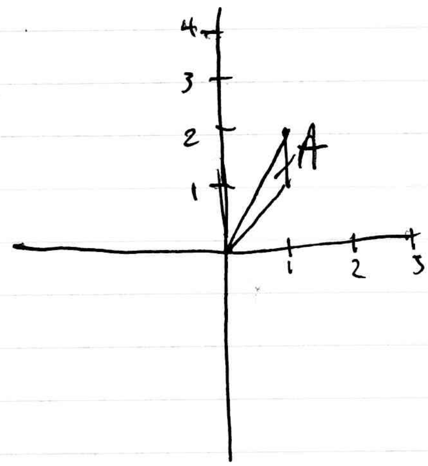
# Exam 3, Problem 1

(a):




(b): By splitting at $y=1$ (where $x=y$ becomes $x=1$),

$$\dots = \int_0^1 \int_{y/2}^{y} dx\, dy + \int_1^2 \int_{y/2}^{1} dx\, dy$$

Exam 3, Problem 2

(a): We see by converting variables that

$$\delta \, \partial A = \frac{r\sin(\theta)}{r^2} \, r \, \partial r \, \partial\theta = \sin(\theta) \, \partial r \, \partial\theta$$

; thus,

$$m = \iint_R \delta \, \partial A = \int_0^{\pi} \int_1^3 \sin\theta \, \partial r \, \partial\theta = \int_0^{\pi} 2\sin\theta \, \partial\theta$$

$$= \Big[ -2\cos \Big]_0^{\pi}$$

$$= 2(1-(-1))$$

$$= 4$$

(b): We see that

$$\overline{x} = \iint x \, \delta \, \partial A = \frac{1}{4} \int_0^{\pi} \int_1^3 r\cos\theta \sin\theta \, \partial r \, \partial\theta$$

But, we see by symmetry across the ~~x~~ axis that

$$\overline{x} = 0$$

Exam 3, Problem 3

(a): For $F$ to be conservative, we need $N_x = M_y$:

$M = 3x^2 - 6y^2 \Rightarrow M_y = -12y$

$N = -12xy + 4y \Rightarrow N_x = -12y$

$F$ is conservative.

(b): We have that

$f_x = 3x^2 - 6y^2 \Rightarrow f = x^3 - 6xy^2 + c(y) \Rightarrow f_y = -12xy + c'(y)$

But by comparing to the given $N = f_y$, we see that $c'(y) = 4y$; thus

$f = x^3 - 6xy^2 + 2y^2 + C$, an integration constant

(c): We see that we can simply evaluate from $(1,0) \to (1,1)$ by ~~plugg~~ finding endpoints:

$f(1,1) - f(1,0) = (1-6+2) - 1 = -4$

Exam 3, Problem 4

(a): We see that

$$\int_C F \cdot \partial r = \int_C (5x+3y)\,\partial x + (1+\cos y)\,\partial y$$

Parametrize with

$x = \cos(\theta)$

$y = \sin(\theta)$

$\partial x = -\sin(\theta)\,\partial\theta$

$\partial y = \cos(\theta)\,\partial\theta$; thus

$$W = \int_0^{2\pi} (5\cos\theta + 3\sin\theta)(-\sin\theta)\,\partial\theta + (1 + \cos(\sin\theta))\cos\theta\,\partial\theta.$$

What a mess.

(b): By Green's theorem, instead evaluate

$$\iint_R \operatorname{curl}(F)\,\partial A = \iint_R -3\,\partial A = -3\cdot(\text{unit circle area})$$

$$= -3\pi$$

# Exam 3, Problem 5

(a): We see by Green's Theorem that

$$\oint_C F \cdot \hat{n} = \iint_R \text{curl}(F)\ dy\ dx$$

$$= \int_0^1 \int_0^4 y + \cos(x)\cos(y) - \cos(x)\cos(y)\ dy\ dx$$

$$= \int_0^1 \int_0^4 y\ dy\ dx$$

$$= \int_0^1 \left[\frac{y^2}{2}\right]_0^4 = 8$$

(b): We see that on $C_4$, $x=0$; thus $F = -\sin y\,\mathbf{j}$ and $\hat{n} = -\mathbf{i}$; thus $F \cdot \hat{n} = 0$ and $\text{flux}(C_4, F) = 0$. Then,

$$\text{Flux}(C_1 + C_2 + C_3) = \text{Flux}(C)$$

# Exam 3, Problem 6

(a): Make the change of variables

$u = 2x - y$

$v = x + y - 1$

$J(dy\,dx \to du\,dv) = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 3$; thus

$du\,dv = 3\,dx\,dy$;

Then,

$$\iint_A \left(4 - (2x-y)^2 - (x+y-1)^2\right) dx\,dy$$

$$\ldots = \iint_A \left(4 - u^2 - v^2\right) \frac{1}{3}\,du\,dv$$

Then note that since

$A = \{u^2 + v^2 < 4\}$,

we can change to polar coordinates quite nicely.

$$\ldots = \int_0^{2\pi}\int_0^2 (4 - r^2)\, r\,dr\,d\theta \cdot \frac{1}{3}$$

$$= \int_0^{2\pi} \left[2r^2 - \frac{r^3}{3}\right]_0^2 \cdot \frac{1}{3}$$

$$= \frac{8\pi}{3}$$