Multivariable Calculus: Tutorial 11

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May 6th, 2019

Progress Update

Over the past week I have been introduced to:

1. Green's Theorem
2. Flux

Green's Theorem

Green's theorem relates the closed line integral of a function to the double integral of the curl over the region enclosed by aforementioned line; more clearly:

$$\oint_C F \cdot dr = \iint_R \mathrm{curl}(F) dy\, dx$$

If $\mathrm{curl}(F) = 0$, then it is clearly true that the closed line integral should be zero; curl being 0 implies the function is conservative.

Flux

Flux is a measure of how much material crosses a boundary in unit time; see that

$$\text{Flux} = \int_C F\hat{n}ds$$

Where $\hat{n}$ is the vector normal to the curve C; we are in effect integrating the component of the force and the normal vector; an amazingly intuitive idea.

Flux Example

Problem: Take C to be the square of side length 1 with opposite vertices at $(0,0)$ and $(1,1)$, directed clockwise. Let $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$; find the flux across C.

Solution: Break the square into 4 sides going counter-clockwise: $\{C_1, C_2, C_3, C_4\}$. Then, the total flux

$$\int_C F \cdot n = \int_{C_1} \cdots \int_{C_4} \tag{1}$$

$$= \int_1^0 dy - \int_1^0 dx = -2 \tag{2}$$