

24.6

For the third time;

$$\begin{aligned}
 (f * f)(z) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left(\frac{-x^2}{2}\right) \exp\left(-\frac{\cancel{(x-z)}^2 (x-z)^2}{2}\right) dx \\
 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left(\frac{-x^2 + (-x^2 + 2zx - z^2)}{2}\right) dx \\
 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left(\frac{-2x^2 + 2zx - z^2}{2}\right) dx \\
 &= 11 \exp\left(-x^2 + zx - \frac{z^2}{4} - \frac{z^2}{4}\right) dx \\
 &= 11 \exp\left(-\left(x^2 - zx + \frac{z^2}{4}\right)\right) \exp\left(-\frac{z^2}{4}\right) dx \\
 &= 11 \exp\left(-\left(x - \frac{z}{2}\right)^2\right) \exp\left(-\frac{z^2}{4}\right) dx \\
 &= \frac{1}{2\pi} \exp\left(-\frac{z^2}{4}\right) \int_{-\infty}^{\infty} \exp\left(-\left(x - \frac{z}{2}\right)^2\right) dx
 \end{aligned}$$

We the n y-sub $y = x - \frac{z}{2} \Rightarrow dy = dx$

$$\begin{aligned}
 &= \frac{1}{2\pi} \exp\left(-\frac{z^2}{4}\right) \int_{-\infty}^{\infty} \exp(-y^2) dy \\
 &= \frac{1}{2\pi} \cdot \sqrt{\pi} \exp\left(-\frac{z^2}{4}\right) \\
 &= \frac{1}{2\sqrt{\pi}} \exp\left(-\frac{z^2}{4}\right)
 \end{aligned}$$