Fastest Notes

Ferman

23.5

(a): We shall be integrating

$$W = \int_{V_a}^{V_b} \frac{N k_b T}{V}\, dV = N k_b T \cdot \left(\log(V_b) - \log(V_a) \right)$$

$$= N k_b T \cdot \log\left(\frac{V_b}{V_a}\right)$$

(b): Solve $\cancel{\left(p + \frac{aW^2}{V^2}\right)(V - Nb) = N k_b T \quad \text{for } V}$

$\cancel{\text{multiply by } \frac{V^2}{V^2}: \frac{(pV^2 + aW^2)(V - Nb)}{V^2} = N k_b T}$

$$\left(p + \frac{aW^2}{V^2}\right)\left(V - Nb\right) = N k_b T \text{ for } p$$

$$pV \cancel{+} - \cancel{pNb} + \frac{aW^2}{V} - \frac{aW^2 Nb}{V^2} = N k_b T$$

$$p = \frac{\frac{aW^2}{V} - \frac{aW^2 Nb}{V^2} - N k_b T}{V - Nb}$$

$$p = \frac{aW^2}{V^2 - Vnb} - \frac{aW^2 Nb}{V^3 - V^2 Nb} - \frac{N k_b T}{V - Nb}$$

then integrate: $-aW^2 \int \frac{1}{V} \cdot \frac{1}{V - nb} + aW^2 Nb \cdot \int \frac{1}{V^2} \cdot \frac{1}{V - Nb}$

$$+ \left[N k_b T \log(V - Nb) \right]_{V_a}^{V_b}$$

equals $\cancel{N k_b T \log\left(\frac{V_b - Nb}{V_a - Nb}\right) - aN^2 \int_{V_a}^{V_b} \frac{1}{V^2 - VNb}\, dV + aN^2 Nb \int_{V_a}^{V_b} \frac{1}{V^3 - V^2 Nb}}$

$$\left(p + \frac{aN^2}{V^2}\right)\left(V - Nb\right) = N k_b T$$

$$p = \frac{N k_b T}{V - Nb} - \frac{aN^2}{V^2}$$

Integrate $p(V)$ from $V_a \to V_b$

$$W(V_a, V_b) = \int_{V_a}^{V_b} \frac{N k_b T}{V - Nb} - \int_{V_a}^{V_b} \frac{aN^2}{V^2}$$

$$= N k_b T \left(\log(V_b - Nb) - \log(V_a - Nb)\right) \ldots$$

$$= N k_b T \log\left(\frac{V_b - Nb}{V_a - Nb}\right) - aN^2 \int_{V_a}^{V_b} \frac{1}{V^2}$$

$$= N k_b T \log\left(\frac{V_b - Nb}{V_a - Nb}\right) + aN^2 \cancel{\log\left(\frac{V_b}{V_a}\right)} \left(\frac{1}{V_b} - \frac{1}{V_a}\right)$$