Problem 1

(a) Draw a picture of the region of integration of $\int_0^1 \int_x^{2x} \, dy \, dx$.

(b) Exchange the order of integration to express the integral in part (a) in terms of integration in the order $dx\,dy$. Warning: your answer will have two pieces.

Solution

Problem 2

(a) Find the mass M of the upper half of the annulus $1 < x^2 + y^2 < 9$, $y \geq 0$, with density $\delta = \frac{y}{x^2+y^2}$.

(b) Express the x-coordinate of the center of mass, $\overline{x}$, as an iterated integral. (Write explicitly the integrand and limits of integration.) Without evaluating the integral, explain why $\overline{x} = 0$.

Solution

Problem 3

(a) Show that $\mathbf{F} = (3x^2 - 6y^2)\mathbf{i} + (-12xy + 4y)\mathbf{j}$ is conservative.

(b) Find a potential function for $\mathbf{F}$.

(c) Let C be the curve $x = 1 + y^3(1-y)^3$, $0 \le y \le 1$. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

Solution

Problem 4

(a) Express the work done by the force field $\mathbf{F} = (5x + 3y)\mathbf{i} + (1 + \cos(y))\mathbf{j}$ on a particle moving counter-clockwise once around the unit circle centered at the origin in the form $\int_a^b f(t)\,dt$. (Do not evaluate the integral; don't even simplify $f(t)$.)

(b) Evaluate the line integral using Green's theorem.

Solution

Problem 5

Consider the rectangle R with vertices $(0,0)$, $(1,0)$, $(1,4)$, and $(0,4)$. The boundary of R is the curve C, consisting of C_1, the segment from $(0,0)$ to $(1,0)$, C_2, the segment from $(1,0)$ to $(1,4)$, C_3, the segment from $(1,4)$ to $(0,4)$, and C_4, the segment from $(0,4)$ to $(0,0)$. Consider the vector field

$$\mathbf{F} = (xy + \sin(x)\cos(y))\mathbf{i} - (\cos(x)\sin(y))\mathbf{j}. \tag{1}$$

(a) Find the flux of $\mathbf{F}$ out of R through C. Show your reasoning.

(b) Is the total flux out of R through C_1, C_2, and C_3 more than, less than, or equal to the flux out of R through C? Show your reasoning.

Solution

Problem 6

Find the volume of the region enclosed by the plane $z = 4$ and the surface

$$z = (2x - y)^2 + (x + y - 1)^2. \tag{2}$$

Suggestion: change of variables.

Solution