Problem 1

Let C be the portion of the cylinder $x^2 + y^2 \leq 1$ lying in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) and below the plane $z = 1$. Set up a triple integral in *cylindrical coordinates* which gives the moment of inertia of C about the z-axis; assume the density to be $\delta = 1$.

(Give the integrand and limits of integration, but do not evaluate.)

Solution

Problem 2

(a) A solid sphere S of radius a is placed above the xy-plane so it is tangent at the origin and its diameter lies along the z-axis. Give its equation in *spherical coordinates*.

(b) Give the equation of the horizontal plane $z = a$ in spherical coordinates.

(c) Set up a triple integral in spherical coordinates which gives the volume of the portion of the sphere S lying *above* the plane $z = a$. (Give the integrand and limits of integration, but do not evaluate.)

Solution

Problem 3

Let $\vec{F} = (2xy + z^3)\mathbf{i} + (x^2 + 2yz)\mathbf{j} + (y^2 + 3xz^2 - 1)\mathbf{k}$.

(a) Show that $\vec{F}$ is conservative.

(b) Using a systematic method, find a potential function $f(x, y, z)$ such that $\vec{F} = \vec{\nabla} f$. Show your work even if you can do it mentally.

Solution

Problem 4
Let S be the surface formed by the part of the paraboloid $z = 1 - x^2 - y^2$ lying above the xy-plane, and let $\vec{F} = x\mathbf{i} + y\mathbf{j} + 2(1 - z)\mathbf{k}$.

Calculate the flux of $\vec{F}$ across S, taking the upward direction as the one for which the flux is positive. Do this in two ways:

(a) by direct calculation of $\int\int_S \vec{F} \cdot \mathbf{n}\, dS$;

(b) by computing the flux across a simpler surface and using the divergence theorem.

Solution

Problem 5

Let $\vec{F} = -2xz\mathbf{i} + y^2\mathbf{k}$.

(a) Calculate $\text{curl}(\vec{F})$.

(b) Show that $\int\int_R \text{curl}(\vec{F}) \cdot \mathbf{n}\, dS = 0$ for any finite portion R of the unit sphere $x^2 + y^2 + z^2 = 1$ (take the normal vector $\mathbf{n}$ pointing outward).

(c) Show that $\oint_C \vec{F} \cdot d\vec{r} = 0$ for any simple closed curve C on the unit sphere $x^2 + y^2 + z^2 = 1$.

Solution