

# A stochastic path tracer implementation

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# Camera Model

$$P(i, j) = \text{center} + v^*V + u^*U$$

$$V = (j + 0.5) - \text{height}/2$$

$$U = (i + 0.5) - \text{width}/2$$

$$\text{ray.p} = P(i, j)$$

$$\text{ray.d} = P(i, j) - E$$

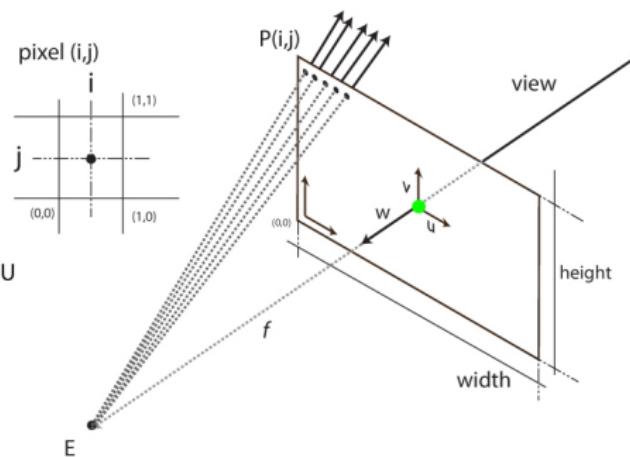
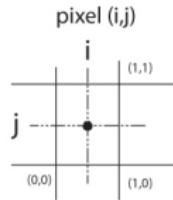


Figure : Perspective projection.

# Camera Model



$$P(i, j) = \text{center} + v*V + u*U$$

$$V = (j + 0.5) - \text{height}/2$$

$$U = (i + 0.5) - \text{width}/2$$

$$\text{ray.p} = P(i, j)$$

$$\text{ray.d} = -1 * w$$

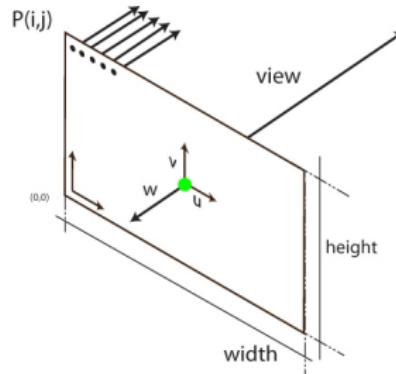


Figure : Parallel projection. It is used if the focus length is 0.

# Camera Model - Examples

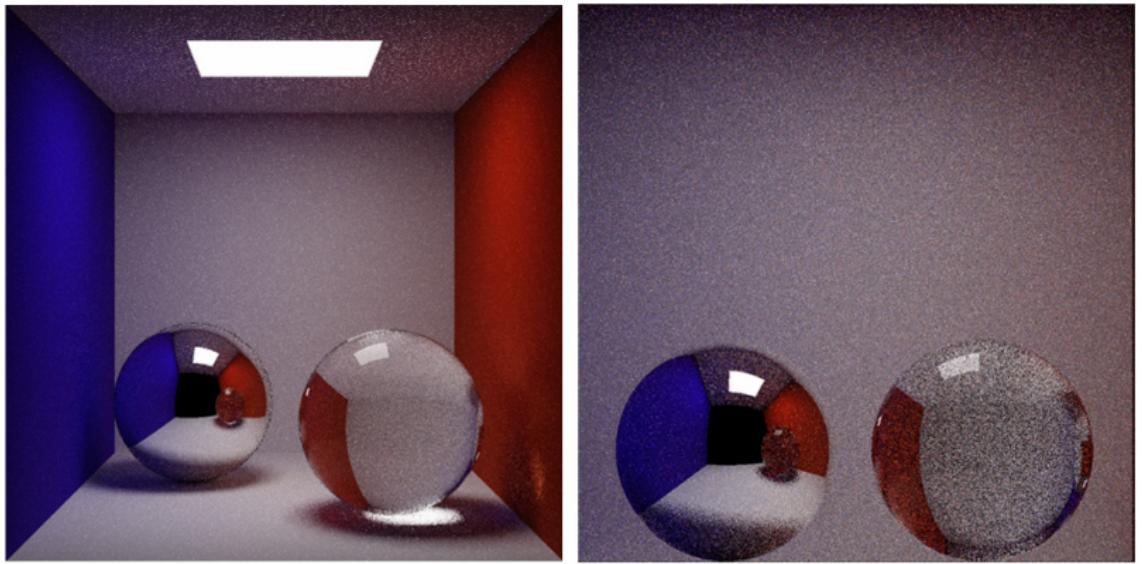


Figure : Perspective and Parallel projection examples.

# Camera Model - Examples

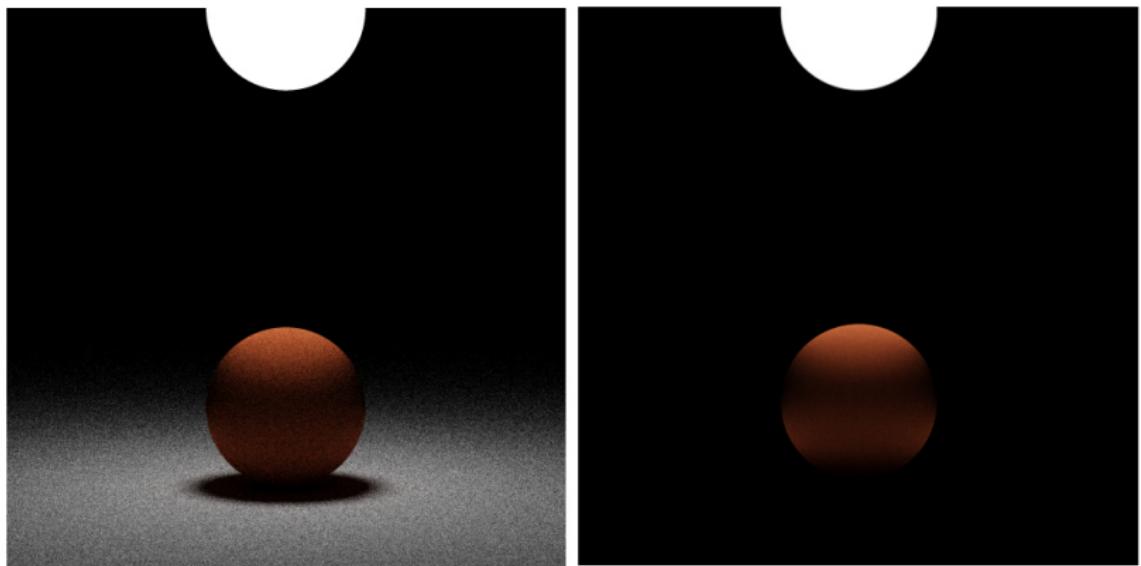


Figure : Perspective and Parallel projection examples.

# Camera Model - Examples

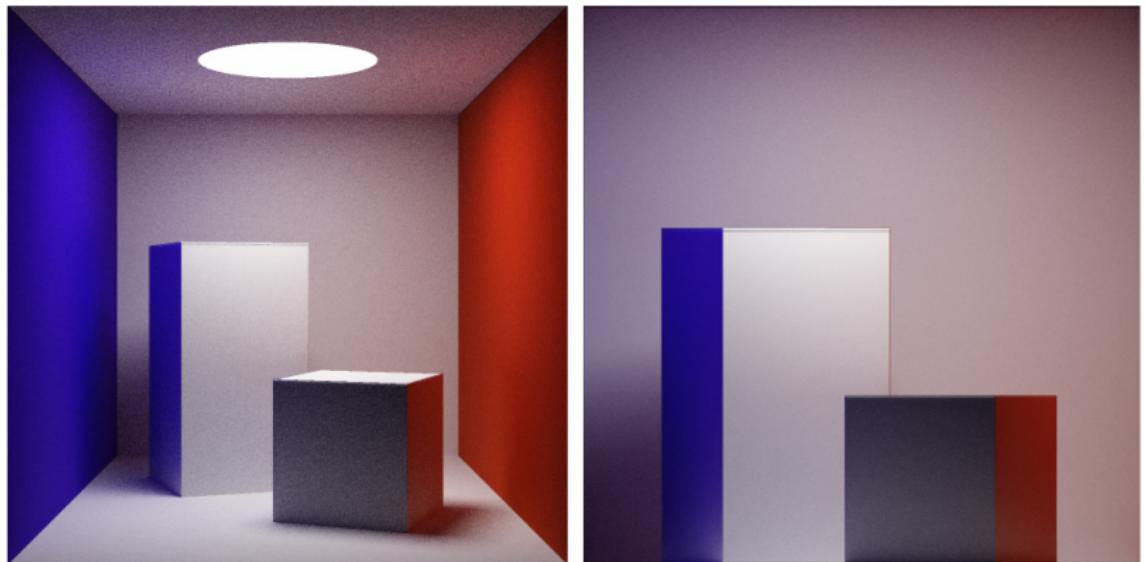


Figure : Perspective and Parallel projection examples.

