1-E4

Write the parametricq equation for the line

$$\overrightarrow{Q_0Q_t} = t\,\overrightarrow{Q_0Q_1}$$

$$\langle x(t)-0, y(t)-1, z(t)-2\rangle = t\langle 2,-1,1\rangle$$

$$x(t) = 2t$$

$$y(t) = -t+1$$

$$z(t) = t+2$$

Then insert into our plane:

$$2t + 4(1-t) + t + 2 = 4$$

$$-t + 6 = 4$$

$$t = 2$$

$$Q_0 + 2\langle 2,-1,1\rangle$$

$$(0,1,2) + \langle 4,-2,2\rangle = \boxed{(4,-1,4)}$$

1E-3c

All lines that are passing through $(1,1,1)$ and in the plane $x+2y-z$ ~~have~~ are perpendicular to the vector $\langle 1,2,-1\rangle$; build the equation

$$\begin{cases} x(t) = at+1 \\ y(t) = bt+1 \\ z(t) = ct+1 \end{cases}$$

and require that $a+2b-c=0$ where $a,b,c \neq 0$.

m

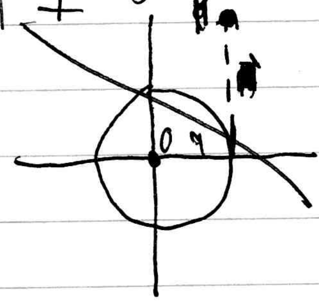
II-3ab

(a): $x(t)=2\cos^2(t)$
$y(t)=\sin^2(t)$
~~From (-800, 800), x(t) va~~
From $-\infty \to \infty$, $x(t)$ outputs on the range $[0,2]$
"   , $y(t)$ outputs on the range $[0,1]$

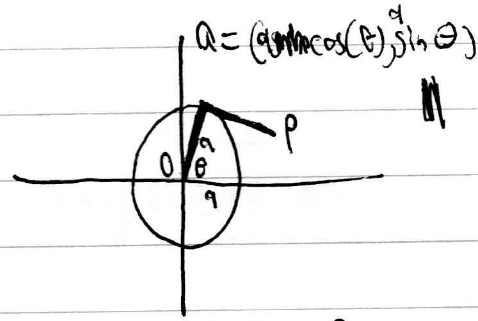
$x+2y=2(\sin^2+\cos^2)$
$x+2y=2$ is the line this position vector points to.

(b): $x(t)=\cos(2t)$
$y(t)=\cos(t)$
Recall the cosine double angle formula
$x=\cos(2z)=\cos^2(\theta)-\sin^2(\theta)=2\cos^2(t)-1$
$\cancel{=(\sin^2+\cos^2(\theta))}=2y^2-1$
Then, this follows the line $x=2y^2-1$ on the domain $(-1,1)$
~~x+~~, $y=\sqrt{1/2(x+1)}$ and range $(-1,1)$

1 I-5



~~Then, $\overrightarrow{OP} = \langle a, y(t) \rangle$.~~



$$|QP| = \frac{\theta}{2\pi} \cdot 2\pi \cdot a = \theta a$$

$$\overrightarrow{OP} = \overrightarrow{OQ} + \overrightarrow{QP}$$

$$QP = \langle a\sin(\theta), \cos(\theta) \rangle \cdot \theta a$$

$$OQ = \langle a\cos\theta, a\sin(\theta) \rangle$$

$$\overrightarrow{OP} = \theta a\sin(\theta) + \cos(\theta)a, \theta a\cos(\theta) + a\sin(\theta)$$

$$\overrightarrow{OP} = ((\theta a\sin(\theta) + \cos(\theta)a)\,\mathbf{i} + (\theta a\cos(\theta) + a\sin(\theta))\,\mathbf{j})$$

$$\boxed{\begin{aligned} x(\theta) &= \theta a\sin(\theta) + a\cos(\theta) \\ y(\theta) &= \theta a\cos(\theta) + a\sin(\theta) \end{aligned}}$$

