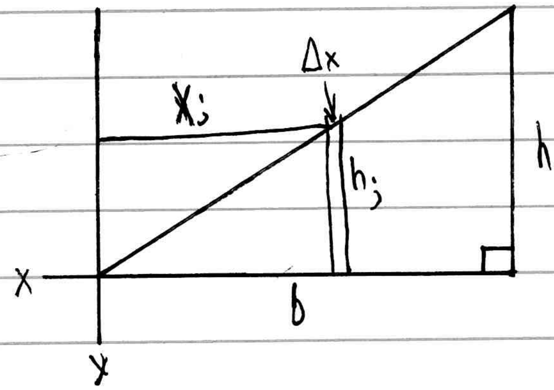
23.7

(a): Consider the triangle (with total mass M)



We see that the center of mass of an infinitely thin strip ~~is~~ x_j from the origin has y location half the height of the strip; by similar triangles

$$\frac{h_j}{x_j} = \frac{h}{b}$$

$$h_j = \frac{x_j h}{b}$$

Then the y center ~~of~~ of mass of this strip is located at $\frac{x_j h}{2b}$. Thus this center of mass has vector pointing $\overrightarrow{O\,Com(x_j)} = \langle x_j, \frac{x_j h}{2b} \rangle = \vec{r}_j$.

(b): We ~~are given that~~ see by inserting given values ~~that~~ $\frac{2}{\rho b h t} \int_0^b \vec{r}_x \cdot \rho t \frac{xh}{b}\, dx$

(c): $= \frac{2}{b^2} \int_0^b \vec{r}_x \cdot x\, dx = i \frac{2}{b^2} \int_0^b x^2\, dx + j \frac{2}{b^2} \int_0^b \frac{x^2 h}{2b}$

$= \frac{2}{b^2}\left(\frac{b^3}{3}\right) i + \cancel{j \frac{h}{b^3} \int_0^b}$

$j \frac{h}{b^3} \int_0^b x^2\, dx$

$$= \frac{2b}{3} i + \frac{h}{b^3}\left(\frac{b^3}{3}\right) j$$

$$= \frac{2}{3} b i + \frac{1}{9} h j$$