kumar-jha-12340390-dav-homework-2

February 5, 2025

0.1 Question 1

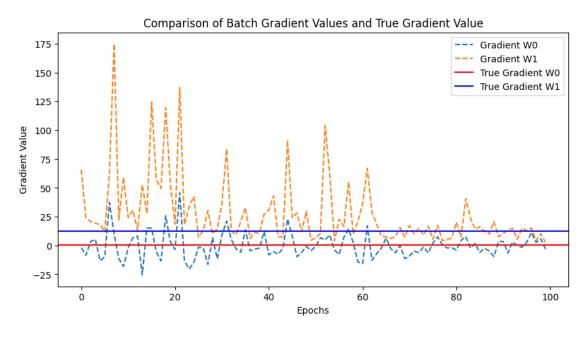
```
[2]: import pandas as pd
     from google.colab import drive
     import numpy as np
     import matplotlib.pyplot as plt
     drive.mount("/content/drive")
     df = pd.read_csv("/content/drive/MyDrive/Course Work/Sem 4/Data Analysis and_
      → Visualization/Homework 2/DIstanceTimeDataset - StudentsHomeTownDistance.csv")
     cols = df.columns.tolist()
     columns to drop = cols[6:]
     df.drop(columns_to_drop, axis=1, inplace=True)
     nan_df = df[df.isna().any(axis=1)]
     rows_to_drop = nan_df.index.tolist()
     df.drop(rows_to_drop, axis=0, inplace=True)
     df.reset_index(drop=True, inplace=True)
     ## Normalizing the features
     time = df[cols[1]]
     distance = df[cols[2]]
     time_normalized = (time - time.mean()) / time.std()
     distance_normalized = (distance - distance.mean()) / distance.std()
     X = time_normalized.values.reshape(-1, 1)
     y = distance_normalized.values.reshape(-1, 1)
     ## Parameters for batch gradient descent
     W = np.array([0, 35], dtype=float)
     learning_rate = 0.01
     num_epochs = 100
     batch_size = 8
     num_samples = X.shape[0]
     gradient_history = []
```

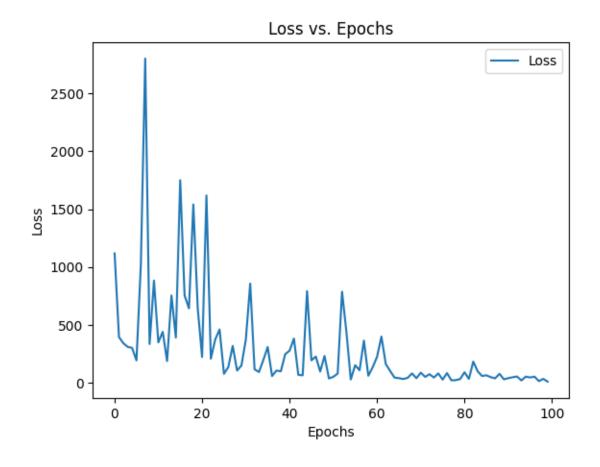
```
loss_history = []
WO_history = []
W1_history = []
gradient_norm_history = []
for epoch in range(num_epochs):
   batch = np.random.choice(df.index, size=8, replace=True)
   X_batch = X[batch]
   y_batch = y[batch]
   y_pred = W[0] + W[1] * X_batch
   loss = np.mean((y_batch - y_pred) ** 2)
   loss_history.append(loss)
   gradient_W0 = -2 * np.sum(y_batch - y_pred) / batch_size
    gradient_W1 = -2 * np.sum((y_batch - y_pred) * X_batch) / batch_size
   gradient = np.array([gradient_W0, gradient_W1])
   gradient_history.append(gradient)
   W = W - learning_rate * gradient
   W0_history.append(W[0])
   W1_history.append(W[1])
   gradient_norm_history.append(np.linalg.norm(gradient))
gradient_history = np.array(gradient_history)
mean_gradient_W0 = np.mean(gradient_history[:, 0])
mean_gradient_W1 = np.mean(gradient_history[:, 1])
mean_gradient = np.array([mean_gradient_W0, mean_gradient_W1])
cov_matrix = np.cov(gradient_history.T)
y_pred_full = W[0] + W[1] * X
true_gradient_W0 = -2 * np.sum(y - y_pred_full) / num_samples
true_gradient_W1 = -2 * np.sum((y - y_pred_full) * X) / num_samples
true_gradient = np.array([true_gradient_W0, true_gradient_W1])
plt.figure(figsize=(10, 5))
plt.plot(gradient_history[:, 0], label="Gradient WO", linestyle='dashed')
plt.plot(gradient history[:, 1], label='Gradient W1', linestyle='dashed')
plt.axhline(true_gradient[0], color='r', linestyle='solid', label='True_u
 Gradient WO')
plt.axhline(true_gradient[1], color='b', linestyle='solid', label='True_
Gradient W1')
plt.xlabel('Epochs')
plt.ylabel('Gradient Value')
plt.legend()
plt.title('Comparison of Batch Gradient Values and True Gradient Value')
```

