18.02 PSET 1

~~$w = c$~~

We let the wind vector ~~$w = c$~~

$$w = c(-i - j)$$

and $|w|^2 = 50^2$

Thus, $|w|^2 = w \cdot w$ since $\cos(\theta) = 1$.

$$\langle -c, -c \rangle \cdot \langle -c, -c \rangle$$

$$|w|^2 = |w|\,|w| = \cancel{\sqrt{c^2+c^2}\,\sqrt{c^2+c^2}} = w \cdot w$$

$$\cancel{\Rightarrow w = \sqrt{2}c}$$

$$c^2 \langle -1, -1 \rangle \cdot \langle -1, -1 \rangle$$

$$2c^2$$

$$|w| = \sqrt{2}c = 50$$

$$\Rightarrow$$

$$c = \frac{50}{\sqrt{2}}$$

$$w = \frac{50}{\sqrt{2}} \langle -1i, -1j \rangle$$

We want $z = \langle 0i, 200j \rangle = \langle \frac{-50}{\sqrt{2}} i, \frac{-50}{\sqrt{2}} j \rangle + v$, thus

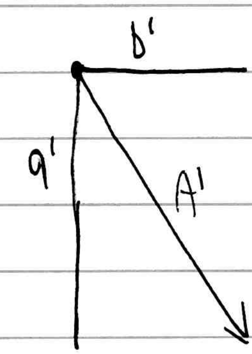
$$v = \langle \frac{+50}{\sqrt{2}} i, (200 + \frac{50}{\sqrt{2}}) j \rangle$$

2

We begin by letting A be placed of the diagonal of a rectangle

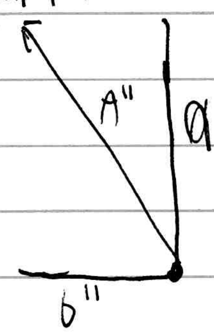
$A = a i + b j$

A: We rotate 90° clockwise about the point:



$A' = b i - a j$

B: Or for the second point:



$\Longrightarrow \quad A'' = -b i + a j$

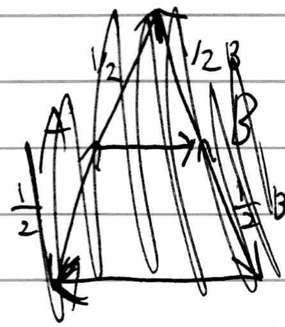
C: We are told $i' = \frac{3i + 4j}{5} = \frac{3}{5} i + \frac{4j}{5}$.

If i' is to be a unit vector, then it must be true $\sqrt{\frac{3}{5}^2 + \frac{4}{5}^2} = 1$

$\sqrt{\frac{9}{25} + \frac{16}{25}} = \sqrt{\frac{25}{25}} = 1$, thus i' is a unit vector.

PSET 3

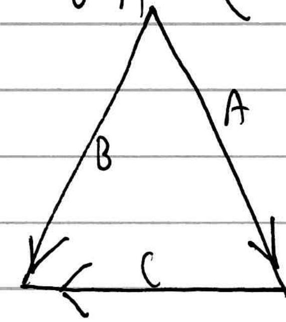
$$A + C - B = 0$$
$$B - A = C$$



$\cancel{B + C - A = 0}$

$\cancel{B - A = -C}$

$\cancel{C = A - B}$



~~We defin~~ Suppose A and B begin at some point. Then we would like to define a vector C such that $C = B - A$. We see that $\frac{1}{2}A$ is the midpoint of A and $1/2\,B$ is the midvector of B; we see that

$$C = B - A$$
$$\Rightarrow$$
$$\frac{1}{2}C = \frac{1}{2}B - \frac{1}{2}A$$

and for further convincing of myself, I see that each of these triangles are scalar multiples of the other; ie similar and share an ~~and~~ angle, thus $\frac{1}{2}C$ and C must be parallel.

PSET 4

We would like for, assuming $A = ci + 2j + -k$

$B = i + -j + 2k$

$$\cos(\theta) \stackrel{!}{=} \frac{A \cdot B}{|A|\,|B|} \stackrel{!}{=} 0$$

$$= \frac{c-4}{\sqrt{6}\cdot\sqrt{6}} = \frac{1}{6}(c-4) = 0$$

$$c = 4$$

If we would like $\cos(\text{theta}) > 0$, then $c > 4$

PSET 5

"Find the component of the force $2i-2j+k$ in the direct-ion of the vector $3i+2j-6k$."

