  - Plot all at once

Text Outputs (Initial and Final)

$$^{\mathrm{ijk}}\mathbf{v}_{\mathrm{ijk}} = {}^{\mathrm{IJK}}\mathbf{v}_{\mathrm{ijk}} + (\boldsymbol{\omega}_{\mathrm{IJK/ijk}})_{\mathrm{ijk}} \times \mathbf{r}_{\mathrm{ijk}}$$

$$^{\mathrm{ijk}}\mathbf{v}_{\mathrm{ijk}} = {}^{\mathrm{IJK}}\mathbf{v}_{\mathrm{ijk}} - \begin{bmatrix} 0 \\ 0 \\ \frac{2\pi}{T_{\oplus}} \end{bmatrix} \times \mathbf{r}_{\mathrm{ijk}}$$

- $\mathbf{r}_{\mathrm{ijk}}$ already calculated
- $^{\mathrm{ijk}}\mathbf{v}_{\mathrm{ijk}}$ requires change of coordinate <u>system</u> and <u>frame</u>

Text Outputs (Initial and Final)

- Already calculated
- If N is an integer, the initial and final latitudes should be the same

Orbital Resonance

$$M T_{\oplus} = N T_{\boxed{\bullet}}$$

$$M T_{\oplus} = N \, 2\pi \sqrt{\frac{a^3}{\mu}}$$

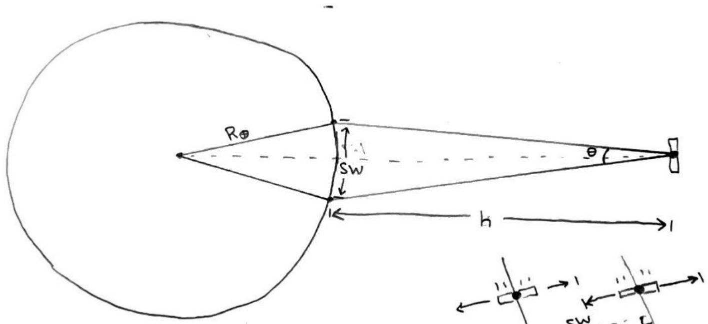
$$\sqrt{\frac{a^3}{\mu}} = \frac{M T_{\oplus}}{2\pi N}$$

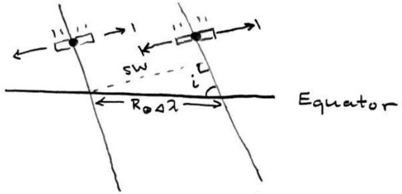
$$a = \mu^{1/3} \left(\frac{M T_{\oplus}}{2\pi N} \right)^{2/3}$$

Orbital Resonance

- Loop through all possible acceptable M/N combinations
  - Neglect very large N

Coverage Angle





$$\tan\left(\frac{\theta}{2}\right) \approx \frac{SW/2}{h}$$

$$SW = 2(r - R_\oplus)\tan\left(\frac{\theta}{2}\right)$$

Note: $a = r$

$$2(a - R_\oplus)\tan\left(\frac{\theta}{2}\right) = 4\pi^2 \frac{R_\oplus}{T_\oplus}\sqrt{\frac{a^3}{\mu}}\sin(i)$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{2\pi^2 R_\oplus}{(a - R_\oplus)T_\oplus}\sqrt{\frac{a^3}{\mu}}\sin(i)$$

$$\theta = 2\arctan\left(\frac{2\pi^2 R_\oplus}{(a - R_\oplus)T_\oplus}\sqrt{\frac{a^3}{\mu}}\sin(i)\right)$$

$$\sin(i) \approx \frac{SW}{R_\oplus \Delta\lambda}$$

$$SW = R_\oplus \Delta\lambda \sin(i)$$

$$SW = R_\oplus \omega_\oplus T \sin(i)$$

$$SW = R_\oplus \frac{2\pi}{T_\oplus}\left(2\pi\sqrt{\frac{a^3}{\mu}}\right)\sin(i)$$

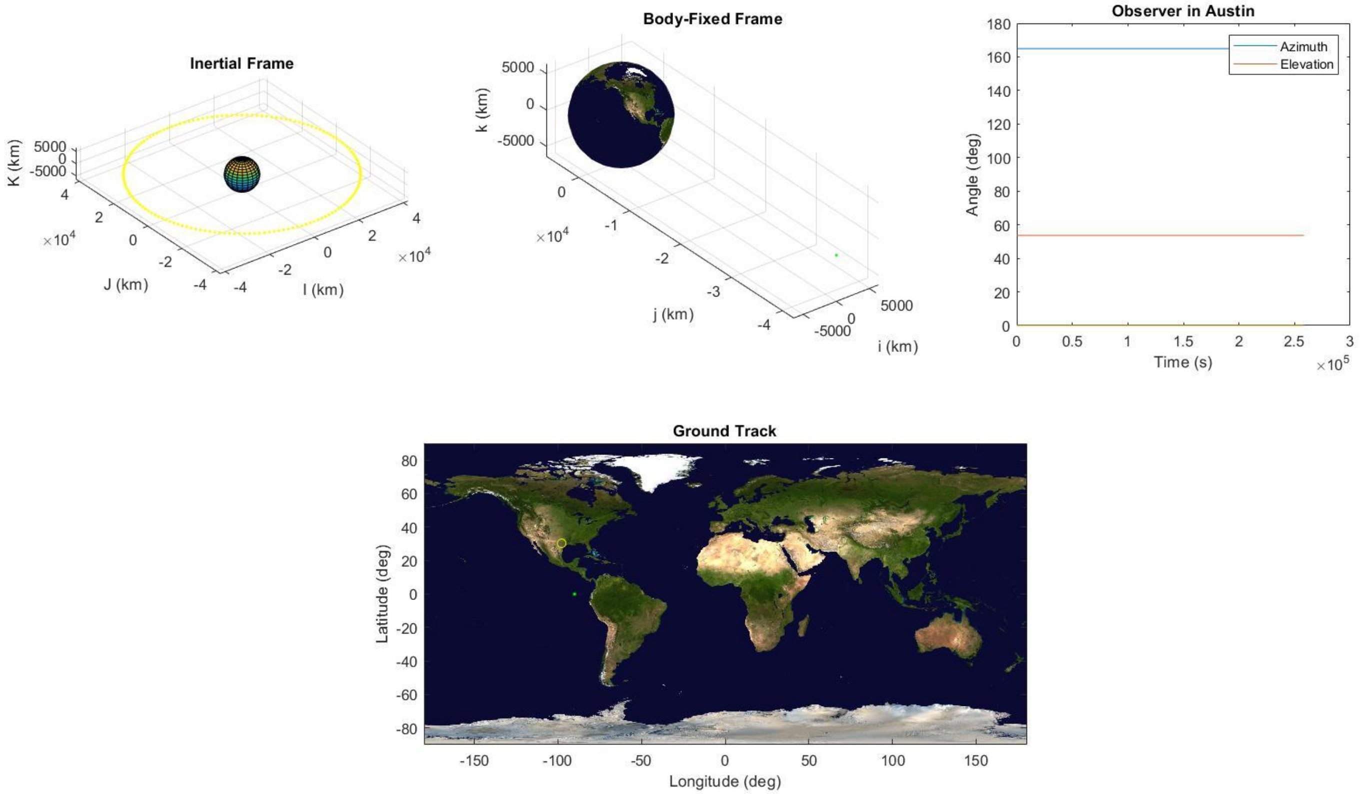
$$SW = 4\pi^2 \frac{R_\oplus}{T_\oplus}\sqrt{\frac{a^3}{\mu}}\sin(i)$$

