31.2

(2). A Fourier series is a sum of trig functions that approximates a ~~sum~~ function, while a Fourier transform is a method of modifying a non periodic function to see its periodic nature. Gibbs phenomenon is the spike in trig functions found when approximating a jump discontinuity.

(3).

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{2\pi n x}{L}\right) + b_n \sin\left(\frac{2\pi n x}{L}\right)\right)$$

(4):

- $$\int_0^L \sin\left(\frac{2\pi n x}{L}\right)\cdot \cos\left(\frac{2\pi m x}{L}\right) dx = 0$$
- $$\int_0^L \cos\left(\frac{2\pi m x}{L}\right)\cdot \cos\left(\frac{2\pi m x}{L}\right) dx = \frac{L}{2}\delta_{nm} \qquad (1)$$
- $$\int_0^L \sin\left(\frac{2\pi n x}{L}\right)\cdot \sin\left(\frac{2\pi m x}{L}\right) dx = \frac{L}{2}\delta_{nm} \qquad (2)$$

The δ_{nm} encodes the information that if $n = m$, then (1) and (2) equal $L/2$; otherwise, (1) and (2) $= 0$.

(5):

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{2\pi n x}{L}\right) dx$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{2\pi n x}{L}\right) dx$$

(7): Since the sawtooth function is odd, $a_n = 0$; then, see that

$$b_n = \frac{2}{L}\int_{-L/2}^{L/2} f(x)\sin\left(\frac{2\pi n x}{L}\right)$$

$$= \frac{2}{L}\int_{-L/2}^{L/2} Ax\sin\left(\frac{2\pi n x}{L}\right)$$

Referring to a table, see that

$$\int x\sin(rx) = \frac{1}{r^2}\sin(rx) - \frac{x}{r}\cos(rx)$$

implies

$$b_n = \left(\left[-x \frac{L}{2\pi n} \cos\left(\frac{2\pi n x}{L}\right)\right]_{-L/2}^{L/2} + \left(\frac{L}{2\pi n}\right)^2 \left[\sin\left(\frac{2\pi n x}{L}\right)\right]_{-L/2}^{L/2}\right.$$

$$= -\frac{AL}{\pi n} \cos(\pi n)$$

$$= (-1)^{n+1} \left(\frac{AL}{\pi n}\right.$$

which the n allows us to see that, by (3),

$$f(x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{AL}{\pi n} \sin\left(\frac{2\pi n x}{L}\right)$$

$$= \frac{AL}{\pi} \cdot \left(\sin\left(\frac{2\pi x}{L}\right) - \frac{1}{2}\left(\sin\left(\frac{4\pi x}{L}\right) + \frac{1}{3}\left(\sin\left(\frac{6\pi x}{L}\right) - \ldots\right)\right.\right.$$