15-2

~~(B)~~:(a): $OP = (1+t^2)^{-1}\,i + t(1+t^2)^{-1}\,j$

$$\vec{V} = \frac{-2t}{(1+t^2)^2}\,i + \left(\frac{1}{1+t^2} + \frac{-t(2t)}{(1+t^2)^2}\right)j$$

$$\quad '' \quad + \left(\frac{1+t^2}{(1+t^2)^2} - \frac{2t^2}{(1+t^2)^2}\right)j$$

$$= \frac{1}{(1+t^2)^2}\left(-2t\,i + (1-t^2)\,j\right)$$

$$\left|\frac{ds}{dt}\right| = \frac{1}{(1+t^2)^2}\sqrt{(-2t)^2+(1-t^2)^2}$$

$$= \;''\; \sqrt{4t^2 + (1-2t^2+t^4)} = ''\sqrt{1+2t^2+t^4}$$

$$= \;''\; \sqrt{(1+t^2)^2}$$

$$= \frac{1}{(1+t^2)}$$

$$T = \hat{v} = \frac{1}{(1+t^2)}\cdot\langle -2t,\, 1-t^2\rangle$$

(b): $\left|\frac{ds}{dt}\right|$ is greatest at $t=0$ and is slowest when $t \to \infty, -\infty$

(c): We see by $\vec{OP}$ that $x(t) = \frac{1}{(1+t^2)}$ and thus $y(t) = t\,x(t)$, $t = \frac{y}{x}$.

Then $x = \frac{1}{\left(1+\frac{y^2}{x^2}\right)} \Rightarrow x^2+y^2-x=0.$

1J-4

(a): A sphere has constant radius a; Thus we can ~~not~~ write that

$$x(t)^2 + y(t)^2 + z(t)^2 = a^2$$

and by differentiating that

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt} = 0.$$

By multiplying these equations we conclude that

$r \cdot r' = 0$ and thus they are perp.

(b): $\frac{d}{dt} r \cdot r = \frac{dr}{dt} \cdot r + \frac{dr}{dt} r = 2\frac{dr}{dt} r$

~~but $|r| = a \Rightarrow |r||r| = a^2 \Rightarrow r \cdot r$~~

We see that $r(t)$ is constant a;

$|r| = r = a \Rightarrow |r|^2 = r \cdot r = a^2$

Take the derivative of $r \cdot r = a^2$ with respect to t

$2r \cdot \frac{dr}{dt} = 0$, then

$v \cdot R = \frac{d}{dt} r \cdot r = 0$ and thus they are perpendicular

(c): By $a \cdot b = |a||b| \cos(\theta)$, $r \cdot r = |r|^2$.

$r \cdot v = 0 \Rightarrow \cancel{r \cdot \frac{dr}{dt} = 0}$ $\frac{d}{dt} r \cdot r = 2r\frac{dr}{dt} = 0$;

~~r(t) is either 0 or a constant~~ must be constant, so

$r \cdot r = C$ where C is some constant

such that $C = |r|^{1/2}$.

1J-6

(a) (1)

$$v = \frac{d}{dt} r = \frac{d}{dt} \langle a\cos(t), a\sin(t), b t \rangle$$

$$= \langle -a\sin(t), a\cos(t), b \rangle$$

(2)

$$a = \frac{d}{dt} v = \frac{d}{dt} \quad ||$$

$$= \langle -a\cos(t), -a\sin(t), 0 \rangle$$

(4)

$$\left|\frac{ds}{dt}\right| = \sqrt{(a\cos(t))^2 + (-a\sin(t))^2 + b^2}$$

$$= \sqrt{a^2(\sin^2(t) + \cos^2(t)) + b^2}$$

$$= \sqrt{a^2+b^2}$$

(3)

$$\hat{T} = \frac{v}{\left|\frac{ds}{dt}\right|} = \frac{1}{\sqrt{a^2+b^2}} \cdot \langle -a\sin(t), a\cos(t), b \rangle$$

(b): By dot product $v \cdot a = 0$; this is also follows from 1J-5 assuming v is constant.

1J-9ab

(a): By taking the dot product

$r \cdot r = |r||r|$

$= \sqrt{9\cos^2(t) + 25\sin^2(t) + 16\cos^2(t)}^{\,2}$

$= \sqrt{25(\sin^2(t) + \cos^2(t))}$

$= 5$

Thus, this ~~sphere~~ graph has constant $|r|^2 = 5$ and thus lies on a sphere.