Results

Part 2

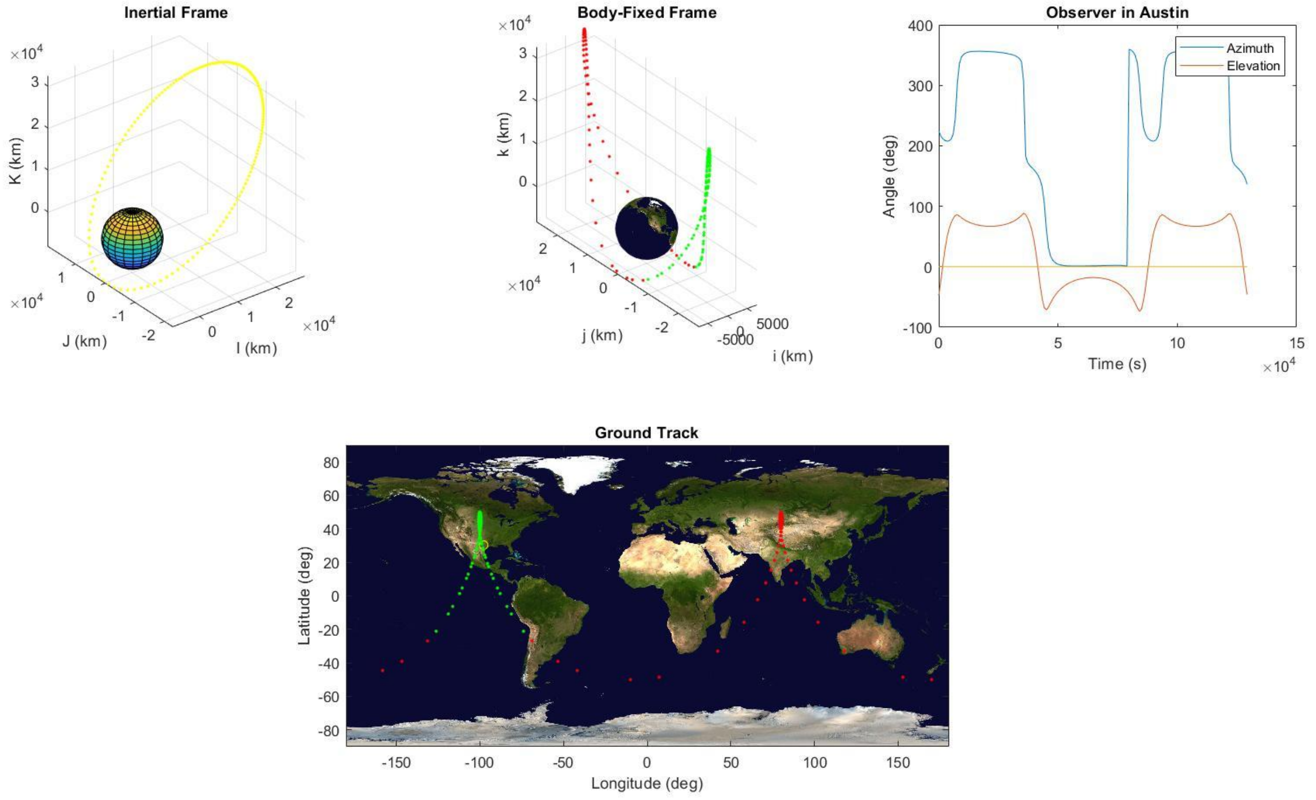
Problem 1(a)

oe = (42164 km, 0.00, 0 deg, -90 deg, 0 deg, 0 deg)

