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27.3

(a): We have $f(x) = \sqrt{1+x}$ and $p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$; then write

$$f(0) = p(0) \qquad 1 = a_0$$
$$f'(0) = p'(0) \Rightarrow \quad 1/2 = a_1$$
$$f''(0) = p''(0)$$
$$f'''(0) = p'''(0)$$

$$f''(x) = \left(\frac{1}{2\sqrt{1+x}}\right)' = \frac{-1}{4\sqrt{1+x}^3} \Rightarrow f''(0) = \frac{-1}{4} = 2a_2$$

$$a_2 = \frac{-1}{8}$$

$$f'''(x) = \left(\left(-\frac{1}{4}\right)(1+x)^{-3/2}\right)^2 \Rightarrow f'''(0) = \frac{3}{8} = 6a_3$$
$$= \frac{3}{8}(1+x)^{-5/2}$$

$$\frac{1}{16} = a_3$$

Thus, $f(x)$ near 0

$$f(x) \approx 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$$

(b): We see that

$$\cos(0.1) \approx \cos(0) + \left(1 - \frac{1}{2}0^2\right)0.1$$
$$\approx 1 - \frac{1}{2}(0.01) = 0.995$$

Then the fractional error

$$\Delta = \frac{|0.995004 - 0.995|}{0.995004} = 0.0004\%. \quad \text{: log wow:}$$

(c): It's Fibonacci.