question 2 views

Daily Challenge 12.6

~~(**Due: Saturday 8/25 at 12:00 noon eastern**)~~
(**Due: Wednesday 8/29 at 12:00 noon eastern**)

I apologize for the tedious "cookbook" exercises, but I think it's worth doing a couple days' worth of challenges that just involve taking derivatives of routine functions.

Admittedly these are not the most interesting problems. Nonetheless, one must practice differentiation until it becomes as automatic as addition or evaluation of trigonometric functions (e.g. you should not have to pause and think to write down $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$) in order to be competent at calculus.

(1) Problem: taking some derivatives.

Find $f'(x)$ in each of the following cases. This should not take you more than 20 minutes if you have become sufficiently fluent with the various derivative rules.

After you have computed all of the derivatives, **choose one function to differentiate in Mathematica** and verify your result. The syntax in Mathematica is

```
D[f[x], x]
```

For instance, if you were checking your result for part (c), you would open Mathematica and type

```
D[(x^2+1) Sin[x], x]
```

and then hit Shift+Enter.

(a) $f(x) = 4x^3 + 3x - 8 - 2x^{-2}$

(b) $f(x) = (3x^2 + 2)^5$

(c) $f(x) = (x^2 + 1)\sin(x)$

(d) $f(x) = \cos(2x)$

(e) $f(x) = e^{2x}$

(f) $f(x) = (5\log(x))\,(1 + \tan(x))$

(g) $f(x) = \log\left(x^3\right)$

(h) $f(x) = x^2 e^x \sec(x)$

daily_challenge

Updated 7 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

a: By the power rule applied to each term, $f'(x) = 12x^2 + 3 + 4x^{-3}$.
b: By the chain rule, $f'(x) = 5(3x^2+2)^4 \times 6x$.
c: By the product rule, $f'(x) = 2x\sin(x) + (x^2+1)\cos(x)$.
d: By the chain rule, $f'(x) = -\sin(2x) \times 2$.
e: By the chain rule, $f'(x) = e^{2x} \times 2$
f: First by the product rule, $f'(x) = (5\log(x))' \cdot (1+\tan(x)) + (5\log(x)) \cdot \sec^2(x)$ which by the product rule the applied to the foremost element equals $\frac{5}{x} \cdot (1+\tan(x)) + (5\log(x)) \cdot \sec^2(x)$ and therein is our answer.
g: By the chain rule, $f'(x) = \frac{1}{x^3} \cdot 3x^2$.
h: By the triple product rule, we see that $f'(x) = 2xe^x\sec(x) + x^2e^x\sec(x) + x^2e^x\sec(x)\tan(x)$ This is the one I shall check with mathematica; And after realizing that parentheses aren't used in mathematica trig functions, my answer has been confirmed;

```
In[6]:= D[x^2 E^x Sec[x], x]
Out[6]= 2 e^x x Sec[x] + e^x x^2 Sec[x] + e^x x^2 Sec[x] Tan[x]
```

Updated 7 months ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

My responses follow.

(a) $f(x) = 4x^3 + 3x - 8 - 2x^{-2}$

Solution. By the power rule, we have $f'(x) = 12x^2 + 3 + 4x^{-3}$.

(b) $f(x) = (3x^2 + 2)^5$

Solution. Using the chain rule and power rule, we see that $f'(x) = 5(3x^2 + 2)^4 \cdot 6x = 30x(3x^2 + 2)^4$.

(c) $f(x) = (x^2 + 1)\sin(x)$

Solution. By the product rule, $f'(x) = 2x\sin(x) + (x^2 + 1)\cos(x)$.

(d) $f(x) = \cos(2x)$

Solution. We apply the chain rule to find $f'(x) = -2\sin(2x)$.

(e) $f(x) = e^{2x}$

Solution. By the chain rule, $f'(x) = 2e^{2x}$.

(f) $f(x) = (5\log(x))(1 + \tan(x))$

Solution. By the product rule, we have $f'(x) = \frac{5}{x}(1 + \tan(x)) + 5\log(x)\sec^2(x)$.

(g) $f(x) = \log\left(x^3\right)$

Solution. We use the chain rule, which yields $f'(x) = \frac{1}{x^3} \cdot 3x^2 = \frac{3}{x}$.

(h) $f(x) = x^2 e^x \sec(x)$

Solution. We apply the product rule twice to find

$f'(x) = (2x)e^x \sec(x) + x^2 e^x \sec(x) + x^2 e^x \sec(x)\tan(x)$.

Below I have verified all of these calculations in Mathematica.

In[9]:= $D[4x^3 + 3x - 8 - 2x^{-2}, x]$

Out[9]= $3 + \frac{4}{x^3} + 12x^2$

In[10]:= $D[(3x^2 + 2)^5, x]$

Out[10]= $30x(2 + 3x^2)^4$

In[3]:= $D[(x^2 + 1)\,\mathrm{Sin}[x], x]$

Out[3]= $(1 + x^2)\,\mathrm{Cos}[x] + 2x\,\mathrm{Sin}[x]$

In[4]:= $D[\mathrm{Cos}[2x], x]$

Out[4]= $-2\,\mathrm{Sin}[2x]$

In[5]:= $D[E^{2x}, x]$

Out[5]= $2e^{2x}$

In[6]:= $D[(5\,\mathrm{Log}[x])\,(1 + \mathrm{Tan}[x]), x]$

Out[6]= $5\,\mathrm{Log}[x]\,\mathrm{Sec}[x]^2 + \frac{5(1 + \mathrm{Tan}[x])}{x}$

In[7]:= $D[\mathrm{Log}[x^3], x]$

Out[7]= $\frac{3}{x}$

In[8]:= $D[x^2 E^x\,\mathrm{Sec}[x], x]$

Out[8]= $2e^x x\,\mathrm{Sec}[x] + e^x x^2\,\mathrm{Sec}[x] + e^x x^2\,\mathrm{Sec}[x]\,\mathrm{Tan}[x]$

Updated 7 months ago by Christian Ferko

followup discussions *for lingering questions and comments*