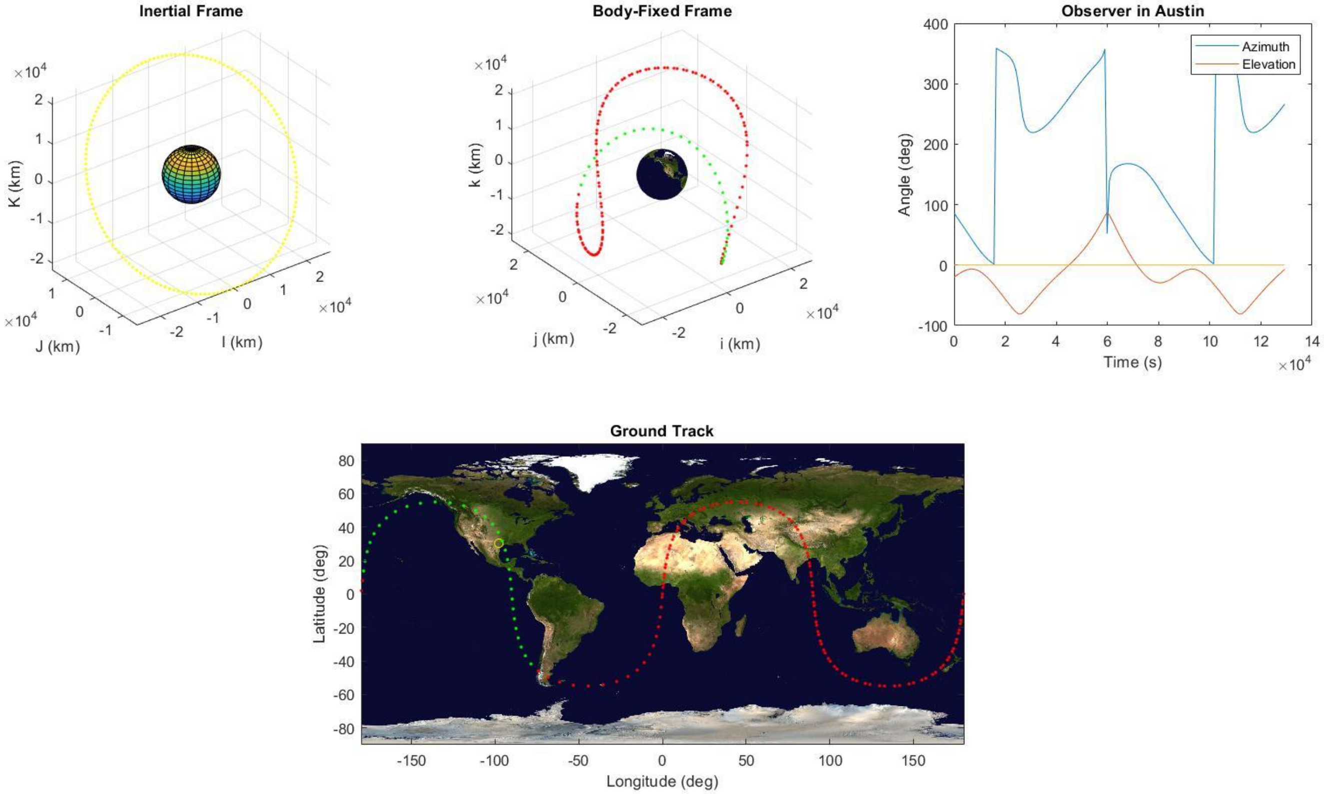


Problem 1(b)

oe = (26562 km, 0.60, 50 deg, -90 deg, -100 deg, 0 deg)