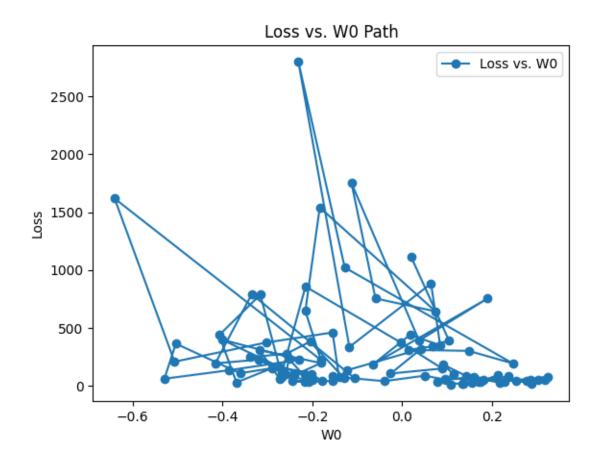
```
plt.show()
# Plot Loss vs. Epochs
plt.figure()
plt.plot(loss_history, label='Loss')
plt.xlabel('Epochs')
plt.ylabel('Loss')
plt.title('Loss vs. Epochs')
plt.legend()
plt.show()
# Plot Loss vs. WO
plt.figure()
plt.plot(WO_history, loss_history, label='Loss vs. WO', marker='o')
plt.xlabel('WO')
plt.ylabel('Loss')
plt.title('Loss vs. WO Path')
plt.legend()
plt.show()
# Plot Loss vs. W1
plt.figure()
plt.plot(W1_history, loss_history, label='Loss vs. W1', marker='o')
plt.xlabel('W1')
plt.ylabel('Loss')
plt.title('Loss vs. W1 Path')
plt.legend()
plt.show()
# Plot WO vs. Epochs
plt.figure()
plt.plot(W0_history, label='W0 Value')
plt.xlabel('Epochs')
plt.ylabel('W0')
plt.title('WO Value Change Over Epochs')
plt.legend()
plt.show()
# Plot W1 vs. Epochs
plt.figure()
plt.plot(W1_history, label='W1 Value')
plt.xlabel('Epochs')
plt.ylabel('W1')
plt.title('W1 Value Change Over Epochs')
plt.legend()
plt.show()
```

