

# Control of Uncertain LTI Systems Based on an Uncertainty and Disturbance Estimator

Qing-Chang Zhong\*

e-mail: zhongqc@ieee.org

David Rees

e-mail: drees@glam.ac.uk

School of Electronics, University of Glamorgan,  
Pontypridd, CF37 1DL, United Kingdom

*This paper proposes a robust control strategy for uncertain LTI systems. The strategy is based on an uncertainty and disturbance estimator (UDE). It brings similar performance as the time-delay control (TDC). The advantages over TDC are: (i) no delay is introduced into the system; (ii) there are no oscillations in the control signal; and (iii) there is no need of measuring the derivatives of the state vector. The robust stability of LTI-SISO systems is analyzed, and simulations are given to show the effectiveness of the UDE-based control with a comparison made with TDC.*  
[DOI: 10.1115/1.1850529]

**Keywords:** Time Delay Control (TDC), Uncertainty and Disturbance Estimator (UDE), Parametric Uncertainty, Robust Control, Interval Plants

## 1 Introduction

About 15 years ago, Youcef-Toumi and Ito [1,2] proposed a robust control method for systems with unknown dynamics. It is called time delay control (TDC). Based on the assumption that a continuous signal remains unchanged during a small enough period, the past observation of uncertainties and disturbances is used to modify the control action directly, rather than to adjust controller gains, such as gain scheduling or to identify system parameters, such as adaptive control. During the last decade, TDC has been widely studied [3–6] and has been applied to a number of industrial applications, such as DC servo motors [7], four-wheel steering systems [8], hybrid position/force control of robots [9], brushless DC motors [10], overhead cranes [11], and robot trajectory control [12].

However, TDC inherently requires that all the states and their derivatives be available for feedback. This imposes very strict limitations on the application. Another inherent drawback is that oscillations always exist in the control signal. Moreover, as is well known in the control community, delay is not good for control in most cases. In particular, it brings difficulties into system analysis and tends to destabilize a system. Although it has been shown in the above-cited papers that TDC has very good potential to improve system performance, it is still desirable to get rid of delays, if possible, so as to simplify system analysis.

The assumption used in TDC is a time-domain assumption. In this paper, we use an assumption in the frequency domain to propose an alternative control strategy to obtain similar performance to TDC. The major part of the controller is called an uncertainty and disturbance estimator (UDE). The two inherent drawbacks of

TDC, namely, there is no need to measure the derivative of the states and there is no oscillation in the control signal, are addressed. Also, an additional benefit is that the delay is removed. The system stability is more easily analyzed. In addition to all these advantages, the system performance is very similar to that obtained using TDC.

This paper focuses on the comparison with the TDC strategy. The reader is referred to [13–15] for other control strategies based on a perturbation or disturbance observer. In order to make a clear comparison with the TDC strategy, a similar structure to that used by Youcef-Toumi and Ito [2] is adopted in this paper. In Sec. 2, the general UDE-based control law for uncertain LTI systems is derived. More aspects of the control strategy are analyzed in Sec. 3 for LTI-SISO systems. Simulation examples are given in Sec. 4 and conclusions are made in Sec. 5.

## 2 UDE-Based Control Law

**2.1 System Description.** The system to be considered is formulated as

$$\dot{\mathbf{x}} = (\mathbf{A} + \mathbf{F})\mathbf{x} + \mathbf{B}\mathbf{u}(t) + \mathbf{d}(t) \quad (1)$$

where  $\mathbf{x} = (x_1, \dots, x_n)^T$  is the state,  $\mathbf{u}(t) = [u_1(t), \dots, u_r(t)]^T$  is the control input,  $\mathbf{A}$  is the known state matrix,  $\mathbf{F}$  is the unknown state matrix,  $\mathbf{B}$  is the control matrix having full column rank, and  $\mathbf{d}(t)$  is the unpredictable external disturbances.

**2.2 Reference Model and Structural Constraint.** Assume that the desired specification can be described by the reference model

$$\dot{\mathbf{x}}_m = \mathbf{A}_m\mathbf{x}_m + \mathbf{B}_m\mathbf{c}(t) \quad (2)$$

where  $\mathbf{c}(t) = [c_1(t), \dots, c_r(t)]^T$  is the desired state. The control objective is to make the state error  $\mathbf{e}$  between the system and the reference model

$$\mathbf{e} = (x_{m1} - x_1 \dots x_{mn} - x_n)^T = \mathbf{x}_m - \mathbf{x} \quad (3)$$

converge to zero. In other words, the error dynamics

$$\dot{\mathbf{e}} = (\mathbf{A}_m + \mathbf{K})\mathbf{e} \quad (4)$$

is stable, where  $\mathbf{K}$  is called the error feedback gain. Since the reference model is often chosen to be stable,  $\mathbf{K}$  may be chosen as 0. If different error dynamics is desired or is required to guarantee robust stability (see Sec. 3.2), then common control strategies, such as pole placement, can be used to choose  $\mathbf{K}$ .

