2.6.7 temp ab

(a) We propose the following temporary argument: Let the ~~rectangle~~ drawn out between points P_1 and P_2 have side ~~lengths~~ lengths $r\,\partial\theta$ and $r\sin\theta\,\partial\phi$; then by a pseudo-pythagoras, the length across the diagonal

$$L = \sqrt{r^2\,\partial\theta^2 + r^2\sin^2\theta\,\partial\phi^2}$$

(b) We are told to find the Euler-Lagrange equation for

$$J[\phi] = \int_{\theta_1}^{\theta_2} \sqrt{1+\sin^2(\theta)\,(\phi'(\theta))^2}.$$

Recall the Euler-lagrange equation for $f(x, y(x), y'(x))$:

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

Let

$$f(\theta, \phi(\theta), \phi'(\theta)) = \sqrt{1+\sin^2(\theta)\,(\phi'(\theta))^2}$$

then, the Euler-Lagrange equation is

$$\frac{d}{d\theta}\left(\frac{\partial f}{\partial \phi'}\right) = 0$$

since there are no occurences of $\phi(\theta)$ for the formula to take the derivative with respect to. Thus

$$\frac{\partial f}{\partial \phi'} = C$$

We see that

$$\frac{\partial f}{\partial \phi'} = (1+\sin^2(\theta)(\phi'(\theta))^2)^{-1/2} \cdot 2\sin^2(\theta)\,\phi'(\theta)$$

and thus by absorbing the 2 into C,

$$C = \frac{\sin^2(\theta)\,\phi'(\theta)}{\sqrt{1+\sin^2(\theta)(\phi'^2\theta)}}$$