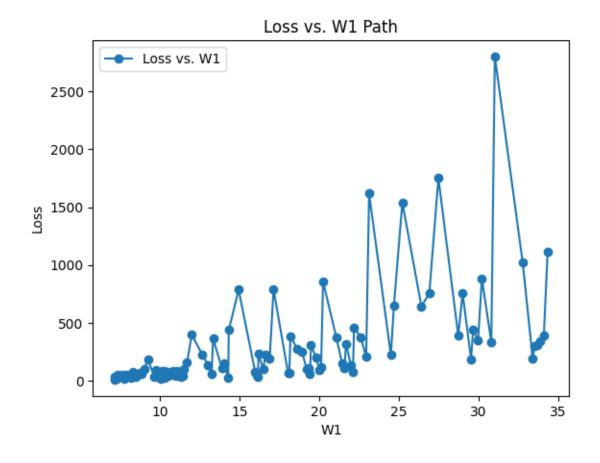
```
# Plot Loss vs. Gradient Norm
plt.figure()
plt.plot(gradient_norm_history, loss_history, label='Loss vs. Gradient Norm')
plt.xlabel('Gradient Norm')
plt.ylabel('Loss')
plt.title('Loss vs. Gradient Norm')
plt.legend()
plt.show()
```

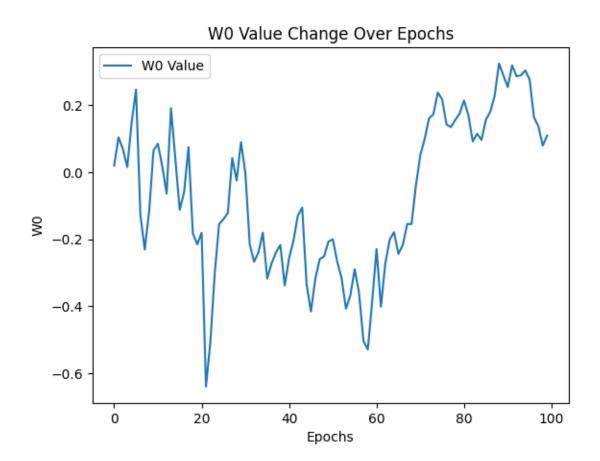
Drive already mounted at /content/drive; to attempt to forcibly remount, call drive.mount("/content/drive", force_remount=True).

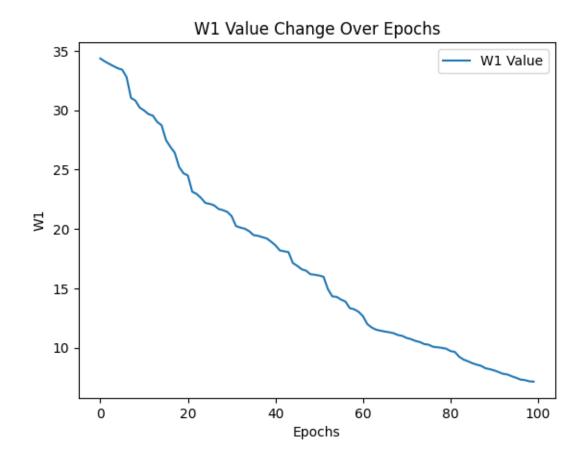




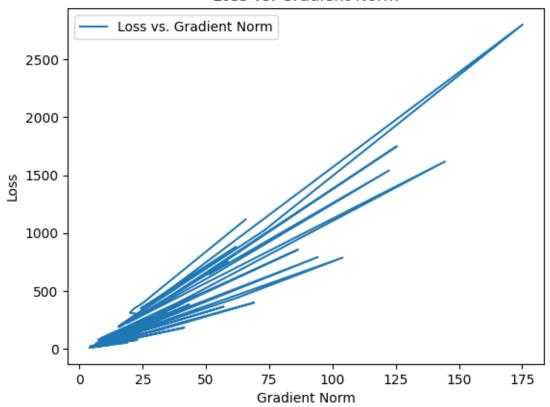












- 1. Random batches are created for 100 number of batches
- 2. Each rabdom batch has a size of 8
- 3. Mean is calculated for both the gradients (gradients of both the parameters involved)
- 4. Covariance matrix is calculated keeping in account both the gradients
- 5. Visualization showing the true gradient values and also the gradient values vs number of epochs, for both the parameters involved
- 6. The path for both the gradient values are shown in the graph plotted
- 7. The path for both the parameter values are shown in the graph plotted, in terms of the number of epochs, and also in terms of the loss function
- 8. Variation of the loss function with the number of epochs is shown
- 9. The plot of the loss function with the gradient norm is shown
- 10. Answer to all the questions asked are present in the latex file submitted