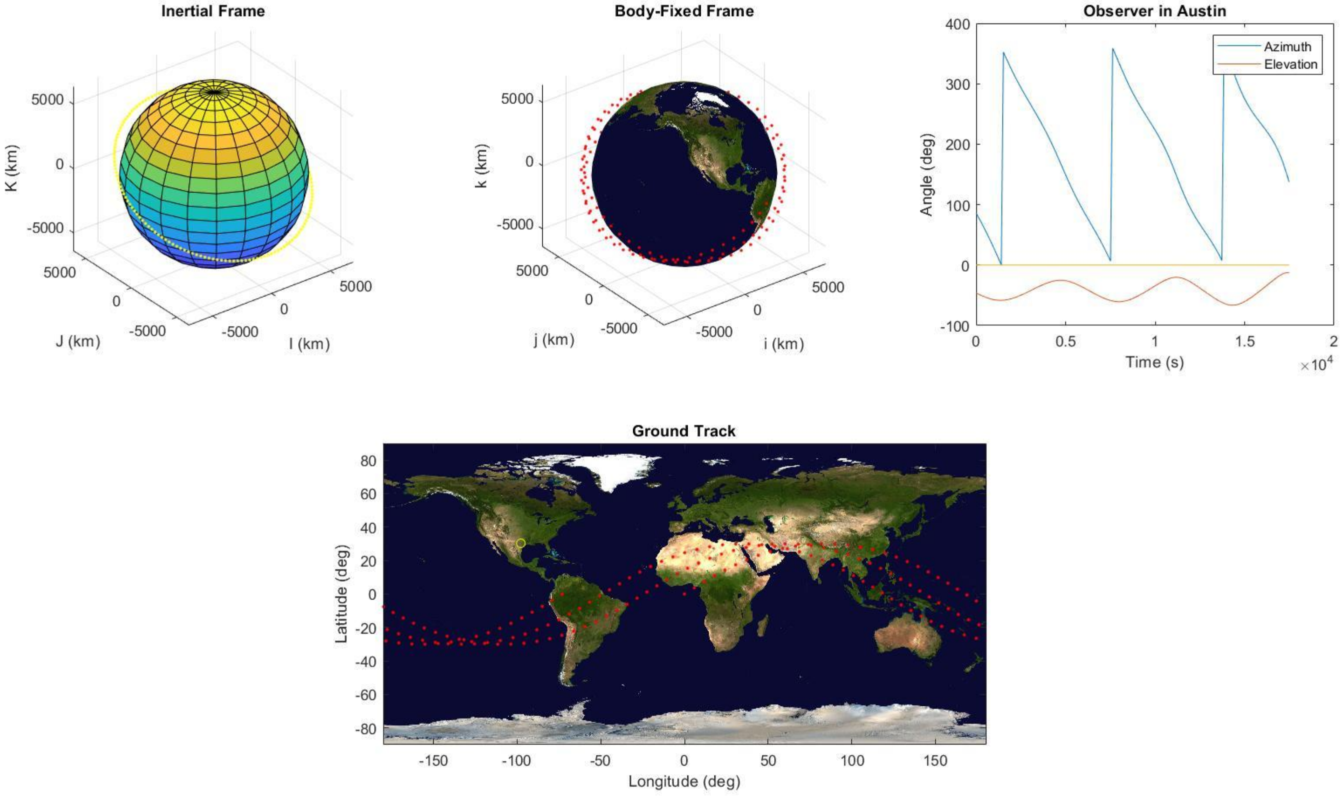


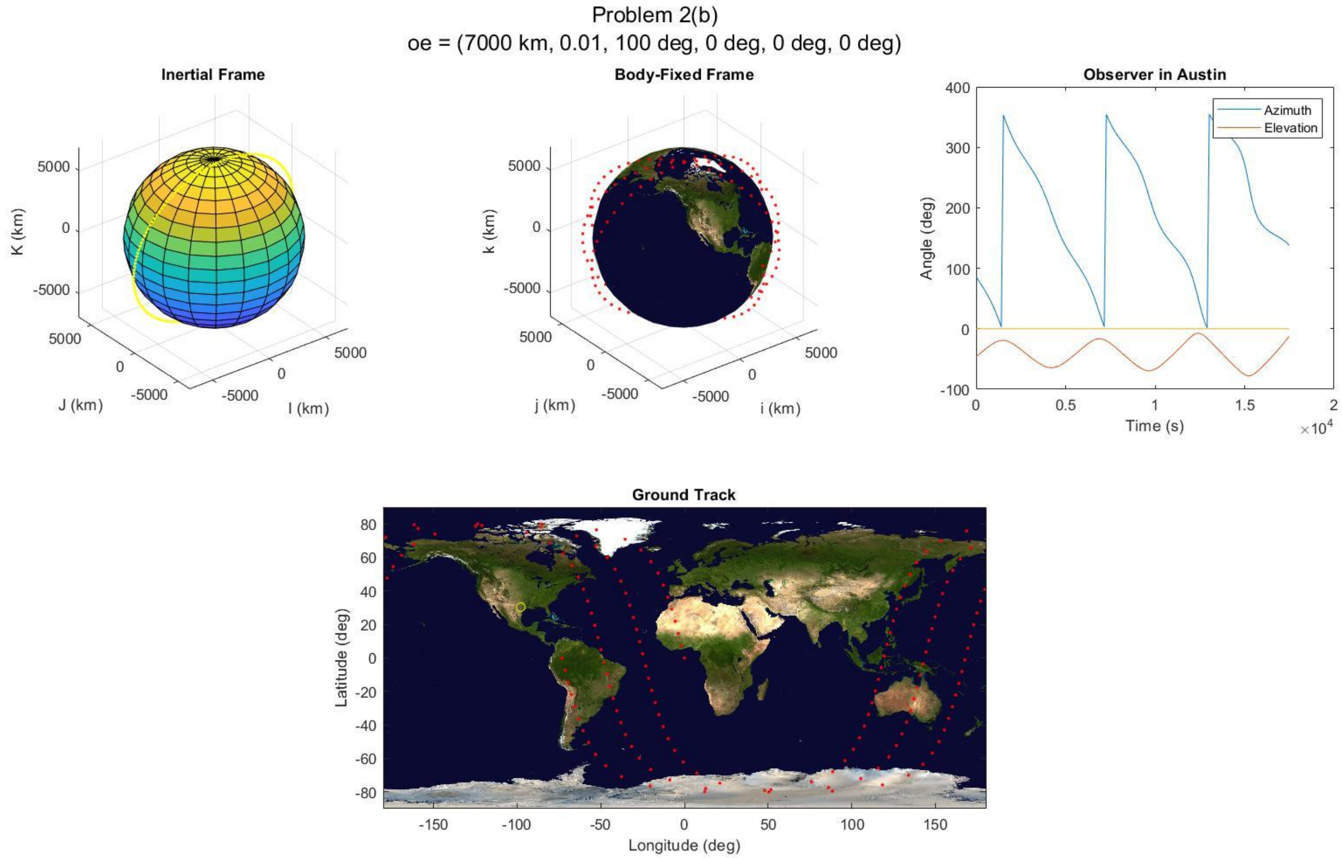
Problem 1(c)
oe = (26562 km, 0.00, 55 deg, 0 deg, 0 deg, 0 deg)



Problem 2(a)

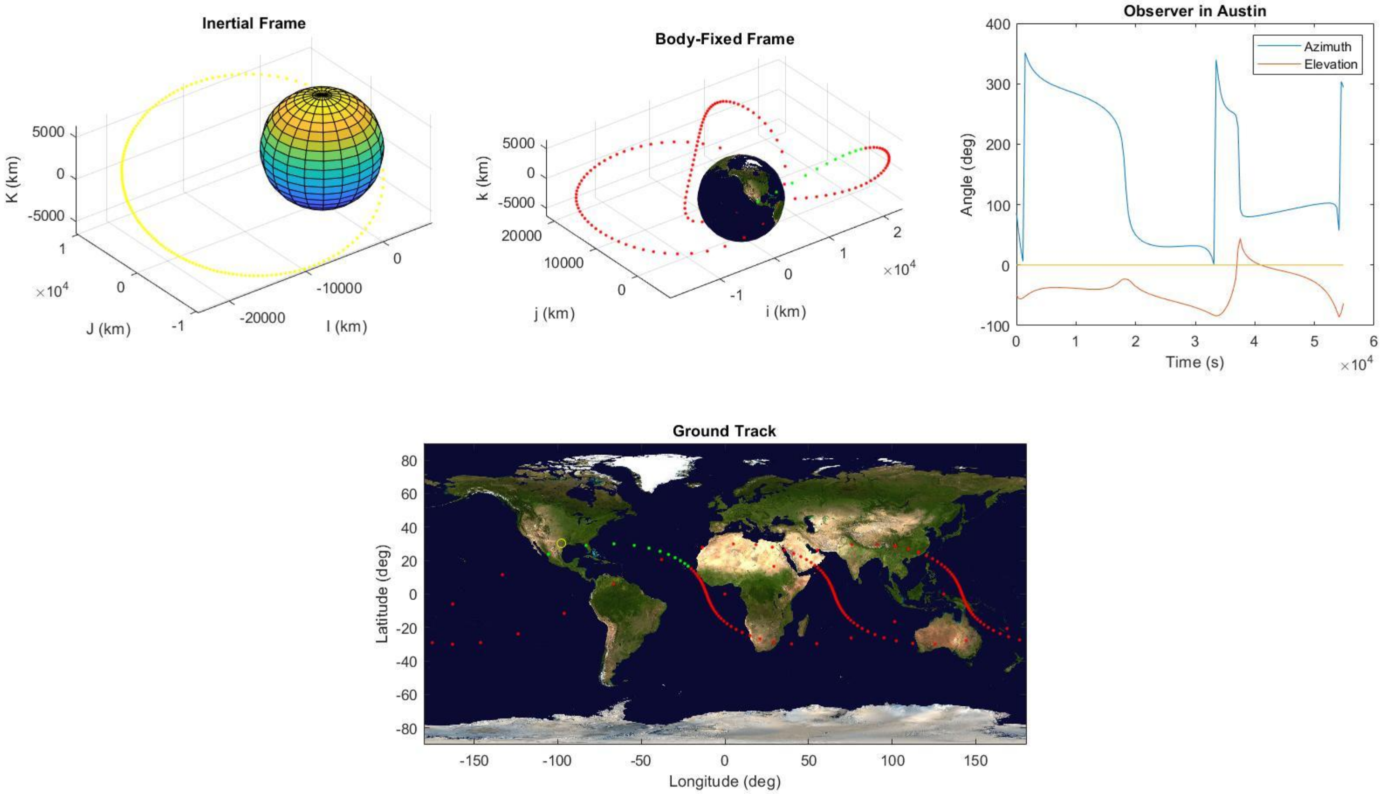
oe = (7000 km, 0.01, 30 deg, 0 deg, 0 deg, 0 deg)




Problem 2(b)
oe = (7000 km, 0.01, 100 deg, 0 deg, 0 deg, 0 deg)
Inertial Frame
K (km)
J (km)
I (km)
Body-Fixed Frame
k (km)
j (km)
i (km)
Observer in Austin
Azimuth
Elevation
Angle (deg)
Time (s)
$\times 10^4$
Ground Track
Latitude (deg)
Longitude (deg)

