question 2 views

Daily Challenge 7.4

(**Due: Friday 6/8 at 12:00 noon Eastern**)

Let's get back on track with daily challenges -- I want to work through a few more revisions for the consolidation document.

On problem set 1, we had the following question about suprema.

Problem. If A and B are bounded subsets of $\mathbb{R}$ with $\sup(A) = \alpha$ and $\sup(B) = \beta$, then what can you say about $\sup(A \cup B)$ and $\sup(A \cap B)$, in terms of α and β? Prove your assertions.

The original response was

Solution 3.1

I can conclude the following based off of the given information: the result of $\sup(A \cup B)$ will be the greater of the two α and β, and this can be proven by the fact that, given two least upper bounds, one is no longer has its status due to the definition of least upper bound, where a number qualifies if it is less than or equal to any and all of the elements in the subset. On the other side, the result of $\sup(A \cap B)$ can be expressed that if α and β are present in both sets, then the proof applied to the previous statement can be applied here. If just α is present in both sets, then it is the resulting lest upper bound, and the same logic can be applied to β. If each are only present

2

Logan Pachulski Problem Set 1 Solutions

in one set, then the least upper bound is just whatever results in that scenario, *shrug*. Also this probably doesn't actually work as a proof but I don't know how to test either, so, nice.

The attempted solution above captures some good intuition about suprema, but is non-rigorous because it does not use the definitions of these terms to construct a formal proof.

Let's answer a very similar question.

New Problem. If A and B are bounded intervals with $A \cap B \neq \emptyset$, $\sup(A) = \alpha$ and $\sup(B) = \beta$, show that $\sup(A \cap B) = \min(\alpha, \beta)$. Prove your assertions.

Try to prove the above, using the definition of suprema appropriately and justifying every step. It may help to use the strategy from our "reading proofs" video to check your own work: namely, as you read the proof you've written, can you explain why every deduction is true given the statements that precede it?

daily_challenge

Updated 10 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

Logan Pachulski:

Exploration: I begin my exploration by recognizing that the ranges of the intervals A and B overlap by at least one element since their intersection isn't a null set. I do not find myself convinced of the truthfulness of this theorem upon reading it, so I decide to create an example. Say the interval A is $(0, 2)$ and B is the interval $(1, 3)$ The supremum of A is 2 and the supremum of B is 3. The supremum of the intersections of these sets is 2, and the minimum of the individual supremums is 2. I want to say I'm convinced but something still seems wrong, like a coincidence. After further thinking I find myself convinced. I believe I can proceed to this proof.

Proof: Suppose that A is the interval (a, b) and B is the interval (c, d). The theorem states that the intersection of A and B is not a null set, therefore they do overlap. I shall define this intersection as C and the resulting interval is (b, c) since the intersection of two intervals is the supremum of the former set and the infimum of the latter. If said set would be empty, then substitute the former set for the latter and vice versa and the result is as desired. We can now define the supremum of A as α and the supremum of B as β. We can now proceed. The supremum of the interval previously defined as C is b, and this is equal to $\min(\alpha, \beta)$ which is in turn b. We have at this point proven that $\sup(A \cap B) = \min(\alpha, \beta)$. $\square$

Updated 10 months ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

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followup discussions *for lingering questions and comments*