note 2 views

Daily Challenge 4.2

You probably have your hands full with making up DC 2.4, 3.6, and 4.1, so let's take another make-up day. No problem, but here's a short reading on more trigonometric functions.

Reading

So far, we've been working entirely with the cosine function $\cos(\theta)$, which labels the x-coordinate of a point at angle θ on a circle of radius 1, and the sine function $\sin(\theta)$, which labels the y-coordinate.

There are four other trigonometric functions that are formed from ratios and multiplicative inverses of sine and cosine. We will need to understand all of them (in chapter 3, for instance, we will learn how to differentiate all six!).

The new functions are:

- The **tangent** function, written $\tan(\theta)$, is the ratio of the sine and cosine:

  $$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}.$$

- The **cotangent** function, denoted $\cot(\theta)$, is the ratio of cosine and sine:

  $$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}.$$

- The **secant**, written $\sec(\theta)$, is the multiplicative inverse of cosine (by this I mean the number 1 divided by $\cos(\theta)$):

  $$\sec(\theta) = \frac{1}{\cos(\theta)}.$$

  Do not confuse the *multiplicative* inverse of cosine, namely $(\cos(\theta))^{-1}$, with the *inverse function* for cosine, $\cos^{-1}(\theta)$! The former is secant; the latter is arccos.

- The **cosecant**, called $\csc(\theta)$, is the multiplicative inverse of sine:

  $$\csc(\theta) = \frac{1}{\sin(\theta)}.$$

Unfortunately, learning these names and definitions is one of the few places in math where one must memorize.

Example. Prove that, for all $\theta \in \mathbb{R}$ such that $\sin(\theta) \neq 0$ and $\cos(\theta) \neq 0$, it is true that $\sec^2(\theta) + \csc^2(\theta) = \sec^2(\theta)\csc^2(\theta)$.

Solution. Identities involving squares usually come from the Pythagorean identity. We begin by writing

$$\sin^2(\theta) + \cos^2(\theta) = 1.$$

Since we have assumed that $\sin(\theta)$ and $\cos(\theta)$ are both nonzero, we can divide both sides of this equation by $\sin^2(\theta)\cos^2(\theta)$:

$$\frac{\sin^2(\theta)}{\sin^2(\theta)\cos^2(\theta)} + \frac{\cos^2(\theta)}{\sin^2(\theta)\cos^2(\theta)} = \frac{1}{\sin^2(\theta)\cos^2(\theta)}.$$

This means

$$\left(\frac{1}{\cos(\theta)}\right)^2 + \left(\frac{1}{\sin(\theta)}\right)^2 = \left(\frac{1}{\cos(\theta)}\right)^2 \left(\frac{1}{\sin(\theta)}\right)^2.$$

We recognize the definitions: $\frac{1}{\cos(\theta)}$ is the definition of $\sec(\theta)$, and $\frac{1}{\sin(\theta)}$ is the definition of $\csc(\theta)$, so this equation becomes

$$\sec^2(\theta) + \csc^2(\theta) = \sec^2(\theta)\csc^2(\theta),$$

which is what we wanted to show. $\square$

daily_challenge

Updated 11 months ago by Christian Ferko

followup discussions *for lingering questions and comments*