22.2

(a) We would like to show that, if $E \subset \mathbb{R}^d$ and $x \in \mathbb{R}^d$, that $E+x = \{y+x \mid y \in E\}$ is elementary.

We see that we can, by definition of E, write

$$E = B_1 \cup B_2 \cup \dots \cup B_n$$

and then that

$$B_1 \cup \dots \cup B_n = [b_1, b_2] \times \dots \times [b_{2-1}, b_2] \cup \dots \cup [b_1', b_2'] \times [\dots]$$

We see that the vector x ~~has~~ shifts each coordinate by a number ~~of values~~ associated with it, thus shifting can be written as

$$[b_1 + x_1, b_2 + x_1] \times \dots [b_{2-1} + x_{2-1}, b_2 + x_2] \cup \dots = E'$$

This still is a finite union of boxes, albeit shifted in space from the original; Thus E' is elementary.

(b) ~~We won~~ Let us begin by noting ~~$E \cap F =$~~ that we can write $E = B_1 \cup \dots \cup B_n$ and $F = B_1' \cup \dots \cup B_m'$. Then

$$E \cap F = (B_1 \cup \dots \cup B_n) \cap (B_1' \cup \dots B_m').$$

We now refer to our chapter 1 result that set intersection is distributive over union. We adopt new notation: \bigcup.

$$\bigcup_{i=1, j=1}^{n, m} (B_i \cap B_j')$$

We just have to prove that the intersection of two boxes is elementary then.

Consider two boxes F and G.

$F = [a_1, b_1] \times \cdots \times [a_n, b_n]$ and $G = [a_1', b_2'] \times \cdots \times [a_n', b_n']$

thus

$F \cap G = [a_1, b_1] \cup [a_1', b_1'] \times \cdots$

due to intersections occuring on ~~just one ax~~ along just one axis. If any of these intersections are zero, ie $(b_n) < (a_n')$, then the ~~box~~ intersection of the boxes is null but elementary. Otherwise, We have a valid product of many intervals, ie a box that is elementary. We conclude that since the inter-section of two boxes is ~~elementary~~ an elementary box. The union of elementary boxes is equal to our beginning $E \cap F$, thus $E \cap F$ is elementary.