Multivariable Calculus: Tutorial 15

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Progress Update

Over the past week I have:

1. Completed the Unit 4 exam.

Example Problem: Statement

Problem: Let $\vec{F} = (2xy + z^3)\mathbf{i} + (x^2 + 2yz)\mathbf{j} + (y^2 + 3xz^2 - 1)\mathbf{k}$.

(a) Show that $\vec{F}$ is conservative.

(b) Using a systematic method, find a potential function $f(x, y, z)$ such that $\vec{F} = \vec{\nabla} f$. Show your work even if you can do it mentally.

Example Problem: Solution

Solution: Recall

$$\text{Curl}(F) = <R_y - Q_z, P_z - R_x, Q_x - P_y> \tag{1}$$
$$= <2y - 2y, 3z^2 - 3z^2, 2x - 2x> \qquad = 0 \tag{2}$$

Thus we have proven the first part; we then see that

$$f_x = 2xy + z^3 \implies f = x^2y + xz^3 + g(y,z) \tag{3}$$

We then see that by taking a y-derivative and comparing to Q that

$$f_y = x^2 + 2yz = x^2 + g'(y,z) \implies g(y,z) = y^2z + h(z) \tag{4}$$

and in turn

$$h'(z) = -1 \implies h(z) = -z \tag{5}$$