35.6

We begin by re writing the system of equations as matrix multipliation (also let kappa = L)

$$m x_1 = (-K-L) x_1 + L x_2$$

$$m x_2 = (-K-L) x_2 + L X_1$$

is encoded by

$$m \frac{\partial^2}{\partial t^2}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -K-L & L \\ L & -K-L \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

we then make the ansatz that

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} e^{rT}; \text{ thus}$$

$$m r^2 \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} -K-L & L \\ L & -K-L \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \quad (1)$$

and thus this substitution works if

$M = \begin{bmatrix} -K-L & L \\ L & -K-L \end{bmatrix}$ has eigenvector $\begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$ and eigenvalue mr^2

We can find the eigenvalues by solving

$$\det\left(\begin{bmatrix} -K-L-\lambda & L \\ L & -K-L-\lambda \end{bmatrix}\right) = 0$$

for λ;

$$(-k-L-\lambda)^2 - L^2 = 0$$

implies

$$\lambda = -L-k \pm L = -K, -k-2L$$

In order for our ansatz to hold, we must provide that by setting that

$$mr^2 = \lambda \Rightarrow r = i\sqrt{\frac{K}{m}} \text{ or } r = \sqrt{\frac{K+2L}{m}}$$

Plugging in these r into (1), and substituting where suitable, we get that

$$L\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ A_2 \end{bmatrix} = 0 \Rightarrow \lambda = -K \Rightarrow \text{eigenvector} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and for $\lambda = -K - 2L$, $L\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ A_1 \end{bmatrix} = 0 \Rightarrow \text{eigenvector} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$