Multivariable Calculus: Week 4

Logan Pachulski

March 18, 2019

Progress Update

Over the past week I have:

1. Reviewed all section 1 topics.
2. Completed Exam 1.

For the remaining slides I shall go through the punchlines of each problem I solved. All work can be found on the GitHub solutions document.

Problem 1

Problem: Let P, Q, R be the points at 1 on the x-axis, 2 on the y-axis, and 3 on the z-axis, respectively.
(a) Express $\overrightarrow{QP}$ and $\overrightarrow{QR}$ in terms of $\hat{\mathbf{i}}, \hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$.
(b) Find the cosine of the angle PQR.

Solution 1

(a) $\overrightarrow{QP} = \mathbf{i} - 2\mathbf{j}$, $\overrightarrow{QR} = -2\mathbf{j} + 3\mathbf{k}$.
(b) $\cos(\theta) = \frac{4}{\sqrt{65}}$.

Problem 2

Problem: Let $P = (1, 1, 1)$, $Q = (0, 3, 1)$, and $R = (0, 1, 4)$.
(a) Find the area of the triangle PQR.
(b) Find the plane through P, Q, and R, expressed in the form $ax + by + cz = d$.
(c) Is the line through $(1, 2, 3)$ and $(2, 2, 0)$ parallel to the plane in part (b)? Explain why or why not.

Solution 2

(a) The area of the triangle is $\frac{7}{2}$.
(b) The plane has formula $6x + 3y + 2z = 11$.
(c) Yes; the cross product of the two is zero.

Problem 3

Problem: A ladybug is climbing on a Volkswagen Bug (=VW). In its starting position, the surface of the VW is represented by the unit semicircle $x^2 + y^2 = 1$, $y \geq 0$ in the xy-plane. The road is represented by the x-axis. At time $t = 0$, the ladybug starts at the front bumper, $(1, 0)$, and walks counterclockwise around the VW at unit speed relative to the VW. At the same time, the VW moves to the right at speed 10.

(a) Find the parametric formula for the trajectory of the ladybug, and find its position when it reaches the rear bumper. (At time $t = 0$, the rear bumper is at $(-1, 0)$.)

(b) Compute the speed of the bug, and find where it is largest and smallest. Hint: it is easier to work with the square of the speed.

Solution 3

(a) It has parametric formula $x(t) = 10 + \cos(t)$ and $y(t) = \sin(t)$, and reaches the rear bumper as position $(10\pi - 1, 0)$
(b) The speed of the bug is given by $\sqrt{101 - 20\sin(t)}$, and is minimized at $t = \pi/2$ and maximized at $t = 0, \pi$.

Problem 4

Problem: Let

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & -1 & -1 \end{pmatrix},$$

$$M^{-1} = \frac{1}{12}\begin{pmatrix} 1 & 1 & 4 \\ a & 7 & -8 \\ b & -5 & 4 \end{pmatrix}. \tag{1}$$

(a) Compute the determinant of M.
(b) Find the numbers a and b in the formula for the matrix M^{-1}.

Problem 4 (continued)

(c) Find the solution $\vec{\mathbf{r}} = \langle x, y, z \rangle$ to

$$\begin{aligned} x + 2y + 3z &= 0, \\ 3x + 2y + z &= t, \\ 2x - y - z &= 3, \end{aligned} \tag{2}$$

as a function of t.
(d) Compute $\frac{d\vec{\mathbf{r}}}{dt}$.

Solution 4

Problem 5

Problem: (a) Let $P(t)$ be a point with position vector $\vec{\mathbf{r}}(t)$. Express the property that $P(t)$ lies on the plane $4x - 3y - 2z = 6$ in vector notation as an equation involving $\vec{\mathbf{r}}$ and the normal vector to the plane.
(b) By differentiating your answer to (a), show that $\frac{d\vec{\mathbf{r}}}{dt}$ is perpendicular to the normal vector to the plane.

Solution 5