question 2 views

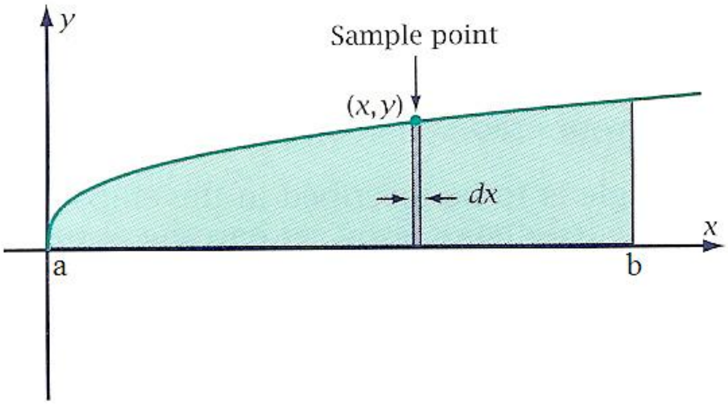
Daily Challenge 21.5

(**Due: Saturday 2/16 at 12:00 noon Eastern**)

Today we'll start computing volumes of so-called *solids of revolution*.

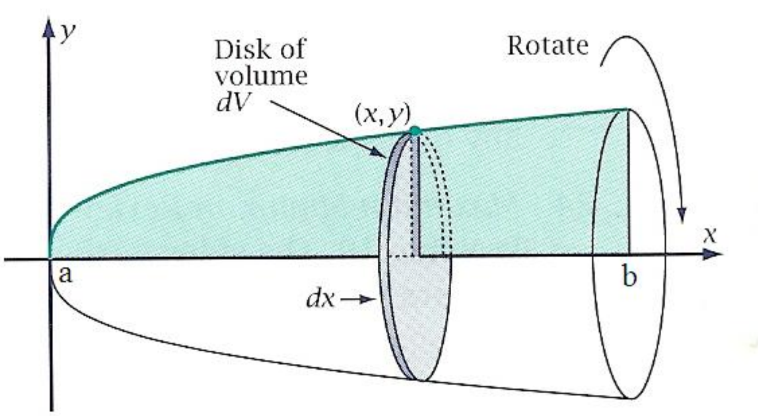
(1) Rotating an area.

Consider the graph of $f(x) = \sqrt[3]{x}$ between $a = 0$ and some right endpoint b.



We already know how to find the **area** under this graph using integrals: we add up the areas of thin rectangles with base lengths $[x_i, x_i + dx]$ and heights $f(x_i) = \sqrt[3]{x_i}$, as shown in the figure above.

Now we will do something more interesting. Imagine revolving the green shaded area about the x axis, which sweeps out a solid volume:



We can chop up this volume into discs, or skinny cylinders. Each disc has an area $A(x) = \pi r^2$, but we can see that the radius of the disc is simply $y = \sqrt[3]{x}$, so the disc area is $A(x) = \pi x^{2/3}$. Using our general result

$$V = \int_0^b A(x)\,dx,$$

we can find the volume of the region as

$$\begin{aligned} V &= \int_0^b \left(\pi x^{2/3}\right)\,dx \\ &= \pi\left[\frac{3}{5}x^{5/3}\right]_0^b \\ &= \frac{3\pi}{5}b^{5/3}. \end{aligned}$$

(2) Problem: some solids of revolution.

Try these straightforward exercises.

(a) Find the volume of the region enclosed by the surface resulting when the curve $y = x^3$ on $[0, 2]$ is rotated about the x-axis. (Check that your answer is $\frac{128\pi}{7}$).

(b) Find the volume of the region enclosed by the surface resulting when the curve $y = \cos(x)$ on $[0, \pi/2]$ is rotated about the x axis. (Make sure you get $\frac{\pi^2}{4}$.

(c) You can plot and visualize surfaces of revolution in Wolfram Alpha using this syntax. Think up another surface of revolution and plot it to get some practice with visualization.

daily_challenge

Updated 1 month ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

(a): The surface area of the circle enclosed is
$A(x) = \pi x^6$.
Thus,
$V = pi \int_0^2 x^6$
$V = pi(\frac{2^7}{7}) = \frac{128\pi}{7}$
:thumbsup:

(b): We see that the surface area a distance from the origin is
$A(x) = \pi \cos^2(x)\, dx$, thus
$V(x) = \pi \int_0^{\pi/2} \cos^2(x)\, dx$
We then see that since $\cos(2a) = 2\cos^2(a) - 1$, then

$$\begin{aligned} \cdots &= \pi/2 \left(\int_0^{\pi/2} \cos(2x) + \int_0^{\pi/2} 1 \right) \\ &= \pi/2(1/2(\sin(\pi) - 1) + \pi/2) \\ &= \frac{\pi}{2} \cdot \frac{\pi}{2} = \frac{\pi^2}{4} \end{aligned}$$

rotate y=cos(x), 0<x<4pi about the y-axis

Browse Examples Surprise Me

Input interpretation:

surface of revolution	$y = \cos(x)$	$x = 0$ to 4π	about the y-axis

Parametric representation of surface:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \cos(\theta) \\ \cos(x_0) \\ x_0 \sin(\theta) \end{pmatrix} \text{ for } 0 < x_0 < 4\pi \text{ and } 0 < \theta < 2\pi$$

Open code

Implicit representation of surface:

$$y = \cos\left(\sqrt{x^2 + z^2}\right) \text{ for } x^2 + z^2 \le 16\pi^2$$

(c):

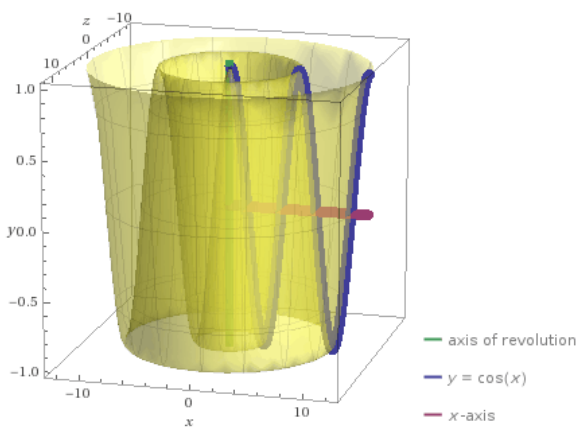
Area of surface:

$$\int_0^{4\pi} 2\pi |x| \sqrt{1 + \sin^2(x)}\, dx = 603.261$$

$|z|$ is the absolute value of z

Plot:

Show solid



Updated 1 month ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

Click to start off the wiki answer

followup discussions *for lingering questions and comments*