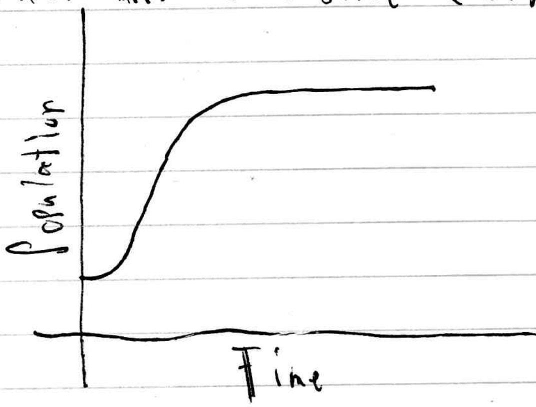
34.3

(a): The model in question tells us that the population will increase until the population N reaches some carrying capacity K.



(b): Recall

$$\frac{dN}{dt} = r\left(1 - \frac{N}{K}\right)N$$

Abusing notation to ~~see~~ ~~m~~ move all terms of N to the left side,

$$\frac{dN}{(1-\frac{N}{K})N} = r\,dt$$

Apply partial fractions to see that

$$\frac{1}{(1-\frac{N}{K})N} = \frac{A}{(1-\frac{N}{K})} + \frac{B}{N}$$

$$\Downarrow$$

$$AN + B - \frac{BN}{K} = 1$$

$$\Downarrow$$

$$B = 1$$

$$\Downarrow$$

$$A = \frac{1}{K}$$

And thus, integrating each side

$$\int \left(\frac{1}{(1-\frac{N}{K})K} + \frac{1}{N}\right) dN = \int r\,dt$$

implies

$$C_1 + -\log(K-N) + \log(N) = rt + C_2$$

$$\Downarrow$$

$$\frac{N}{K-N} = e^{rt+C} \qquad (1)$$

(0: ~~Now~~ let's solve for N as seen in (1)

$$N = (K-N)(e^{rt+C})$$

$$= K(e^{rt}+C) - N(e^{rt}+C)$$

$$\Downarrow$$

$$\cancel{N(1+e^{rt}+C) = K(e^{rt}+C)}$$

$$N + N(e^{rt}+C) = K(e^{rt}+C)$$

$$N(1+e^{rt}+C) = \quad "$$

$$N = \frac{K(e^{rt}+C)}{(1+e^{rt}+C)}$$

Attempt no. 2:

$$N = (K-N)e^{rt+C}$$

$$\Downarrow$$

$$N = \frac{Ke^{rt+C}}{1+e^{rt+C}}$$

multiplying by $1 = e^{-rt-C}/e^{-rt-C}$,

$$N = \frac{K}{1+e^{-rt-C}} = \frac{K}{1+Ce^{-rt}}$$

Then, for $N(0) = N_0$,

$$N_0(1+C) = K$$

$$C = \frac{K-N_0}{N_0}$$

and thus we conclude

$$N(t) = \frac{K}{1+\left(\frac{K}{N_0}-1\right)e^{-rt}}.$$