1 Question 2

2 Submitted in latex, because those questions were Math Based

2.1 Question 3

[8]: cols

```
[3]: import pandas as pd
     from google.colab import drive
     import numpy as np
     import matplotlib.pyplot as plt
     from sklearn.linear model import LinearRegression
     from sklearn.metrics import mean_squared_error
     from sklearn.model selection import train test split
[4]: drive.mount("/content/drive")
    Drive already mounted at /content/drive; to attempt to forcibly remount, call
    drive.mount("/content/drive", force_remount=True).
[5]: df = pd.read_csv("/content/drive/MyDrive/Course Work/Sem 4/Data Analysis and_
      -Visualization/Homework 2/DSAICourseInterestRelevanceSurvey - Original.csv")
[6]: df.head()
[6]:
       Unnamed: 0
                   MAL100 MAL101 MAL403 EEL101 ECL101
                                                            BML101 CSL100
                                                                             CSL201 \
     0 Student 1
                      4.0
                               3.0
                                       4.0
                                               1.0
                                                       1.0
                                                                1.0
                                                                        4.0
                                                                                5.0
     1 Student 2
                      3.0
                                       3.0
                                               1.0
                                                       1.0
                                                                1.0
                                                                                5.0
                               3.0
                                                                        4.0
     2 Student 3
                      4.0
                               4.0
                                       3.0
                                               3.0
                                                       4.0
                                                                2.0
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     3 Student 4
                      3.0
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                                       4.0
                                               1.0
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                                                                1.0
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     4 Student 5
                      3.0
                                       4.0
                                               3.0
                                                       3.0
                              3.0
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        CSL202 DSL201
                        Unnamed: 11
                                      Unnamed: 12 Please fill values {1,2,3,4,5}
     0
           5.0
                   5.0
                                 3.3
                                              NaN
                                                                           Legend
           5.0
                   5.0
                                 3.1
                                              NaN
     1
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     2
           4.0
                                 3.7
                                                                                2
                   NaN
                                              NaN
     3
           5.0
                   2.0
                                 3.1
                                              NaN
                                                                                3
           5.0
                   4.0
                                 3.6
                                              NaN
                                               Unnamed: 14
     0
                                                       NaN
     1
                         Neither interesting nor relevant
     2 Little interesting, but relevance not clear. 0...
     3
               Somewhat interesting and somewhat relevant
     4
                                 Interesting and Relevant
[7]: cols = df.columns.tolist()
```

```
[8]: ['Unnamed: 0',
       'MAL100',
       'MAL101',
       'MAL403',
       'EEL101',
       'ECL101',
       'BML101',
       'CSL100',
       'CSL201',
       'CSL202',
       'DSL201',
       'Unnamed: 11',
       'Unnamed: 12',
       'Please fill values {1,2,3,4,5}',
       'Unnamed: 14']
 [9]:
      df.rename(columns={cols[0]:"Student ID"}, inplace=True)
[10]:
      columns_to_drop = cols[12:]
[11]: df.drop(columns_to_drop, axis=1, inplace=True)
[12]:
      df.head()
[12]:
        Student ID
                                                       ECL101
                     MAL100
                              MAL101
                                      MAL403
                                               EEL101
                                                                BML101
                                                                         CSL100
                                                                                  CSL201 \
      0 Student 1
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      1 Student 2
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                                 3.0
                                          3.0
                                                  1.0
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      2 Student 3
                        4.0
                                 4.0
                                          3.0
                                                  3.0
                                                           4.0
                                                                    2.0
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                                                                                     5.0
      3 Student 4
                        3.0
                                 4.0
                                          4.0
                                                  1.0
                                                           1.0
                                                                    1.0
                                                                            5.0
                                                                                     5.0
      4 Student 5
                        3.0
                                 3.0
                                          4.0
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         CSL202 DSL201
                          Unnamed: 11
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                     5.0
                                   3.3
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            5.0
                     5.0
                                   3.1
      2
            4.0
                     NaN
                                   3.7
      3
             5.0
                     2.0
                                   3.1
            5.0
                     4.0
                                   3.6
[13]:
      df
[13]:
          Student ID
                       MAL100
                                MAL101
                                        MAL403
                                                 EEL101
                                                          ECL101
                                                                  BML101
                                                                           CSL100 \
                           4.0
           Student 1
                                   3.0
                                            4.0
                                                     1.0
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      0
      1
           Student 2
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                                   3.0
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                                                     1.0
                                                             1.0
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                                                                              4.0
                                   4.0
      2
           Student 3
                          4.0
                                            3.0
                                                     3.0
                                                             4.0
                                                                      2.0
                                                                              4.0
      3
           Student 4
                           3.0
                                   4.0
                                            4.0
                                                                      1.0
                                                                              5.0
                                                    1.0
                                                             1.0
      4
           Student 5
                           3.0
                                   3.0
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                                                     3.0
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                                                                      2.0
                                                                              4.0
      5
           Student 6
                           3.0
                                   3.0
                                            5.0
                                                     1.0
                                                             1.0
                                                                      1.0
                                                                              4.0
```

