question 2 views

Daily Challenge 17.3

(**Due: Wednesday 11/7 at 12:00 noon Eastern**)

Let's fill in one gap from our proof that $\int_a^b (f+g) = \int_a^b f + \int_a^b g$

(1) Problem: infima of a sum.

Let $P = \{t_0, \cdots, t_n\}$ be a partition of $[a, b]$ and let f, g be integrable on $[a, b]$. Define the infima as we did in session 42:

$$m_i = \inf\left(\{f(x) + g(x) \mid t_{i-1} \le x \le t_i\}\right),$$
$$m_i' = \inf\left(\{f(x) \mid t_{i-1} \le x \le t_i\}\right),$$
$$m_i'' = \inf\left(\{g(x) \mid t_{i-1} \le x \le t_i\}\right),$$

You may like to refer to slide 23.

Prove that

$m_i' + m_i'' \le m_i.$

daily_challenge

Updated 5 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

lgoanasdfxzvcpsadfchulsfawefiki:
Exploration:
Begin by rewording what we want to prove; We want to prove that the infimum of two added functions is greater than or equal to the infimum of the functions individually. It is not guaranteed that the infimum of each occurs at the some point in our subset of the domain.

Proof: I believe we can do this fairly simply by casework; We see that $m_i' = f(a) \text{ where } a \in [t_{i-1}, t_i]$, and similarly $m_i'' = g(b) \text{ where } b \in [t_{i-1}, t_i]$. We then break this into cases dependent on the equality of a and b;
1: $a = b$ We then see that by our definition $m_i = m_i' + m_i''$, since the infimum of $f(x)$ and $g(x)$ occurs at a.

2: $a \ne b$ This means one of two things, $\inf\left(f(x) + g(x)\right)$ on our interval either occurs at a or b. Assume without loss of generality that the infimum of the addition occurs at a; We see a few things;] before we really begin operating; by definition of infimum it must be true that $g(b) < g(a)$ and that $f(a)$ is the infimum of this range; We know that $g(a) + f(a) = g(a) + m_i' = m_i$, so simply by substituting our $g(b)$ for $g(a)$ we get the inequality that $m_i'' + m_i' < m_i$

We add these statements we gain by casework to see $m_i' + m_i'' \le m_i$. :wow:

Updated 5 months ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

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followup discussions *for lingering questions and comments*