question 2 views

Daily Challenge 22.4

(**Due: Saturday 2/23 at 12:00 noon Eastern**)

In session 51, I will describe an alternative (and more rigorous) way of defining trigonometric functions directly from the integral.

Although I will skip some details, this construction is spelled out explicitly in chapter 15 of Spivak.

(1) Problem: proving cosine angle addition a third time.

Download Spivak's book (make sure you have a DJVU reader like SumatraPDF) and read chapter 15. Try to at least read the statements of theorems carefully, but you can skim/gloss over the proofs.

Now look at Theorem 5 on page 314. Spivak proves the first equation, the angle addition formula for sine, but leaves the second formula to you as an exercise. Do this exercise: that is, prove that

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

by using a strategy almost identical to the one that Spivak uses for the sine formula.

More precisely: define a function $f(x) = \cos(x+y)$ and show that it satisfies the hypotheses of Spivak's Theorem 4, which means that $f = b\sin(x) + a\cos(x)$ where $a = f(0)$ and $b = f'(0)$.

As a side comment, you now know of three ways to prove the cosine angle addition formula:

1. from Euler's formula;
2. using vector methods;
3. using Spivak's Theorem 4.

Answer on Overleaf here: https://www.overleaf.com/1231232126rckrscxfchyf

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Updated 1 month ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

green boi

Updated 1 month ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

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