question 4 views

Daily Challenge 2.7

(**Due: Monday 5/7 at 12:00 noon Eastern**)

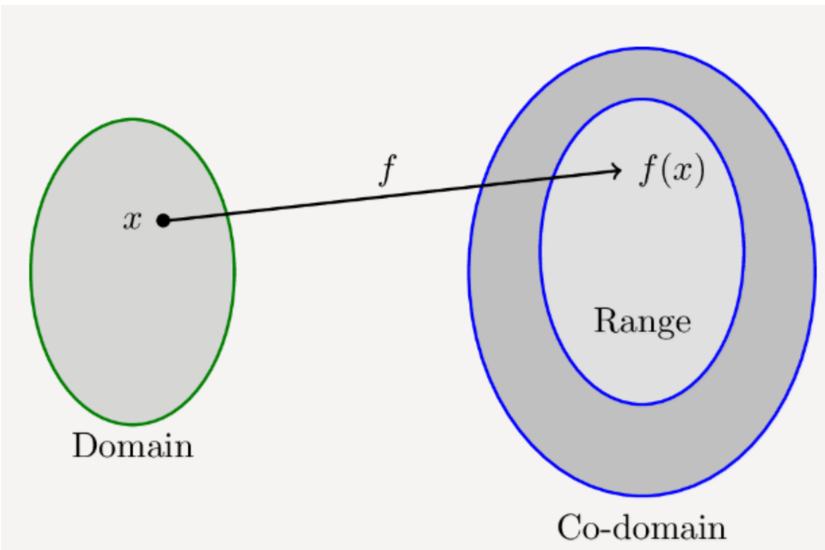
Another week down! Weekly skips replenish tomorrow.

Review

In our last meeting, we gave a more formal definition of "function" than you may be used to. Roughly speaking, this definition views a function as a machine which eats elements of some set A and spits out elements of a second set B.

Definition. A **function** from a set A to a set B, denoted $f : A \to B$, associates to each $a \in A$ an element $f(a) \in B$. The set A is called the **domain** of f and the set B is called the **codomain** of f.

Notice that the codomain is not, in general, the same as the range.



The range consists of all elements in the codomain which the function actually outputs. More precisely,

Definition. The **range** of a function $f : A \to B$, denoted $\mathrm{Rng}(f)$, is

$$\mathrm{Rng}(f) = \{y \in B \mid y = f(x) \text{ for some } x \in \mathrm{Dom}(f)\} .$$

This can be written in abbreviated form as $\mathrm{Rng}(f) = \{f(x) \mid x \in \mathrm{Dom}(f)\}$; it is the set of all outputs of the function.

For example, consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$. Although the codomain is $\mathbb{R}$, this function only outputs nonnegative values, so the range is $\mathrm{Rng}(f) = [0, \infty)$.

Problem

Answer the following questions about domain and range. Proof is not required, but rules (2) and (3) for submitting solutions do apply ("...write solutions in complete sentences..." and "Include explanations of your reasoning.")

(a) Find the domain and range of $f(x) = \sqrt{2 - x - x^2}$. [Hint: the domain is restricted because we cannot take the square root of a negative number in $\mathbb{R}$, so you will need to find the zeros of the quadratic. The range is constrained because the parabola has a maximum; find it.]

(b) What are the domain and range of the function $f(x) = \sin(x)$?

daily_challenge

Updated 11 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

Solutions (Corbin). Your results go here.

(a)

(b)

Solutions (Logan). Your results go here.

(a) The domain of $f(x)=\sqrt{2-x-x^2}$ is $[-2,1]$ because an x value of 1 is the largest value that evaluates true and -2 is the lowest x value that evaluates true. The range is $[0,1]$ because the point half-way between -2 and 1 (0.5) gives a y-value of 1 and the square root of a number can't be less than zero while being real.
(b) The domain of $\sin(x)$ is $[-\infty,+\infty]$ because one can "go around" a circle an infinite amount of degrees clockwise or counter-clockwise. The range of $\sin(x)$ is $[-1,+1]$ because the x-value of a circle with radius 1 centered at the origin can't exceed those values.

Updated 11 months ago by Logan Pachulski and Christian Ferko

the instructors' answer, *where instructors collectively construct a single answer*

Solutions (Christian).

(a) It will be convenient to re-write the quadratic by completing the square. We have

$$\begin{aligned}-x^2-x+2&=-\left(x^2+x+\frac{1}{4}\right)+\frac{9}{4}\\&=-\left(x+\frac{1}{2}\right)^2+\frac{9}{4},\end{aligned}$$

We see that the maximum occurs at $x=-\frac{1}{2}$, where the quadratic takes the value $\frac{9}{4}$. The two roots are found by solving

$$\left(x+\frac{1}{2}\right)^2=\frac{9}{4},$$

which has solutions $x=1$ and $x=-2$.

Now consider the function

$$f(x)=\sqrt{-\left(x+\frac{1}{2}\right)^2+\frac{9}{4}}.$$

The domain is the set of all reals where the quantity under the square root is nonnegative, so

$$\mathrm{Dom}(f)=[-2,1]$$

The range is the set of all values output by the function. The largest such value occurs where the parabola under the square root has its maximum of $\frac{9}{4}$, which gives a square root of $\frac{3}{2}$. We can never achieve negative outputs of $f(x)$, so the range is

$$\mathrm{Rng}(f)=\left[0,\ \frac{3}{2}\right].$$

(b) The sine function $\sin(x)$ is defined as the y coordinate of the point on the unit circle obtained by going counter-clockwise by an angle x. This angle can be any real number; if it is negative, we simply go clockwise, and if it is greater than 360°, we simply loop around the circle and keep going. Thus

$$\mathrm{Dom}(f)=\mathbb{R}.$$

However, all points on the unit circle have y coordinates between -1 and 1, since the circle has radius 1. Thus

$$\mathrm{Rng}(f)=[-1,1].$$

Updated 11 months ago by Christian Ferko

followup discussions *for lingering questions and comments*