25.6

(a): Recall the definition of Fourier Transform;

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx} f(x)\, dx$$

thus if

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

then

$$F(k) = \cancel{\frac{1}{\sqrt{2\pi}}} \int_{0}^{\infty} \lambda e^{-\lambda x} e^{-ikx}\, dx$$

$$= \cancel{\frac{\lambda}{\sqrt{2\pi}}} \int_{0}^{\infty} e^{-(x(\lambda+ik))}\, dx \cdot \lambda$$

~~u-sub $u = \sqrt{x(\lambda+ik)} \rightarrow u^2 = x(\lambda+ik)$~~

$$= \lambda \left[\frac{-1}{\lambda+ik} e^{-x(\lambda+ik)}\right]_0^{\infty}$$

$$= \lambda\left(0 - 1 \cdot \frac{-1}{\lambda+ik}\right) = \frac{\lambda}{\lambda+ik}$$

(b): From ~~the previous p~~ (a),

$$F(k) = \frac{\lambda}{\lambda+ik}, \quad F_k(k) = (\lambda(\lambda+ik)^{-1})' = -i\lambda(\lambda+ik)^{-2}$$

$$F_{kk}(k) = -\lambda 2(\lambda+ik)^{-3}$$

$$F_{kkk} = i6\lambda(\lambda+ik)^{-4}$$

thus

$$E[x^1] = \frac{1}{-i} \cdot \frac{-i\lambda}{\lambda^2} = \frac{1}{\lambda}$$

$$E[x^2] = -1 \cdot \frac{-2\lambda}{\lambda^3} = \frac{+2}{\lambda^2}$$

$$E[x^3] = \frac{1}{i} \cdot \frac{i\lambda 6}{\lambda^4} = \frac{6}{\lambda^3}$$