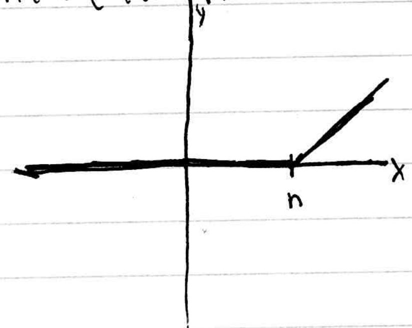
30.1

(a): Let's sketch the sceanario



Ie; we have a line at zero until it connects to a line with x-intercept n. We then see that

$$\lim_{n\to\infty} f_n(x) = f(x) = 0$$

since we can ~~continue to change n t~~ make n large enought hat $n > b \Rightarrow f([a,b]) = 0$; this of course does not apply for the interval $[-\infty, \infty]$, since we can't make n large enough for that rightend point to be handled; ~~so this So~~ Thus $f_n(x)$ has pointwise limit 0 ~~for~~ over any $[a,b]$ s.t $a, b \in \mathbb{R}$. It converges uniformly acoarse -statt as well.

(b): ~~T~~ We see that $\lim (x^n - x^{2n}) = 0 - 0$ for $0 < x < 1$, thus the pointwise limit is 0. ~~Let's find the maximum of~~

~~$|f - f_n|$ on $[0,1]$;~~

~~$\cdots = |0 - f_n| = f_n$ since $f_n > 0$ for all n, then the max occurs where~~

~~$f_n' = 0$; $f_n' = nx^{n-1} - 2nx^{2n-1} = 0$~~

~~$nx^{n-1} = 2nx^{2n-1}$~~

~~$\frac{x^{n-1}}{x^{2n-1}} = 2$~~

~~$\frac{1}{x^n} = 2$; $x^n = \frac{1}{2}$~~

~~$x = \sqrt[n]{\frac{1}{2}} = \frac{1}{2}e^{1/n}$~~

~~Thus the maximum on this interval occurs at~~
~~$x = \frac{1}{2} e^{1/n}$~~

(c): ~~Let $M = x + \frac{1}{2}$ [illegible] $(1-x)$. Then notice that $x < M \Rightarrow x^n = f_n(x) < M^n$; Since $M < 1$, then M is a convergent geometric series. Then conclude that since $x < M$, then~~

Let's try writing this again for mental clarity; We have some

$f_n(x) = x^n$

and we want to ~~show it converges~~ show

$\sum_{n=1}^{\infty} x^n$ converges for $x \in (-1, 1)$ and thus $(-\rho, \rho]$ for $0 \le \rho < 1$.

Let $M = x + 1/2(1-x) = \frac{1}{2}x + \frac{1}{2}$; then for $|x| \le 1$, $x < M \Rightarrow x^n < M^n$.

Since $M^n < 1$ if $x < 1$, then M^n ~~can~~ can represent a convergent geometric series in the argument by Weierstrass that since $f_n(x) < M_n$, ~~then~~ and M_n is convergent, then the series $\sum_{n=0}^{\infty} x^n$ converges for $x \in (-1, 1)$, and thus for ~~ρ~~ $(-\rho, \rho)$ s.t. ~~ρ~~ $0 \le \rho < 1$.

(a): See in what remarks of the original (b) why the pointwise limit goes to zero; now we want to demonstrate that

$f_n(x) = x^n - x^{2n}$ converges uniformly.

The hint tells us to begin by finding the maximum of $|f(x) - f_n(x)|$ on $[0,1]$.

Let's do so by ~~to~~ first noting that the maximum occurs where the derivative is zero. See that $f(x) = 0$ by the pointwise argument, then

$$\frac{\partial}{\partial x}\left|0 - f_n(x)\right| = \frac{\partial}{\partial x}|f_n(x)|$$

then since $f_n(x)$ is strictly non-negative (small number - smaller number),

$$\ldots = \frac{\partial}{\partial x} f_n(x) = n x^{n-1} - 2n x^{2n-1} \overset{!}{=} 0$$

$$n x^{n-1} = 2n x^{2n-1}$$

$$\frac{x^{n-1}}{x^{2n-1}} = 2$$

$(n-1) + (-2n+1) = -n$;

$$\frac{1}{x^n} = 2$$

$$x = \frac{1}{\sqrt[n]{2}}$$

Then plug into

$$f\left(\frac{1}{\sqrt[n]{2}}\right) = \left(\left(\frac{1}{2}\right)^{1/n}\right)^n - \left(\left(\frac{1}{2}\right)^{1/n}\right)^{2n} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

The pointwise limit of zero cannot possibly be made ε-close to this 1/4 jump that must occur, so ~~to~~ $f_n(x)$ does not converge uniformly.