question 2 views

Daily Challenge 13.3

~~**(Due: Wednesday 9/29 at 12:00 noon eastern)**~~
(**Due: Sunday 9/2 at 12:00 noon eastern**)

Another CD 3 problem, but this one is more computational

(1) Problem: tangent lines.

(a) Find the equation of the tangent line to the graph $y = (x-1)^3 + 2$ at the point $(3, 10)$.

(b) Let $f(x) = x^3 + ax + b$, with $a \neq b$, and suppose that the tangent lines to the graph of f at $x = a$ and at $x = b$ are parallel. What is $f(1)$?

daily_challenge

Updated 7 months ago by Christian Ferko

the instructors' answer, *where instructors collectively construct a single answer*

(a) Differentiating, we have $y'(x) = 3(x-1)^2$, so the tangent line slope at $x = 3$ is $y'(3) = 3(3-1)^2 = 12$. Meanwhile, the line with slope 12 passing through $(3, 10)$ is just $g(x) = 12x - 26$.

(b) If the tangent lines to f at a and b are parallel, then $f'(a) = f'(b)$. But we see $f'(x) = 3x^2 + a$, so this means

$$3a^2 + a = 3b^2 + a,$$

which implies $a = \pm b$. The problem statement says that $a \neq b$, so we must have $a = -b$. Then this means $f(1) = 1^3 + a + b = 1$.

Updated 7 months ago by Christian Ferko

followup discussions *for lingering questions and comments*