question

2 views

Daily Challenge 23.5

(**Due: Tuesday 3/5 at 12:00 noon Eastern**)

We defined *work* as the integral of a force applied over a distance,

$$W = \int_{x_a}^{x_b} F(x)\,dx.$$

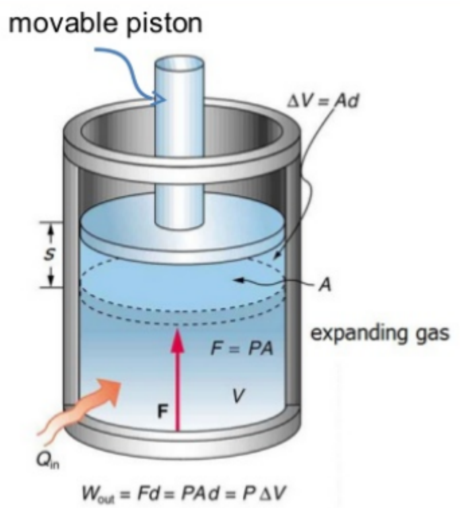
Now consider a gas in a container held at pressure P. The container expands slightly, increasing in volume by an amount ΔV. I claim that **work has been done by the expanding gas**.

One way to see this is by looking at the units: pressure has units of force per area, while volume has units of meters-cubed, so

$$\begin{aligned}[P\Delta V] &= \frac{\text{Newtons}}{\text{meters}^2}\cdot\text{meters}^3 = \text{Newtons}\cdot\text{meters}\\ &= \text{Joules}\\ &= [W],\end{aligned}$$

where by $[x]$ I mean the units of quantity x.

Perhaps more helpful is to look at a picture:



Let the area of the top surface in the container be A, as in the figure. Since pressure is force per unit area, the total force acting on this surface is $F = PA$. If the surface moves upward by a distance s, then the total work done is

$$W = \int F(x)\,dx = \int_0^s P(x)A\,dx$$

and in the special case where the pressure $P(x) = P$ is constant, we find

$$W = PAs = P\,\Delta V,$$

since $\Delta V = As$ is simply the volume of a cylinder with height s and cross-sectional area A.

Problem, part (a). Of course, the pressure is not constant in a realistic expansion process. Rather, it approximately satisfies the ideal gas law

$$P(V) = \frac{Nk_BT}{V},$$

where I write $P = P(V)$ to emphasize that the pressure is a function of the volume (this is not multiplication by V!).

Assume that an ideal gas is held at constant temperature T and expands from volume V_A to volume V_B. Compute the total work

$$W = \int F(x)\,dx = \int_{V_A}^{V_B} P(V)\,dV$$

by using the ideal gas law.

You should find the answer

$$W = Nk_BT\log\left(\frac{V_B}{V_A}\right).$$

Problem, part (b). Real gases are not ideal gases. The leading correction to the ideal gas law is expressed in the Van der Waals equation,

$$\left(P + \frac{aN^2}{V^2}\right)(V - Nb) = Nk_BT,$$

where b is a constant that accounts for the fact that gas particles are not point-particles but have some finite size, and a is a constant which expresses the fact that gas particles interact with one another (the ideal gas law assumed *no* interactions).

Solve the Van der Waals equation for $P = P(V)$ as a function of V, then compute the work done as a Van der Waals gas expands from V_A to V_B:

$$W = \int_{V_A}^{V_B} P(V)\,dV.$$

You should find the result

$$W = Nk_BT\log\left(\frac{V_B - Nb}{V_A - Nb}\right) - aN^2\left(\frac{1}{V_A} - \frac{1}{V_B}\right).$$

Notice that, when $a = 0$ and $b = 0$, your formula for the work reduces to the expression from the ideal gas case (as it must).

daily_challenge

Updated 1 month ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

That green might be for me

~ An instructor (Christian Ferko) endorsed this answer ~

Updated 1 month ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

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