Multivariable Calculus: Week 2

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Progress Update

Over the past week I have been introduced to the following matrix topics:

1. Multiplication.
2. Inverse finding.
3. Applications to solving systems of equations.

The Matrix

A matrix is an array of vectors. Consider a 3×3 matrix M; In Python notation,

$$M = \{\{a_1, a_2, a_3\}, \{b_1, b_2, b_3\}, \{c_1, c_2, c_3\}\} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_2 \\ c_1 & c_2 & c_3 \end{bmatrix} \tag{1}$$

Where $a_1, \cdots, c_3$ are scalars. A $N \times M$ matrix can act on any other with $M \times Z$; ie, a matrix can act on, through matrix multiplication, another matrix or vector where the width of the first equals the height of the second.

Matrix Multiplication

Matrix multiplication is taking the dot product of the rows and columns where the desired output overlaps; it is much clearer through LaTeX:

$$\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \begin{bmatrix} c_1 & c_2 \\ d_1 & d_2 \end{bmatrix} = \begin{bmatrix} a_1 c_1 + a_2 d_1 & a_1 c_2 + a_2 d_2 \\ b_1 c_1 + b_2 d_1 & b_1 c_2 + b_2 d_2 \end{bmatrix} \tag{2}$$

Matrix inverse

Suppose we want to solve the equation (and this will double as my worked PSET problem):

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} \tag{3}$$

for x_1, x_2, x_3. We need a matrix that undoes M, ie an M^{-1}.

Matrix Inverse (continued)

We have a 4 step method to finding the inverse matrix M, assuming $\det(M) \neq 0$ (a matrix with $\det(M) = 0$ is non-invertible):
(1) Minors:

$$\begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \\ -2 & 1 & 1 \end{bmatrix} \tag{4}$$

(2) Checkerboard:

$$\begin{bmatrix} 3 & -1 & 1 \\ 1 & 3 & -1 \\ -2 & -1 & 1 \end{bmatrix} \tag{5}$$

Matrix Inverse (continued)

(3) Swap rows and columns:

$$\begin{bmatrix} 3 & 1 & -2 \\ -1 & 3 & -1 \\ 1 & 2 & 1 \end{bmatrix} \tag{6}$$

(4) Divide by $\det(M)$:

$$\frac{1}{5} \cdot \begin{bmatrix} 3 & 1 & -2 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} = M^{-1} \tag{7}$$

Matrix Inverse (Solution)

Then, since

$$M^{-1}Mx = M^{-1}b \tag{8}$$

$$x = \frac{1}{5} \cdot \begin{bmatrix} 3 & 1 & -2 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} \tag{9}$$

$$x = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \tag{10}$$

Thus, $x_1 = 1, x_2 = -1, x_3 = 1$.

Solving systems of equations

We are given the following system of equations:

$$3x + 2y + 4z = 1 \tag{11}$$

$$5x - y - 3z = -7 \tag{12}$$

$$4x + 3y + z = 2 \tag{13}$$

We can now write a matrix multiplication to represent this system:

$$\begin{bmatrix} 3 & 2 & 4 \\ 5 & -1 & -3 \\ 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ 2 \end{bmatrix} \tag{14}$$

and then apply the matrix inverse method seen above.