34, 5

(a): Make the exponential ansatz

$$y = e^{rx}$$

to see that

$$r^2 e^{rx} + r e^{rx} - 6 e^{rx} = 0$$

$$r^2 + r - 6 = 0$$

then

$$r = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot -6}}{2} = \frac{-1 \pm 5}{2} = 2, -3$$

and we have general solution

$$y = C_1 e^{2x} + C_2 e^{-3x}$$

(b): This time, make the ansatz

$$y = x e^{rx}$$

and ~~plug in~~ note

$$y' = e^{rx} + x r e^{rx}$$

$$y'' = r e^{rx} + r(e^{rx} + x r e^{rx})$$

$$= 2 r e^{rx} + x r^2 e^{rx}, \text{ thus}$$

$$a 2 r e^{rx} + a x r^2 e^{rx} + b e^{rx} + b x e^{rx} + x e^{rx} = 0$$

$$2ar + a x r^2 + b + bx + x = 0$$

applying the quadratic formula to find r,

$$r = \frac{-2a \pm \sqrt{4a^2 - 4 \cdot}}{}$$

(b): Evaluate y' and y'':

$$y' = e^{rx} + rxe^{rx}$$

$$y'' = re^{rx} + re^{rx} + r^2xe^{rx},$$

thus

$$ay'' + by' + cy = 0$$

becomes

$$2are^{rx} + ar^2e^{rx} + be^{rx} + brxe^{rx} + cxe^{rx} = 0$$

~~put~~ dividing out all e^{rx} (since nox has $e^{rx} = 0$),

$$2ar + ar^2x + b + brx + cx = 0,$$

the nplugging in $r = \frac{-b}{2a}$,

$$-b + \frac{b^2}{4a}x + b + \frac{-b^2}{2a}x + cx = 0$$

b's cancel and we cam make the equation true by setting $x = a$,
Thus, we have general solution

$$y = C_1e^{rx} + C_2xe^{rx}.$$

(c): Given

$$y = C_1e^{(a+i\beta)x} + C_2e^{(a-i\beta)x},$$

seperate powers ~~to factor~~ and factor to have

$$y = e^{ax}\left(C_1e^{i\beta x} + C_2e^{-i\beta x}\right).$$

Recall Eulers Formula,

$$e^{ix} = \cos(x) + i\sin(x),$$

Then

$$y = e^{ax}\left(C_1\left(\cos(\beta x) + i\sin(\beta x)\right) + C_2\left(\cos(\beta x) - i\sin(\beta x)\right)\right)$$

Factor and distribute a lot of bois:

$$y = e^{ax}\left((C_1 + C_2)\cos(\beta x) + i(C_1 - C_2)\sin(\beta x)\right)$$

Or redefining

$$K_1 = C_1 + C_2$$

$$K_2 = i(C_1 - C_2),$$

$$y = e^{ax}\left(K_1 \cos(\beta x) + K_2 \sin(\beta x)\right).$$