(b): First find $v = \frac{d}{dt} r$

$= \frac{d}{dt} \langle 3\cos(t), 5\sin(t), 4\cos(t) \rangle$

$= \langle -3\sin(t), 5\cos(t), -4\,\cancel{\cos}\sin(t) \rangle$

Then

$|v| = \left|\frac{ds}{dt}\right| = \sqrt{9\sin^2(t) + 25\cos^2(t) + 16\sin^2(t)}$

$= \sqrt{25(\sin^2(t) + \cos^2(t))}$

$= 5$

and thus speed is constant.

1J-9bc

(c): Since $a = \frac{d}{dt} v$

$= \frac{d}{dt} \langle -3\cos(t), -5\sin(t), -4\cos(t) \rangle$

$= -r$, thus accelalation is always towards the center

(d): All ~~these points~~ the points of points to exist on the plane $4x - 3z = 0$ which also has solution $0 = (0, 0, 0)$.

(e): This point orbits through the intersection of a sphere with radius 5 centered on the origin and the interse-ction with the plane seen in (d).

1K-3

We see that since f is central, we can write the force pulling ~~it~~ our point towards the center as some constant times the position vector;

$$F = cr$$

Recall Newton's Law

$$F = \cancel{MA}\ ma$$

$$\frac{F}{m} = a;$$

Then $$\frac{c}{m} r = a$$

Consider again

$$F = cr;$$

then $$r = r$$

and by cross producting each side,

$$r \times a = r \times \frac{c}{m} r$$

$$\cancel{r \times a = 0}\ r \times a = 0$$

$$\frac{d}{dt}(r \times v) = v \times v + r \times a$$

$$\frac{d}{dt}(r \times v) = r \times a = 0$$

$r \times v = K$ where K is some constant vector by the result of 1K-2. We are given that

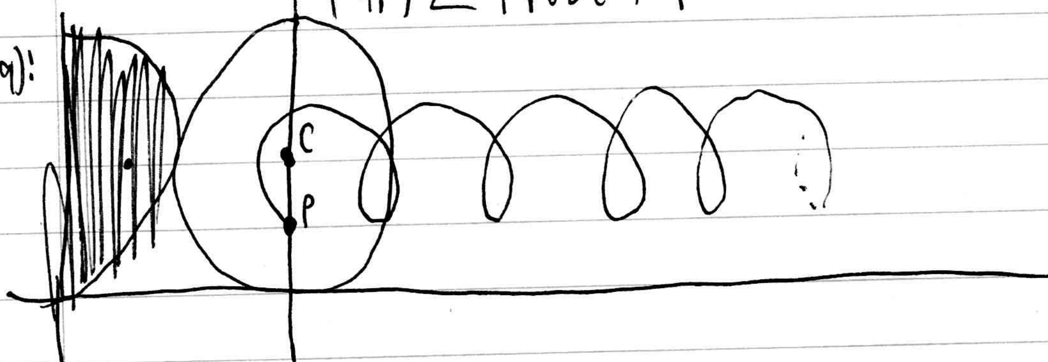
$$|r \times v| = 2\frac{dA}{dt}, \text{ thus}$$

$$\frac{1}{2}|K| = \frac{dA}{dt}$$

and the area swept is constant

Part 2 Problem 1

(a):



(b): We shall write this vector as an addition of vectors:

$$\vec{OP} = \vec{OC} + \vec{CP}$$

$$\vec{OC} = \langle 4\pi \tfrac{\theta}{2\pi}, 2 \rangle = \langle 2\theta, 2 \rangle$$

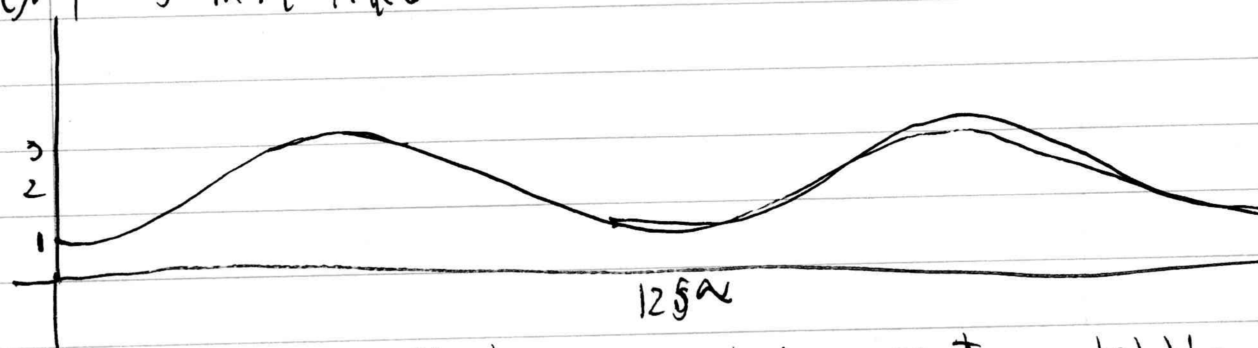
$$\vec{CP} = \langle -\sin(\theta), -\cos(\theta) \rangle$$