Combining Eqs. (1)–(4), we obtain

$$\mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c}(t) - \mathbf{A}\mathbf{x} - \mathbf{F}\mathbf{x} - \mathbf{B}\mathbf{u}(t) - \mathbf{d}(t) = \mathbf{K}\mathbf{e} \quad (5)$$

Then the control action  $\mathbf{u}(t)$  is obtained as

$$\mathbf{u}(t) = \mathbf{B}^+ [\mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c}(t) - \mathbf{A}\mathbf{x} - \mathbf{F}\mathbf{x} - \mathbf{d}(t) - \mathbf{K}\mathbf{e}] \quad (6)$$

where  $\mathbf{B}^+ = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T$  is the pseudoinverse of  $\mathbf{B}$ . Since  $\mathbf{u}(t)$  given in (6) is only an approximate solution of (5) rather than the accurate solution, (4) and (5) will only be met fully under the following structural constraint:

$$[\mathbf{I} - \mathbf{B}\mathbf{B}^+] \cdot [\mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{c}(t) - \mathbf{A}\mathbf{x} - \mathbf{F}\mathbf{x} - \mathbf{d}(t) - \mathbf{K}\mathbf{e}] = 0 \quad (7)$$

Obviously, if  $\mathbf{B}$  is invertible, the above structural constraint is always met. If not, the choice of the reference model and the error feedback gain matrix will be somewhat restricted. However, as shown in [2], the systems described in canonical form meet this constraint. The following assumes that this constraint is satisfied.

**2.3 UDE and the Control Law.** The control law in (6) can be represented in the  $s$ -domain by using Laplace transforms (assuming zero-initial states) as

$$\begin{aligned} \mathbf{U}(s) = & \mathbf{B}^+ [\mathbf{A}_m\mathbf{X}(s) + \mathbf{B}_m\mathbf{C}(s) - \mathbf{K}\mathbf{E}(s) - \mathbf{A}\mathbf{X}(s)] \\ & + \mathbf{B}^+ [-\mathbf{F}\mathbf{X}(s) - \mathbf{D}(s)] \end{aligned} \quad (8)$$

\*Corresponding author. <http://come.to/zhongqc>

Contributed by the Dynamic Systems, Measurement, and Control Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS for publication in the ASME JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL. Manuscript received by the ASME Dynamic Systems and Control Division November 27, 2002; final revision, March 29, 2004. Review conducted by N. Olga.

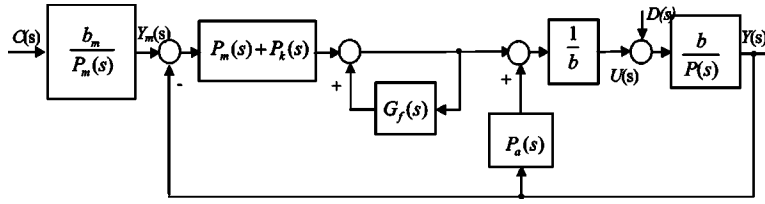


Fig. 1 The equivalent structure of UDE-based LTI-SISO control systems

where the Laplace transformation of a signal is denoted by the corresponding capital letter. The first part in (8) is known while the second part, denoted hereafter by  $\mathbf{U}_d(s)$ , includes the uncertainties and the external disturbance. According to the system dynamics (1),  $\mathbf{U}_d(s)$  can be rewritten as

$$\mathbf{U}_d(s) = \mathbf{B}^+ [-\mathbf{F}\mathbf{X}(s) - \mathbf{D}(s)] = \mathbf{B}^+ [(\mathbf{A} - s\mathbf{I})\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s)] \quad (9)$$

In other words, the unknown dynamics and the disturbances can be observed by the system states and the control signal. However, it cannot be used in the control law directly. TDC adopts an estimation of this signal by using a small delay in the time domain [2]. Here, we will use a different estimation strategy by looking at this problem in the frequency domain.

Assume that  $G_f(s)$  is a strictly proper low-pass filter with unity steady-state gain and broad enough bandwidth, then  $\mathbf{U}_d(s)$  can be accurately approximated to **UDE**, where

$$\mathbf{UDE} = \mathbf{B}^+ [(\mathbf{A} - s\mathbf{I})\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s)] G_f(s) \quad (10)$$

This is called the uncertainty and disturbance estimator (UDE). The UDE only uses the control signal and the states to observe the uncertainties and the unpredictable disturbances. Hence,

$$\begin{aligned} \mathbf{U}(s) &= \mathbf{B}^+ \cdot [\mathbf{A}_m \mathbf{X} + \mathbf{B}_m \mathbf{C} - \mathbf{K}\mathbf{E} - \mathbf{A}\mathbf{X}] + \mathbf{UDE} \\ &= \mathbf{B}^+ \cdot [\mathbf{A}_m \mathbf{X} + \mathbf{B}_m \mathbf{C} - \mathbf{K}\mathbf{E} - \mathbf{A}\mathbf{X}(1 - G_f) - sG_f \mathbf{X} + G_f \mathbf{B}\mathbf{U}] \end{aligned}$$

The **UDE**-based control law is then derived as

$$\begin{aligned} \mathbf{U}(s) &= (\mathbf{I} - \mathbf{B}^+ \mathbf{B} G_f)^{-1} \mathbf{B}^+ \cdot [\mathbf{A}_m \mathbf{X} + \mathbf{B}_m \mathbf{C} - \mathbf{K}\mathbf{E} - \mathbf{A}\mathbf{X}(1 - G_f) \\ &\quad - sG_f \mathbf{X}] \end{aligned} \quad (11)$$

The control signal is formed by the state, the low-pass filter, the reference model, and the error feedback gain. It has nothing to do with the uncertainty and the disturbance. Since  $G_f$  is strictly proper,  $sG_f$  is physically implementable and there is no need for measuring the derivative of states.

