Exam 2

Problem 1

(A): $\int_0^{\pi/2} \int_0^1 \int_0^1 r^3 \, dz \, dr \, d\theta$

Problem 2

(A): (a): $\rho = 2a \cos\phi$

(b): $\rho = a \sec\phi$

(c): $\displaystyle\int_0^{2\pi}\int_0^{\pi/4}\int_{a\sec\phi}^{2a\cos\phi} \rho^2 \, d\rho \, \sin\phi \, d\phi \, d\theta$

Problem 3

(a): To show that F is conservative, we must show that

$\text{curl}(\vec{F}) = 0$;

$$\text{curl}(\vec{F}) = \vec{F} \times \vec{\nabla} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x & \cdots & \end{vmatrix}$$

$$= \langle ((2y) - (2y)), ((3z^2) - (3z^2)), ((2x) - (2x))$$

$$= 0.$$

(b): ~~See that f~~

$\cancel{f_x = f_x = 2xy + z^3 \Rightarrow f = x^2 y + z^3 x + g(y,z)}$

$\cancel{f_y = x^2 + 2yz \Rightarrow f = x^2 y + y^2 z + h(z,x)}$

$\cancel{\Downarrow}$

$\cancel{f = x^2 y + z^3 x + y^2 z}$

$\cancel{f_z = 2yz +}$

We see that

$$f_x = 2xy + z^3 \Rightarrow f = x^2 y + z^3 x + g(y,z)$$

$$\Downarrow$$

$$f_y = x^2 + \frac{\partial}{\partial y} g(y,z) = x^2 + 2yz$$

$$\Downarrow$$

$$g = y^2 z + h(z)$$

$$\Downarrow$$

$$f_z = 3z^2 x + y^2 + h'(z) = y^2 + 3z^2 x - 1$$

$$\Downarrow$$

$$h(z) = -z$$

$$\Downarrow$$

$$f = x^2 y + z^3 x + y^2 z - z + C$$

Problem 4

(a): We see that S is the graph of

$z = f(x,y) = 1 - x^2 - y^2$; then,

$\hat{n} \cdot dS = \langle 2x, 2y, 1 \rangle \, dA$.

In turn

$\iint_S \vec{F} \cdot \hat{n} \, dS = \iint_S 4x^2 + 4y^2 \, dA$

make a change of variables to spherical to see that

$\ldots = 2\pi$

Problem 5

(a): $\langle 2y, -2x \rangle$

(b): Consider the unit sphere with $\hat{n} = \langle x, y, z \rangle$; then
$\text{Curl}(\vec{F}) \cdot \hat{n} = 2xy - 2yx = 0$

(c): By Stoke's Theorm we can set it equal to the statement of (b) and get $\emptyset$.