Problem 1
Let $f(x, y) = x^2y^2 - x$.

(a) Find ∇f at $(2, 1)$.

(b) Write the equation for the tangent plane to the graph of f at $(2, 1, 2)$.

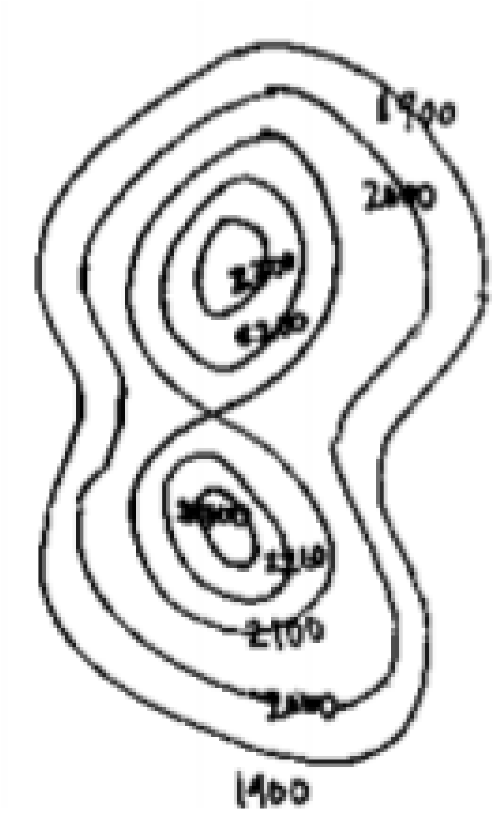
(c) Use a linear approximation to find the approximate value of $f(1.9, 1.1)$.

(d) Find the directional derivative of f at $(2, 1)$ in the direction of $-\hat{\mathbf{i}} + \hat{\mathbf{j}}$.

Solution

Problem 2

On the contour plot below, mark the portion of the level curve $f = 2000$ on which $\frac{\partial f}{\partial y} \geq 0$.



Solution

Problem 3
(a) Find the critical points of

$$w = -3x^2 - 4xy - y^2 - 12y + 16x \tag{1}$$

and say what type each critical point is.

(b) Find the point of the first quadrant $x \geq 0, y \geq 0$ at which w is largest. Justify your answer.

Solution

Problem 4

Let $u = \frac{y}{x}$, $v = x^2 + y^2$, $w = w(u, v)$.

(a) Express the partial derivatives w_x and w_y in terms of w_u and w_v (and x and y).

(b) Express $xw_x + yw_y$ in terms of w_u and w_v. Write the coefficients as functions of u and v.

(c) Find $xw_x + yw_y$ in case $w = v^5$.

Solution

Problem 5

(a) Find the Lagrange multiplier equations for the point of the surface

$$x^4 + y^4 + z^4 + xy + yz + zx = 6 \tag{2}$$

at which x is largest. (Do not solve.)

(b) Given that x is largest at the point (x_0, y_0, z_0), find the equation for the tangent plane to the surface at that point.

Solution

Problem 6

Suppose that $x^2 + y^3 - z^4 = 1$ and $z^3 + zx + xy = 3$.

(a) Take the total differential of each of these equations.

(b) The two surfaces in part (a) intersect in a curve along which y is a function of x. Find $\frac{dy}{dx}$ at $(x, y, z) = (1, 1, 1)$.

Solution