Assume that the frequency range of the system dynamics and the external disturbance is limited by  $\omega_f$ , then the ideal low-pass filter  $G_f$  has a low-frequency gain ( $\omega < \omega_f$ ) of 1 and a high-frequency gain ( $\omega > \omega_f$ ) of zero. Hence, the estimation error of the uncertainty and the disturbance

$$\mathbf{B}^+ [(\mathbf{A} - s\mathbf{I})\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s)](1 - G_f)$$

is approximately zero for all the frequency range because  $1 - G_f = 0$  for the low-frequency range and  $(\mathbf{A} - s\mathbf{I})\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s) = 0$  for the high-frequency range. A practical low-pass filter is

$$G_f(s) = \frac{1}{Ts + 1} \quad (12)$$

where  $T = 1/\omega_f$ . Although this will cause some error in the estimation, the steady-state estimation error is always zero because  $G_f(0) = 1$  (the initial value of  $G_f(s)$  can always be chosen to be zero).

### 3 Special Case: LTI-SISO Systems

In this section, we consider LTI-SISO systems with uncertainties and disturbances described in the canonical form, i.e., the corresponding matrices in (1) have the following partitions:

$$\mathbf{A} = \begin{pmatrix} 0 & \mathbf{I}_{n-1} \\ \mathbf{A}_1 \end{pmatrix}; \quad \mathbf{F} = \begin{pmatrix} 0 \\ \mathbf{F}_1 \end{pmatrix}; \quad \mathbf{B} = \begin{pmatrix} 0 \\ b \end{pmatrix}; \quad \mathbf{d}(t) = \begin{pmatrix} 0 \\ d(t) \end{pmatrix} \quad (13)$$

where  $\mathbf{A}_1 = (-a_1, -a_2, \dots, -a_n)$  and  $\mathbf{F}_1 = (-f_1, -f_2, \dots, -f_n)$  are  $1 \times n$  row vectors, and  $f_i \leq \bar{f}_i$  ( $i = 1, \dots, n$ ) are uncertain parameters.  $\mathbf{I}_{n-1}$  is the  $(n-1) \times (n-1)$  identity matrix and  $b \neq 0$ . The output of the system is assumed to be  $y = x_1$ .

**3.1 Control Scheme.** The reference model and the error feedback gain matrix are partitioned as

$$\mathbf{A}_m = \begin{pmatrix} 0 & \mathbf{I}_{n-1} \\ \mathbf{A}_{m1} \end{pmatrix}; \quad \mathbf{B}_m = \begin{pmatrix} 0 \\ b_m \end{pmatrix}; \quad \mathbf{K} = \begin{pmatrix} 0 \\ \mathbf{K}_1 \end{pmatrix} \quad (14)$$

where  $\mathbf{A}_{m1} = (-a_{m1}, -a_{m2}, \dots, -a_{mn})$  and  $\mathbf{K}_1 = (-k_1, -k_2, \dots, -k_n)$  are  $1 \times n$  row vectors.

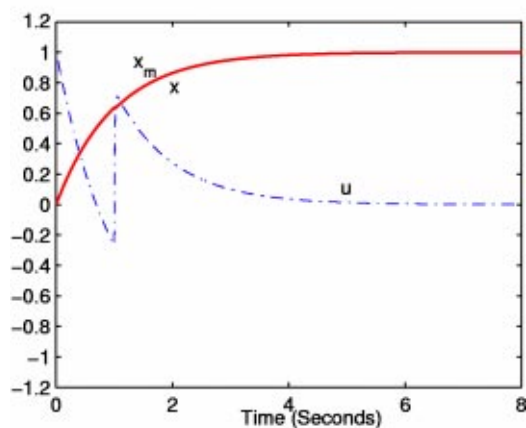
Substituting the matrices into (11), the control law is

$$\begin{aligned} U(s) &= \frac{1}{b[1 - G_f(s)]} \left( -\sum_{i=1}^n a_{mi} X_i + b_m C + \sum_{i=1}^n k_i E_i - sG_f X_n \right) \\ &\quad + \frac{1}{b} \sum_{i=1}^n a_i X_i \end{aligned} \quad (15)$$

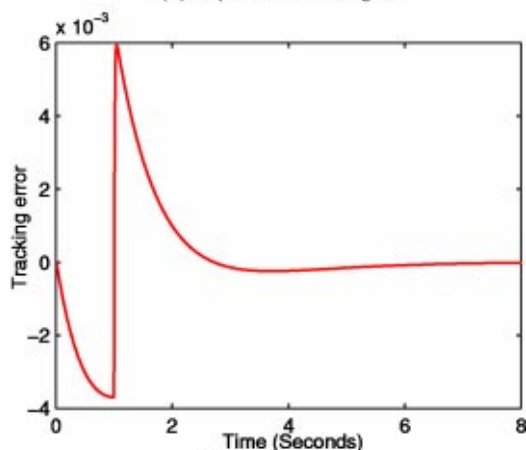
It can be further simplified as follows:

