question 2 views

Daily Challenge 21.1

Evaluate each of the following integrals using substitutions of the form $x = \sin(u)$, $x = \cos(u)$, etc. You might need to use that

$$\int \sec(x)\,dx = \log(\sec(x) + \tan(x)),$$
$$\int \csc(x)\,dx = -\log(\csc(x) + \cot(x)).$$

(a) $\int \frac{dx}{\sqrt{x^2-1}}$.

(b) $\int \frac{dx}{x\sqrt{x^2-1}}$.

(c) $\int \frac{dx}{x\sqrt{1+x^2}}$.

(d) $\int \sqrt{1+x^2}\,dx$.

daily_challenge

Updated 4 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

As suggested by Christian, I look up https://en.wikipedia.org/wiki/Trigonometric_substitution .

(a): We let $x = \sec(\theta)$ and thus $dx = \sec\theta\tan\theta\,d\theta$ we then see that

$$\begin{aligned}\int \frac{dx}{\sqrt{x^2-1}} &= \int \frac{\sec\theta\tan\theta}{\sqrt{\sec^2(\theta)-1}}\,d\theta\\ &= \int \frac{\sec\theta\tan\theta}{\tan(\theta)}\,d\theta\\ &= \int \sec(\theta)\,d\theta\\ &= \log(\sec(\theta)+\tan(\theta)).\end{aligned}$$

We need to substitute something for theta, so we shall assume that there does somehow exist a function arcsec such that $y = \operatorname{arcsec}\sec(y + k2\pi)$ where $k \in \mathbb{Z}$. Then $\theta = \operatorname{arcsec}(x)$ and

$$\int \frac{dx}{\sqrt{x^2-1}} = \log(x + \tan(\operatorname{arcsec}(x))).$$

(b): Once again let $x = \sec(\theta)$ and thus $dx = \sec\theta\tan\theta\,d\theta$. Then we have (omitting the step involving substitution of $\sec(\theta)$ for x and simplifying)

$$\begin{aligned}\int \frac{dx}{x\sqrt{x^2-1}} &= \int \frac{\sec\theta\tan\theta}{\sec(\theta)\tan(\theta)}\,d\theta\\ &= \int 1\,d\theta\\ &= \theta\\ &= \operatorname{arcsec}(x).\end{aligned}$$

(c): Let $x = \tan(\theta)$ and thus $dx = \sec^2(\theta)\,d\theta$. We then see that

$$\begin{aligned}\int \frac{dx}{x\sqrt{1+x^2}} &= \int \frac{\sec^2(\theta)}{\tan(\theta)\sqrt{1+\tan(\theta)^2}}\,d\theta\\ &= \int \frac{\sec(\theta)}{\tan(\theta)}\,d\theta = \int \frac{1}{\cos(\theta)}\cdot\frac{\cos(\theta)}{sin(\theta)}\\ &= \int \csc(\theta) = -\log(\csc(\theta)+\cot(\theta))\end{aligned}$$

We then see that we can substitute $\theta = \arctan(x)$ and conclude that

$$\int \frac{dx}{x\sqrt{1+x^2}} = -\log(\csc(\arctan(x)) + 1/x)$$

(d): Let $x = \tan(\theta)$ and thus $dx = \sec^2(\theta)\,d\theta$. We then see that

$$\int \sqrt{1+x^2}\,dx = \int \sec(\theta)\,d\theta = \log(\sec(\arctan(x)) + x)$$

Updated 2 months ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

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followup discussions *for lingering questions and comments*