First we must find the unit vector in the direction of $3i+2j-6k$; thus we must solve

$$\sqrt{9a^2+2a^2+6a^2} = 1$$

$$9a^2+2a^2+6a^2 = 1$$

$$a^2 = \frac{1}{17}$$

$$a = \frac{1}{\sqrt{17}}$$

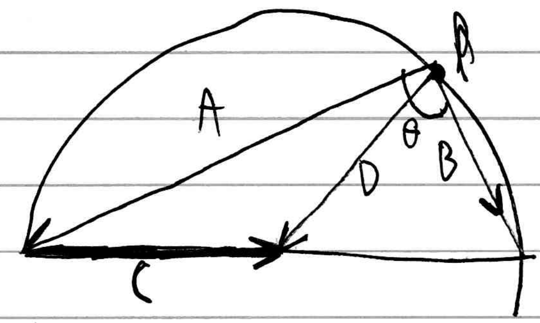
Thus we have the unit vector $\frac{3}{\sqrt{17}}i+\frac{2}{\sqrt{17}}j-\frac{6}{\sqrt{17}}k$.

and thus $C=(2i-2j+k)\cdot\left(\frac{3}{\sqrt{17}}i+\frac{2}{\sqrt{17}}j-\frac{6}{\sqrt{17}}k\right)$

$$=\frac{6}{\sqrt{17}}-\frac{4}{\sqrt{17}}-\frac{6}{\sqrt{17}}$$

$$=\frac{-4}{\sqrt{17}}$$

PSET 6



We see by drawing a semicircle and defining a vector $\vec{C}$ going to half the diameter / the center that we can connect our point P that A and B originate from to the center using a vector:

$$D = A + C = B - C$$

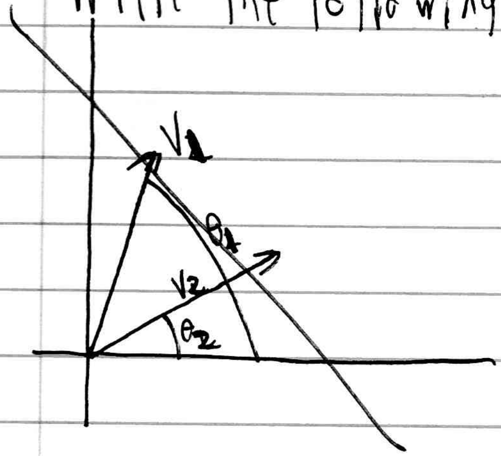
We want to find $\cos(\theta) = \dfrac{A \cdot B}{|A|\,|B|} \stackrel{!}{=} 0.$

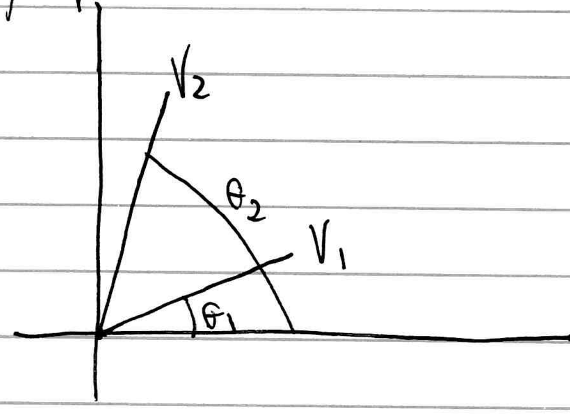
We see that $A = D - C$, $B = D + C$ and thus

$$\checkmark\ 0 = \frac{0}{|A||B|} \qquad \frac{(D-C)(D+C)}{|A||B|} = \frac{D^2 - C^2}{|A||B|} = \frac{|D|^2 - |C|^2}{|A||B|}$$

PSET 7

~~Write~~ Draw the following graphs





$V_1 \cdot V_2 = |V_1||V_2| \cos(\theta_2 - \theta_1)$ since $\cos(-x) = \cos(x)$

