question 2 views

Daily Challenge 19.7

(**Due: Tuesday November 27 at 12:00 noon Eastern**)

In tonight's meeting we'll learn about integration by parts and some other techniques for finding anti-derivatives. This process is a bit computational, so it can feel boring, but it requires more ingenuity than taking derivatives: integration is more of an art form.

Like differentiation, however, the only way to learn is to grind through a lot of examples.

(1) Problem: practicing integration by parts.

Integrate by parts to evaluate each of the following.

(a) $\int x\sqrt{x+1}\,dx$. [Hint: $f=x$, $g'=\sqrt{x+1}$].

(b) $\int x^2e^x\;dx$. [Hint: integrate by parts twice. In the first step, let $f=x^2$ and $g'=e^x$. Proceed]

(c) $\int \sec^3(x)\;dx$. This is a famous and important integral. Try to integrate by parts in a way which expresses the integral in terms of itself (the "recursive definition trick"). You may use without proof that $\int \sec(x)\;dx = \log(\tan(x)+\sec(x))$.

If you aren't able to solve this yourself, look up the solution and rewrite it in your own words.

daily_challenge

Updated 4 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

(a) As the hint suggests, we let $f=x$ and $g'=\sqrt{x+1}$, and thus $g=\frac{2}{3}(x+1)^{\frac{3}{2}}$; We then see that $\int x\sqrt{x+1}\;dx = \frac{2x}{3}(x+1)^{\frac{3}{2}} - \int \frac{2}{3}(x+1)^{\frac{3}{2}}$. We then see by an inverse power rule that overall $\int x\sqrt{x+1}\;dx = \frac{2x}{3}(x+1)^{\frac{3}{2}} - \frac{4}{15}(x+1)^{\frac{5}{2}}$.

(b) We see by integrating by parts, once again taking a hint and letting $f=x^2$ and $g'=e^x$, and thus $g=e^x$; We see that $\int x^2e^x\;dx = x^2e^x - \int 2xe^x$. We then apply integration by parts again, where $f=2x$ and $g=e^x$ to see that $\int x^2e^x\;dx = x^2e^x - 2xe^x + 2e^x$.

(c) At first I believed there was no way to write this as multiplication, then I remembered the definition of power. We see that we can write the original integral as $\int sec(x)\cdot\sec^2(x)dx$. We let $\sec^2(x)=g'$ and we let $\sec(x)=f$. I will have to scan my work in once parents leave, but the result through a serious of steps as gorgeous as beana is $\int \sec^3(x) = \frac{\sec(x)\tan(x)+\log(tan(x)+\sec(x)}{2}$.

log_19.7.jpg

Updated 2 months ago by Christian Ferko and Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

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