3G.1

Begin by solving

$$\det\left(\begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix}\right) = 0$$

$$(1-\lambda)(2-\lambda) - 6 = 0$$

$$2 + \lambda^2 - 3\lambda - 6 = 0$$

$$\frac{3 \pm \sqrt{9-(-16)}}{2} = 0 \Rightarrow \lambda = -1, 4$$

Now, for $\lambda = -1$, we want

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ a_1 \end{bmatrix} = -\begin{bmatrix} 1 \\ a_1 \end{bmatrix} \Rightarrow -a_1 = 3 + 2a_1 \Rightarrow a_1 = -1$$

and thus for $\lambda = -1$, we have eigenvector

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

Now for $\lambda = 4$,

$4a_1 = 3 + 2a_1 \Rightarrow a_1 = \frac{3}{2}$, and thus we have eigenvector $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

Now we must solve the system of equations associated with

$$x(t) = c_1 e^{-t}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{4t}\begin{bmatrix} 2 \\ 3 \end{bmatrix}; \text{ given } x(0) = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

$$0 = c_1 + 2c_2$$

$$-4 = -c_1 + 3c_2$$

$$-4 = 5c_2 \Rightarrow c_2 = \frac{-4}{5} \Rightarrow c_1 = \frac{+8}{5}$$