$= \cos(\theta_1 - \theta_2)$

$V_1 \cdot V_2$

$V_1 = \cos(\theta_1)\, i + \sin(\theta_1)\, j$

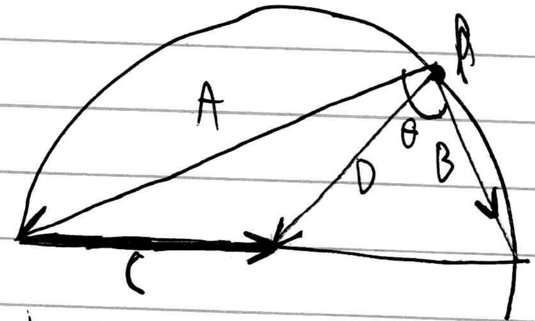
$V_2 = \cos(\theta_2)\, i + \sin(\theta_2)\, j$

$= \cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)(\sin(\theta_2))$

Thus,

$$\cos(\theta_1 - \theta_2) = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2$$

PSET 6



We see by drawing a semicircle and defining a vector $\vec{C}$ going to half the diameter / the center that we can connect our point P that A and B originate from to the center using a vector:

$$D = A + C = B - C$$

We want to find $\cos(\theta) = \frac{A \cdot B}{|A||B|} \overset{!}{=} 0.$

We see that $A = D - C$ and thus
$B = D + C$

$$\sqrt{0} = \frac{0}{|A||B|} \quad \frac{(D-C)(D+C)}{|A||B|} = \frac{D^2 - C^2}{|A||B|} = \frac{|D|^2 - |C|^2}{|A||B|}$$

PSET 8

(a): $$\begin{vmatrix} -1 & 0 & 4 \\ 1 & 2 & 2 \\ 3 & -2 & 1 \end{vmatrix} = -1\begin{vmatrix} 2 & 2 \\ -2 & 1 \end{vmatrix} - 0|\dots| + 4\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix}$$

$$= -1(2+4) + 4(-2-6)$$

$$= -6 + -32$$

$$= -38$$

(c): $$\begin{vmatrix} -1 & 0 & 4 \\ 1 & 2 & 2 \\ 3 & -2 & 1 \end{vmatrix} = -1\begin{vmatrix} 2 & 2 \\ -2 & 1 \end{vmatrix} - 1\begin{vmatrix} 0 & 4 \\ -2 & 1 \end{vmatrix} + 3\begin{vmatrix} 0 & 4 \\ 2 & 2 \end{vmatrix}$$

$$\cancel{\quad + 7 \quad + 3(}$$

$$= -6 + -8 + 3(-8)$$

$$= -38$$

PSET 9

Consider the determinant

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

We multiply the ~~bottom by~~ 2nd row by a scalar multiple of the first; let the scalar be z; the

$$\begin{vmatrix} a & b \\ c+az & d+bz \end{vmatrix} = a(d+bz) - b(c+az)$$

$$= ad + abz - bc - abz$$

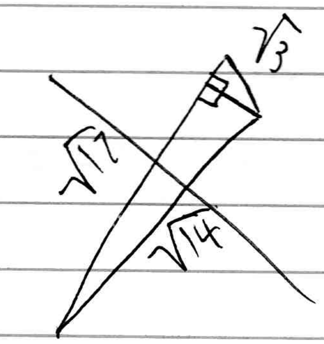
$$= ad - bc$$

PSET 10

~~$\left|\overrightarrow{PQ} = 1i + 1j + -1K\right| = \sqrt{3}$~~

~~$\left|\overrightarrow{PR} = -3i + 1j + 2K\right| = \sqrt{14}$~~

~~$\left|\overrightarrow{QR} = -4i + 0j + -1k\right| = \sqrt{17}$~~



We have the points $(2, 0, 1)$

$(3, 1, 0)$

$(-1, 1, -1)$.

