question 2 views

Daily Challenge 25.1

(Due dates:

- **Parts (a) - (c) due Thursday 3/21 at 12:00 noon Eastern.**
- **Parts (d) and (e) due Friday 3/22 at 12:00 noon Eastern.**
- **Parts (f) - (h) due Saturday 3/23 at 12:00 noon Eastern.)**

I'd like to take a brief detour as we begin to discuss statistics. The most important idea in statistical inference is *Bayes' theorem*.

If you're not already familiar with Bayesian reasoning, I want you to read about it in a simplified context that only involves discrete events and no calculus.

The best introduction to Bayes' theorem on the internet is, in my opinion, this interactive guide. I'm going to ask you to read through it carefully over the span of three days and think deeply about its implications.

Please make a genuine effort to read all of every page, even parts which I do not ask questions about (seriously; this will make you a more rational thinker).

Do the following:

1. Open the link and read the first page. Be sure to chuckle/smile at bullet point three.

2. Choose option 3, "I want the basics, but I'm also interested in reading more about the theoretical implications and the reasons why Bayes' rule is considered so important." Start reading.

3. Read the first page on frequency diagrams.

   (**Part a**) Write out a solution/explanation (handwritten or $\LaTeX$) to the problem at the end of the first page, namely "What's the probability that a student who does *not* turn the tongue depressor black - a student with a negative test result - has Diseasitis?"

   For this part, use a counting argument (similar to the one earlier on this page) rather than applying Bayes' theorem as a formula.

   By "solution/explanation," I mean to write a sequence of logically valid English sentences presenting each step in the calculation with reasoning. Say *why* things are true rather than just writing a string of equations.

4. Read the second page on waterfalls. I *love* this geometric intuition for Bayes' rule.

   (**Part b**) Write out a solution/explanation to the problem at the bottom, "What percentage of sparking widgets are bad?"

   For this part, write out the argument of multiplying the prior probability by the likelihood, then dividing to convert to a final probability.

   Again, write out your justification in complete sentences.

5. Read the third page, on Bayes' rule in odds form.

   (**Part c**) First, give an explanation of the difference between "odds form" and "probability form" of Bayes' rule, and how to convert between them, using the fact that the probabilities must sum to 1.

   Next, write out the proof of Bayes' rule at the bottom of this page, explaining each step. The second step is obvious (divide top and bottom by $\mathbb{P}(e_0)$).

   In your proof, be sure to explain: why are the first step and third step true? Make sure you understand the notation (what does $\wedge$ mean?).

6. Read the fourth page on probability interpretation.

   (**Part d**) Write out an solution/explanation to the socks-dresser problem.

   Note that this is rather like the Monty Hall problem -- the answer is *not* $\frac{1}{3}$, even though there are three possibilities left (two drawers, plus the possibility that it is not in any drawer). The point is that we have removed the probability mass from the first six drawers, and now must renormalize our PDF.

7. Read the fifth page, on extraordinary claims.

   (**Part e**) The final section of that page links to a Python script on Github. Run the code in a Jupyter cell. Modify the code to find the percentage of runs which, at some point, support the false hypothesis that the coin comes up heads **90% of the time** at the 100:1 level. (You should find that it is around 0.65% of runs.)

   Add a comment (or comments) to the parts of the code that you're changing, explaining what the modification does. Please also add a text cell at the top of the notebook explaining that the original author of this code is Nate Soares from MIRI and that you're just playing around with it. Push your notebook to Github.

8. Read the sixth page, on ordinary claims.

   (**Part f**) Give an example from your life experience of "motivated skepticism", that is, an instance where you or someone else was tempted to demand extraordinary evidence for an ordinary claim, in contradiction to Bayesian reasoning. (For me, many conspiracy theories and religious people demanding "proof that god does not exist" come to mind.)

9. Read the seventh page.

   (**Part g**) Define "Bayesian surprise," both intuitively and mathematically. (By "mathematically" I mean introduce a scenario involving a set of symbols and write an actual equation which defines the surprise.)

   The word "bit" is being used as a unit measuring evidence; what, precisely, is meant by the statement that an observation "provides one bit of evidence"? What do you think it means for a measurement to provide one nat of evidence?

10. Read the eighth and final page. This is mostly about the philosophy of science, which is not an especially quantitative subject, but I still think it's worthwhile for you to understand why science works.

The page uses some examples of scientific theories, like DNA theory and gravitational theory. I will ask you to repeat their analysis in a different case.

(**Part h**) Write a few paragraphs in response to the following prompt:

Explain why a Bayesian/Popperian would argue that quantum mechanics is a "good" scientific theory. Give examples which support the claim that quantum mechanics, as a theory, has some of the virtues described on this page.

You may choose any specific experiments or phenomena that you're familiar with, like the photoelectric effect or the spectrum of hydrogen. Interpret the outcomes of any examples using Bayesian vocabulary (likelihood ratios, etc.).

Feel free to use any references, like Wikipedia or Atkins, to refresh your memory on quantum mechanics and quantum chemistry if necessary.

daily_challenge

Updated 22 days ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

woah

Updated 18 days ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

yellow boi

Updated 21 days ago by Christian Ferko

followup discussions *for lingering questions and comments*