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Daily Challenge 25.4

(**Due: Wednesday 3/27 at 12:00 noon Eastern**)

In this problem, you will show that the Fourier transform of a Gaussian is another Gaussian. For simplicity, take the mean to be $\mu = 0$, and write our Gaussian as

$$f(x) = e^{-a^2 x^2}.$$

Compute the Fourier transform $\tilde{f}(k)$. The algebra will be easier if you use the definition of the Fourier transform with no 2π in the exponent, i.e.

$$\tilde{f}(k) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx}\, dx.$$

Hint/Solution. combine the two exponents to get $\exp\left(-ikx - a^2x^2\right)$. Then complete the square, using

$$-ikx - a^2x^2 = -a^2\left(x + \frac{ik}{2a^2}\right)^2 - \frac{k^2}{4a^2}.$$

Then "change variables" in the integral to $u = x + \frac{ik}{2a^2}$ and evaluate using the usual Gaussian integral. This u-sub is technically illegal, since we have changed variables to a *complex* number (and we never proved anything about how integrals work in this case), but assume that everything behaves in the same way as it would if u were real.

Then we have

$$\begin{aligned}\tilde{f}(k) &= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{k^2}{4a^2}\right) \int_{-\infty}^{\infty} e^{-a^2u^2}\, du \\ &= \frac{1}{a\sqrt{2}} \exp\left(-\frac{k^2}{4a^2}\right).\end{aligned}$$

This is the result. Importantly, if the original Gaussian was very narrow (large a^2), then the Fourier-transformed Gaussian is very wide.

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Updated 18 days ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

dingus

Updated 17 days ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

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