Let us apply the vector $-2i + 0j - 1K$ to all the points

$(0, 0, 0)$

$(2, 1, -1)$

$(-3, 1, -2)$

We then must find

$$\frac{1}{2}\left\|\begin{vmatrix} i & j & K \\ 2 & 1 & -1 \\ -3 & 1 & -2 \end{vmatrix}\right\| = \frac{1}{2}\left| i\begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} - j\begin{vmatrix} 2 & -1 \\ -3 & 2 \end{vmatrix} + K\begin{vmatrix} 2 & 1 \\ -3 & 1 \end{vmatrix}\right|$$

$$\frac{3}{2}\sqrt{3} \qquad = \frac{1}{2}\left| -i \quad -(1j) + K(5) \right|$$

PSET 11

(a) Recall that $|A \times B| = |A|\,|B| \sin(\theta)$. We are told that $|A \times B| = |A||B|$. Thus we can conclude that $\sin(\theta) = 1$, which implies that these two angles are seperated by 90°.

(b) We see that $|A \times B| = |A||B| \sin(\theta)$, meanwhile
$A \cdot B = |A||B| \cos(\theta)$, thus

$$|A||B| \sin(\theta) = |A||B| \cos(\theta) \Rightarrow \sin(\theta) = \cos(\theta) \Rightarrow \theta = \frac{\pi}{4}.$$

These two vectors are seperated by 45°.

PSET 12

Shift ~~one~~ all of our points $(1,0,1)$ by the vector $-i+0j-k$
$(-1,1,2)$
$(0,0,2)$
$(3,1,-1)$

to see that we have the points $(0,0,0)$
$(-2,1,1)$
$(-1,0,1)$
$(2,1,-2)$

This tetrahedron has volume

$$\frac{1}{6}\left(\det(A,B,C)\right) \text{ where } \begin{matrix} A=-2i+j+k \\ B=-1i+0j+k \\ 2i+j-2k \end{matrix}$$

$$\det(A,B,C)=\begin{vmatrix} -2 & 1 & 1 \\ -1 & 0 & 1 \\ 2 & 1 & -2 \end{vmatrix} = -2\begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} - \begin{vmatrix} -1 & 1 \\ 2 & -2 \end{vmatrix} + \begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix}$$

$$=2-(0)+(-1)$$

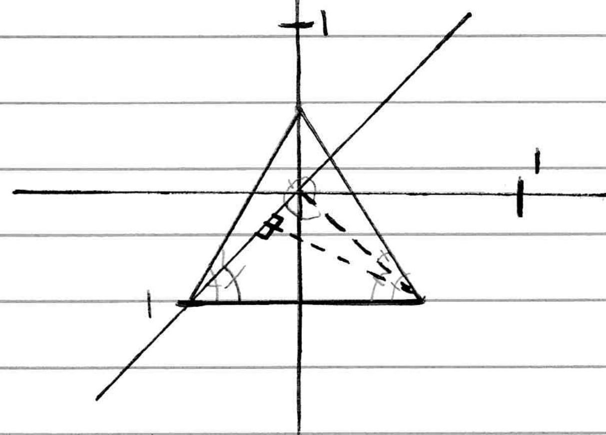
$$=1$$

Thus $\left(\frac{1}{6}\det(A,B,C)\right)=\frac{1}{6}$

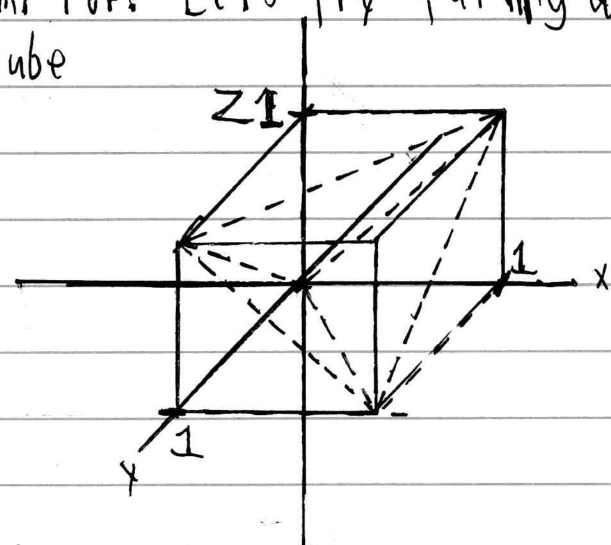
Personal Calc

PSET 13

We must find some vectors to represent a regular tetrahedron. Start by assuming side length = 1.



