question 2 views

Daily Challenge 6.4

(**Due Thursday 6/7 at 11:59 pm Eastern**)

I'd like to make a few comments about next steps, now that we've finished chapter 1.

1. I will write a list of *learning goals*, which will be statements of the form "After finishing chapter 1, students will be able to...".

   The idea is to explicitly enumerate all of the things I expect you to know how to do before we move on. For instance, the list will contain entries like "Recall the values of $\sin(\theta)$ and $\cos(\theta)$ for $\theta \in \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi\right\}$ from memory" and "Prove that two sets are equivalent by showing that each set is a subset of the other."

2. We'll take a short period, maybe a week and two in-person meetings, to review any learning goals that we might be shaky on.

3. Rather than an exam, I'll ask you to write up a *consolidation document*. This will be a longer assignment in Overleaf, similar in form to the problem sets we had before switching to daily challenges.

   The purpose of this document is twofold: it (a) acts as a persuasive document, convincing the instructor that you've mastered the learning goals, and (b) it gives you a reference to look back at later, not unlike a study guide, if you want to refresh your memory about any chapter 1 topics.

   Also unlike an exam, which is submit-once-grade-once, I will give feedback on your the consolidation document to point out any errors and you'll have a chance to correct them (and we'll continue iterating rounds of feedback until everything is totally right).

4. Finally, to conclude chapter 1, I'd like to do a session modeled on the British-style tutorial used at Cambridge and Oxford (and several of my classes here): we'll have a meeting where you present some solutions from your consolidation document and then I'll ask you questions and discuss.

I think this procedure should take about 3 meetings and will give a nice sense of closure to the first leg of the journey. I'll post more details on the learning goals and consolidation document soon.

Problem

To begin the consolidation procedure, please pick either

- one daily challenge that you couldn't solve correctly or used a weekly skip on, or
- one of the review problems with solutions that I posted

Try to choose a problem on a skill you haven't mastered yet (e.g. trig identities or images and inverse images).

In the student answer below, (re-)submit a solution to the problem you chose. Try to make everything as clean, precise, and correct as possible; we will ultimately transplant it to your consolidation document.

daily_challenge

Updated 10 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

(Time to do this for the second time! (flipping drawing tablet))
I have chosen to revise my non-existent response to daily challenge 1.4, which I shall post here for reference:
Theorem. Let m be an integer. Then the number $n = (m-1) \times m \times (m+1)$is divisible by 3

Proof: Assume m is an integer. For the suggested equation to be divisible by 3 (evenly), first it must be a multiple of 3. For this to be true, then one of three scenarios must be true:
A) $m-1$ is a multiple of 3.
B) m is a multiple of 3.
or C) $m+1$ is a multiple of 3.

Due to the logistics of this, one of these scenarios must be true for any given value m, and as such the result will be a multiple of 3. Since any integer that is a multiple of 3 is divisible by 3 due said operations being inverse, then n must be divisible by 3.

(This is walking the line of what I can assume is true I've noticed)

Updated 10 months ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

It seems unfair of me to ask you to do something which I am not doing myself, but I have not used any weekly skips, so instead I will solve a new problem from the textbook that I haven't done before.

Exercise 1.4.3. Show that the converse of the Vertical Line Test is true: that is, if $A \subset \mathbb{R} \times \mathbb{R}$ satisfies the vertical line test, then A must be the graph of some real-valued function f.

Solution. Let $A \subset \mathbb{R} \times \mathbb{R}$ be a subset of the plane which satisfies the Vertical Line Test. By definition (given on page 17 of AoPS), this means that any line of the form $x = b$ intersects the set A in at most one point.

We will construct a function as follows: let $\mathrm{Dom}(f) = \{x \in \mathbb{R} \mid (x, y) \in A \text{ for some } y\}$. That is, since A consists of a set of ordered pairs, define the domain of our function f to be the set of all real numbers which appear in the first position of an ordered pair in A.

For each $x \in \mathrm{Dom}(f)$, the Vertical Line Test guarantees that there is a single element $(x, y) \in A$ with first element x. Define this value y to be the output $f(x)$ of our function.

The object thus defined is indeed a function, since it outputs a single element $y = f(x)$ for each $x \in \mathrm{Dom}(f)$, and by construction the graph $\{(x, f(x)) \mid x \in \mathrm{Dom}(f)\}$ of f is precisely A. This proves the converse of the Vertical Line Test, as desired. $\square$

Updated 10 months ago by Christian Ferko

followup discussions *for lingering questions and comments*

Resolved Unresolved

Christian Ferko 10 months ago
Aside: the consolidation document structure is what I personally use when learning things for "real life", i.e. research stuff.