AE-675/AE-675A: Introduction to Finite Element Methods
Computer Assignment No. 1

Figure 1 shows an elastic bar under traction load and constrained at the ends A and B. Develop a generic finite element code to get the approximate solution to the resulting governing differential equation for the bar shown in Figure 1.

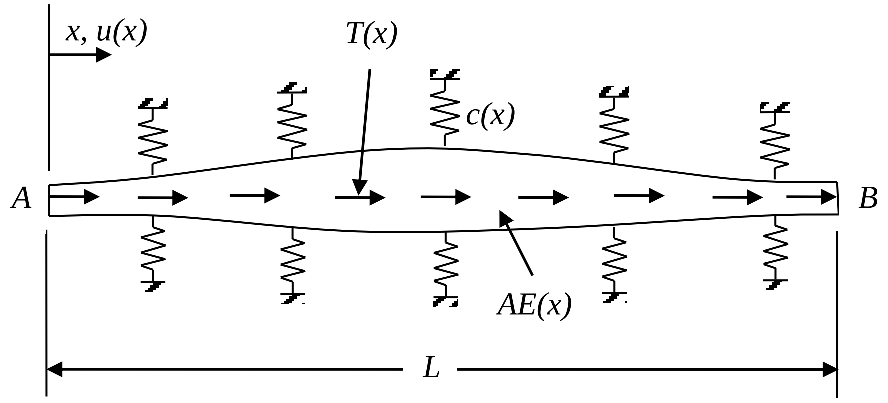

Figure 1: Elastic bar loaded in traction and constrained at ends *A* and *B*.

The code should have the following capabilities:

1. Boundary conditions/End Constraints: Both ends can be constrained by specifying (a) primary variable (Dirichlet/Displacement/Essential), (b) secondary variable or force (Force/Neumann/Natural) and (c) springs (Mixed/Robin)
2. The variables $T(x), c(x)$ and $AE(x)$ can vary from a constant to a quadratic function.
3. The length L and the number of elements will be input values. Discretize the domain into given number of elements with equal lengths.
4. There should be a provision to put at least one concentrated load at any given location (excluding the ends).
5. Use of either Lagrange interpolation or hierarchic shape functions upto quartic order should be possible.
6. Postprocessing must be able to represent the primary, secondary and other variables over the domain either continuously or discretely as required.

Produce a detailed report to include the following:

1. Do the patch test for the following cases:

   $AE(x) = 1$ and $c(x) = 0$ with $u(x)|_{x=0} = 0$ and $\left.\frac{du}{dx}\right|_{x=1} = 0$. When $T(x) = 1$, use 1, 2, 5, 10 and 100 number of linear and quadratic elements and when $T(x) = x$ use 1, 2, 5, 10 and 100 number of linear, quadratic and cubic elements and superimpose your solutions with the respective exact solutions. Plot the error in the solution. Also plot the derivative of the exact solution and finite element solutions. Discuss the results.

2. For the problem in Point 1, plot the strain energy of the exact and finite element solution against the number of elements in the mesh for all the cases. Also plot the strain energy of the solution. Discuss the results.
3. Take $AE(x) = 1$, $c(x) = 1$ and $T(x) = 1$ with $u(x)|_{x=0} = 0$ and $\left.\frac{du}{dx}\right|_{x=1} = 0$. Obtain the finite element solution with linear, quadratic, cubic and quartic elements respectively for 1, 10, 20, 40, 80 and 100 number of elements. For these cases:
    a) Plot the exact and finite element solutions together for these cases.
    b) Plot the error in the solution for these cases.
    c) Plot the strain energy of the finite element and exact solution as a function of number of elements.
    d) Plot the strain energy of the error as a function of number of elements.
    e) Plot the log of the relative error in the energy norm versus the log of number of elements.
    f) Try to estimate the convergence rate.

   Discuss the results.
4. Take $AE(x) = 1$, $c(x) = 0$ and $T(x) = \sin\frac{\pi}{L}x$ with $AE\left.\frac{du}{dx}\right|_{x=0} = \frac{1}{\pi}$ and $AE\left.\frac{du}{dx}\right|_{x=1} = k_L\big(\delta_L - u(L)\big)$ with $k_L = 10$ and $\delta_L = 0$. Then repeat the exercise given a) through f) in Point 3.

Note: 1) In all above points take $L = 1$.

2) All of the above should be done with both Lagrange and hierarchic shape functions.

3) Do not us inbuilt capabilities like integration and differentiation if you are going to use MATLAB for coding. You will be penalized for that.