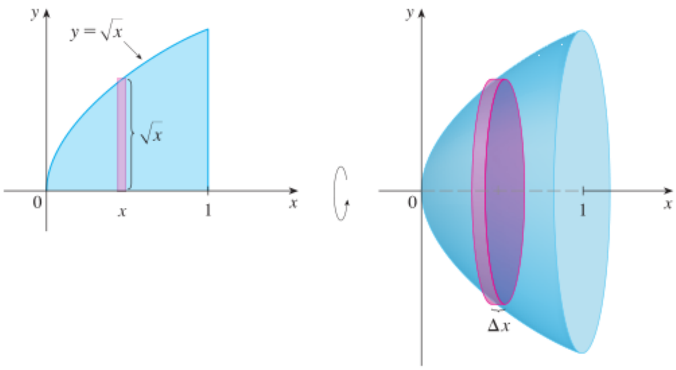
question 2 views

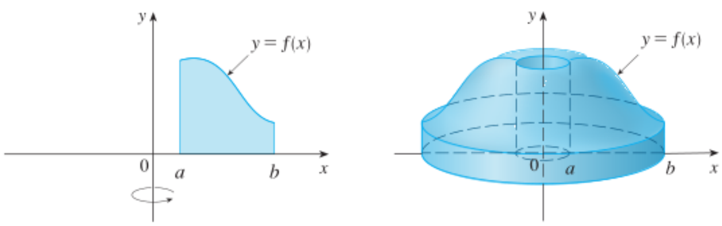
Daily Challenge 21.7

(**Due: Monday 2/18 at 12:00 noon Eastern)**

So far, we've seen two ways to compute volumes of solids. One way is to slice into discs with volumes $A_{\text{disc}} = \pi r_{\text{disc}}^2$, as in this picture:



Another is to slice the region into cylindrical shells with areas $A_{\text{shell}} = 2\pi r_{\text{shell}} h_{\text{shell}}$ as below:



Today we'll see a slight variant of the disc method which involves rings.

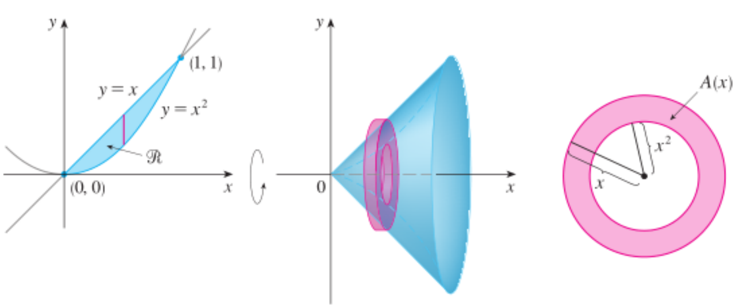
(1) Volumes involving rings.

Some regions have a cross-section with the shape of an annulus, or ring, rather than a solid disk. An annulus with outer radius R and inner radius r has area

$$A_{\text{annulus}} = \pi(R^2 - r^2),$$

which you can see by finding the area of the outer disc and subtracting the area of the inner disc.

For example, suppose we want to find the volume of the region obtained by taking the area between $y = x$ and $y = x^2$, then rotating about the x axis.



As we see from the figure, each cross-section is an annulus with outer radius x and inner radius x^2:

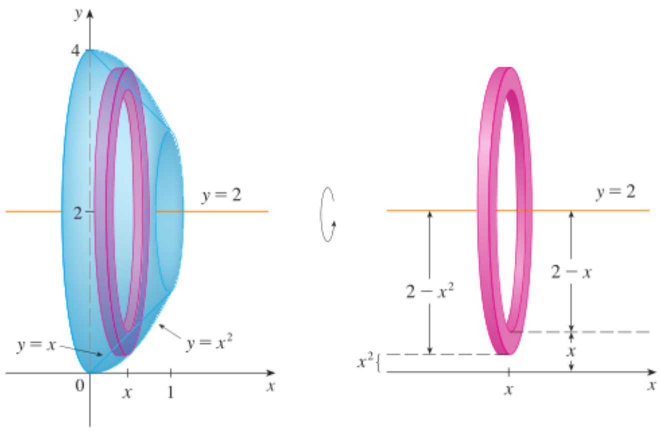
$$A(x) = \pi x^2 - \pi\left(x^2\right)^2 = \pi(x^2 - x^4).$$

To find the volume of the region, we integrate the areas of the annuli from the left intersection point $x = 0$ to the right intersection point $x = 1$:

$$
\begin{aligned}
V &= \int_0^1 \pi(x^2 - x^4)\,dx \\
&= \pi\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 \\
&= \frac{2\pi}{15}.
\end{aligned}
$$

(2) Problem: rotating about a different line.

Consider the same area as in our example above, namely the area between $y = x$ and $y = x^2$. This time, rotate the area about the line $y = 2$ rather than the x axis, as in the following figure:



Find the volume of the resulting solid. Make sure your computation yields $\frac{8\pi}{15}$. Answer in Overleaf.

daily_challenge

Updated 1 month ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

Sent pdf of work and completed on overleaf.

Updated 1 month ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

Enter your solutions in Overleaf here: https://www.overleaf.com/1231232126rckrscxfchyf

Updated 1 month ago by Christian Ferko

followup discussions *for lingering questions and comments*