$$\begin{aligned} U(s) &= \frac{1}{b[1 - G_f(s)]} \left[ -\sum_{i=1}^n (a_{mi} + k_i) X_i + b_m C + \sum_{i=1}^n k_i X_{mi} \right. \\ &\quad \left. - s^n G_f(s) X_1 \right] + \frac{1}{b} \sum_{i=1}^n a_i X_i \\ &= \frac{1}{b[1 - G_f(s)]} \left[ -\sum_{i=1}^n (a_{mi} + k_i) s^{i-1} X_1 + b_m C \right. \\ &\quad \left. + \sum_{i=1}^n k_i s^{i-1} X_{mi} - s^n G_f(s) X_1 \right] + \frac{1}{b} \sum_{i=1}^n a_i X_i \\ &= \frac{1}{b[1 - G_f(s)]} \left\{ -[P_m(s) + P_k(s)] Y + b_m C \right. \\ &\quad \left. + P_k(s) \frac{b_m}{P_m(s)} C + s^n [1 - G_f(s)] Y \right\} + \frac{1}{b} \sum_{i=1}^n a_i s^{i-1} X_i \\ &= \frac{P_m(s) + P_k(s)}{b[1 - G_f(s)]} \left[ \frac{b_m}{P_m(s)} C - Y \right] + \frac{s^n}{b} Y + \frac{1}{b} \sum_{i=1}^n a_i s^{i-1} Y \\ &= \frac{P_m(s) + P_k(s)}{b[1 - G_f(s)]} \left[ \frac{b_m}{P_m(s)} C - Y \right] + \frac{P_a(s)}{b} Y \end{aligned} \quad (16)$$

where  $G_m(s) = b_m/P_m(s) = b_m/(s^n + a_{mn}s^{n-1} + \dots + a_{m1})$  is the transfer function of the reference model,  $P_a(s) = s^n + \sum_{i=1}^n a_i s^{i-1}$  is the characteristic polynomial of the known system dynamics, and  $P_k(s) = k_n s^{n-1} + \dots + k_1$  is the error feedback polynomial.

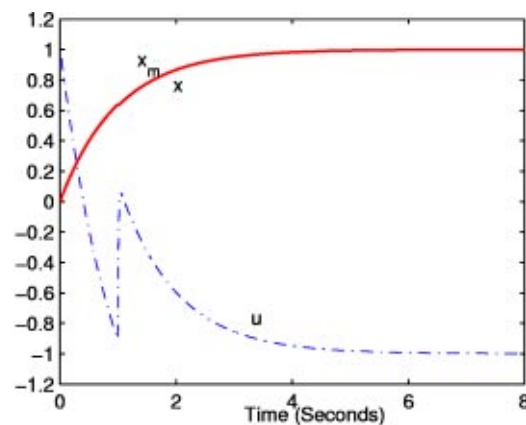


(a) output and control signal

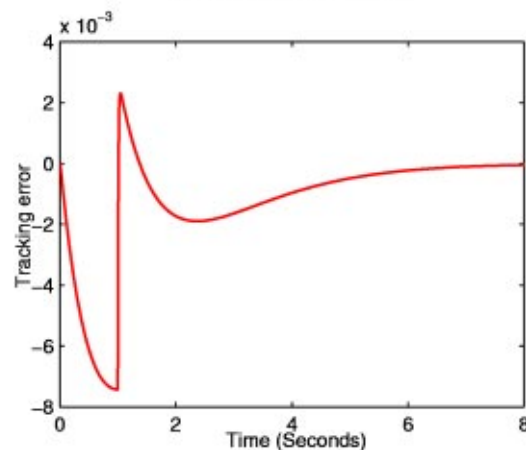


(b) the tracking error

Fig. 2 Example 1: Nominal response



(a) output and control signal



(b) the tracking error

Fig. 3 Example 1: Response when  $f=2$

The equivalent structure of the UDE-based control scheme is shown in Fig. 1. The UDE controller can be divided into four parts: the reference model  $b_m/P_m(s)$  to generate a desired trajectory; a local positive-feedback loop via the low-pass filter (behaving like a PI controller); the polynomial  $P_m(s)+P_k(s)$  for the tracking error and the polynomial  $P_a(s)$  for the output. The last two perform the derivative effect. Obviously, the UDE controller uses more derivative information than the common PID controller, and, hence, UDE-based control has the potential to obtain better performances than the common PID controller. It is worth noting that this structure is the *equivalent* of a UDE-based control, but not the structure for implementation. The control law should be implemented according to (15) under the assumption that all the states are available for feedback.

**3.2 Stability Analysis.** The transfer function of the LTI-SISO plant given by (13) is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b}{s^n + (a_n + f_n)s^{n-1} + \dots + (a_1 + f_1)} = \frac{b}{P(s)} \quad (17)$$

From (16), the closed-loop transfer function of the system is derived to be

$$\frac{Y(s)}{C(s)} = \frac{b_m}{P_m(s)} \frac{P_m + P_k}{P_m + P_k + (1 - G_f)(P - P_a)} \quad (18)$$

Since the reference model is always chosen to be stable, only the stability of the second part in the above equation needs to be

discussed hereafter. If the low-pass filter is chosen as given in (12), then the characteristic polynomial of the closed-loop system is

$$\begin{aligned} P_c(s) &= (Ts + 1)(P_m + P_k) + Ts(P - P_a) \\ &= (P_m + P_k) + Ts(P_m + P_k + P - P_a) \end{aligned} \quad (19)$$

The system stability depends on the reference model, the error feedback gain, the time constant of the filter, and, of course, the plant itself. The reference model is chosen to obtain the desired specifications, and the time constant of the low-pass filter is chosen to have broad enough bandwidth. Hence, the error feedback gain  $\mathbf{K}$  can be used to guarantee the robust stability of the closed-loop system. Intuitively, if  $T$  is small enough, the system is always stable. However, this might be difficult to implement, and, hence, the value of  $T$  has to be a compromise. The robust stability of the polynomial (19) can be guaranteed by the following theorem, which is obtained directly using the box theorem [16] (a generalization of the well-known Kharitonov's theorem [17]) for parametric uncertain LTI systems.