# Luminaires

## Light emitting objects

- Only diffuse luminaires were implemented.
- All luminaries have an area ( $A$ ) and generate points on the surface with density probability  $p=1/A$ .
- Only object luminaries emitted radiance  $L_e > 0$ .
- The light emission value is a RGB vector (affects all wavelengths).

# Luminaires - Examples

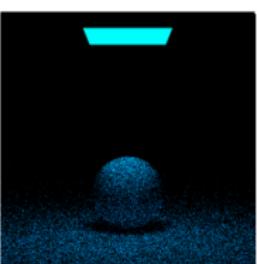
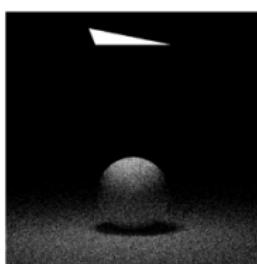
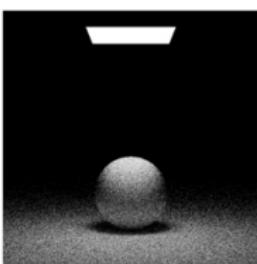
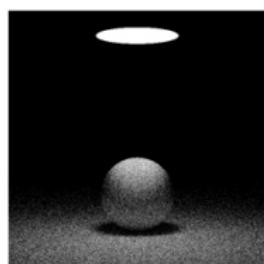
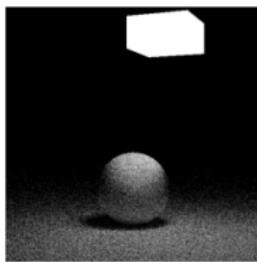
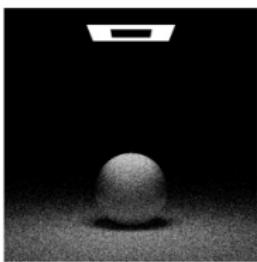
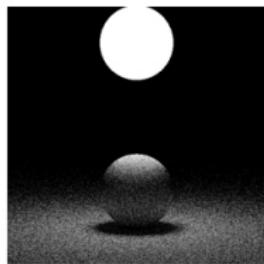


Figure : Object luminaires and different emitted radiance.

# The rendering equation

## Approximation with Monte Carlo integration

$$L_s(k_0) \approx \frac{\rho(k_i, k_o)L_f(k_i)\cos(\theta_i)}{p(k_i)}$$

- $L_s$  is surface outgoing radiance at a given point
- $L_f$  is the incoming radiance at the point.
- $k_i$  is randomly chosen with density p.
- Using  $p(k_i) = \frac{\rho(k_i, k_o)\cos(\theta_i)}{R(k_o)}$
- $R$  is the directional hemispherical reflectance.

## Radiance of a Surface

$$L_s(k_0) \approx R(k_0)L_f(k_i)$$

# The rendering equation

## Radiance seen along a ray

$$L = L_{e1} + R_1(L_{e2} + R_2(L_{e3} + R_3(\dots)))$$

- $L_{ei}$  is the emitted radiance at  $x_i$
- $R_i$  is the reflectance.
- use of Russian Roulette to stop the recursion.

```
RGB radiance (ray r)
    if r hits at x then
        generate direction b  (According to surface)
        ray rn = x + tb
        return Le(x) + R(x)*radiance(rn)
    else
        return background(r)
```

# The rendering equation

```
for each pixel(i,j):
    RGB r = (0,0,0).
    for s = 0; s < spp (samples per point); s ++:
        r += radiance (ray r)/spp ;
```

## Indirect Illumination - Examples

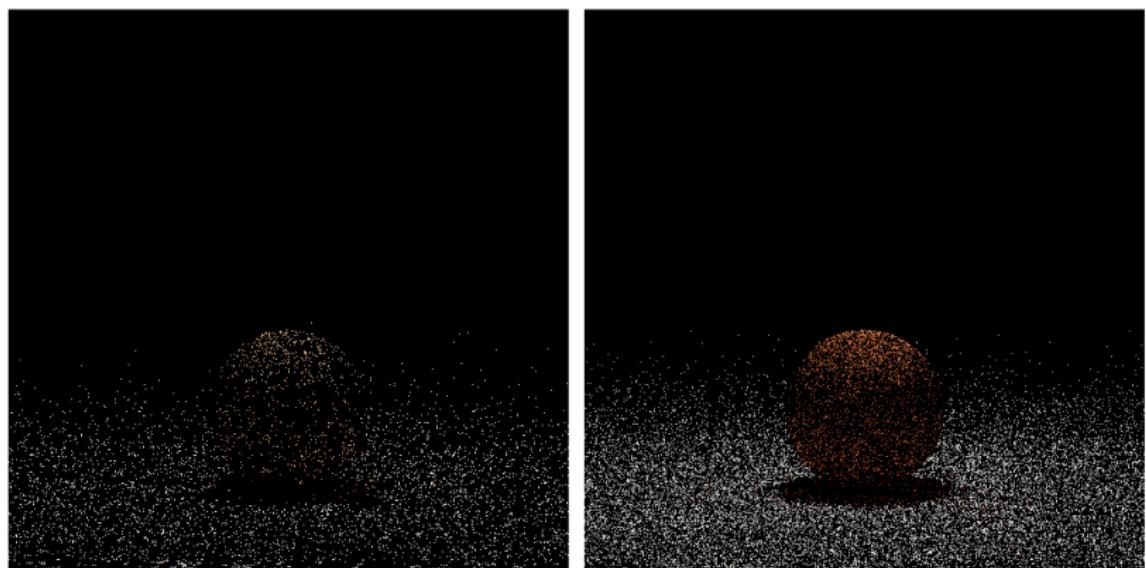


Figure : Lambertian surfaces (left: 5 spp. right: 10 spp).

## Indirect Illumination - Examples

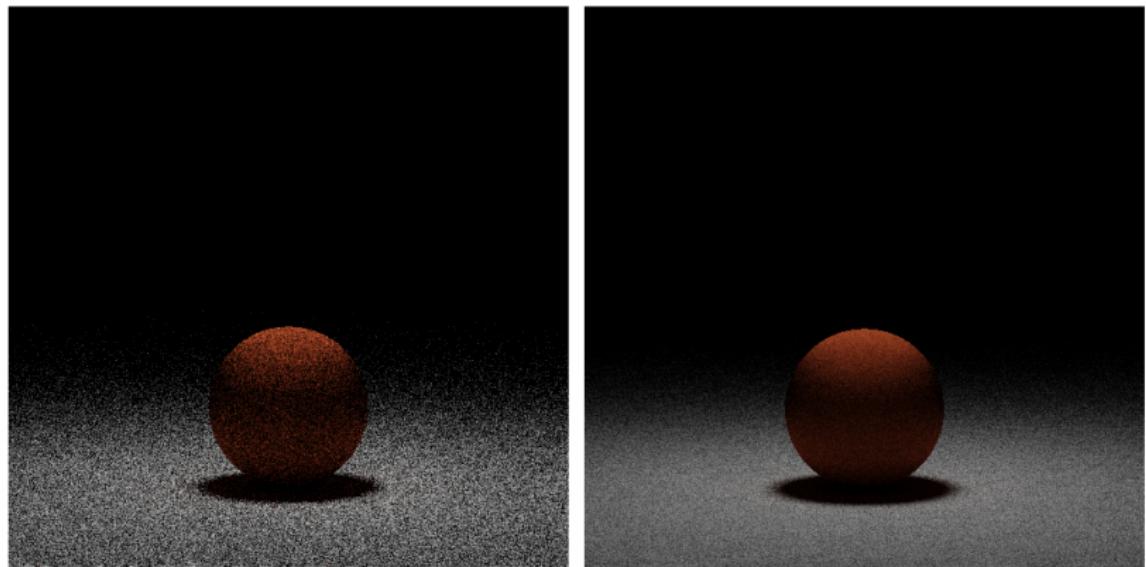


Figure : Lambertian surfaces (left: 200 spp. right: 1600 spp).

