29.1

(a): we see that $\sin(n\theta) \leq 1$ for all $n \in \mathbb{N}$ and likewise $n \in \mathbb{R}$

thus

$$\frac{\sin(\theta n)}{n^2} \leq \frac{1}{n^2}$$; apply the comparison test with

$$a_n = \frac{\sin(\theta_n)}{n^2} \leq b_n = \frac{1}{n^2}$$; $\frac{1}{n^2}$ is summable thus a_n is summable.

(b): See by Leibniz's Theorem that we have

$a_1 = 1 \geq a_2 = \frac{1}{3} \geq a_3 = \frac{1}{5} \geq \dots$; clearly $\lim(a_n) = 0$; then

we conclude that the alternating series

$a_1 - a_2 + a_3 - \dots \to L$; it converges

(c): we see that

$$\frac{1}{\sqrt[3]{n^2-1}} > \frac{1}{n^{2/3}}$$; it is known that since $n^{-2/3}$ has power >-1,

then it diverges by the contrapositive of the comparison test. Since the series in question is strictly greater than a divergent series, it must diverge.

(d): Apply the ratio test; evaluate the limit

$$\lim\left(\frac{(n+1)^2/(n+1)!}{(n)^2/(n)!}\right) = \lim\left(\frac{(n+1)^2}{n^2(n+1)}\right) = \lim\left(\frac{n+1}{n^2}\right) = \lim\left(\frac{1}{n} + \frac{1}{n^2}\right) = 0$$

(e): write down the integral we want to show exists:

$$\int_2^\infty \frac{1}{x\log(x)}\,dx$$ by seeing one to a function; then u-sub $u = \log(x)$, $du = \frac{1}{x}dx$;

$$\dots = \left[\cancel{\log(\log(x))}\right]_2^\infty = \int_{\log(2)}^\infty \frac{1}{u}\,du = \log(\log(\infty)) - \log(\log(2))$$

$$= \lim_{c\to\infty} \log(\log(c)) - \log(\log(2))$$

The integral diverges, so the sum diverges.