note 2 views

Daily Challenge 4.3

No problem today, to leave time for DC 3.6 and 4.1.

Instead, here is a short practice exercise on our new trigonometric functions introduced yesterday.

Exercise. What are the domain and range of the tangent function?

Solution. Recall that $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$, and we cannot divide by zero, so the domain of the tangent function is all angles θ for which $\cos(\theta) \neq 0$. But we know that $\cos(\theta) = 0$ if $\theta \in \{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \cdots\}$, so

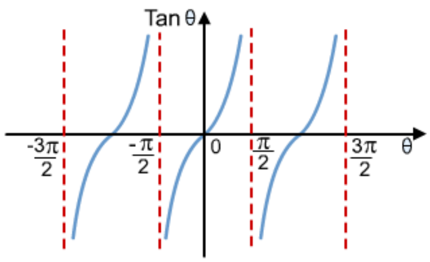
$$\text{Dom}(\tan) = \mathbb{R} \setminus \left\{ \pm\frac{n\pi}{2} \mid n \text{ is an odd integer} \right\}.$$

Now for the range. Near the points where tangent is undefined, like $\theta = \pm\frac{\pi}{2}$, the cosine is getting very close to zero while the sine is getting very close to ± 1. Thus, near these points, the tangent is dividing ± 1 by a very small number, so the output is either a very large positive number or a very large negative number. We can make this output as large as we like by going closer and closer to zero, so we can achieve any output in $(-\infty, \infty)$ that we like.

Although this reasoning was non-rigorous, the conclusion is true! One has

$$\text{Rng}(\tan) = (-\infty, \infty).$$

To visualize why tangent can output arbitrarily large positive or negative numbers, it helps to look at the graph:



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Updated 11 months ago by Christian Ferko

followup discussions *for lingering questions and comments*