## Indirect Illumination - Examples

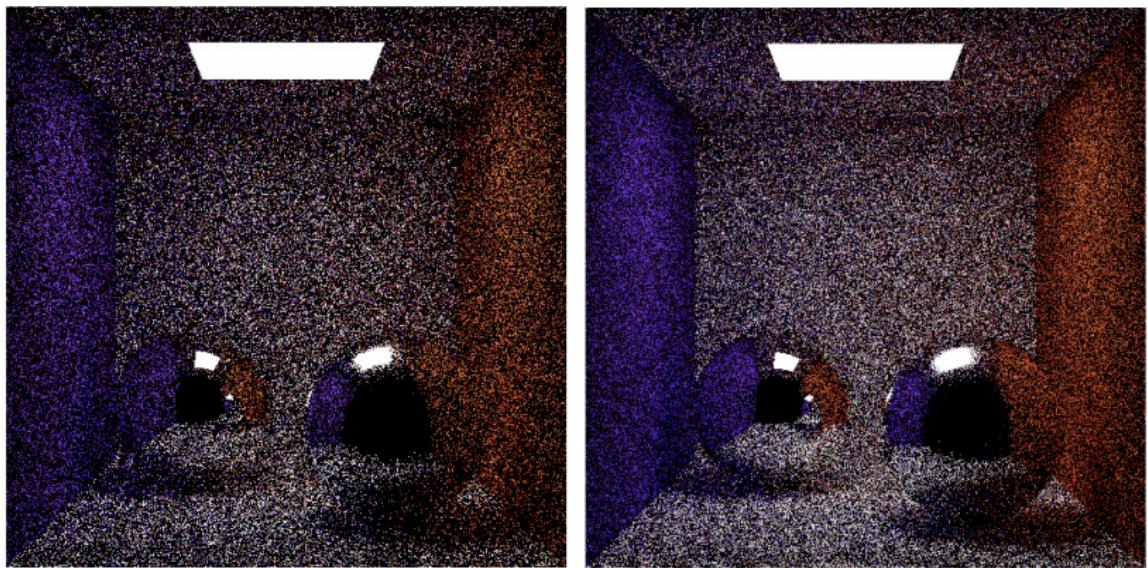


Figure : Lambertian and Specular surfaces (left: 5 spp. right: 10 spp).

## Indirect Illumination - Examples

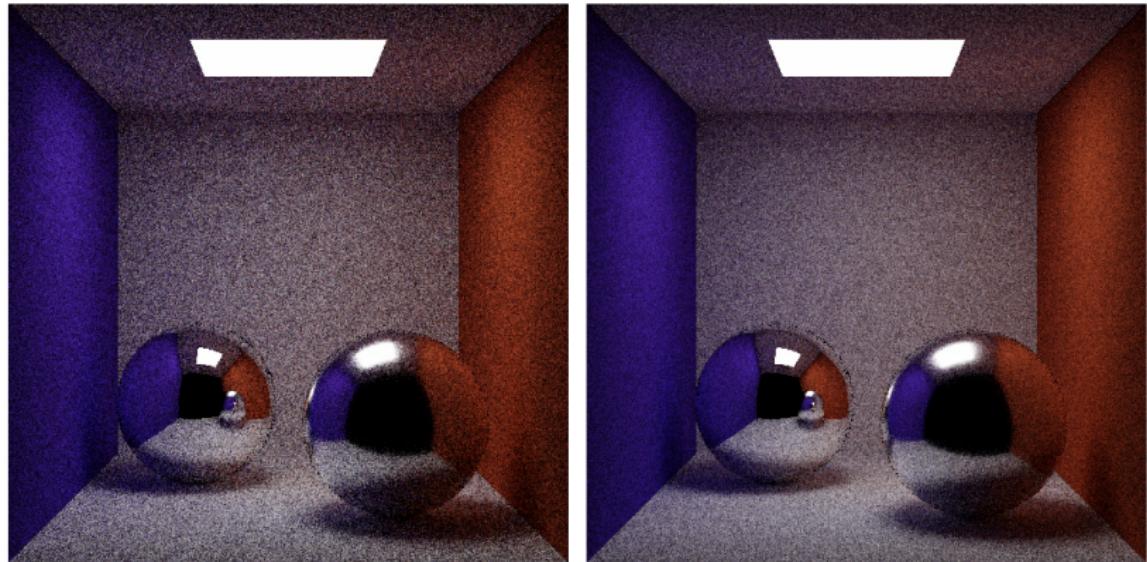


Figure : Lambertian and Specular surfaces (left: 100 spp. right: 200 spp).

## Russian Roulette - Examples

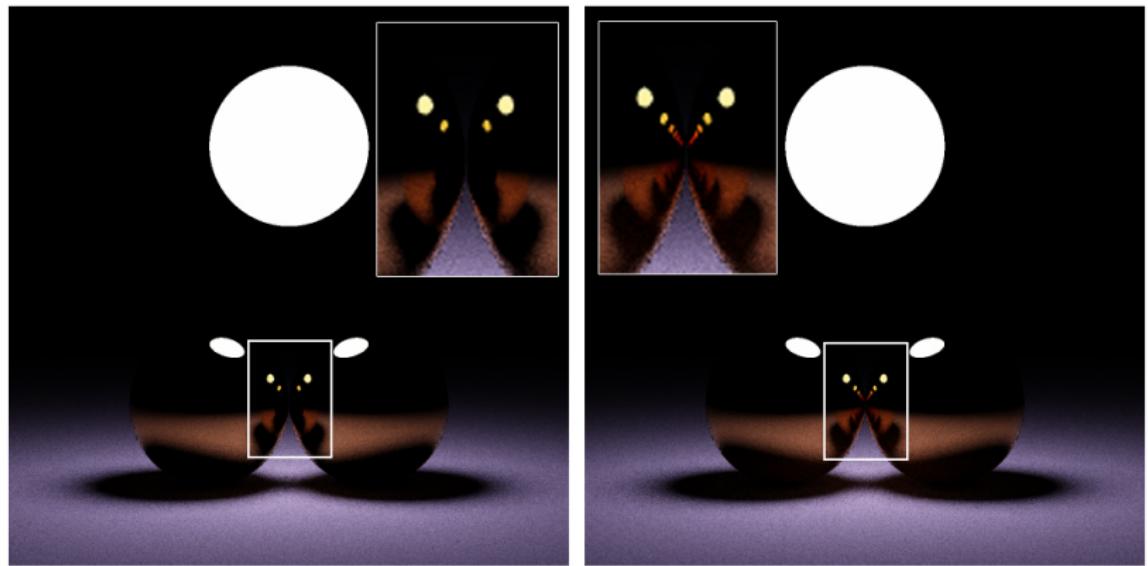


Figure : Specular surfaces at 400spp (left: 2 bounces. right: Russian Roulette).

