Multivariable Calculus: Tutorial 13

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Progress Update

Over the past week I have learned about:

1. Flux in 3 dimensions.
2. The divergence theorem.

Flux in 3 dimensions

Recall from a few tutorials ago the definition of flux - flux is a measure of how much "material" or "volume" crosses across a boundary in unit time; In 2 dimensions, we integrate over a curve C

$$\text{Flux} = \int_C F \cdot \hat{n} ds. \tag{1}$$

But in 3 dimensions, we integrate over a surface S:

$$\text{Flux} = \iint_S F \cdot \hat{n} ds. \tag{2}$$

In both cases, the direction of $\hat{n}$ determines which direction across the boundary the flux is measured.

Divergence theorem

For a closed surface S, define

$$\operatorname{div}(F) = \operatorname{div}(P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}) = P_x + Q_y + R_z. \tag{3}$$

Then,

$$\oiint_S F \cdot \hat{n} ds = \iiint_D \operatorname{div}(F) dv. \tag{4}$$

Example problem (statement)

Problem: By using the divergence theorem, evaluate the surface integral giving the flux of $\mathbf{F} = x\mathbf{i} + z^2\mathbf{j} + y^2\mathbf{k}$ over the tetrahedron with vertices at the origin and the three points on the positive coordinate axes at distance 1 from the origin.

Example problem (solution)

Solution: By the divergence theorem,

$$\begin{aligned}
\oiint_S F \cdot \hat{n} ds &= \iiint_D \mathrm{div}(F) dv \\
&= \iiint_D (1 + 0 + 0) dv \\
&= \iiint_D dv \\
&= \frac{1}{3} \cdot \frac{1}{2} \cdot 1 \\
&= \frac{1}{3} \cdot (\text{base}) \cdot (\text{height})
\end{aligned} \tag{5}$$