**Theorem (Robust stability).** The closed-loop system is stable if and only if the four polynomials  $P_m(s) + P_k(s) + TsP_{pq}(s)$  ( $p, q = 1, 2$ ) are stable, in which  $P_{pq}(s) = P_p(s) + Q_q(s)$  ( $p, q = 1, 2$ ) are the Kharitonov polynomials of the uncertain polynomial  $P_m + P_k + P - P_a = s^n + \sum_{i=1}^n (a_{mi} + k_i + f_i)s^{i-1}$  with

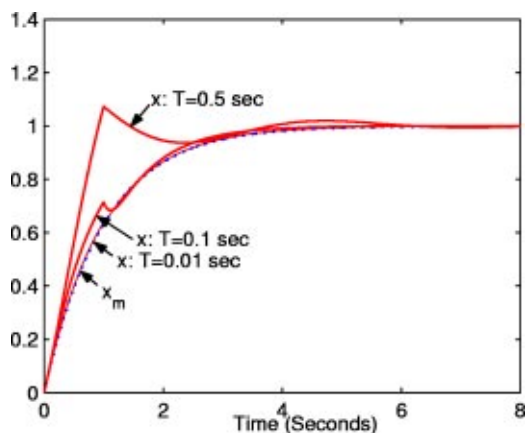


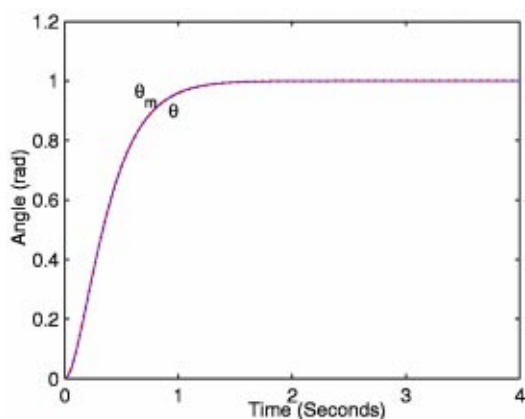
Fig. 4 Example 1: Responses when  $f=2$  for different  $T$

$$P_1(s) = (\beta_1 + \underline{f}_1) + (\beta_3 + \bar{f}_3)s^2 + (\beta_5 + \underline{f}_5)s^4 + \dots$$

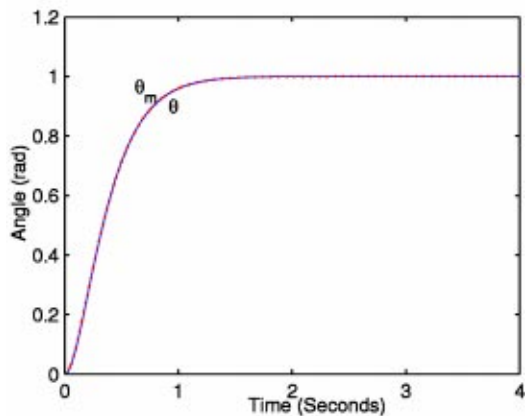
$$= \sum_{i=0, \text{even}}^n (\beta_{1+i} + j^i \cdot \min\{j^i \underline{f}_{1+i}, j^i \bar{f}_{1+i}\}) \cdot s^i$$

$$P_2(s) = (\beta_1 + \bar{f}_1) + (\beta_3 + \underline{f}_3)s^2 + (\beta_5 + \bar{f}_5)s^4 + \dots$$

$$= \sum_{i=0, \text{even}}^n (\beta_{1+i} + j^i \cdot \max\{j^i \underline{f}_{1+i}, j^i \bar{f}_{1+i}\}) \cdot s^i$$

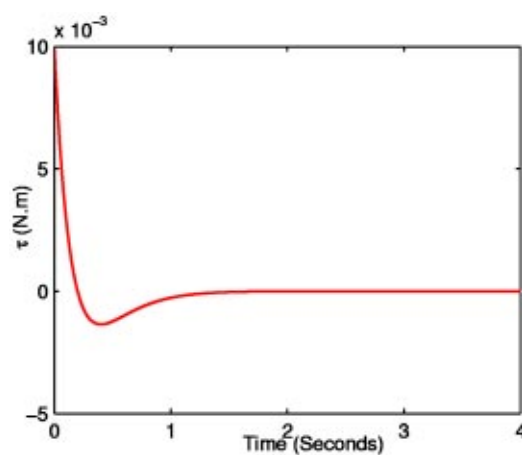


(a) UDE-based control

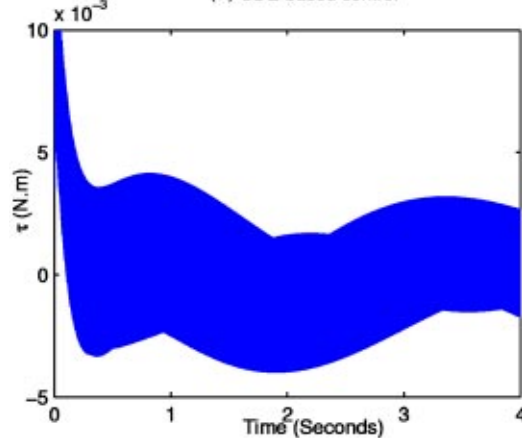


(b) Time-delay control

Fig. 5 Example 2: System responses in the nominal case



(a) UDE-based control



(b) Time-delay control

Fig. 6 Example 2: Control signals in the nominal case

$$Q_1(s) = (\beta_2 + \underline{f}_2)s + (\beta_4 + \bar{f}_4)s^3 + (\beta_6 + \underline{f}_6)s^5 + \dots$$

