question 2 views

Daily Challenge 20.3

Let's get some practice with u-substitution. The following integrals can all be done with simple substitutions, although you can probably do some of them in your head.

(a) $\int e^x \sin(e^x)\, dx$

(b) $\int xe^{-x^2}\, dx$

(c) $\int e^{e^x} e^x\, dx$

(d) $\int x\sqrt{1-x^2}\, dx$

daily_challenge

Updated 4 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

(a) The formatting of this problem leads me to believe that we can let $u = e^x$ and thus $\frac{du}{dx} = e^x \implies du = e^x dx$. We then see by u-substitution that we can similarly compute the integral as $\int \sin(u) du$ which we see to be equal to $-\cos(u) + C = -\cos(e^x) + C$. :wow:

(b) Let $u = -x^2$ and thus $du = -2x\, dx$. We see that we get the original integral is equal to $\int \frac{-1}{2} \cdot e^u du = \frac{-(e^u)}{2} = \frac{-(e^{-x^2})}{2}$.

(c) Once again we let $u = e^x$; we then see that $\frac{du}{dx} = e^x \implies du = e^x dx$, and in turn we have to work with $\int e^u du = e^u + C = e^{e^x} + C$.

(d) We let $u = x^2$ and thus $du = 2x$; we then see that the original integral is equal to $\int \frac{\sqrt{1-u}}{2} du$. Let's think about this. Start with $(1-u)^{\frac{3}{2}}$, which then differs from our ideal result by a sign and a 3 out front, so we place in front of our ongoing idea a $\frac{-1}{3}$ to get that $\int \frac{\sqrt{1-u}}{2} du = \frac{-1}{3} \cdot (1-x^2)^{\frac{3}{2}}$.

Updated 4 months ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

(a) Let $u = e^x$ and $du = e^x\, dx$ so $\int e^x \sin(e^x)\, dx = \int \sin(u)\, du = -\cos(u) + C = -\cos(e^x) + C$

(b) Let $u = x^2$ and $du = 2x\, dx$ so $\int xe^{-x^2}\, dx = \frac{1}{2} \int e^{-u}\, du = -\frac{1}{2} e^{-x^2} + C$.

(c) Let $u = e^x$ and $du = e^x\, dx$ so $\int e^{e^x} e^x\, dx = \int e^u\, du = e^u + C = e^{e^x} + C$.

(d) Let $u = 1 - x^2$ and $du = -2x\, dx$. Then $\int x\sqrt{1-x^2}\, dx = -\frac{1}{2} \int \sqrt{u}\, du = -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} = -\frac{1}{3}(1-x^2)^{3/2} + C$

Updated 4 months ago by Christian Ferko

followup discussions *for lingering questions and comments*