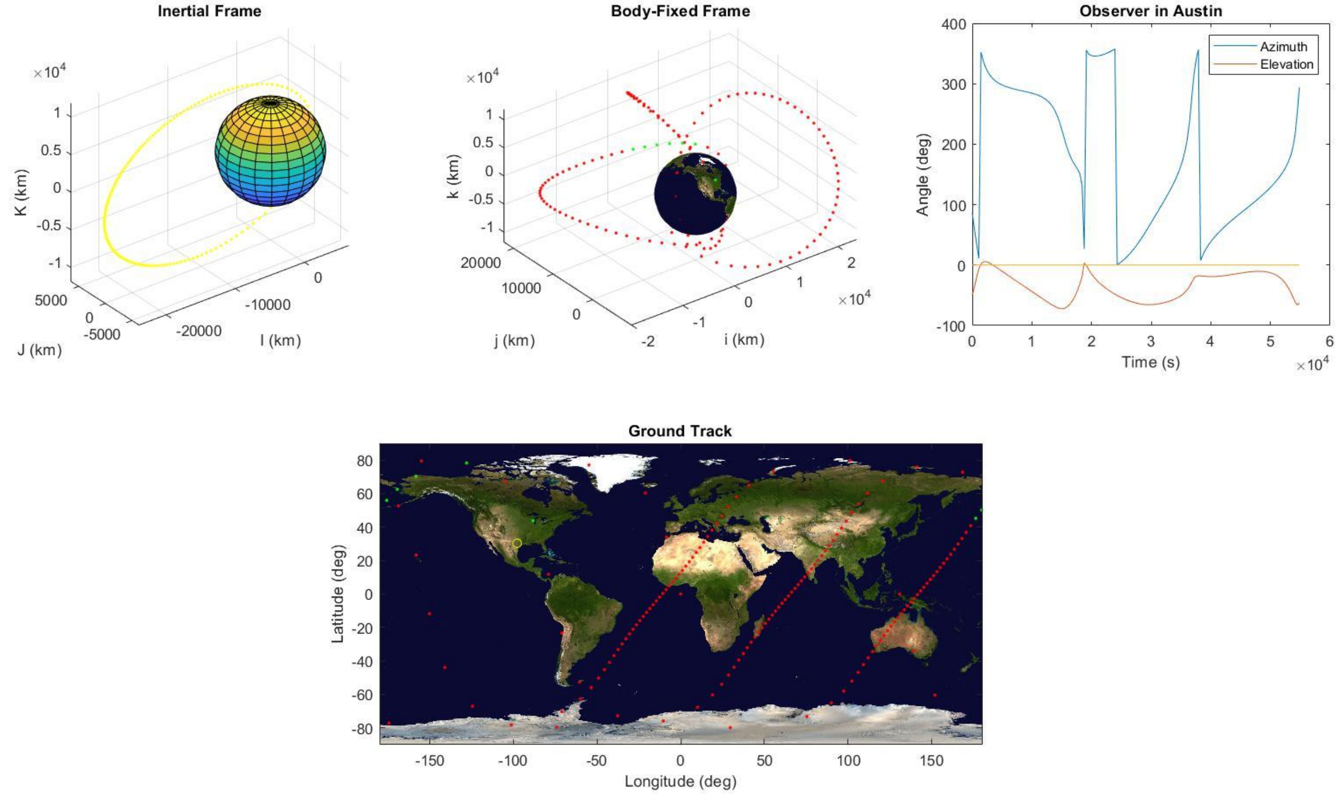
Problem 2(c)

oe = (15000 km, 0.60, 30 deg, 0 deg, 0 deg, 0 deg)

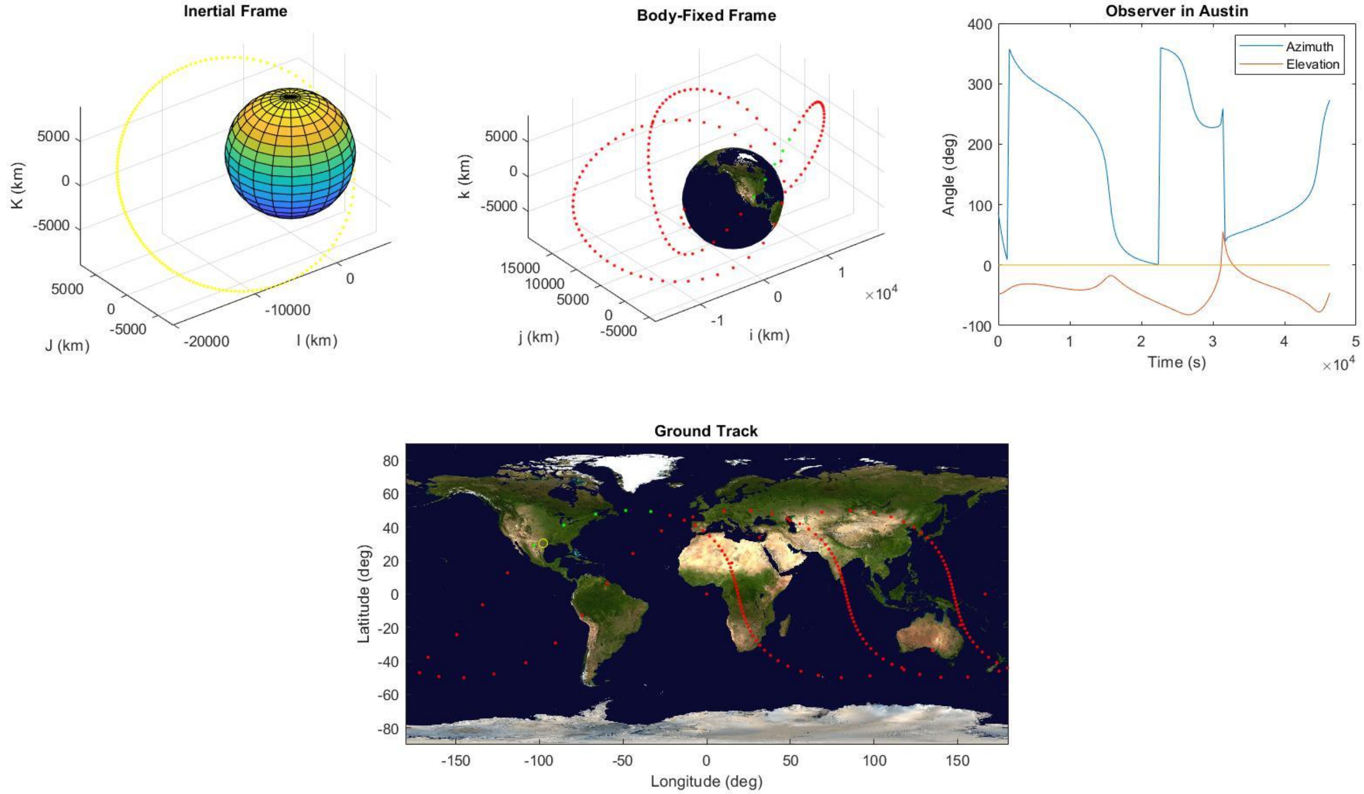


Problem 2(d)

oe = (15000 km, 0.60, 100 deg, 0 deg, 0 deg, 0 deg)

