question 2 views

Daily Challenge 13.4

~~**(Due: Thursday 8/30 at 12:00 noon eastern)**~~
~~**(Due: Monday 9/3 at 12:00 noon eastern)**~~
(**Optional**)

This one is a more interesting CD 3 problem; you'll have to think a bit about what the derivative means in a physical context.

Recall that Newton's motivation for defining the derivative was to compute the instantaneous velocity of a moving particle. For instance, if a particle moves along the x axis with position $x_1(t) = \sin(t)$, then its instantaneous velocity is $v_1(t) = \frac{dx}{dt} = \cos(t)$.

If a second particle moves along the y axis with position $y_2(t) = t^2$, then its velocity is $v_2(t) = 2t$.

Of course, by Pythagoras, we know that the distance between the two particles is $r(t) = \sqrt{x_1(t)^2 + y_2(t)^2} = \sqrt{\sin^2(t) + t^4}$, so the distance between the particles is changing at a rate

$$\frac{dr}{dt} = \frac{1}{2\sqrt{\sin^2(t) + t^4}} \cdot \left(2\sin(t)\cos(t) + 4t^3\right)$$

by the chain rule.

You will perform a similar analysis in today's problem, although the situation will be more complicated because one particle moves along a curve rather than along one of the coordinate axes.

Hint: if you need more help, take a look at pages 149-150 of Spivak (note that the PDF page numbering does not match the numbering in the book; the desired pages are numbered 162-163 in the PDF).

(1) Problem: an application to kinematics.

Particle A moves along the positive x axis while particle B moves along the graph of $f(x) = -\sqrt{3}x$ for $x \neq 0$. At a certain time t_0, A is at the point $(5, 0)$ and is moving with speed $3\ \frac{\mathrm{m}}{\mathrm{s}}$, while B is at a distance of $3\ \mathrm{m}$ from the origin and is moving with speed $4\ \frac{\mathrm{m}}{\mathrm{s}}$. At what rate is the distance between A and B changing?

daily_challenge

Updated 7 months ago by Christian Ferko

the instructors' answer, *where instructors collectively construct a single answer*

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followup discussions *for lingering questions and comments*