question 4 views

Daily Challenge 1.7

(**Due: Monday 4/30 at 12:00 noon Eastern.**)

Last challenge of the week! Weekly skips replenish tomorrow.

Review

We've seen two ways to write down a set. One is to explicitly list all of the elements inside curly braces, like

$$A = \{2, 3, 5, 7\}\,.$$

We could instead define the set by asking for all elements that satisfy a certain condition. The same set A could be written as

$$A = \{x \in \mathbb{Z} \mid 0 < x < 10 \text{ and } x \text{ is prime }\}\,.$$

The vertical bar is read as "such that", so this line is spoken aloud as: "A is the set of all integers x such that x is between 0 and 10 and x is prime," or for short, "A is the set of all primes between 0 and 10."

In the second way of defining the set A, the variable x is called a **dummy variable**. It is used only for writing the condition and can be replaced by any other symbol without changing the meaning. For instance, the expression

$$A = \{\eta \in \mathbb{Z} \mid 0 < \eta < 10 \text{ and } \eta \text{ is prime }\}\,.$$

defines the same set.

It may be helpful to think of this way of defining sets like list comprehensions in Python. For instance, if I wanted to define

$$B = \{3, 6, 9\} = \{x \in \mathbb{Z} \mid 0 < x < 10 \text{ and } x \text{ is divisible by } 3\}\,,$$

I could create a list with these elements in Python as follows:

```
In [5]: mults_of_three = [x for x in range(1, 10) if x%3==0]
        mults_of_three

Out[5]: [3, 6, 9]

In [6]: other_mults_of_three = [David for David in range(1, 10) if David%3==0]
        other_mults_of_three

Out[6]: [3, 6, 9]
```

Note that replacing x by another dummy, like David, gives the same result.

Most of our sets will be written using the "such that" notation, rather than the "listing the elements" notation. One reason is that it is more powerful: for writing a complicated set, like the image of a function, listing the elements would be difficult. Another reason is that it is less ambiguous. For example, if I wrote the set

$$C = \{3, 5, 7, \cdots\}$$

it might be unclear what I meant: did I mean

$$C \stackrel{?}{=} \{x \in \mathbb{Z} \mid x > 1 \text{ and } x \text{ is odd }\}$$

or

$$C \stackrel{?}{=} \{x \in \mathbb{Z} \mid x > 1 \text{ and } x \text{ is prime}\}\,?$$

Problem

Write each of the following sets using the "such that" notation, i.e. by writing an expression like $\{x \in \text{some set} \mid x \text{ satisfies some condition }\}$. If possible, also write the set by listing its elements.

I'll do the first one for you, as an example.

(a) The set of all positive integers strictly less than 6.

Answer: This is $\{x \in \mathbb{Z} \mid 0 < x < 6\}$, which can also be written as $\{1, 2, 3, 4, 5\}$.

(b) The set of all integers whose square is less than 17.

(c) The closed interval $[2, 6]$.

(d) The set of solutions to the equation $(x-1)^2 = 0$.

(e) The half-open interval $(-1, 9]$.

(f) The set of rational numbers less than -1.

daily_challenge

Updated 11 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

Proof (Corbin). Your answers go here.

1. $\{x \in \mathbb{Z} \mid x^2 < 17\}$
2. $\{x \in \mathbb{R} \mid 2 \leq x \leq 6\}$
3. $\left\{x \in \mathbb{R} \mid (x-1)^2 = 0\right\}$
4. $\{x \in \mathbb{R} \mid -1 < x \leq 9\}$
5. $\{x \in \mathbb{Q} \mid x < 1\}$

Proof (Logan) -

1. $\{x \in \mathbb{Z} \mid x^2 < 17\}$
2. $\{x \in \mathbb{R} \mid 2 \leq x \leq 6\}$
3. $\{x \in \mathbb{R} \mid x = 1\}$
4. $\{x \in \mathbb{R} \mid -1 < x \leq 9\}$
5. $\{x \in \mathrm{q} \mid x < 1\}$

Updated 10 months ago by Corbin and 2 others

the instructors' answer, *where instructors collectively construct a single answer*

(b) The set of all integers whose square is less than 17 is

$$\{x \in \mathbb{Z} \mid x^2 < 17\}.$$

We can also write this explicitly as $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

(c) The closed interval $[2, 6]$ is

$$\{x \in \mathbb{R} \mid 2 \leq x \leq 6\}.$$

(d) The set of solutions to the equation $(x-1)^2 = 0$ is

$$\{x \in \mathbb{R} \mid (x-1)^2 = 0\}.$$

Of course, this can be written more simply as the single-element set $\{1\}$.

(e) The half-open interval $(-1, 9]$ is

$$\{x \in \mathbb{R} \mid -1 < x \leq 9\}.$$

(f) The set of rationals less than -1 is

$$\{x \in \mathbb{Q} \mid x < -1\}.$$

Updated 11 months ago by Christian Ferko

followup discussions *for lingering questions and comments*