6	Student 7	3.0	4.0	4.0	1.0	1.0	1.0	4.0
7	Student 8	3.0	4.0	2.0	2.0	3.0	2.0	4.0
8	Student 9	3.0	4.0	4.0	2.0	3.0	1.0	4.0
9	Student 10	4.0	4.0	3.0	3.0	2.0	2.0	4.0
10	Student 11	4.0	5.0	4.0	2.0	3.0	3.0	5.0
11	Student 12	3.0	3.0	3.0	1.0	1.0	2.0	5.0
12	Student 13	2.0	3.0	3.0	1.0	1.0	2.0	4.0
13	Student 14	3.0	3.0	4.0	2.0	2.0	2.0	4.0
14	Student 15	4.0	4.0	5.0	3.0	3.0	1.0	4.0
15	Student 16	3.0	3.0	4.0	1.0	1.0	1.0	4.0
16	Student 17	4.0	4.0	3.0	1.0	1.0	1.0	5.0
17	Student 18	3.0	4.0	3.0	2.0	1.0	3.0	4.0
18	Student 19	3.0	3.0	4.0	1.0	2.0	1.0	4.0
19	Student 20	1.0	5.0	3.0	1.0	1.0	2.0	1.0
20	Student 21	3.0	4.0	3.0	2.0	3.0	1.0	4.0
21	Student 22	4.0	5.0	5.0	1.0	1.0	2.0	4.0
22	Student 23	2.0	2.0	2.0	1.0	1.0	4.0	4.0
23	Student 24	3.0	5.0	5.0	2.0	1.0	1.0	4.0
24	NaN	3.1	3.7	3.6	1.6	1.8	1.7	4.0

	CSL201	CSL202	DSL201	Unnamed: 11
0	5.0	5.0	5.0	3.3
1	5.0	5.0	5.0	3.1
2	5.0	4.0	NaN	3.7
3	5.0	5.0	2.0	3.1
4	5.0	5.0	4.0	3.6
5	5.0	5.0	5.0	3.3
6	4.0	4.0	5.0	3.1
7	4.0	4.0	5.0	3.3
8	4.0	4.0	4.0	3.3
9	4.0	5.0	4.0	3.5
10	5.0	5.0	4.0	4.0
11	4.0	3.0	5.0	3.0
12	4.0	3.0	3.0	2.6
13	5.0	4.0	4.0	3.3
14	5.0	4.0	5.0	3.8
15	4.0	4.0	4.0	2.9
16	5.0	5.0	5.0	3.4
17	4.0	4.0	5.0	3.3
18	3.0	5.0	5.0	3.1
19	5.0	4.0	3.0	2.6
20	4.0	4.0	5.0	3.3
21	1.0	5.0	3.0	3.1
22	2.0	2.0	4.0	2.4
23	4.0	4.0	5.0	3.4
24	4.2	4.3	4.3	3.2

