Multivariable Calculus: Tutorial 6

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Progress Update

Over the past week I have learned about:

1. Total differentials.
2. The multivariable chain rule.
3. Gradient of a function.
4. The directional derivative.

The total differential

The total differential of some function $f(x, y, z)$ is written as

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz, \tag{1}$$

or where we use the compact notation where $\frac{\partial f}{\partial x} = f_x$,

$$df = f_x dx + f_y dy + f_z dz. \tag{2}$$

This loosely encodes the idea of how much the function f changes for a certain change in $x, y,$ or z.

The multivariable chain rule

The multivariable chain rule allows us to calculate the derivative of a multivariable function where all of the variables are based upon a single variable; for example $f(x(t), y(t), z(t))$. Such a function would have

$$\frac{df}{dt} = f_x x_t + f_y y_t + f_z z_t. \tag{3}$$

The idea behind gradient shall follow from the above formula.

Gradient

If we let

$$\nabla f = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k} \text{ and} \tag{4}$$

$$\frac{dr}{dt} = x_t\mathbf{i} + y_t\mathbf{j} + z_t\mathbf{k}, \tag{5}$$

then for some function $f(x(t), y(t), z(t))$

$$\frac{df}{dt} = \nabla f \cdot \frac{dr}{dt}. \tag{6}$$

We define ∇f as the *gradient* of this function.

The directional derivative

The directional derivative of some function at a point, in the direction of a vector **u**; is given by

$$\left.\frac{df}{ds}\right|_{\hat{u}} = \nabla f \cdot \hat{u} \tag{7}$$

where $\hat{u} = \frac{\mathbf{u}}{|\mathbf{u}|}$. Let's solve a PSET problem with this. What's the directional derivative of $f(x, y) = 4 - x - 4y$ in the direction $\mathbf{w} = -2\mathbf{i} - \mathbf{j}$?

$$(-\mathbf{i} + -4\mathbf{j}) \cdot (-2\mathbf{i} - \mathbf{j}) = 2 + 4 = 6 \tag{8}$$