23.1

[redacted]

(a) $\int_1^n \log(x) \cdot 1 \, dx = [x \log(x)]_1^n - \int_1^n \frac{1}{x} \cdot x \, dx$

$- \int_1^n 1 \, dx$

$= n \log(n) - n + C$

When $n = 1$

$0 \stackrel{!}{=} n \log(n) - n + C$

$0 = -1 + C$

$C = 1$

Thus,

$\int_1^n \log(x) dx = n \log(n) - n + 1$

(b) We apply the trapezoidal rule and see

$\int_1^n \log(x) \approx \sum_{k=1}^{n-1} \frac{\log(k) + \log(k+1)}{2} + C = \frac{1}{2}\left(\log(1) + \log(2) + \log 2 + \log 3 \ldots\right) + C$

$= \frac{1}{2}\left(\log((n-1)!^2 n)\right) + C$

$= \frac{1}{2}\left(2\log((n-1)!) + \log(n)\right) + C$

$= \log((n-1)!) + \frac{1}{2}\log(n) + C$

$\log\left(\frac{n!}{n}\right) + \frac{1}{2}\log(n) + C$

$\log(n!) - \log(n) + \frac{1}{2}\log(n) + C$

$\log(n!) - \frac{1}{2}\log(n) + C$

(c) ~~Then $\log((n-1)!) + \log(n) + C = n \log(n) - n + 1$~~

$\log(n!) - \frac{1}{2}\log(n) + C = n \log(n) - n + 1$

$\log(n!) = (n + \frac{1}{2})\log(n) - n + 1 + C$

We assumed when n was large that $n+1 \approx n$, thus

$\log(n!) = \frac{1}{2}\log(n) + n\log(n) - n + C$

then exp each side.

$n! = \sqrt{n} \cdot n^n \cdot e^{-n} \cdot e^C$