25.2

(a): We must integrate

$$\int_{-\infty}^{\infty} f_P(p)\, dp = \int_0^1 1 + \sin(2\pi p)\, dp$$

$$= \left[p - \cos(2\pi p) \cdot \frac{1}{2\pi} \right]_0^1$$

$$= \cancel{(1 - 0) - \left(0 - \frac{1}{2\pi}\right)}$$

$$= \left(1 - \frac{1}{2\pi}\right) - \left(0 - \frac{1}{2\pi}\right)$$

$$= 1$$

We must find the EV of our PDF over its domain:

$$\int_{-\infty}^{\infty} p\, f_P(p)\, dp = \int_0^1 p\,(1 + \sin(2\pi p))\, dp$$

$$= \int_0^1 p\, dp + \int_0^1 p \sin(2\pi p)$$

$$= \left[\frac{p^2}{2}\right]_0^1 + \left[-p \cdot \cos(2\pi p) \cdot \frac{1}{2\pi}\right]_0^1 + \int_0^1 \cos(2\pi p) \cdot \frac{1}{2\pi}$$

$$= \frac{1}{2} + \frac{-1}{2\pi} + \left[\sin(2\pi p) \cdot \frac{1}{(2\pi)^2}\right]_0^1$$

$$= \frac{\pi - 1}{2\pi}$$

(b): Recall the 'Bayes' rule for a continuous variable on a discrete outco

$$f_{P|A}(p|A) = \frac{\mathbb{P}(A | P = p)\, f_P(p)}{\mathbb{P}(A)}$$

We insert the known values here;

$$\dots = \frac{\rho_p \cdot (1+\sin(2\pi\rho))}{(\pi-1)/(2\pi)}$$

Where of course we are assuming $\rho \in [0,1]$ since $f_p(\rho)$ returns non-zero values only in that range.

(c): We integrated our updated PDF:

$$\int_0^1 \rho\left(\frac{\rho\cdot(1+\sin(2\pi\rho))}{(\pi-1)(2\pi)}\right)d\rho = \frac{2\pi}{\pi-1}\int_0^1 \rho^2+\rho^2\sin(2\pi\rho)$$

$$\dots = \frac{2\pi}{\pi-1}\left(\left[\frac{\rho^3}{3}\right]_0^1 + \left(\left[-\rho^2\cos(2\pi\rho)\cdot\frac{1}{2\pi}\right]_0^1 + \frac{1}{2\pi}\int_0^1 \rho\cos(2\pi\rho)\,d\rho\right)\right)$$

$$=\frac{2\pi}{\pi-1}\left(\frac{1}{3}+\left(\frac{-\cancel{\pi}1}{2\pi}+\frac{1}{2\pi}\left(\left[\rho\frac{1}{2\pi}\sin(2\pi\rho)\right]_0^1 - \int_0^1 \sin(2\pi\rho)\,d\rho\right)\right)\right)$$

$$= \left(\quad \left(\quad \| \quad \left(\quad + \left[\cos 2\pi\rho\right]_0^1\cdot\frac{1}{2\pi}\right)\right)\right)$$

$$\frac{2\pi}{\pi-1}\left(\frac{1}{3}+\left(\frac{-\cancel{\pi}1}{2\pi}+\frac{1}{2\pi}(0)\right)\right)$$

$$=\frac{2\pi}{\pi-1}\left(\frac{1}{3}+\frac{-\cancel{\pi}1}{2\pi}\right)$$

~~Which means that we made a mistake.~~ Mistake found, first boundary term