question 2 views

Daily Challenge 16.4

(**Due: Wednesday 10/31 at 12:00 noon Eastern**)

Time for integration! I haven't written CD 5 yet so let's work in Piazza for now.

(1) Finishing a lemma.

Suppose $P \subset Q$ (that is, every point of the partition P is also in the partition Q). Prove that $U(f,P) \geq U(f,Q)$.

This is the second half of our lemma from session 40 which I did not prove; you can mimic the argument for lower sums.

daily_challenge

Updated 5 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

We begin by supposing that Q has just one more point than P. We shall take note of this as
$P = \{t_0, \cdots, t_n\}$
$Q = \{t_0, \cdots, t_{k-1}, t_k, t_{k+1}, \cdots, t_n\}$.

We define
$m' = \sup\left(\{f(x) \mid t_{k-1} \leq x \leq u\}\right)$
$m'' = \sup\left(\{f(x) \mid u \leq x \leq t_k\}\right)$.
We can then see by the definition of upper sums that

$$U(f,P) = \sum_{i=1}^{n} m_i\left(t_i - t_{i-1}\right),$$

$$U(f,Q) = \sum_{i=1}^{k-1} m_i(t_i - t_{i-1}) + m'(u - t_{k-1}) + m''(t_k - u)$$
$$+ \sum_{i=k+1}^{n} m_i(t_i - t_{i-1}).$$

And we see that we would like to prove that
$m_k(t_k - t_{k-1}) \geq m'(u - t_{k-1}) + m''(t_k - u)$.

To do so, we note that the interval
$\{f(x) | t_{k-1} \leq x \leq t_{k+1}\}$
contains all numbers in
$\{f(x) | t_k \leq x \leq t_{k+1}\}$
and a few greater, then the least upper bound of the first set is greater than or equal to the least upper bound of the lower set, and therefore $m_k \geq m'$, and identically $m_k \geq m''$, therefore
$m_k(t_k - t_{k-1}) \geq m'(u - t_{k-1}) + m''(t_k - u)$.

We then see that this is true for all sizes of partition, simply by a case of repeated application of adding a point; we loosely define a few intermediate partitions in the manner
$P = P_1 \subset P_2 \subset \cdots \subset P_a = Q$
where each P_{n+1} has one point more than P_n. Then
$U(f,P) = U(f,P_1) \geq U(f,P_2) \geq \cdots \geq U(f,P_\alpha) = U(f,Q)$

Updated 5 months ago by Logan Pachulski

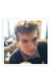
the instructors' answer, *where instructors collectively construct a single answer*

Click to start off the wiki answer

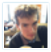
followup discussions *for lingering questions and comments*

Resolved Unresolved


Logan Pachulski 5 months ago
Needs student answer section :blobguns:


Christian Ferko 5 months ago Sorry; will fix when I get home

 **Logan Pachulski** 5 months ago Np at all

 **Christian Ferko** 5 months ago done