8/26/2019

35.4

~~We~~ begin by finding the values λ such that

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}1 & 1\\ 4 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \lambda\begin{bmatrix}x\\ y\end{bmatrix}$$

Subtracting from each side,

$$\begin{bmatrix}1 & 1\\ 4 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} - \lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = 0$$

$$\begin{bmatrix}1-\lambda & 1\\ 4 & -2-\lambda\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = 0$$

thus

$$\det\left(\begin{bmatrix}1-\lambda & 1\\ 4 & -2-\lambda\end{bmatrix}\right) = -2+\lambda^2-\lambda+2\lambda-4$$

$$= \lambda^2+\lambda-6 = 0$$

implies

$$\lambda = -3, 2$$

Then to find eigenvector, we desire v_1 s.t.

$$\begin{bmatrix}4 & 1\\ 4 & 1\end{bmatrix} v_1 = 0, \text{ and}$$

v_2 s.t.

$$\begin{bmatrix}-1 & 1\\ 4 & -4\end{bmatrix} v_2 = 0$$

Consider $v_1 = \begin{bmatrix}a\\ b\end{bmatrix}$,

$$4a+b \cancel{+4a+b} = 0 \Rightarrow v_1 = c\begin{bmatrix}1/4\\ -1\end{bmatrix}$$

and $v_2 = \begin{bmatrix}a\\ b\end{bmatrix}$

$$-a+b+4a-4b=0 \Rightarrow v_2 = c\begin{bmatrix}1\\ 1\end{bmatrix},$$

In turn,

$$X(t) = c_1\begin{bmatrix}1\\ 1\end{bmatrix}e^{2t} + c_1\begin{bmatrix}1/4\\ -1\end{bmatrix}e^{-3t}$$

35.4 (continued)

Recall the definition of an eigen vector; a vector V is an eigen-vector of a matrix M if, for some associated eigenvalue λ,

$$M V = \lambda V.$$

Then, for

$$M = \cancel{\begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix}, \lambda = -3}, \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}, \lambda = -3$$

solve for b_1 in

$$V_1 = \begin{bmatrix} 1 \\ b_1 \end{bmatrix}:$$

$$\cancel{\begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ b_1 \end{bmatrix} = -3 \begin{bmatrix} 1 \\ b_1 \end{bmatrix} \Rightarrow 4 + b_1 = -3b}$$

$$\begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ b_1 \end{bmatrix} = -3 \begin{bmatrix} 1 \\ b_1 \end{bmatrix} \Rightarrow b_1 + 1 = -3$$

$$\underline{b_1 = -4}$$

implies the pairing of

$$\lambda_1 = -3, \quad V_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

Now,

$$M = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}, \quad \lambda_2 = 2, \quad V_2 = \begin{bmatrix} 1 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ b_2 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ b_2 \end{bmatrix} \Rightarrow b_2 + 1 = 2b_2$$

$$\Downarrow$$

$$b_2 = 1,$$

thus

$$\lambda_2 = 2, \quad V_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Setting $t=0$ in

$$\mathbf{X}(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ -4 \end{bmatrix} e^{-3t} \qquad (1)$$

nets you

$$X(0) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

implies

$c_1 = 1$

$c_2 = 1$;

Then by noting the "concatenation" that occured in (1), see that

$$x(t) = e^{2t} + e^{-3t}$$

$$y(t) = e^{2t} - 4e^{-3t}$$