Problem 1

Let P, Q, R be the points at 1 on the x-axis, 2 on the y-axis, and 3 on the z-axis, respectively.

(a) Express $\overrightarrow{QP}$ and $\overrightarrow{QR}$ in terms of $\hat{\mathbf{i}}, \hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$.

(b) Find the cosine of the angle PQR.

Solution

Problem 2

Let $P = (1, 1, 1)$, $Q = (0, 3, 1)$, and $R = (0, 1, 4)$.

(a) Find the area of the triangle PQR.

(b) Find the plane through P, Q, and R, expressed in the form $ax + by + cz = d$.

(c) Is the line through $(1, 2, 3)$ and $(2, 2, 0)$ parallel to the plane in part (b)? Explain why or why not.

Solution

Problem 3

A ladybug is climbing on a Volkswagen Bug (=VW). In its starting position, the surface of the VW is represented by the unit semicircle $x^2 + y^2 = 1$, $y \geq 0$ in the xy-plane. The road is represented by the x-axis. At time $t = 0$, the ladybug starts at the front bumper, $(1, 0)$, and walks counterclockwise around the VW at unit speed relative to the VW. At the same time, the VW moves to the right at speed 10.

(a) Find the parametric formula for the trajectory of the ladybug, and find its position when it reaches the rear bumper. (At time $t = 0$, the rear bumper is at $(-1, 0)$.)

(b) Compute the speed of the bug, and find where it is largest and smallest. Hint: it is easier to work with the square of the speed.

Solution

Problem 4
Let

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & -1 & -1 \end{pmatrix},$$
$$M^{-1} = \frac{1}{12}\begin{pmatrix} 1 & 1 & 4 \\ a & 7 & -8 \\ b & -5 & 4 \end{pmatrix}. \tag{1}$$

(a) Compute the determinant of M.

(b) Find the numbers a and b in the formula for the matrix M^{-1}.

(c) Find the solution $\vec{\mathbf{r}} = \langle x, y, z \rangle$ to

$$\begin{aligned} x + 2y + 3z &= 0, \\ 3x + 2y + z &= t, \\ 2x - y - z &= 3, \end{aligned} \tag{2}$$

as a function of t.

(d) Compute $\frac{d\vec{\mathbf{r}}}{dt}$.

Solution

Problem 5

(a) Let $P(t)$ be a point with position vector $\vec{\mathbf{r}}(t)$. Express the property that $P(t)$ lies on the plane $4x - 3y - 2z = 6$ in vector notation as an equation involving $\vec{\mathbf{r}}$ and the normal vector to the plane.

(b) By differentiating your answer to (a), show that $\frac{d\vec{\mathbf{r}}}{dt}$ is perpendicular to the normal vector to the plane.

Solution