### Multivariable Calculus: Week 2

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## Progress Update

Over the past week I have been introduced to the following matrix topics:

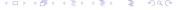
- Multiplication.
- Inverse finding.
- Applications to solving systems of equations.

#### The Matrix

A matrix is an array of vectors. Consider a  $3 \times 3$  matrix M; In Python notation,

$$M = \{\{a_1, a_2, a_3\}, \{b_1, b_2, b_3\}, \{c_1, c_2, c_3\}\} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_2 \\ c_1 & c_2 & c_3 \end{bmatrix}$$
(1)

Where  $a_1, \dots, c_3$  are scalars. A  $N \times M$  matrix can act on any other with  $M \times Z$ ; ie, a matrix can act on, through matrix multiplication, another matrix or vector where the width of the first equals the height of the second.



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## Matrix Multiplication

Matrix multiplication is taking the dot product of the rows and columns where the desired output overlaps; it is much clearer through LATEX:

$$\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \begin{bmatrix} c_1 & c_2 \\ d_1 & d_2 \end{bmatrix} = \begin{bmatrix} a_1c_1 + a_2d_1 & a_1c_2 + a_2d_2 \\ b_1c_1 + b_2d_1 & b_1c_2 + b_2d_2 \end{bmatrix}$$
(2)

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### Matrix inverse

Suppose we want to solve the equation (and this will double as my worked PSET problem):

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$
 (3)

for  $x_1, x_2, x_3$ . We need a matrix that undoes M, ie an  $M^{-1}$ .



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## Matrix Inverse (continued)

We have a 4 step method to finding the inverse matrix M, assuming  $det(M) \neq 0$  (a matrix with det(M) = 0 is non-invertible): (1) Minors:

$$\begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \\ -2 & 1 & 1 \end{bmatrix} \tag{4}$$

(2) Checkerboard:

$$\begin{bmatrix} 3 & -1 & 1 \\ 1 & 3 & -1 \\ -2 & -1 & 1 \end{bmatrix} \tag{5}$$

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# Matrix Inverse (continued)

(3) Swap rows and columns:

$$\begin{bmatrix} 3 & 1 & -2 \\ -1 & 3 & -1 \\ 1 & 2 & 1 \end{bmatrix} \tag{6}$$

(4) Divide by det(M):

$$\frac{1}{5} \cdot \begin{bmatrix} 3 & 1 & -2 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} = M^{-1} \tag{7}$$

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# Matrix Inverse (Solution)

Then, since

$$M^{-1}Mx = M^{-1}b (8)$$

$$x = \frac{1}{5} \cdot \begin{bmatrix} 3 & 1 & -2 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$
 (9)

$$x = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \tag{10}$$

Thus,  $x_1 = 1, x_2 = -1, x_3 = 1$ .

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## Solving systems of equations

We are given the following system of equations:

$$3x + 2y + 4z = 1 \tag{11}$$

$$5x - y - 3z = -7 (12)$$

$$4x + 3y + z = 2 (13)$$

We can now write a matrix multiplication to represent this system:

$$\begin{bmatrix} 3 & 2 & 4 \\ 5 & -1 & -3 \\ 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ 2 \end{bmatrix}$$
 (14)

and then apply the matrix inverse method seen above.

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