26.6

(a): For rotations about the x-axis, we have that

$$A_x = \int_a^b 2\pi x f(x) \sqrt{1+f'(x)^2}\, dx$$

and for rotations about the y-axis,

$$A_y = \int_a^b 2\pi x \sqrt{1+f'(x)^2}\, dx$$

(b): We must solve

$$\int_{-1}^{1} 2\pi \sqrt{4-x^2}\, \sqrt{1+((4-x^2)^{-1/2} \cdot -2x)^2}$$

$$(4-x^2)(1+((4-x^2)^{-1/2} \cdot -2x)^2)$$

$$=(4-x^2)(1+((4-x^2)^{-1} \cdot 4x^2))$$

$$=4+16((4-x^2)^{-1} \cdot x^2) - x^2 + 4x^4 (4-x^2)^{-1}$$

$$\Rightarrow$$

$$2\pi \int_{-1}^{1} \sqrt{4+\frac{16x^2}{4-x^2} - x^2 + \frac{4x^4}{4-x^2}}$$

This is staggering ugly to work with and I've convinced myself that there must be a better solution. Let's try again. Recall

$$A_x = \int_a^b 2\pi f(x) \sqrt{1+f'(x)^2}$$

We have that $a=-1$, $b=1$, and $f(x)=\sqrt{4-x^2}$ and thus

$$f'(x) = (4-x^2)^{-1/2} \cdot \frac{1}{2} \cdot 2x = \frac{x}{\sqrt{4-x^2}}$$

Then,

$$f'(x)^2 = \frac{x^2}{4-x^2} \Rightarrow f'(x)^2+1 = \frac{x^2+4-x^2}{4-x^2} = \frac{4}{4-x^2}$$

$$2\pi \int_{-1}^{1} \sqrt{4-x^2} \cdot \sqrt{\frac{4}{4-x^2}}\, dx = 2\pi \int_{-1}^{1} 2\, dx = 4 \cdot 2\pi = 8\pi$$

(c): We solved an ~~nearly~~ identical problem in 26.5; we found that $f(x) = a\cosh\left(\frac{x-a}{b}\right)$ successfully minimized th. 26.5 and thus the same applies here