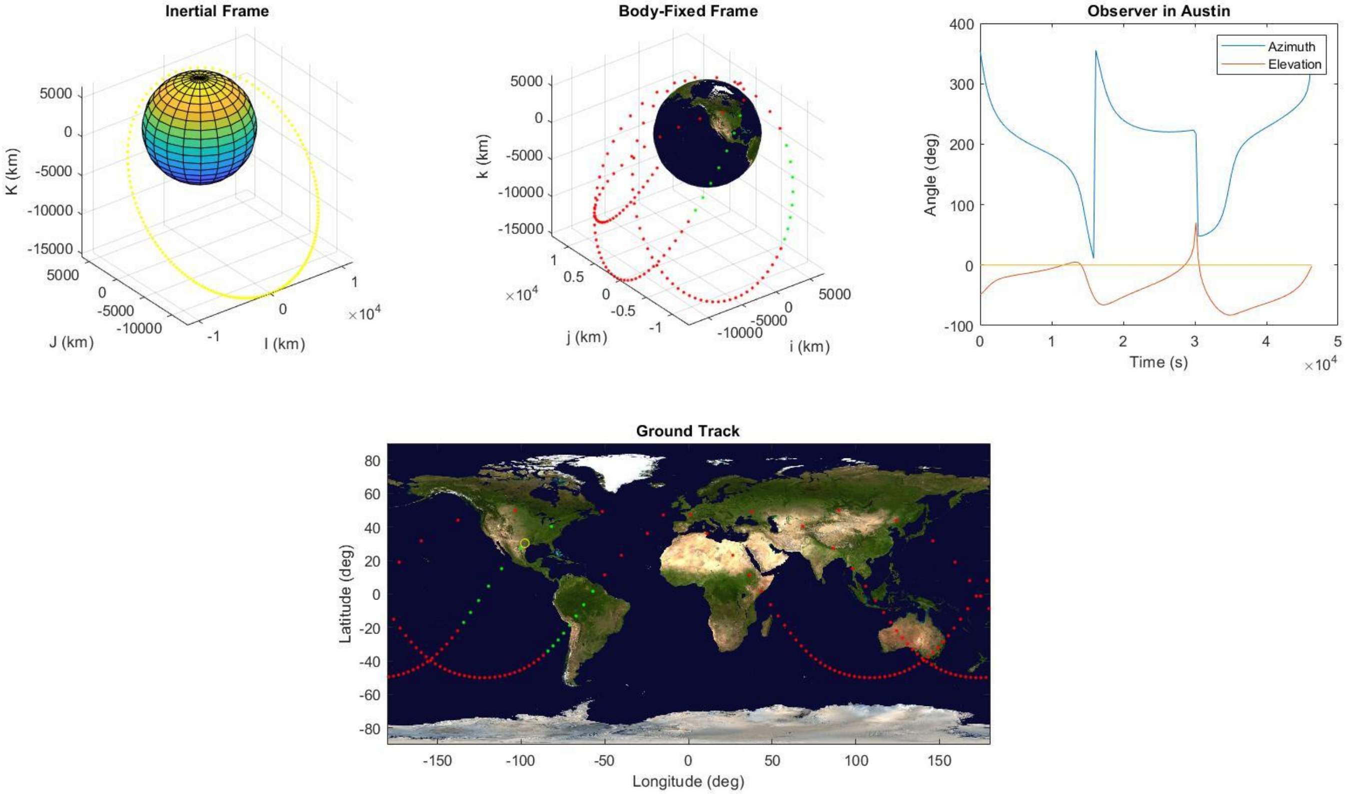


Problem 2(e)
oe = (13400 km, 0.50, 50 deg, 0 deg, 0 deg, 0 deg)



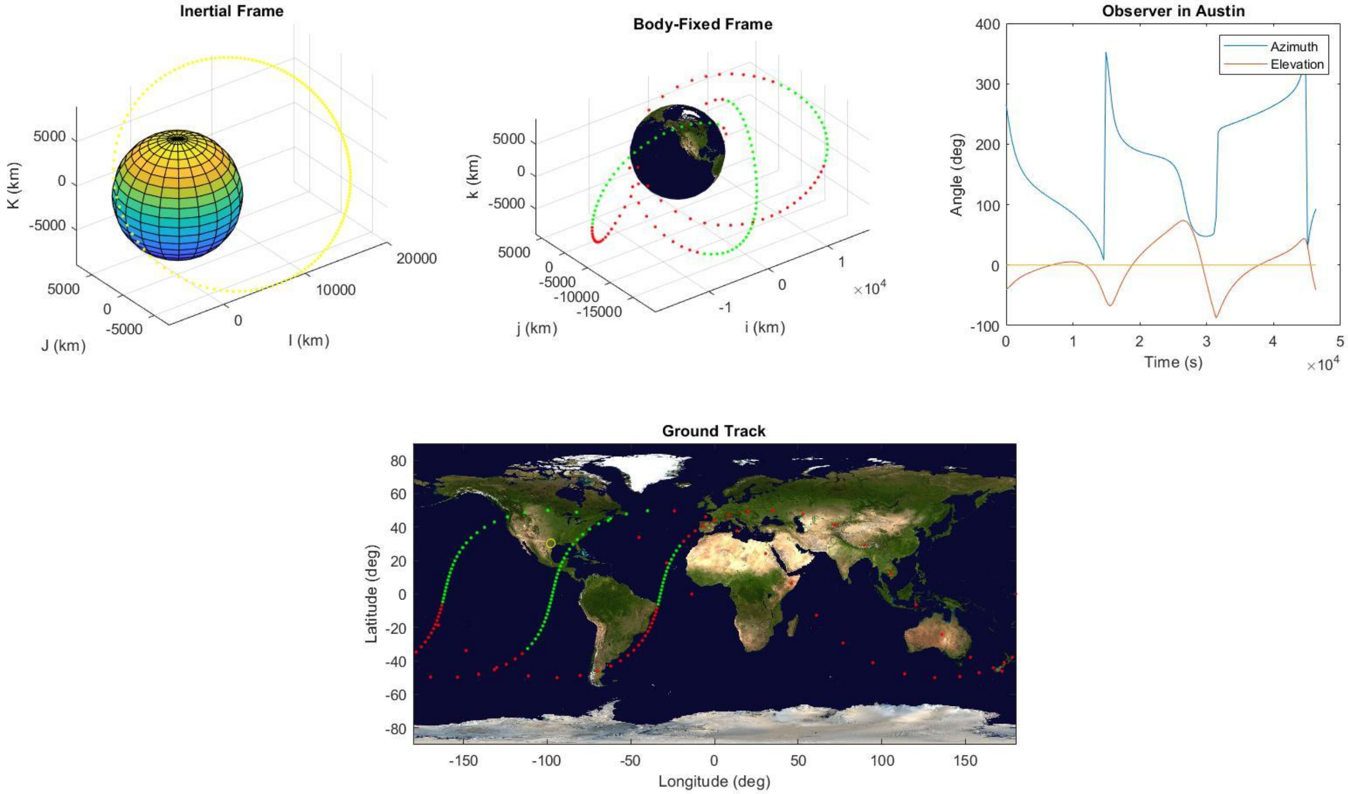
Problem 2(f)