## Russian Roulette - Examples

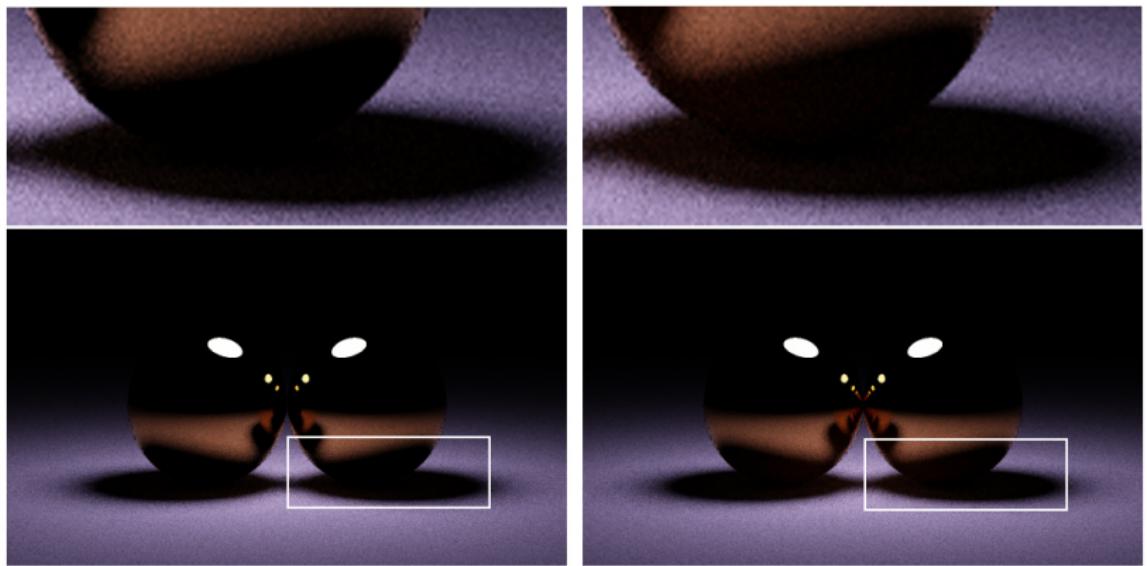
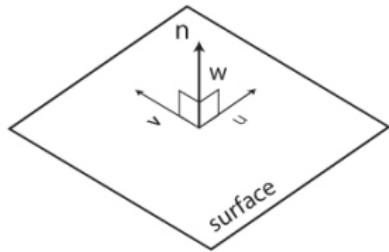


Figure : Specular surfaces at 400spp (left: 2 bounces. right: Russian Roulette).

## Creating ONB from outward normal vector

Creating an orthonormal base  $\mathbf{uvw}$ , from an outward normal vector. ( $\mathbf{u}, \mathbf{v}, \mathbf{w}$  are unit length vectors, mutually perpendicular, and right handed ( $\mathbf{u} \times \mathbf{v} = \mathbf{w}$ ) ).



### ONB from $w$

$$w = n$$

$$m = (0, 1, 0)$$

$$n = (1, 0, 0)$$

$$u = w \times m$$

$$\text{if } \|u\|^2 < \epsilon$$

$$u = w \times n$$

$$v = w \times n$$

# Lambertian Surfaces

Cosine density relative to the surface normal at hit point

- Using:  $\rho(\theta, \phi) = \frac{1}{\pi} \cos(\theta).$
- Generate:  $r_1$  and  $r_2$  canonical random numbers.
- $\cos(\theta) = \sqrt{1 - r_1}.$
- $\phi = 2\pi r_2.$
- Generate orthonormal basis **uvw** for the surface where w points in the direction of the outward surface.

Direction of the ray

$$r_n.b = \cos(\phi)\sin(\theta)u + \sin(\phi)\sin(\theta)v + \cos(\theta)w.$$

# Lambertian Surfaces - Examples

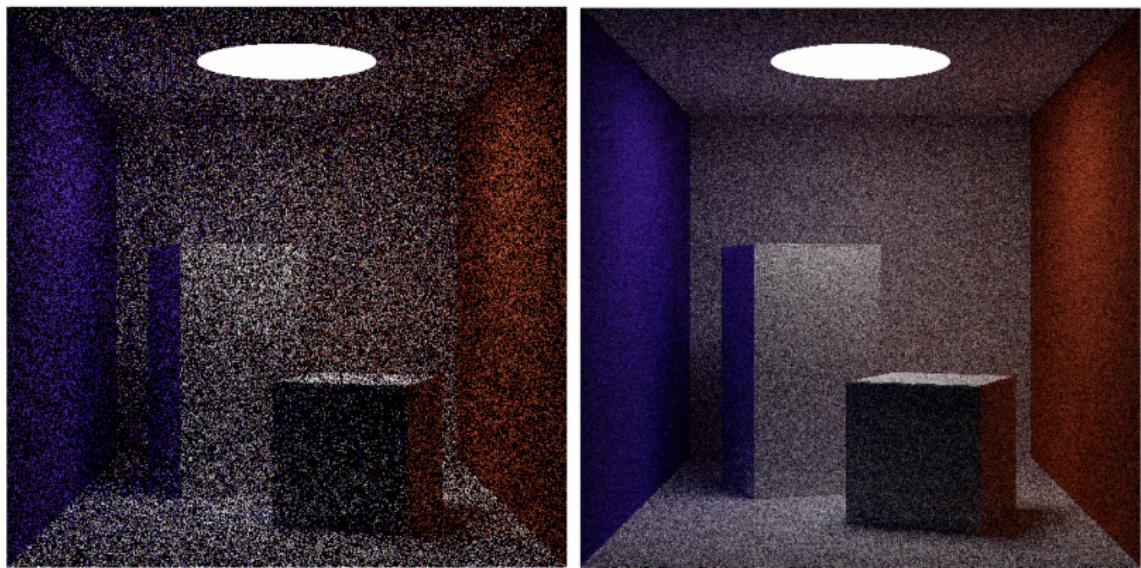


Figure : Lambertian surfaces (left: 10 spp. right: 100 spp).

# Lambertian Surfaces - Examples

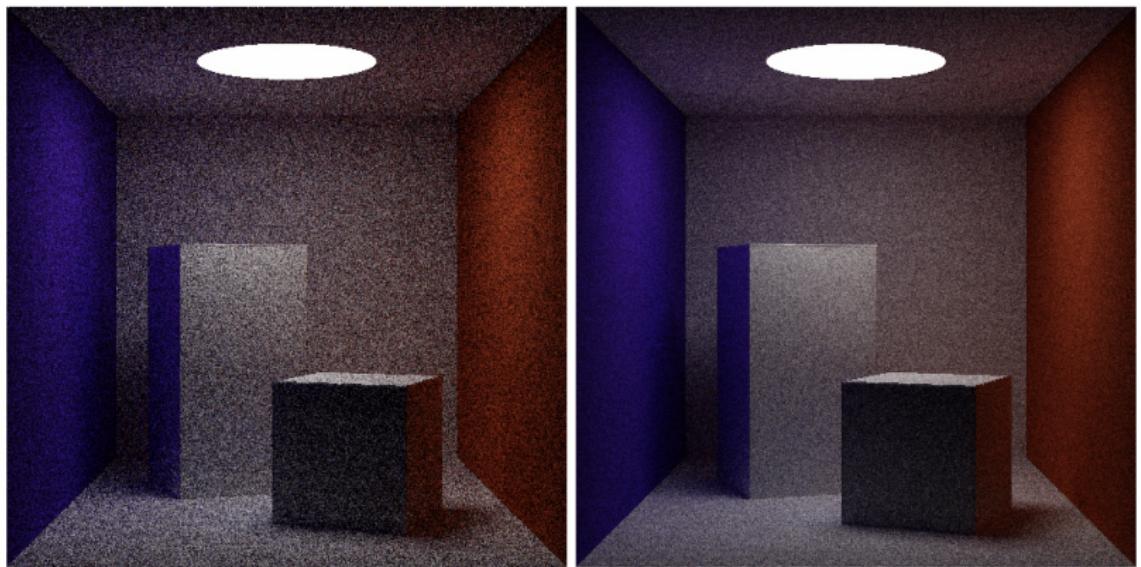


Figure : Lambertian surfaces (left: 100 spp. right: 400 spp).

