question 2 views

Daily Challenge 22.5

(**Due: Sunday 2/24 at 12:00 noon Eastern**)

In session 51, I will describe an alternative (and more rigorous) way of defining the exponential and logarithmic functions directly from the integral.

Although I will skip some details, this construction is spelled out explicitly in chapter 18 of Spivak.

---

(1) Problem: proving a log rule.

Download Spivak's book (make sure you have a DJVU reader like SumatraPDF) and read chapter 18. Try to at least read the statements of theorems carefully, but you can skim/gloss over the proofs.

Now look at Corollary 1 on page 342. Spivak leaves the proof to you as an exercise. Do this exercise: that is, prove that

$$\log(x^n) = n\log(x),$$

for $N \in \mathbb{N}_+$ by using induction.

You may only use the definition of the logarithm in terms of an integral and theorems that Spivak has proven earlier in this chapter, but *not* any other logarithm rules that we've proven before.

The base case $n = 0$ is trivial, since

$$\log(1) \equiv \int_1^1 \frac{1}{t}\,dt = 0$$

using the definition on page 341. To handle the inductive step, assume the formula is true for $n = k$ and prove that it is also true for $n = k + 1$ by applying Spivak's Theorem 1, stated at the bottom of page 341.

Answer on Overleaf here: https://www.overleaf.com/1231232126rckrscxfchyf

daily_challenge

Updated 1 month ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

green boi (now)

Updated 1 month ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

Click to start off the wiki answer

followup discussions *for lingering questions and comments*