Multivariable Calculus: Tutorial 10

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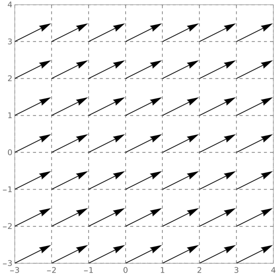
Progress Update

Over the past week I have been introduced to:

1. Vector fields
2. Gradient fields
3. Line integrals

Vector fields

A vector field is a function that takes in a coordinate and spits out a vector; in this chapter, we refer to any vector field that takes in an (x, y) coordinate and returns a $\mathbf{i}, \mathbf{j}$ vector. Consinder the following constant vector field:



This field has $F = \mathbf{i} + \frac{1}{2}\mathbf{j}$.

Gradient fields

A gradient field is a vector field where the function is defined as the gradient of some function that exists in 3-space: Consider the function $f(x,y) = \log(\sqrt{x^2+y^2})$; thus the gradient field has formula

$$\nabla f = \frac{x}{x^2+y^2}\mathbf{i} + \frac{y}{x^2+y^2}\mathbf{j}$$

The line integral (which we shall talk about next) has remarkable properties under gradient functions, including path independence.

Line integral

The purpose of the line integral is to calculate the work done to a particle along a trajectory by a vector field; the vector field may provide some assistance/resistance that we shall calculate using the formula

$$W = \int_C F \cdot dr$$

Line integral example

Suppose you want to find the work done by the constant vector field $F = \mathbf{i} + \frac{1}{2}\mathbf{j}$ on a particle moving from $(0,1)$ to $(1,1)$ in a straight line. We see that

$$W = \int_C 1dx + \frac{1}{2}dy$$

We shall *parametrize* and let $x = x$, $y = 1$, $dx = dx$, $dy = 0$ over the domain $[0,1]$. Then

$$\cdots = \int_0^1 dx = 1.$$

Parametrization enables complicated paths.