29.1

Refer to page 2 (a): ~~we see that $\sin(n\theta) \le 1$ for all $n \in \mathbb{N}$ and likewise $n \in \mathbb{R}$ thus~~

~~$\frac{\sin(\theta n)}{n^2} \le \frac{1}{n^2}$; apply the comparison test with~~

~~$a_n = \frac{\sin(\theta n)}{n^2} \le b_n = \frac{1}{n^2}$; $\frac{1}{n^2}$ is summable thus a_n is summable.~~

√ (b): See by Leibniz's Theorem that we have

$a_1 = 1 \ge a_2 = \frac{1}{3} \ge a_3 = \frac{1}{5} \ge \cdots$; clearly $\lim(a_n) = 0$; then

we conclude that the alternating series

$a_1 - a_2 + a_3 - \cdots \to L$; it converges

Refer to page 2 (c): ~~we see that~~

~~$\frac{1}{\sqrt[3]{n^2-1}} > \frac{1}{n^{2/3}}$; it is known that since $n^{-2/3}$ has power > -1,~~

~~then it diverges by the contrapositive of the comparison test. Since the series in question is strictly greater than a divergent series, it must diverge.~~

√ (d): Apply the ratio test; evaluate the limit

$$\lim\left(\frac{(n+1)^2/(n+1)!}{(n)^2/(n)!}\right) = \lim\left(\frac{(n+1)^2}{n^2(n+1)}\right) = \lim\left(\frac{n+1}{n^2}\right) = \lim\left(\frac{1}{n} + \frac{1}{n^2}\right) = 0$$

√ (e): Write down the integral we want to show exists:

$\int_2^\infty \frac{1}{x\log(x)}\,dx$ by sequence to a function; then u-sub $u = \log(x)$

$du = \frac{1}{x}\,dx$;

$$\cdots = \left[\cancel{\log(\log(x))}\right]_2^\infty = \int_{\log(2)}^\infty \frac{1}{u}\,du = \cancel{\log(\log(\infty)) - \log(\log(2))}$$

$$= \lim_{C\to\infty} \log(\log(C)) - \log(\log(2))$$

The integral diverges, so the sum diverges.

29.1 fixes

(a): As it stands, we cannot apply the ~~as~~ comparison test, since there exist values of $n\theta$ such that $\sin(n\theta) < 0$; to counteract this, we shall work instead with

$\displaystyle\sum_{n=1}^{\infty}\left|\frac{\sin(n\theta)}{n^2}\right|$ since this would imply the convergence/divergence of its counter-parts. We see that

$$\left|\frac{\sin(n\theta)}{n^2}\right| \leq \frac{1}{n^2},$$ since $|\sin(n\theta)|$ has range $[0,1]$

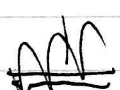
But we know that $\frac{1}{n^2}$ is convergent, so by the comparison test

$\displaystyle\sum_{n=1}^{\infty}\left|\frac{\sin(n\theta)}{n^2}\right|$ converges. We see by assumption (5) that it must be true then that

$\displaystyle\sum_{n=1}^{\infty}\frac{\sin(n\theta)}{n^2}$ converges. $\square$

(c): We begin by noting that, since

~~$(n^2)^{2/3}$ $(n^2)^{2/3}$, then~~



$$(n^2-1)^{1/3} < n^{2/3},$$ then

$$\frac{1}{\sqrt[3]{n^2-1}} > \frac{1}{n^{2/3}}.$$

We then see by the "All power law sums at once" proof in session 63 that the sum

$\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^{2/3}}$ diverges due to $p = 2/3 \leq 1$.

Then, by the contrapositive of the Comparison theorem (since the sum in question has sequence greater than another ~~divergent~~ sequence that diverges under summing), that

$$\frac{1}{n^{2/3}} \text{ diverges} \Rightarrow \frac{1}{\sqrt[3]{n^2-1}} \text{ diverges.}$$