# Specular Surfaces

Computing the ray direction in specular surfaces (mirrors)

- Law of reflection .
- Fresnel equation (Schlick Approx.)  
$$R(\theta) \approx R_0 + (1 - R_0)(1 - \cos(\theta))^5.$$

Direction of the ray

$$r_n.b = d - 2(d \bullet n)n.$$

# Specular Surfaces - Examples

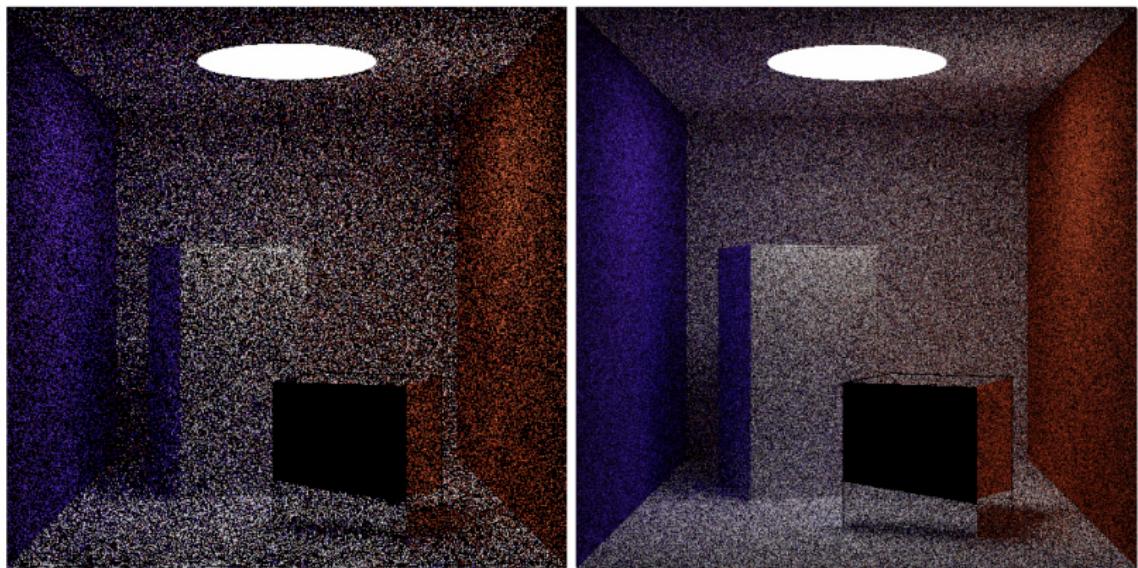


Figure : Specular surfaces (left: 10 spp. right: 100 spp).

# Specular Surfaces - Examples

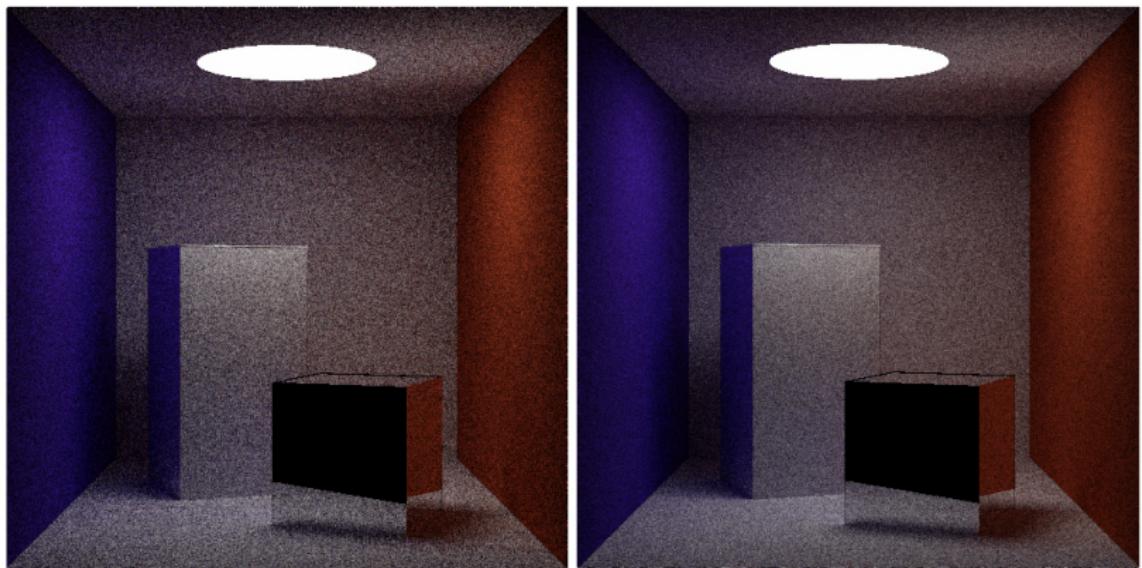


Figure : Specular surfaces (left: 200 spp. right: 400 spp).

# Imperfect Specular Surfaces

Cosine density relative to the surface normal at hit point

- Using:  $\rho(k) = \frac{n+1}{2\pi} (r \bullet k)^n$ .  $r$ : mirror direction normalized.
- Generate:  $r_1$  and  $r_2$  canonical random numbers.
- $\cos(\theta) = r \bullet k = (1 - r_1)^{\frac{1}{n+1}}$ .
- $\phi = 2\pi r_2$ .
- Generate orthonormal basis **uvw** for the surface where w points in the direction of the outward surface.

Direction of the ray

$$r_n \cdot b = \cos(\phi) \sin(\theta) u + \sin(\phi) \sin(\theta) v + \cos(\theta) w.$$

check  $\cos(\theta) < 0$  regenerate ray above the surface.

# Imperfect Specular Surfaces - Examples

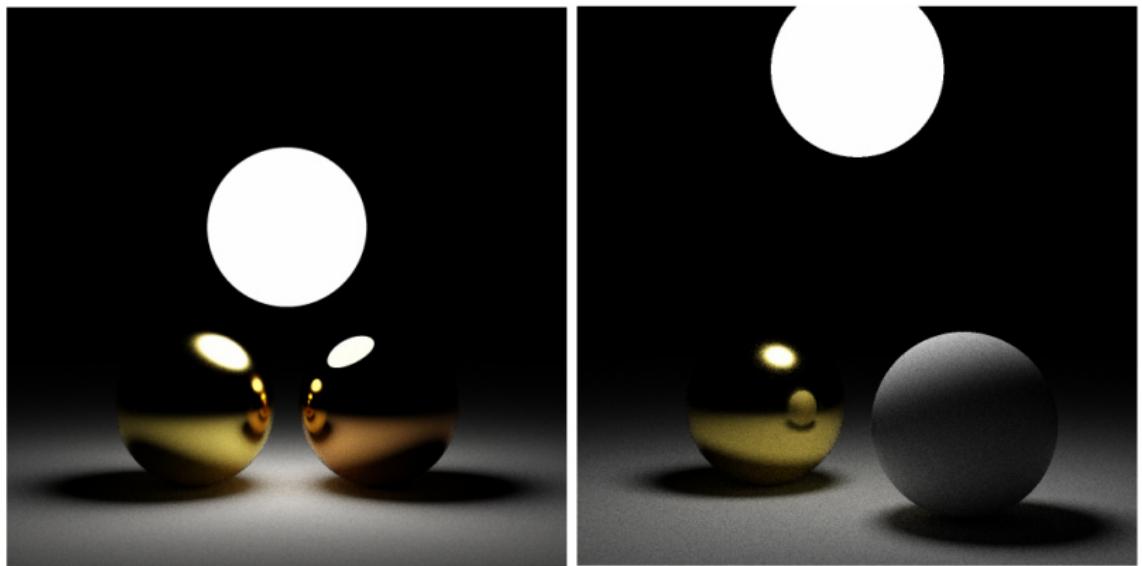


Figure : Imperfect specular gold, specular copper and lambertian surfaces (left: 800 spp. right: 400 spp).