$$\vec{OP} = \langle 2\theta - \sin(\theta), 2 - \cos(\theta) \rangle$$

$$x(\theta) = 2\theta - \sin(\theta)$$

$$y(\theta) = 2 - \cos(\theta)$$

(c): I was more like



I believed my first was correct because I expected the inner rotation to outdo the outer rotation somehow.

Part 2, Problem 2

(a): Unfortunately this is not auto magically true. We see that

$r = \langle \cos(t), \sin(t) \rangle$

and thus

$$r \cdot r = \sqrt{\sin^2(t) + \cos^2(t)}^{\,2} = 1 = |r||r| \cos(\theta)$$
$$= |r|^2$$

$r(t) = \sin^2(t) + \cos^2(t) \equiv 1$ and thus sweeps the unit circle.

(b): The first unit vector to consider must be perpendicular to the normal vector of our plane; let $n = \langle 1, 2, 1 \rangle$

$u = \langle p, q, r \rangle$; then require $p + 2q + r = 0$.

$u = \langle 1, -1, 1 \rangle$ is a solution; $\hat{u} = \frac{1}{\sqrt{3}} \langle 1, -1, 1 \rangle$

We then define

$$v = \cancel{n \times \hat{v}} = n \times u = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= 3i + -3k$$
$$= 3\langle 1, 0, 1 \rangle$$

Then,

$$\hat{v} = \frac{1}{\sqrt{2}} \langle 1, 0, 1 \rangle$$

We build the function for $\hat{u}$ and $\hat{v}$ components

$$r(t) = \frac{1}{\sqrt{3}} \langle 1, 2, 1 \rangle \cos(t) + \frac{1}{\sqrt{2}} \langle 1, 0, 1 \rangle \sin(t)$$

Part2 Problem3

(a): Any line on this plane must be normal to the plane;
$L = \langle a, b, c \rangle$ such that $a + 2b + c = 0$