$$= \sum_{i=1, \text{odd}}^n (\beta_{1+i} + j^{i-1} \cdot \min\{j^{i-1} \underline{f}_{1+i}, j^{i-1} \bar{f}_{1+i}\}) \cdot s^i$$

$$Q_2(s) = (\beta_2 + \bar{f}_2)s + (\beta_4 + \underline{f}_4)s^3 + (\beta_6 + \bar{f}_6)s^5 + \dots$$

$$= \sum_{i=1, \text{odd}}^n (\beta_{1+i} + j^{i-1} \cdot \max\{j^{i-1} \underline{f}_{1+i}, j^{i-1} \bar{f}_{1+i}\}) \cdot s^i$$

where  $\beta_i = a_{mi} + k_i$  ( $i = 1, \dots, n$ ),  $\beta_{n+1} = 1$ ,  $\underline{f}_{n+1} = \bar{f}_{n+1} = 0$  and  $j = \sqrt{-1}$ .

## 4 Examples

**4.1 A First-Order Uncertain LTI System.** Consider the following first-order LTI system:

$$\dot{x} = fx + u + d$$

where  $0.5 \leq f \leq 2$  is an uncertain parameter with the nominal value  $f=1$ , and  $d$  is a unit-step disturbance, acting at 1 s.

The reference model is chosen as  $\dot{x}_m = -x_m + c$ , and the error feedback gain is chosen as  $k=0$ . The low-pass filter is chosen as  $G_f(s) = 1/(Ts + 1)$  with  $T=0.01$  s. According to (11), the control law is

$$U(s) = \frac{1}{1 - G_f(s)} [-X(s) + C(s) - sG_f(s)X(s)]$$



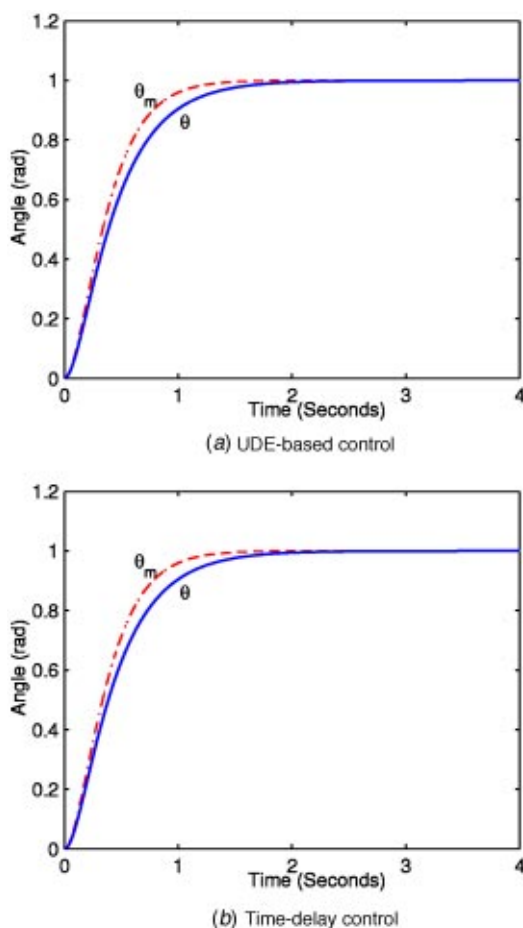


Fig. 7 Example 2: Robust performance ( $L=5$  ms or  $T=5$  ms)

The measurement of  $\dot{x}$  is not needed. In the simulations,  $c(t)$  is chosen to be the step signal  $1(t)$ . The nominal response is shown in Fig. 2, and the response when  $f=2$  is shown in Fig. 3. The system state  $x$  tracks the desired state  $x_m$  very well. When the uncertain parameter  $f$  is changed to  $f=2$ , the performance does not degrade significantly because the controller can well estimate the uncertainty. It can also be seen from the control signal  $u$  (there is a jump in  $u$  when the disturbance starts at  $t=1$  s) that the controller estimates the disturbance very quickly.

The bandwidth (time constant) of the low-pass filter is very important for the system performance. The responses of three cases with different time constants are shown in Fig. 4. The smaller the time constant (the broader the bandwidth), the better the tracking (i.e., the better the performance). However, in practice, the time constant might be limited by the computation capability and the measurement noise.

**4.2 A Servomotor Positioning System.** In order to make a clear comparison with TDC, we take the servo motor positioning system studied in [2] as the second example.