oe = (13400 km, 0.50, 50 deg, 90 deg, 0 deg, 0 deg)



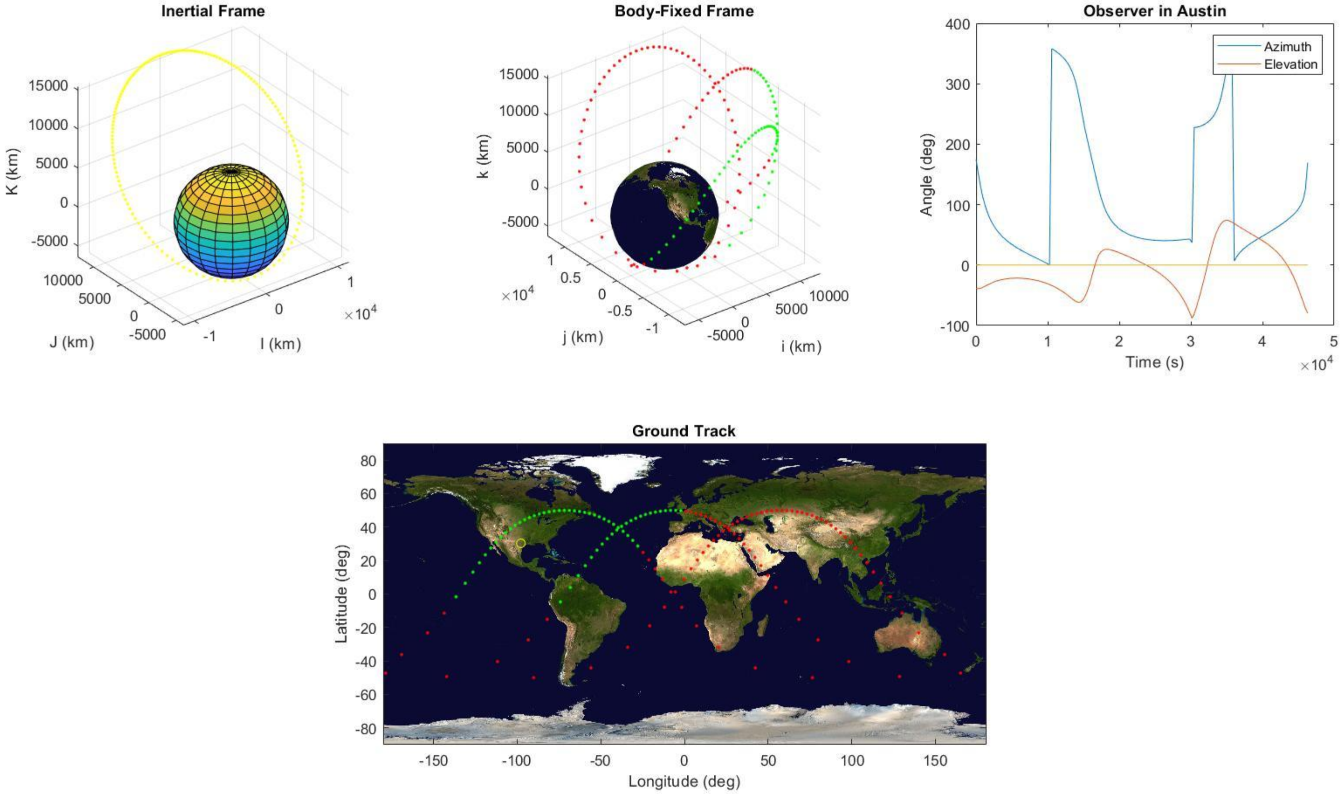
Problem 2(g)

oe = (13400 km, 0.50, 50 deg, 180 deg, 0 deg, 0 deg)