Hmm, this tetrahedron is clear but very hard to find the top point for. Let's try putting a tetrahedron inside a 1x1x1 cube



$P = (0,0,0)$	We would like to find the angle between
$Q = (1,1,0)$	the vectors connecting $(0.5, 0.5, 0) \to (1,0,1)$
$R = (1,0,1)$	and $(0.5, 0.5, 0) \to (0,1,1)$
$S = (0,1,1)$	$(0.5i + -0.5j + k) \cdot (-0.5i + 0.5j + 1k)$

$$=$$

$$\cos(\theta) = \frac{1}{3} \longleftarrow \frac{-1}{4} + \frac{-1}{4} + 1 = \frac{1}{2} = \cancel{\sqrt{\tfrac{1}{4} + \tfrac{1}{4} + 1} \cdot}$$

$$= \frac{6}{4} \cos(\theta)$$

$$\theta = 70.5$$

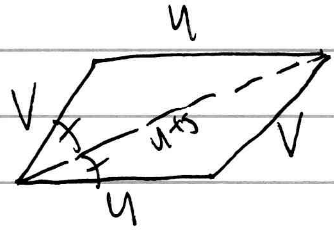
PSET 14

(A): We see that

$$|u+v|^2 = |u+v||u+v| = (u+v)(u+v) = u^2 + 2uv + v^2$$
$$- \; |u-v|^2 = |u-v||u-v| = (u-v)(u-v) = u^2 - 2uv + v^2$$

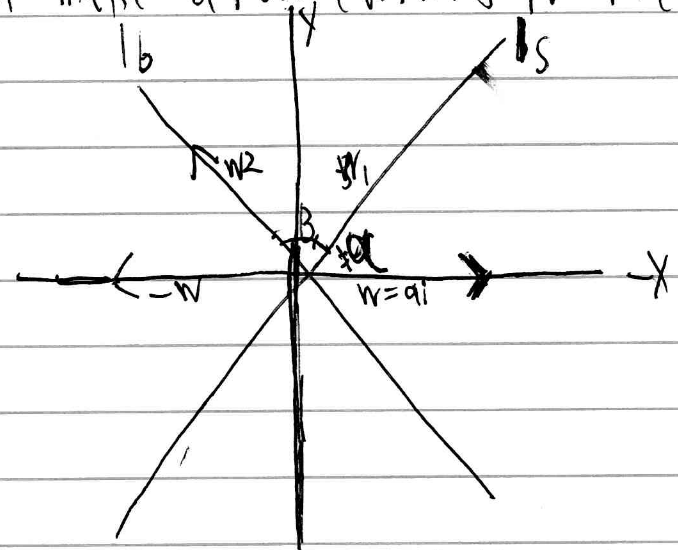
$$|u+v|^2 - |u-v|^2 = 4uv$$
$$\frac{1}{4}\left(|u+v|^2 - |u-v|^2\right) = uv$$

(B): Assume u and v have ~~equal~~ length 1 ~~length 1~~. Then $u+v$ bisects u and v.



PSET 15

(A): We shall make a few editions to the sketch ~~of~~ given.



"Let w_1 be the projection of w ontoline l_s. Show that W_1 does not have a non-zero component in the direction of $-w$."

This is intuitively and mathematically true. The component of w onto l_s represents the portion of the vector along W_1. Then, taking the component along the opposite, $-W$, yields zero since ~~no values i or j remain positi~~ this cancels all remaining terms.

(B): We let the direction of travel on our graph be represented by W_2. Then, we note that $|W_1| = w \cdot \hat{l}_s = |w| \cos(\alpha)$

$$= a\cos(\alpha)$$

And $|W_2| = \cos(\beta)\,|W_1| = \cos(\beta)\, a\cos(\alpha)$

We then see by carefully inspecting our sketch ~~the~~ + algebra that

$$W_2 = a\cos(\beta)\cos(\alpha)\left(\cos(\alpha+\beta)\,i + \sin(\alpha+\beta)\,k\right)$$

and in turn W_2 has component along i

$$W_2 \cdot \hat{i} = a\cos(\beta)\cos(\alpha)\cos(\alpha+\beta)$$

and thus observationally

$$W_2 \cdot \hat{i} < 0 \text{ when } \alpha+\beta < \frac{\pi}{2}$$