# Imperfect Specular Surfaces - Examples

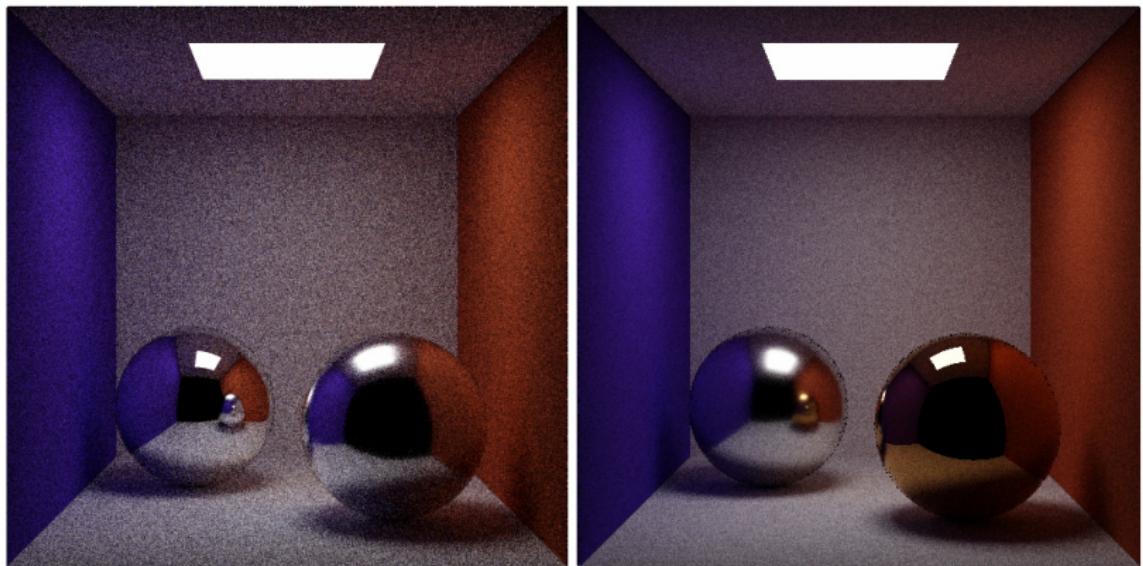


Figure : Imperfect specular and specular surfaces (left: 400 spp. right: 800 spp).

# Imperfect Specular Surfaces - Examples

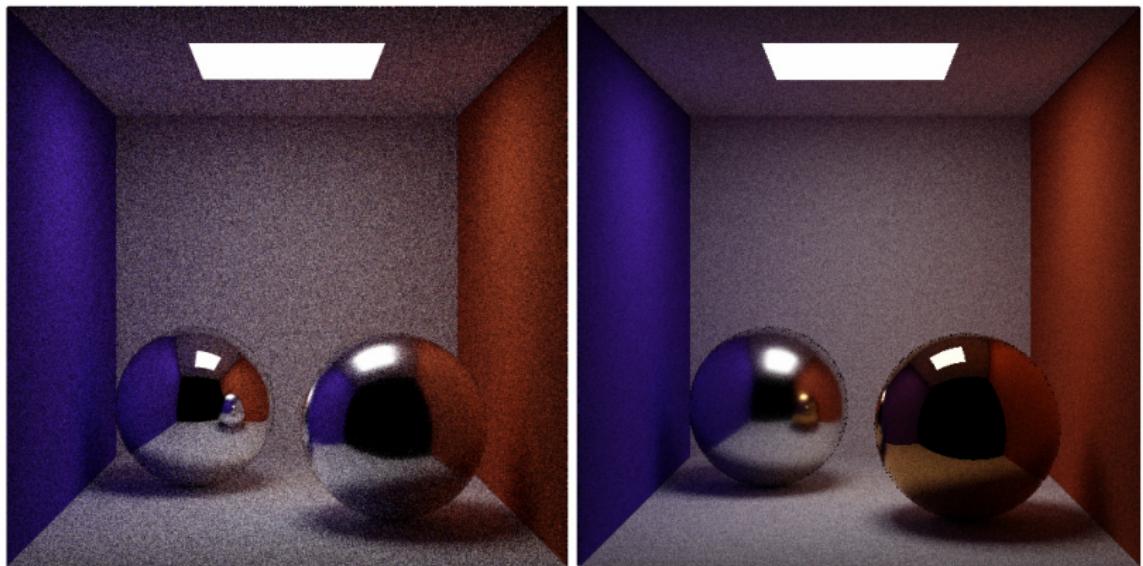


Figure : Imperfect specular and specular surfaces (left: 400 spp. right: 800 spp).

## Other Surfaces - Examples

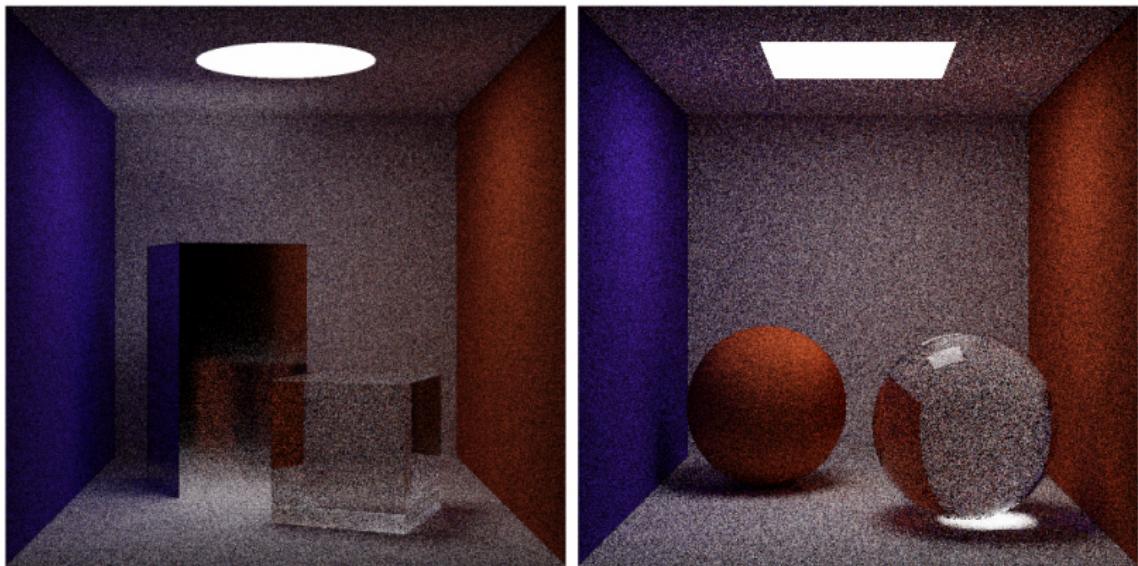


Figure : Smooth Dielectrics (refractive index: 1.5).

# Direct Illumination

Approximation with Monte Carlo integration

$$L_s(x, k_0) \approx \frac{\rho(k_i, k_o) L_e(x', x - x') \cos(\theta_i) \cos(\theta')}{p(x') ||x - x'||^2}$$

# Direct Illumination - Examples

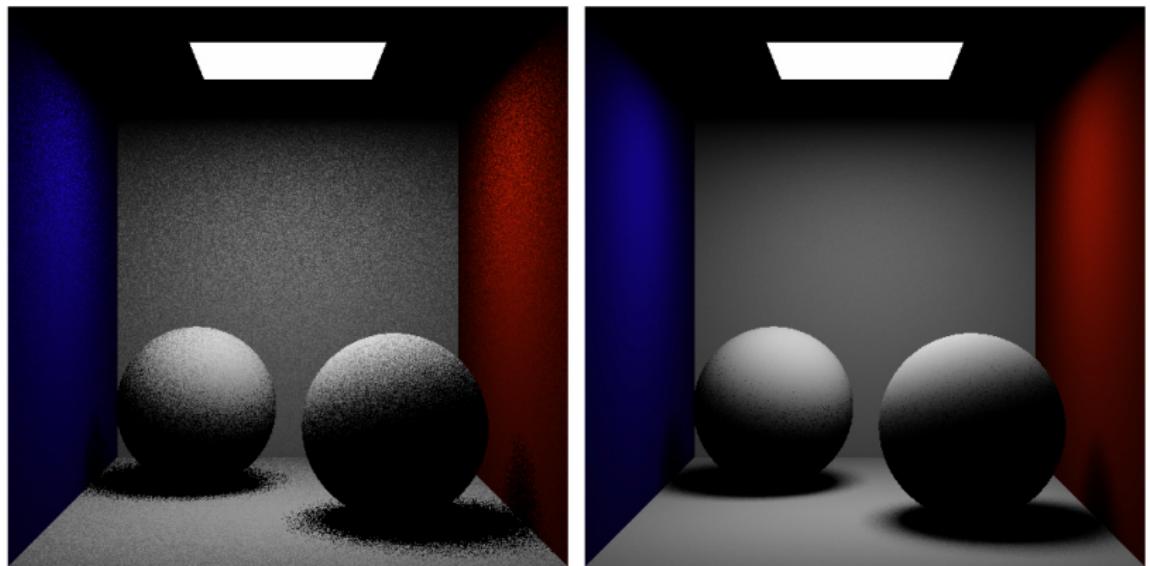


Figure : Lambertian surfaces ( Left: 2spp , Right: 200spp).

