22.7

(A) $V(x,R) = \dfrac{\pi^{x/2} \; R^x}{\Gamma(\frac{x}{2}+1)}$

$$\log(V(x,R)) = \log((\sqrt{\pi}\,R)^x) - \log(\Gamma(\tfrac{x}{2}+1))$$

$$= x\log(\sqrt{\pi}\,R) - \log(\Gamma(\tfrac{x}{2}+1))$$

$$\frac{d}{dx}\Big(\log(V(x,R))\Big) = \log(\sqrt{\pi}\,R) - \left(\frac{1}{\Gamma(\frac{x}{2}+1)} \cdot \Gamma'(\tfrac{x}{2}+1) \cdot \frac{1}{2}\right)$$

(B) $0 \overset{!}{=} \log(\sqrt{\pi}\,R) - \frac{1}{2}\psi(\frac{x}{2}+1)$ $\qquad \psi = \dfrac{\Gamma'(2)}{\Gamma(2)}$

$$\log(\sqrt{\pi}\,R) = \frac{1}{2}\psi(\tfrac{x}{2}+1)$$

$$2\cdot(\tfrac{1}{2}\log(\pi) + \log(R)) = \psi(\tfrac{x}{2}+1$$

$$\log(\pi) + 2\log(R) = \psi(\tfrac{x}{2}+1)$$

(C) Yes, $\psi(\frac{x}{2}+1)$ intersects $\log(\pi)$ at $(5.257, 1.145)$.