4/28/2019

27.6

(1): The mathematical ~~property~~ statement, concerning a sequence a_n,

$$\lim_{n\to\infty} a_n = L$$

is ~~the statemen~~ defined as the following: Given an $\epsilon > 0$, the limit exists if there is a positive N ~~$> n$ such tha~~ such that

if $n > N$, then $|a_n - L| < \epsilon$.

(2): To show that the limit

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(-1)^{n+1}}{n} \stackrel{!}{=} 0,$$

we must show how to pick an N such that, given $\epsilon > 0$

if $n > N$, then $|a_n - L| < \epsilon$.

We see that we need

$$\left|\frac{(-1)^{n+1}}{n} - 0\right| < \epsilon \Rightarrow \left|\frac{(-1)^{n+1}}{n}\right| < \epsilon$$

Then due to symmetry about the x-axis and the absolute value, this becomes

$$\frac{1}{n} < \epsilon$$

or, we see that we need

$$N = \frac{1}{\epsilon}$$

for this to be satisfied. Thus, the limit exists.