question

2 views

Daily Challenge 15.5

(**Due: Saturday 9/29 at 12:00 noon Eastern**)

Now that we've seen the technique of taking time derivatives of static equations to obtain dynamical equations ("related rates"), let's apply it to a nice meaty problem. This is problem 8 on CD 4.

(1) Problem: concentric circles.

The area between two varying concentric circles is at all times $9\pi \text{ in}^2$. The rate of change of the area of the larger circle is $10\pi \frac{\text{in}^2}{s}$. How fast is the circumference of the smaller circle changing when it has area $16\pi \text{ in}^2$?

daily_challenge

Updated 6 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

We can let the plural radius of the larger and smaller circle be $R(t)$ and $r(t)$ (what an excellent usage of capitalization) respectively; we then see we can write the difference in area as $\pi R^2 - \pi r^2 = 9\pi in^2$. We then see by the chain rule that the derivative, with respect to time, of each side of this is $\pi 2RR' - \pi 2rr' = 0$. We then see that due to the structure of the former element, we can simply substitute into this our given rate of change for the area of the larger circle; then $10\pi \frac{\text{in}}{s^2} - \pi 2rr' = 0$ or $10 \frac{\text{in}}{s^2} = 2rr'$. We are also told that the area of the smaller circle at the time in questions is $16\pi \text{in}^2$ and therefore it has a radius of 4in. Therefore $10 \frac{\text{in}}{s^2} = 2 \cdot 4r'$, and therefore at the time in question $\frac{5}{4} \frac{\text{in}}{s^2} = r'$, and we see by taking the first derivative of the formula for circumference ($C' = 2\pi r'$), then by inserting our changing radius in, the circumference is inscreasing by $\frac{5\pi}{2} \frac{\text{in}}{s^2}$.

Updated 6 months ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

Let the radius of the larger circle be $R(t)$ and the radius of the smaller circle be $r(t)$. The area between the two is fixed, so

$$\pi \left(R^2 - r^2 \right) = 9\pi \text{ in}^2.$$

Differentiating both sides with respect to time, one finds

$$\pi \left(2R\dot{R} - 2r\dot{r} \right) = 0.$$

We consider the time at which the rate of change of the *area* of the larger circle is $10\pi \frac{\text{in}^2}{\text{s}}$. That is,

$$\dot{A} = 2\pi R\dot{R} = 10\pi \frac{\text{in}^2}{\text{s}},$$

which means that $2R\dot{R} = 10 \frac{\text{in}^2}{\text{s}}$. We are also told that, at the time in question, the smaller circle has area $\pi r^2 = 16\pi \text{ in}^2$, which means that its radius at this time is 4 in.

Plugging in all of this data to our equation $2R\dot{R} = 2r\dot{r}$, one finds

$$10 \frac{\text{in}^2}{\text{s}} = 2 \cdot (4 \text{ in}) \cdot \dot{r},,$$

from which we have that

$$\dot{r} = \frac{5}{4} \frac{\text{in}}{\text{s}}.$$

Finally, note that we are asked how fast the *circumference* of the smaller circle is changing. If $C = 2\pi r$, then $\dot{C} = 2\pi\dot{r}$, so

$$\dot{C} = \frac{5\pi}{2} \frac{\text{in}}{\text{s}}.$$

Updated 6 months ago by Christian Ferko

followup discussions *for lingering questions and comments*