The inherent unknown dynamics in this system are the viscous and dry frictions. Additional uncertainty is simulated by an elastic spring that is physically attached to the motor load. The dynamic equation of the system is

$$\ddot{\theta} = -\frac{k_s}{J}\theta - \frac{b_s}{J}\dot{\theta} - d_s \operatorname{sgn}(\dot{\theta}) + \frac{1}{J}\tau, \quad (20)$$

where  $\theta$  is the motor rotation angle,  $\tau$  is the motor torque,  $k_s$  is the unknown spring constant,  $b_s$  is the unknown viscous friction coefficient,  $d_s$  is the unknown dry friction coefficient, and  $J=4.0 \times 10^{-4}$  kg m<sup>2</sup> is the total inertia. This is a nonlinear system. Al-

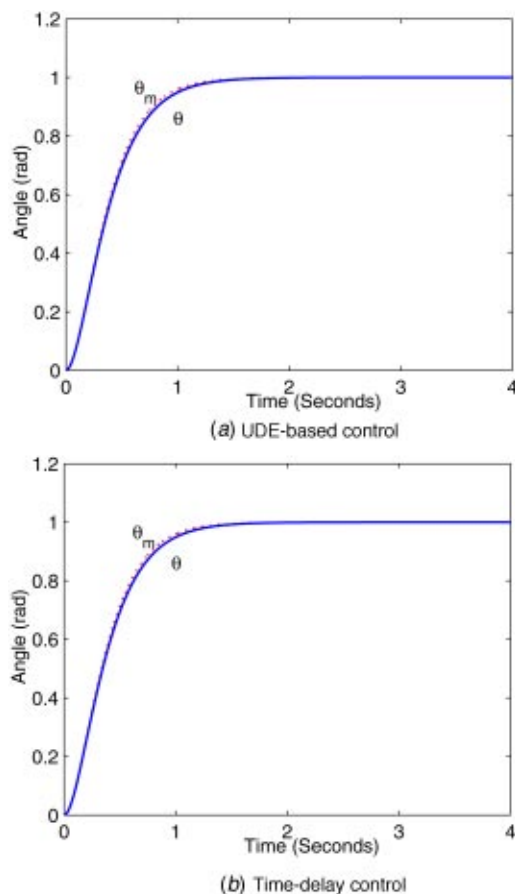


Fig. 8 Example 2: Robust performance ( $L=1$  ms or  $T=1$  ms)

though the strategy developed in this paper cannot be directly applied to arbitrary nonlinear systems, the idea can still be applied to this system because the nonlinearity only appears in  $\operatorname{sgn}(\dot{\theta})$  and  $d_s$  is unknown.

The reference model is chosen as the following second-order system:

$$\begin{pmatrix} \dot{\theta}_m \\ \ddot{\theta}_m \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{pmatrix} \begin{pmatrix} \theta_m \\ \dot{\theta}_m \end{pmatrix} + \begin{pmatrix} 0 \\ \omega_n^2 \end{pmatrix} c \quad (21)$$

where  $\omega_n=5$  rad/s and  $\zeta=1$  are the same as those in [2] and the command  $c=1$  rad. The error feedback gain is chosen to be 0. The UDE-based control law is

$$\tau(s) = \frac{1}{1-G_f(s)} J[-25\theta(s) - 10\dot{\theta}(s) + 25C(s) - sG_f(s)\dot{\theta}(s)] \quad (22)$$

where the low-pass filter  $G_f(s)$  is  $1/(Ts+1)$  with  $T=0.005$  s. Obviously, the angular acceleration signal is not needed for feedback control, unlike the case of time delay control [2]. In order to compare with TDC, simulations are also done for the case using the following TDC control law given in [2]:

$$\tau(t) = \tau(t-L) + J \left[ -25\theta(t) - 10\dot{\theta}(t) + 25c(t) - \frac{\dot{\theta}(t) - \dot{\theta}(t-L)}{L} \right]$$

where  $L=0.005$  s. The derivative of  $\dot{\theta}(t-L)$  is approximated by  $[\dot{\theta}(t) - \dot{\theta}(t-L)]/L$  here.

**4.2.1 Nominal Performance.** When the frictions are omitted and the spring is not attached to the motor load,  $k_s=0$ ,  $b_s=0$ ,  $d_s=0$ , the responses of  $\theta$  under the UDE-based control and the

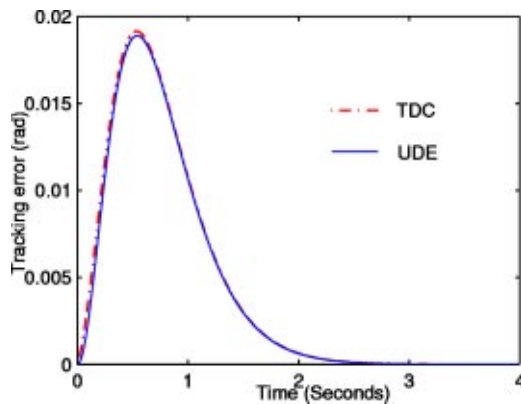


Fig. 9 Tracking errors for Figs. 8(a) and 8(b)

time delay control are shown in Fig. 5. They are almost the same as the response of the reference model (noted as  $\theta_m$  in the figures). The control signals of both cases are shown in Fig. 6. There are oscillations in the control signal for time delay control, but not in the control signal for UDE-based control.

