21.7

Find the volume of the function

$$x^2 \le y \le x$$

rotated 360° about a $y=2$ axis.

Recall the area formula for an annulus:

$A = \pi(R^2 - r^2)$ where R is the outer radius and r is the outer radius. Then the area of our annulus on the range $[0,1]$ can ~~be represented as~~

$\cancel{A(x) = \pi(x^4 - x^2)}$,

~~thus, we write the integral~~

$\cancel{\pi \int x^4 - x^2 \, dx = V(x)}$

False, recall that the internal radius is $\cancel{(x-2)}$ and OR is $\cancel{(x^2-2)}$
$(2-x)$ and OR is $(2-x^2)$

$$A(x) = \pi\left((x^2-2)^2 - (x-2)^2\right)$$
$$\left((x^4-4x^2+4) - (x^2-4x+4)\right)$$
$$\pi(x^4-5x^2-4x)$$

implies

$$V(x) = \pi \int_0^1 x^4 - 5x^2 - 4x \, dx$$
$$= \pi\left[\frac{x^5}{5} - \frac{5x^3}{3} - 2x^2\right]_0^1$$
$$= \pi\left(\frac{1}{5} - \frac{5}{3} - 2\right)$$
$$= \pi\left(\frac{3}{15} - \frac{25}{15} - \frac{30}{15}\right)$$

Retry:

$$A(x) = \pi\left((-x^4+4x^2-4) - (-x^2+4x-4)\right)$$
$$= -x^4 + 5x^2 - 4x$$

At this point I realize that my IR and OR are of the wrong sign.

Let me try again from the start; new page

IR: $2-x$

OR: $2-x^2$

$(2-x^2)(2-x^2) = 4 - 2x^2 - 2x^2 + x^4$

$(2-x)(2-x) = 4 - 4x + x^2$

$$A(x) = \pi\left((2-x^2)^2 - (2-x)^2\right)$$

$$\pi\left((4-4x^2+x^4) - (4-4x+x^2)\right)$$

$$\pi\left(-5x^2 + 4x + x^4\right)$$

Thus, we must solve the integral

$$V(x) = \pi \int -5x^2 + 4x + x^4$$

$$= \pi\left[\frac{-5x^3}{3} + 2x^2 + \frac{x^5}{5}\right]_0^1$$

$$= \pi\left(\frac{-5}{3} + 2 + \frac{1}{5}\right)$$

$$= \pi\left(\frac{-25}{15} + \frac{30}{15} + \frac{3}{15}\right)$$

$$= \pi \frac{8}{15}$$