22.3

(a): ~~We are told that~~

We are given that for a non-null set A, we let

$$A_- = \{-x \mid x \in A\}$$

then we must prove that $A_- \neq \emptyset$, A_- is bounded above, and that $-\sup(A) = \inf(A_-)$

First recall that if some point $c \in A$, then $-c \in A_-$ by the definition of A_-. Thus, $A \neq \emptyset$.

Second we let some value $d \leq a \in A$. then this implies by passing through the definition of A_- that

$-d \geq -a \quad \forall\, a \in A$ which implies that

$-d \geq b \quad \forall\, b \in A_-$ once again by the definition of A_-.

Thus $-d$ is an upper bound for A_-.

Third we must prove that $-\sup(A) = \inf(A_-)$

We begin by letting $g = \sup(A_-)$; then

$g \geq a \quad \forall\, a \in A$. Then by multiplying each side and ~~referr~~ referring to the definition of A_-:

$$-g \leq b \quad \forall\, b \in A$$

$$\Longrightarrow$$

$$-\sup(A_-) = \inf(A)$$

I log now i

(b): I shall focus on clear handwriting for this page.

"If $A \neq \emptyset$ is bounded below, let B be the set of all lower bounds of A. ~~Then~~ Show that $B \neq \emptyset$, that B is bounded above, and that $\sup(B) = \inf(A)$."

Show that $B \neq \emptyset$: We are told that A is bounded below; thus there exists x such that

$$x \leq a \quad \forall\, a \in A$$

But by definition of B, then this $x \in B$ and thus

$$B \neq \emptyset$$

Show that B is bounded above: We are told that

$$B = \{x \mid x \leq a \;\; \forall\, a \in A\}$$

We let $a \in A$. Then, assuming $\text{len}(A) \neq 1$, (python notation) then $a \geq b \;\; \forall\, b \in B$ and B has an upper bound.

Show that $\sup(B) = \inf(A)$. To do so, we let

$$d = \inf(A).$$

and we must show $\quad d \stackrel{?}{=} \sup(B).$

To ~~do~~ so we must prove that:

d is a upper bound of B: We ~~are t~~ defined that $d = \inf(A)$. ~~If we let b~~ Then, if $b \in B$, then $b \leq d$.

d is the least upper bound of B: Suppose by way of contradiction there was some e such that $e > d$ and both are upper bounds of B. Then $e = \inf(A)$; but we defined $d = \inf(A)$, thus it must be true that $\sup(B) = \inf(A)$