question 2 views

Daily Challenge 20.5

Try these integrals from the 2017 Integration Bee. I've included the answers, so you can check your work, but of course the real challenge is figuring out the steps!

(a) $\int_1^\infty \frac{\log(x)}{x^2}\,dx = 1.$

(b) $\int x^3 e^{x^2}\,dx = \frac{1}{2}(x^2-1)e^{x^2}.$

(c) $\int \frac{(2+x)e^{-x}}{x^3}\,dx = -\frac{e^{-x}}{x^2}.$

daily_challenge

Updated 2 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

(a) We shall apply integration by parts. Let $a' = \frac{1}{x^2}$, $a = \frac{-1}{x}$, and $b = \log(x)$. Then

$$\begin{aligned}\int \frac{\log(x)}{x^2} &= \frac{-\log(x)}{x} + \int \frac{1}{x^2} \\ &= \frac{-log(x)}{x} + \frac{-1}{x}.\end{aligned}$$

We then apply the FTC:

$$\left(\frac{-log(\infty)}{\infty} + \frac{-1}{\infty}\right) - \left(\frac{-log(1)}{1} + \frac{-1}{1}\right)$$
$$(0) - (-1)$$
$$1$$

(b) We shall apply u-sub; let $u = x^2$ and thus $du = 2x\,dx$. We then see that

$$\int x^3 e^{x^2} = \int xue^u\,dx = \frac{1}{2}\int ue^u$$

Then, by integration by parts where $a' = e^u$, $a = e^u$, and $b = u$:

$$\begin{aligned}\frac{1}{2}\int ue^u &= \frac{1}{2}(ue^u - e^u) \\ &= \frac{1}{2}e^u(u-1) \\ &= \frac{1}{2}e^{x^2}(x^2-1)\end{aligned}$$

(c)We split the original integral into two;

$$\begin{aligned}\int \frac{(2+x)e^{-x}}{x^3}\,dx &= 2\int \frac{e^{-x}}{x^3}\,dx + \int \frac{xe^{-x}}{x^3}\,dx \\ &= 2\int \frac{e^{-x}}{x^3}\,dx + \int \frac{e^{-x}}{x^2}\,dx\end{aligned}$$

We integrate by parts the left term, where $a' = \frac{1}{x^3}$, $a = \frac{-1}{2x^2}$, and $b = e^{-x}$ to see that

$$\int \frac{e^{-x}}{x^3}\,dx = \frac{-e^{-x}}{2x^2} - \int \frac{e^{-x}}{2x^2}$$

We multiply everything here by the seen 2 to pull the 2 out of the denominators, and conclude that

$$\frac{-e^{-x}}{x^2} - \int \frac{e^{-x}}{x^2} + \int \frac{e^{-x}}{x^2} = \frac{-e^{-x}}{x^2},$$

the result we desired.

log_20.5.pdf

Updated 2 months ago by Logan Pachulski and Christian Ferko

the instructors' answer, *where instructors collectively construct a single answer*

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