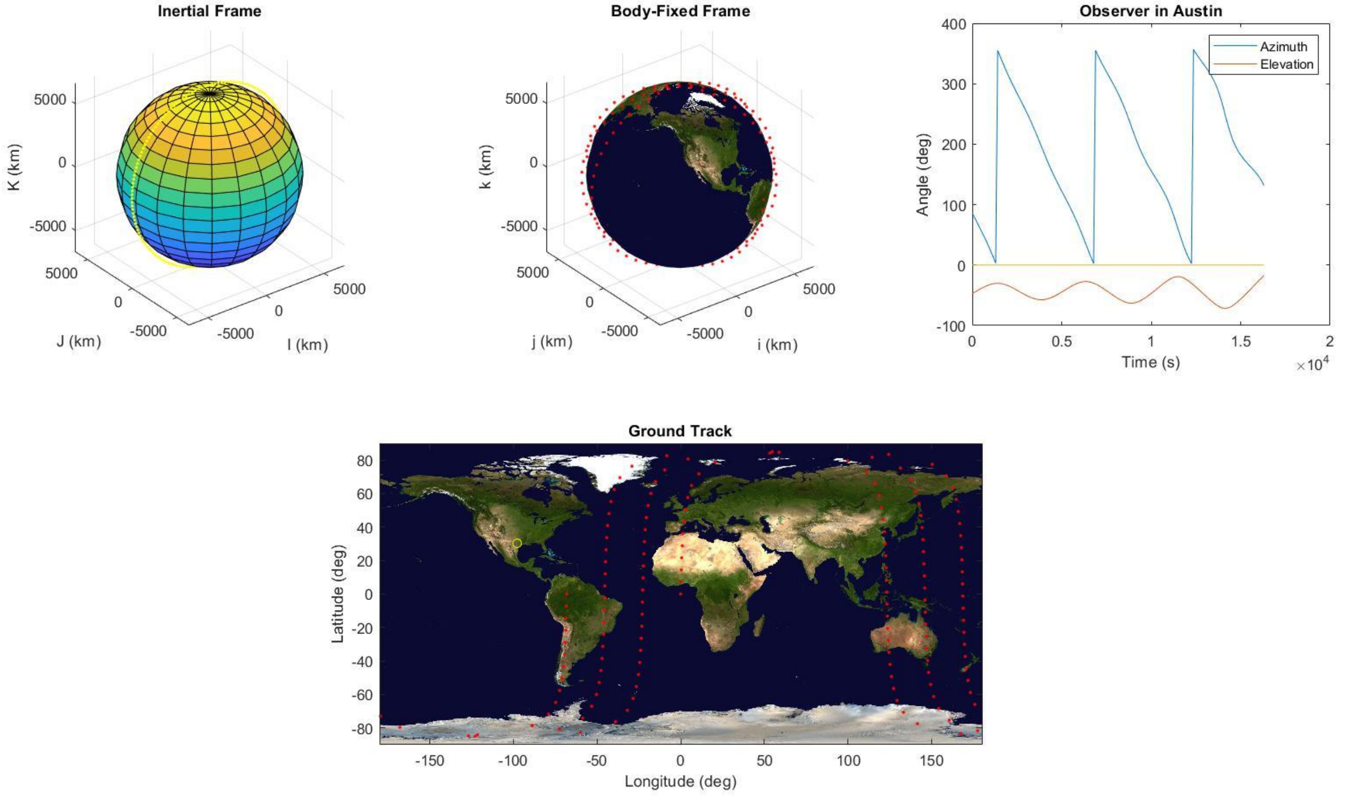
Problem 2(h)

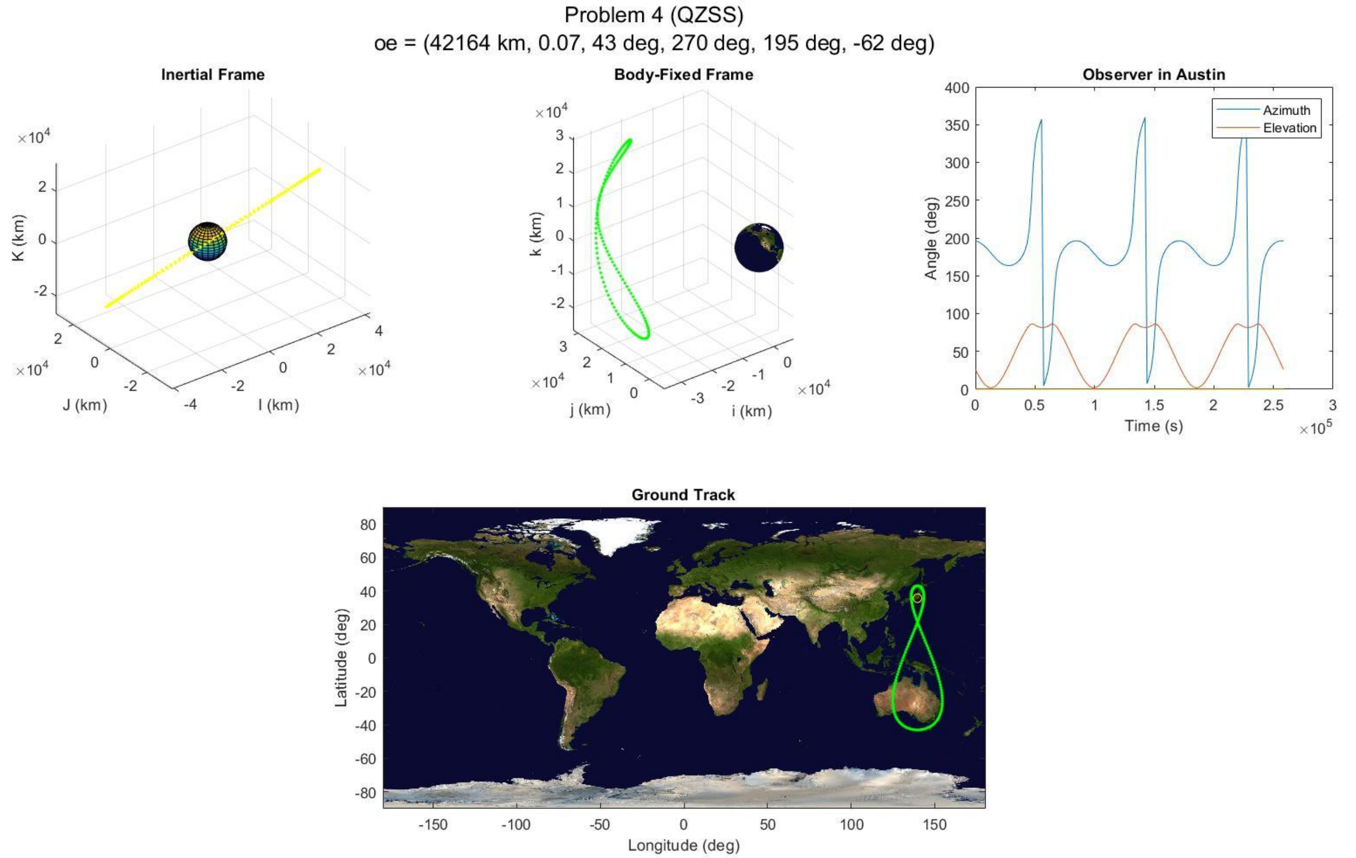
oe = (13400 km, 0.50, 50 deg, 270 deg, 0 deg, 0 deg)



Problem 3

oe = (6680 km, 0.00, 85 deg, 0 deg, 0 deg, 0 deg)





Source: https://en.wikipedia.org/wiki/Quasi-Zenith_Satellite_System

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