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Daily Challenge 28.5

(a): Let

$\cancel{a_n = \sum_{k=1}^{m} S_n}$

$$S_n = \sum_{k=1}^{n} a_k$$

and then by the problem statement

$$\lim S_n = \sum_{k=1}^{\infty} a_k = L$$

Now, see that

~~that~~ $a_k = \sum_{n=1}^{k} a_n - \sum_{n=1}^{k-1} a_n = S_k - S_{k-1}$

Then,

$$\lim(a_n) = \lim(S_n) - \lim(S_{n-1}) \overset{(1)}{\underset{\text{weakly}}{\approx}} \sum_{n=1}^{\infty} a_n - \sum_{n=1}^{\infty - 1} a_n$$

$$= L - L$$

$$\lim(a_n) = 0$$

where (1) is true because for $n \gg 1$, $n \approx n-1$.

~~(b): We see ~~that~~ since $-1 = -1$, and $\sin^2(n^3)$ is at most 1, then~~

$$\cancel{\frac{1}{2^n - 1 + \sin^2(n^3)} \leq \frac{1}{2^n}}$$

~~Then since~~

$$\cancel{\frac{1}{2^n}}$$

~~converges, as a geometric series, then by the comparison test~~

$$\cancel{\sum_{n=1}^{\infty} \frac{1}{2^n - 1 + \sin^2(n^3)}}$$

~~converges.~~

~~(c): See that~~

(c): ~~Take the limit of~~

$$\cancel{\lim\left(\frac{n+1}{n^2+1}\right) = \lim \frac{\lim(n)+1}{\lim(n^2)+1}}$$

(b): We shall apply the limit comparison test; let $a_n = \frac{1}{2^n}$ and $b_n = (2^n - 1 + \sin^2(n^3))$; then evaluate

$$\lim_{n\to\infty}\left(\frac{2^n-1+\sin^2(n^3)}{2^n}\right) = \lim\left(\frac{2^n}{2^n}\right) - \lim\left(\frac{1}{2^n}\right) + \lim\left(\frac{\sin^2(n^3)}{2^n}\right)$$

$$= 1 - 0 + \lim\left(\frac{\sin^2(n^3)}{2^n}\right)$$

Since $\sin^2$'s range is only $[0,1]$,

$$\ldots = 1 - 0 + 0$$

$$= 1 \blacksquare$$

thus, since $\frac{1}{2^n}$ is known to converge, then the sum in question converges.

(c): Apply the limit comparison test with the known divergent $1/n$. Evaluate

$$\cancel{\lim\left(\frac{n+1}{n^2+1} \cdot \left(\frac{1}{n}\right)^{-1}\right) = \lim\left(\frac{n^2+n}{n^2+1}\right)}$$

$$\lim\left(\frac{n^2+1}{n+1} \cdot \frac{1}{n}\right) = \lim\left(\frac{n^2+1}{n^2+n}\right)$$

$$= \lim\left(\frac{1+1/n^2}{1+1/n}\right)$$

$= 1$, hait, sifili, eber, isin, sifr, efes,

uno, mann, eins, oon, unu, von, jeden

Thus the series ~~converges~~ diverges.

(d): ~~Begin with $-1<r<1$; then~~ Evaluate for $r>0$

$$\lim\left(\frac{(n+1)\,r^{(n+1)}}{n\,r^n}\right) = \lim\left(\frac{(n+1)}{n}\,r\right) = \lim\left(r + \frac{r}{n}\right)$$

which tells us that for $0<r<1$, this converges, and for $r>1$, it diverges. Now to handle $r<0$.