Multivariable Calculus: Tutorial 3

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Progress Update

Over the past week I have:

1. Identified and solved problem with parametric equations.
2. Learned about the geometry of the cycloid and its parametric equation.
3. Applied derivatives to parametric equations to find velocity and acceleration of particles following a function.

Parametric equations

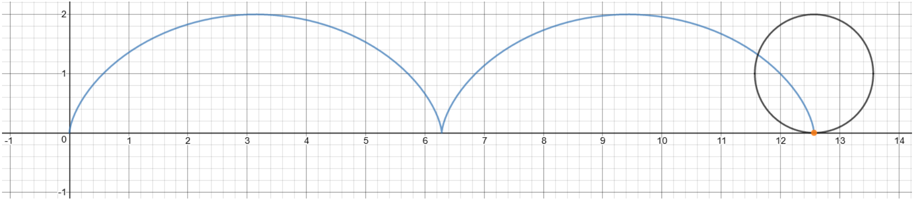
A parametric function is one based off of a single variable that returns a point; consider the function $y = x^2$; we can write a pair of parametric function for this:

$$x(t) = t \tag{1}$$
$$y(t) = (x(t))^2 \tag{2}$$

The path this function draws is equal to $y = x^2$, but the third variable allows us to build more complicated functions.

The cycloid

The cycloid is an early example of a function that is easier to draw with a parametric equation;



The cycloid has parametric equations

$$x(t) = 1 - \sin(t) \tag{3}$$
$$y(t) = 1 - \cos(t) \tag{4}$$

Derivatives of parametric functions

Consider a particle with position $P(t) = (a(t), b(t))$. Then the velocity

$$V(t) = |a'(t)\mathbf{i} + b'(t)\mathbf{j}|. \tag{5}$$

Similarly, the acceleration

$$(t) = |a''(t)\mathbf{i} + b''(t)\mathbf{j}|. \tag{6}$$

Example problem (statement)

Consider the following modified PSET problem: **Problem:** What is the acceleration of the function defined by

$$x(t) = \frac{1}{1+t^2} \qquad y(t) = \frac{t}{1+t^2} \tag{7}$$

at $t = 14$?

Example problem (solution)

Solution: