Multivariable Calculus: Tutorial 7

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Progress Update

Over the past week I have learned about:

1. Lagrange Multipliers
2. Constrained Differentials

Lagrange Multipliers

To minimize or maximimize functions, we must develop a new technique; this technique is referred to as lagrange multipliers. Given a function and some constraint on the function's independent variables, we can find the critical points. Find the point such that

$$\nabla f = \lambda \nabla g \tag{1}$$

Or in the case of a function $f(x, y, z)$,

$$\begin{aligned} f_x &= \lambda g_x &(2)\\ f_y &= \lambda g_y &(3)\\ f_z &= \lambda g_z &(4)\\ g(x, y, z) &= c &(5) \end{aligned}$$

Problem: A rectangular produce box is to be made of cardboard; the sides of single thickness, the ends of double thickness, and the bottom of triple thickness, with the top left open. Its volume is to be 1 cubic foot; what should be its proportions in order to use the least cardboard?

Lagrange Multipliers (Solution)

Solution: We have that

$$f(x,y,z) = 4xz + 3xy + 2zy \text{ and} \tag{6}$$
$$g(x,y,z) = xyz = 1 \tag{7}$$

and thus

$$3y + 4z = \lambda yz \implies \lambda = 3/z + 4/y \tag{8}$$
$$3x + 2z = \lambda xz \implies \lambda = 3/z + 2/x \tag{9}$$
$$2y + 4x = \lambda xy \implies \lambda = 2/x + 4/y \tag{10}$$

Set these resulting lambda's equal to eachother and make a ratio; multiply by the GCF of the denominators to get the integer ratio 2 : 4 : 3.

Constrained Differentials

Constrained differentials is a simple algebraic manipulation to find the derivative of one variable with respect to another; take $g(x, y, z) = c$; then

$$dg = 0 = g_x dx + g_y dy + g_z dz \tag{11}$$

and in turn

$$dz = \frac{g_x dx + g_y dy}{g_z}, \tag{12}$$

allowing us to conclude

$$\frac{\partial z}{\partial x} = \frac{g_x}{g_z} \,\&\, \frac{\partial z}{\partial y} = \frac{g_y}{g_z} \tag{13}$$