[15]: df [15]: MAL403 CSL100 \ Student ID MAL100 MAL101 EEL101 ECL101 BML101 Student 1 4.0 4.0 0 4.0 3.0 1.0 1.0 1.0 1 Student 2 3.0 3.0 3.0 1.0 1.0 1.0 4.0 2 Student 3 4.0 4.0 3.0 3.0 4.0 2.0 4.0 3 Student 4 3.0 4.0 4.0 1.0 1.0 1.0 5.0 4 Student 5 3.0 3.0 4.0 3.0 3.0 2.0 4.0 4.0 5 Student 6 3.0 3.0 5.0 1.0 1.0 1.0 6 4.0 1.0 1.0 4.0 Student 7 3.0 4.0 1.0 7 Student 8 3.0 4.0 2.0 2.0 3.0 2.0 4.0 8 Student 9 3.0 4.0 4.0 2.0 3.0 1.0 4.0 9 Student 10 4.0 4.0 3.0 3.0 2.0 2.0 4.0 3.0 5.0 10 Student 11 4.0 5.0 4.0 2.0 3.0 Student 12 3.0 3.0 3.0 1.0 1.0 2.0 5.0 11 2.0 3.0 3.0 1.0 2.0 4.0 12 Student 13 1.0 Student 14 3.0 3.0 4.0 2.0 2.0 2.0 4.0 13 14 Student 15 4.0 4.0 5.0 3.0 3.0 1.0 4.0 15 Student 16 3.0 3.0 4.0 1.0 1.0 1.0 4.0 16 Student 17 4.0 4.0 3.0 1.0 1.0 1.0 5.0 Student 18 3.0 17 3.0 4.0 2.0 1.0 3.0 4.0 18 Student 19 3.0 3.0 4.0 1.0 2.0 1.0 4.0 19 Student 20 1.0 5.0 3.0 1.0 1.0 2.0 1.0 4.0 4.0 20 Student 21 3.0 3.0 2.0 3.0 1.0 21 4.0 1.0 2.0 4.0 Student 22 5.0 5.0 1.0 22 Student 23 2.0 2.0 2.0 1.0 1.0 4.0 4.0 23 Student 24 3.0 5.0 5.0 2.0 1.0 1.0 4.0 CSL201 CSL202 DSL201 Unnamed: 11 0 5.0 5.0 5.0 3.3 1 5.0 5.0 5.0 3.1 2 5.0 4.0 NaN 3.7 3 3.1 5.0 5.0 2.0 4 5.0 5.0 4.0 3.6 5 5.0 5.0 5.0 3.3 6 4.0 4.0 5.0 3.1 7 4.0 3.3 4.0 5.0 8 4.0 4.0 4.0 3.3 9 4.0 5.0 4.0 3.5 5.0 4.0 4.0 10 5.0 11 4.0 3.0 5.0 3.0 4.0 12 3.0 3.0 2.6 13 5.0 4.0 4.0 3.3 14 5.0 4.0 5.0 3.8

[14]: df.drop([24],axis=0,inplace=True)