**4.2.2 Robust Performance.** In this case, we assume that  $k_s = 0.17 \text{ N}\cdot\text{m}/\text{rad}$ ,  $b_s = 0.01 \text{ N}\cdot\text{m}\cdot\text{s}/\text{rad}$  and  $d_s = 0.1 \text{ s}^{-1}$ . The responses are shown in Fig. 7. They are all degraded because the time delay or the time constant is not small enough. When the time constant or time delay is chosen as  $0.001 \text{ s}$ , the responses match the reference response very well and are almost not affected by the uncertainties, as shown in Fig. 8. Again, there are oscillations in the control signal for TDC but, in the case with UDE-based control, the control signal is very smooth (the figures are omitted due to page limitation). The tracking errors for both cases are shown in Fig. 9. It clearly shows that, in addition to the advantages of UDE over TDC, the tracking error of the UDE-based control is slightly smaller than that of TDC.

## 5 Conclusions

This paper presents an uncertainty and disturbance estimator (UDE) for LTI systems with uncertainties and disturbances. Similar performance as obtained by using TDC are obtained without the use of a delay element. Moreover, the inherent drawbacks of the TDC have been eliminated. There is no need for measuring the derivative of the states and no oscillation in the control signal. It is also easier to analyze the stability of a UDE-based control system than to analyze the stability of a TDC control system. Further work should be done to show if the delay is not required for the input-output linearization using time delay control, as reported in, e.g., [18,19].

## Acknowledgment

The authors would like to thank Dr. Leonid Mirkin, for proposing this issue, and Professor Nejat Olgac and the anonymous reviewers for their useful comments.

## References

- [1] Youcef-Toumi K., and Ito, O., 1987, "Controller Design for Systems With Unknown Nonlinear Dynamics," *Proc. of ACC*, Minnesota, pp. 836–844.
- [2] Youcef-Toumi, K., and Ito, O., 1990, "A Time Delay Controller for Systems With Unknown Dynamics," *ASME J. Dyn. Syst., Meas., Control*, **112**(1), pp. 133–142.
- [3] Chen, W., Zhang, H. G., and Yin, C. W., 1997, "An Output Tracking Control for Nonlinear Systems With Uncertainties and Disturbances Using Time Delay Control," *Cybernetics*, **40**(3), pp. 229–237.
- [4] Chin, S. M., Lee, C. O., and Chang, P. H., 1994, "An Experimental Study on the Position Control of an Electro-Hydraulic Servo System Using Time Delay Control," *Control Eng. Pract.*, **2**(1), pp. 41–48.
- [5] Elmali, H., and Olgac, N., 1992, "Sliding Mode Control With Perturbation Estimation (SMCPE): A New Approach," *Int. J. Control*, **56**(4), pp. 923–941.
- [6] Zhong, Q.-C., 1999, "Time Delay Control and Its Applications," Ph.D. thesis, Shanghai Jiao Tong University, Shanghai, China.
- [7] Jong, H. P., and You, M. K., 1999, "Time Delay Sliding Mode Control for a Servo," *ASME J. Dyn. Syst., Meas., Control*, **121**(1), pp. 143–148.
- [8] Song, J. G., and Yong, Y. S., 1998, "Feedback Control of Four-Wheel Steering Using Time Delay Control," *Int. J. Veh. Des.*, **19**(3), pp. 282–298.
- [9] Chang, P. H., Park, B. S., and Park, K. C., 1996, "Experimental Study on Improving Hybrid Position/Force Control of a Robot Using Time Delay Control," *Mechatronics*, **6**(8), pp. 915–931.
- [10] Chang, P. H., Park, S. H., and Lee, J. H., 1999, "A Reduced Order Time Delay Control for Highly Simplified Brushless DC Motor," *ASME J. Dyn. Syst., Meas., Control*, **121**(3), pp. 556–560.
- [11] Cheng, C. C., and Chen, C. Y., 1996, "Controller Design for an Overhead Crane System With Uncertainty," *Control Eng. Pract.*, **4**(5), pp. 645–653.
- [12] Chang, P. H., and Lee, J. W., 1996, "A Model Reference Observer for Time-Delay Control and Its Application to Robot Trajectory Control," *IEEE Trans. Control Syst. Technol.*, **4**(1), pp. 2–10.
- [13] Moura, J. T., Elmali, H., and Olgac, N., 1997, "Sliding Mode Control With Sliding Perturbation Observer," *ASME J. Dyn. Syst., Meas., Control*, **119**, pp. 657–665.
- [14] Li, H. X., and Van Den Bosch, P. P. J., 1993, "A Robust Disturbance-Based Control and Its Application," *Int. J. Control*, **58**(3), pp. 537–554.
- [15] Zhong, Q.-C., 2003, "Control of Integral Processes With Dead-Time. Part 3: Deadbeat Disturbance Response," *IEEE Trans. Autom. Control*, **48**(1), pp. 153–159.
- [16] Chapellat, H., and Bhattacharyya, S. P., 1989, "A Generalization of Khartourov's Theorem: Robust Stability of Interval Plants," *IEEE Trans. Autom. Control*, **34**(3), pp. 306–311.
- [17] Minnichelli, R. J., and Anagnost, J. J., 1989, "An Elementary Proof of Khartourov's Stability Theorem With Extensions," *IEEE Trans. Autom. Control*, **34**(9), pp. 995–998.
- [18] Youcef-Toumi, K., and Wu, S. T., 1992, "Input-Output Linearization Using Time Delay Control," *ASME J. Dyn. Syst., Meas., Control*, **114**(1), pp. 10–19.
- [19] Lee, J. W., and Chang, P. H., 1998, "Input/Output Linearization Using Time Delay Control and Time Delay Observer," *Proc. of ACC*, Vol. 1, Philadelphia, pp. 318–322.