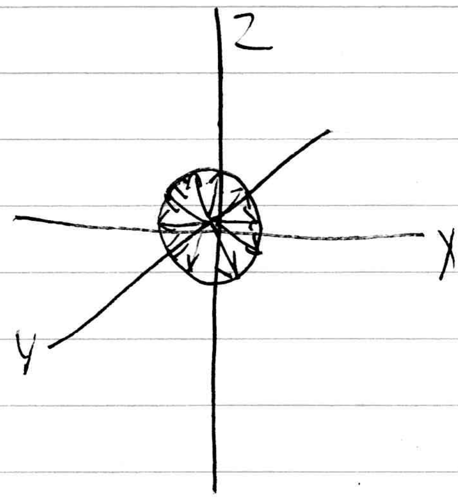
$$c = -(a+2b)$$

We then can write $x = at$

$$y = bt$$

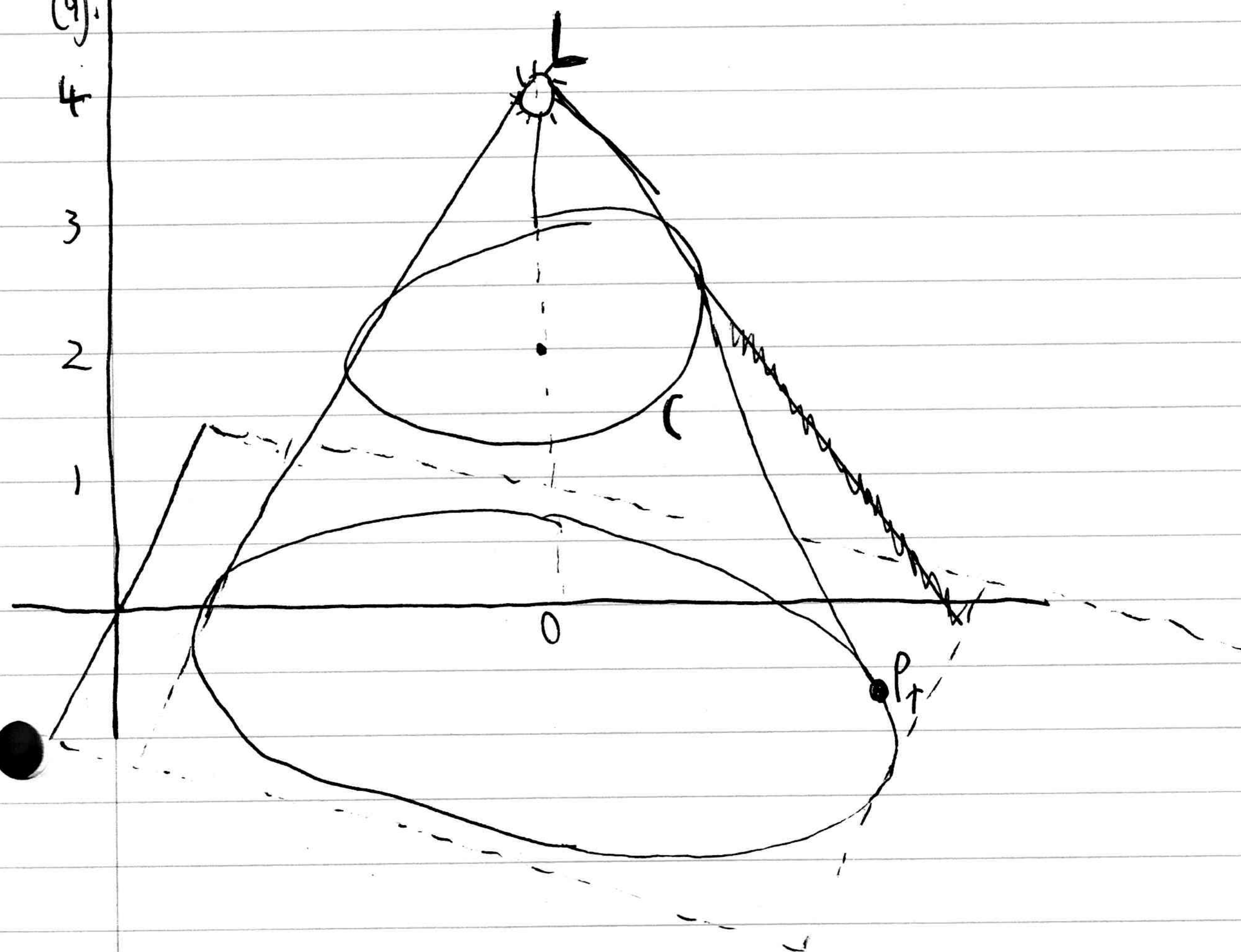
$$z = -(a+2b)t$$

(b): All of the lines on this plane ~~can b~~ can be represented by and can be parallel to any unit vector with solution coefficients.



Part 2 Problem 4

(a):



$P_\sigma = my + z = 0$

(b): We let P_t be the line along
$(0,0,4) \to (\cos(t), \sin(t), 2) \to P_\sigma$

Then we let
$\overrightarrow{LP_t} = \overrightarrow{R_t(u)} = \langle 0,0,4\rangle + u\langle \cos(t), \sin(t), -2\rangle$
such that u connects the L and P_σ.
Them we substitute the plane equation
~~our~~ $My+z=d$
~~the~~ $x = u\cos(t)$
$x = \sin(t)$
$z = -2u + 4$

to get

$$m\,y\sin(t) + (4-2y) = 0$$

$$\Rightarrow$$

$$y = \frac{4}{2-m\sin(t)}$$

and after algebra, ie inserting into our parametric formulas

$$P_t = \left(\frac{4\cos(t)}{2-m\sin t}, \frac{4\sin t}{2-m\sin t}, \frac{-4m\sin t}{2-m\sin t}\right)$$

(c): If $\alpha = 0$ then $m=0$;

$$P_t = \left(2\cos(t), 2\sin(t), 0\right)$$

We see that

$$|~~R_{ort}~~\ R_{ot}| = 2\sqrt{5}\ m \left|\frac{\sin t}{2-m\sin t}\right|$$

And the maximum distortion is $\frac{2\sqrt{5}\ m}{2-m}$.