15

4.0

4.0

4.0

2.9

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5.0
                                5.0
                                              3.4
      16
                       5.0
      17
              4.0
                       4.0
                                5.0
                                              3.3
              3.0
                                              3.1
      18
                       5.0
                                5.0
      19
              5.0
                       4.0
                                3.0
                                              2.6
      20
              4.0
                       4.0
                                5.0
                                              3.3
              1.0
                                3.0
                                              3.1
      21
                       5.0
      22
              2.0
                       2.0
                                4.0
                                              2.4
      23
              4.0
                       4.0
                                5.0
                                              3.4
[16]: cols = df.columns.tolist()
      n = len(cols)
      column_to_drop = cols[n-1]
      df.drop([column_to_drop],axis=1,inplace=True)
[17]: df
                                                                             CSL100 \
[17]:
           Student ID
                       MAL100
                                MAL101
                                         MAL403
                                                  EEL101
                                                           ECL101
                                                                    BML101
      0
            Student 1
                           4.0
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      1
            Student 2
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            Student 3
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      3
            Student 4
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      4
            Student 5
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      5
            Student 6
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      6
            Student 7
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      7
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            Student 8
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                                    4.0
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      8
            Student 9
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                                             4.0
                                                               3.0
                                                                        1.0
           Student 10
                           4.0
                                    4.0
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      9
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      10
           Student 11
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                                             4.0
                                                      2.0
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           Student 12
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                                             3.0
                                                      1.0
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      11
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                                    3.0
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      12
          Student 13
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      13
           Student 14
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      14
           Student 15
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                                                               3.0
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                                                      1.0
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      15
           Student 16
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      16
           Student 17
                           4.0
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                                                      1.0
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      17
           Student 18
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      18
           Student 19
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           Student 20
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                                                                        2.0
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      19
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      20
           Student 21
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                                                                        1.0
                                                                                 4.0
      21
           Student 22
                           4.0
                                    5.0
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                                                               1.0
                                                                        2.0
                                                                                4.0
      22
           Student 23
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      23
           Student 24
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           CSL201
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3

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      22
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             4.0
      23
                     4.0
                             5.0
[18]: df.drop(["Student ID"],axis=1,inplace=True)
[19]: def introduce_missing_data(df, missing_percentage):
          df_missing = df.copy()
          num_missing = int(df.size * missing_percentage)
          missing_indices = np.random.choice(df.size, num_missing, replace=False)
          df_missing.values.ravel()[missing_indices] = np.nan
          return df_missing
[20]: def predict_missing_values(df):
          df_filled = df.copy()
          mse_scores = []
          for col in df.columns:
              if df[col].isnull().any():
                  # Separate data into training and missing sets
                  known_data = df_filled[df_filled[col].notnull()]
                  unknown_data = df_filled[df_filled[col].isnull()]
                  # Features and targets
```

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4

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X_known = known_data.drop(columns=[col])

Train a Linear Regression model

X_unknown = unknown_data.drop(columns=[col])

y_known = known_data[col]

```
model = LinearRegression()
                  model.fit(X_known, y_known)
                  # Predict missing values
                  predicted_values = model.predict(X_unknown)
                  df_filled.loc[unknown_data.index, col] = predicted_values
                  # Evaluate using MSE (if possible)
                  X_train, X_test, y_train, y_test = train_test_split(X_known,_

y_known, test_size=0.2, random_state=42)
                  model.fit(X_train, y_train)
                  y_pred = model.predict(X_test)
                  mse = mean_squared_error(y_test, y_pred)
                  mse_scores.append(mse)
          return df_filled, mse_scores
[21]: def visualize_results(original, predicted):
          plt.figure(figsize=(10, 6))
          plt.scatter(original, predicted, alpha=0.6, edgecolors='k')
          plt.xlabel('Actual Values')
          plt.ylabel('Predicted Values')
          plt.title('Actual vs Predicted Values')
          plt.grid(True)
          plt.show()
[22]: missing_percentages = [0.2, 0.4, 0.6, 0.8]
      for missing_percentage in missing_percentages:
          df_missing = introduce_missing_data(df, missing_percentage)
          df_filled, mse_scores = predict_missing_values(df_missing)
          visualize_results(df_missing, df_filled)
```