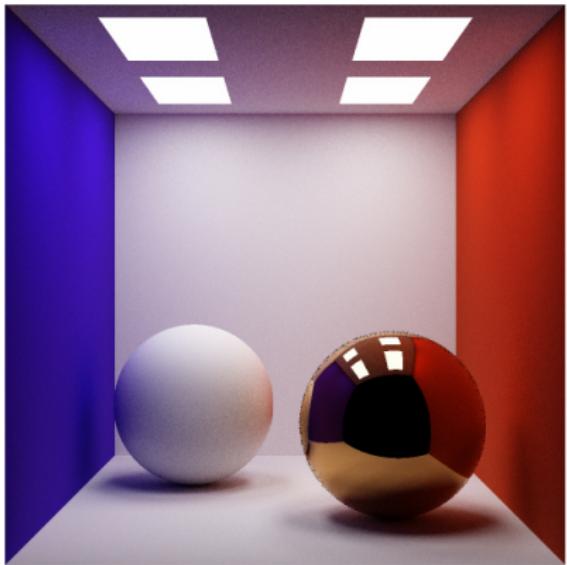
# Multiple Lights

## Choosing weights

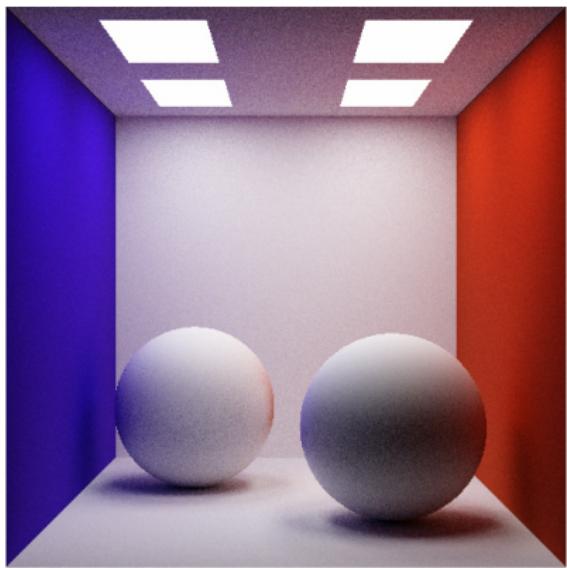
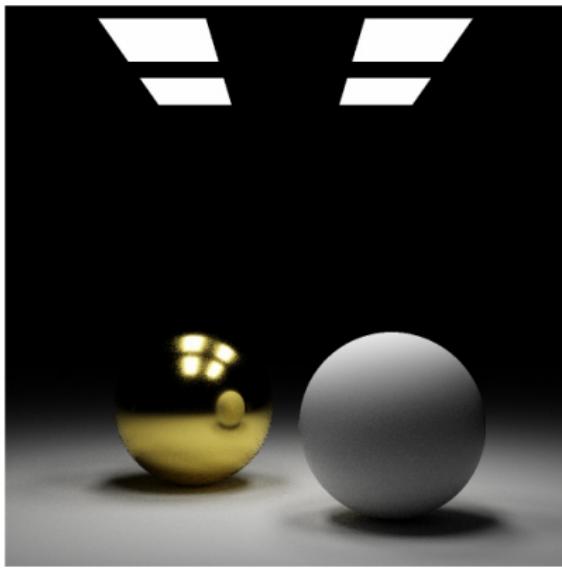
In scenes with multiple lights, I assumed all the lights have the same area and same  $L_i$  (emission).

The contribution of each light was  $\frac{1}{N_l}$  (  $N_l$  is the amount of light objects).

# Multiple Lights - Examples



# Multiple Lights - Examples



# Supersampling

A simple approach to reduce aliasing

Take more samples per pixel.

Split pixel in equally spaced cells (vertically and horizontally).

The pixel radiance is the mean of the values.

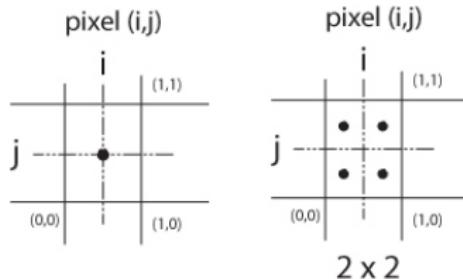


Figure : 4x supersampling.

# Antialiasing - Examples

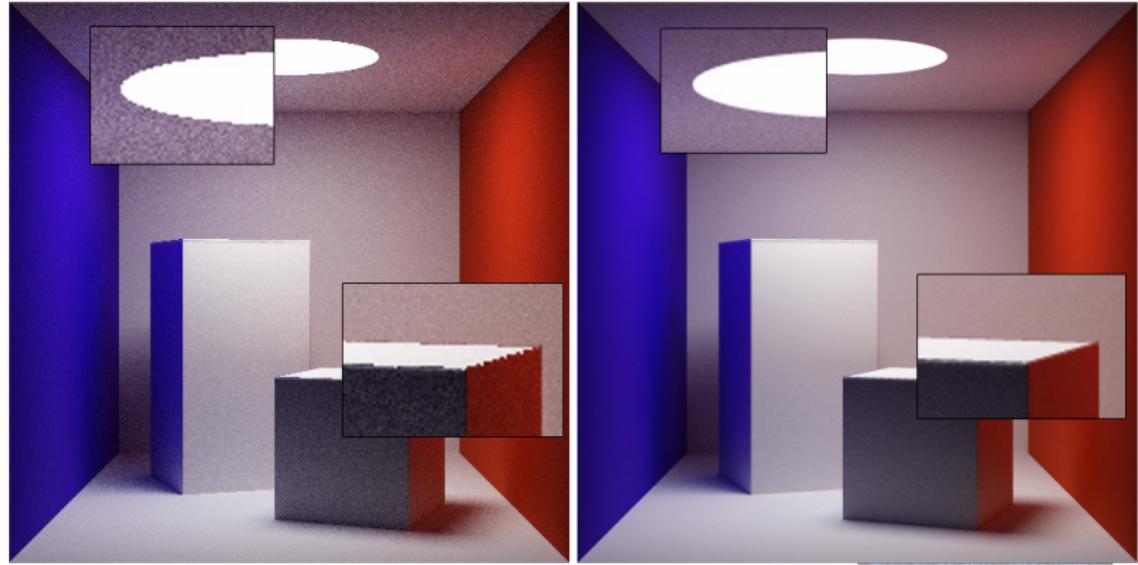


Figure : 16x (4x4) supersampling ( Left: 100spp , Right: 50spp).

