question 2 views

Daily Challenge 23.2

(**Due: Thursday 2/28 at 12:00 noon Eastern**)

In the past couple of sessions we have met some interesting new functions. We defined the *gamma function*

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-z}\, dt,$$

the *beta function*,

$$B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}\, dt,$$

which are related by the equation

$$B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}.$$

We also met the *digamma function*

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

which is related to the harmonic numbers $H_n = \sum_{j=1}^n \frac{1}{j}$ by

$$\psi(n) = -\gamma + H_{n-1} \quad , \quad n \in \mathbb{N}.$$

Just as sine and cosine satisfy certain identities like $\sin^2(x) + \cos^2(x) = 1$, these new functions satisfy some relations. I'll ask you to prove some of them.

(a) Show directly from the definition $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}\, dt$ that the beta function is *symmetric*, i.e. $B(x, y) = B(y, x)$. (This is obvious if you use its expression in terms of gammas, but use the integral). Hint: change variables to $s = 1 - t$.

(b) Prove that

$$\frac{d}{dx} B(x, y) = B(x, y) \cdot (\psi(x) - \psi(x+y))$$

by differentiating the relation $B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$ and carefully applying the chain rule. Note that y is considered a constant since we are differentiating with respect to x.

(c) Check numerically in Python that

$$\Gamma(1-z)\Gamma(z) = \frac{\pi}{\sin(\pi z)}$$

by plugging in some values of z and seeing whether they agree. Include a screenshot.

Use scipy.special.gamma.

daily_challenge

Updated 1 month ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

pain.png

20190301_120404.jpg

Updated 1 month ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

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