# Antialiasing - Examples

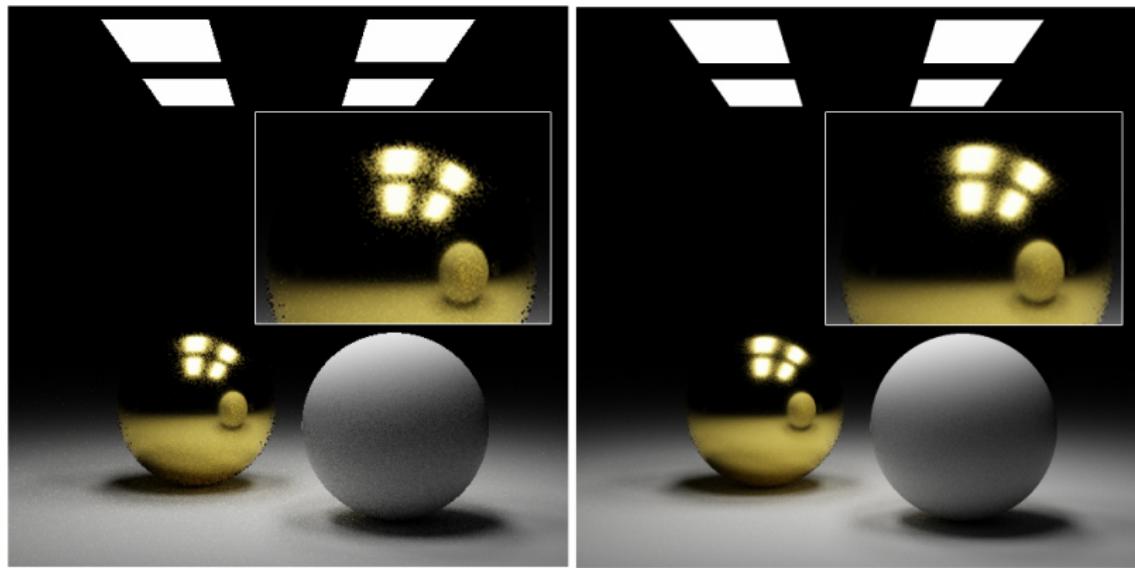


Figure : 4x (2x2) supersampling ( Left: 100spp , Right: 100spp).

# Parallel Programming - OpenMP

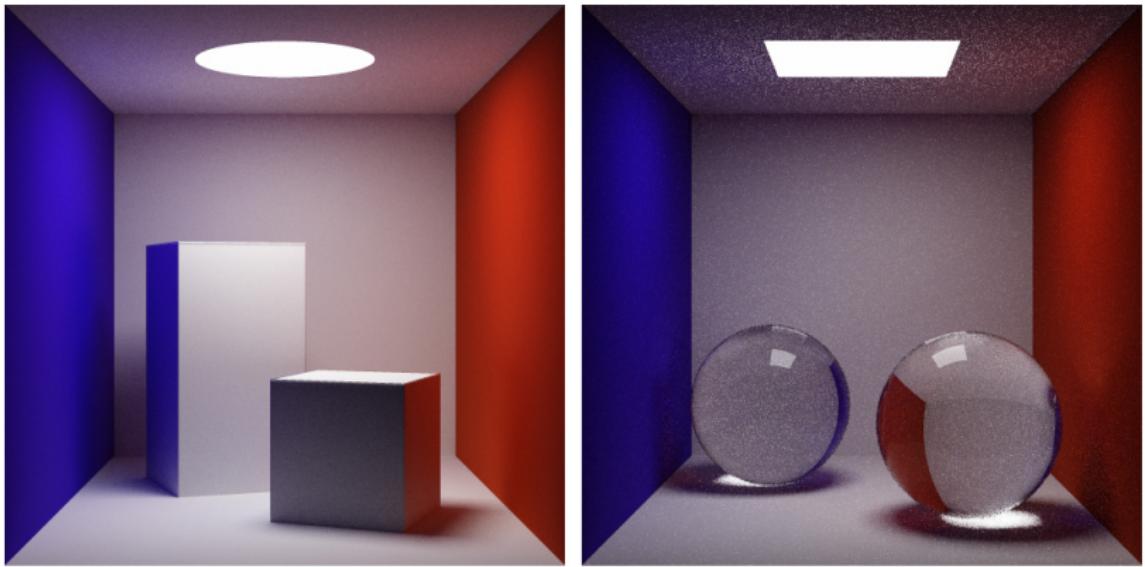
## OpenMP

Path tracing is a highly parallel algorithm.

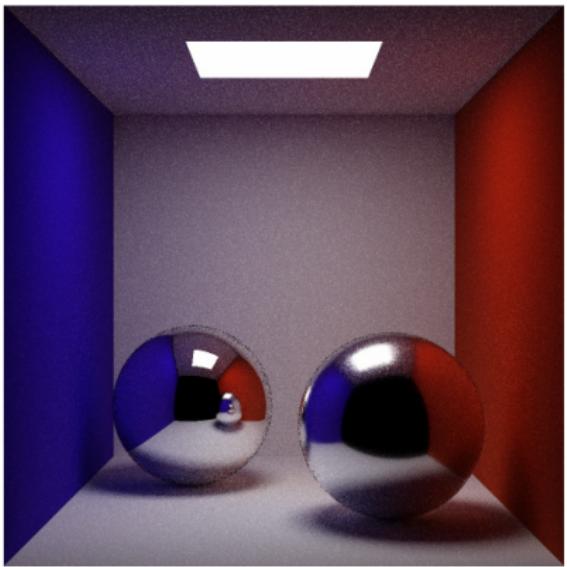
- GNU-Linux: compile and link with `-fopenmp`
- Windows: compile and link with `/openmp`
- Mac: I wish I had one :), should be like GNU-Linux

```
for (int y = 0; y < HEIGHT; y++)
    #pragma omp parallel for schedule(dynamic, 1)
    for (int x = 0; x < WIDTH; x++)
        . . .
```

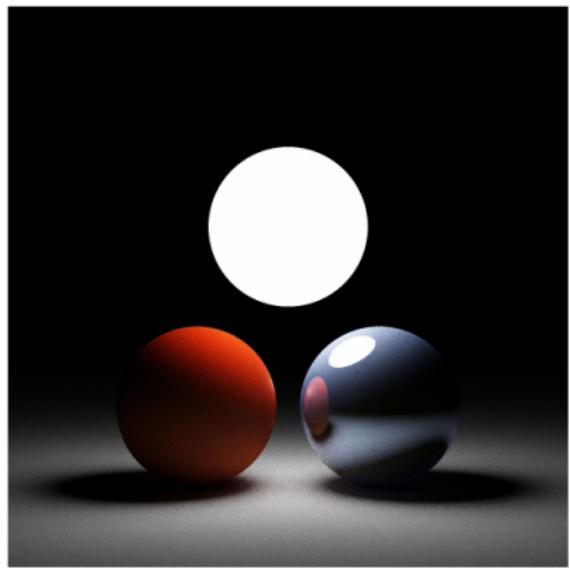
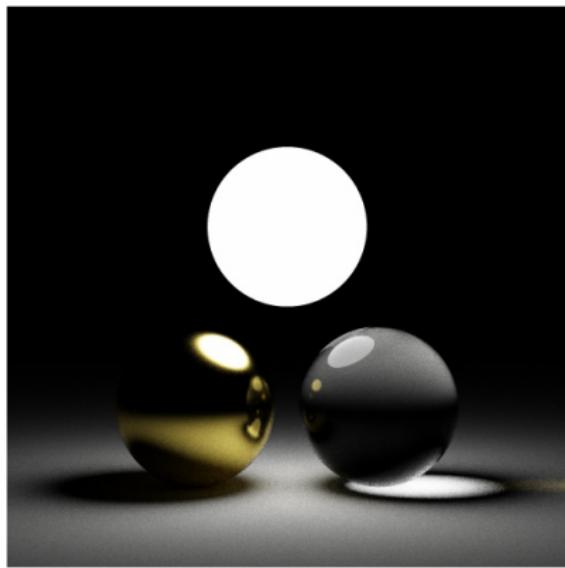
## More examples



## More examples



## More examples



-  PETER SHIRLEY AND R. KEITH MORLEY (2003). "Realistic Ray Tracing", *A K Peters/CRC Press*, 2nd edition, 2003.
-  PETER SHIRLEY AND STEVE MARSCHNER (2009). "Fundamentals of Computer Graphics" , *AK Peters Ltd.*, 3rd edition, 2009.
-  PHILIP DUTRÉ, KAVITA BALA, AND PHILIPPE BEKAERT. (2006). "Advanced Global Illumination", *AK Peters Ltd.*, 2nd edition, 2006.

# Thank You!!

Questions??