25.3

(a) We have the prior PDF

$$f_P(p) = \frac{p^{h_0}(1-p)^{t_0}}{B(h_0+1, t_0+1)} \qquad (1)$$

then by the continuous on discrete form of the Bayes theorem, which states

$$f_P(p \mid H=h') = \frac{f_P(p) \cdot P(H=h' \mid p)}{P(H=h')}$$

we see by (1) the value of $f_P(p)$; and that

$$P(H=h' \mid p) = \binom{N'}{h'} p^{h'}(1-p)^{t'}$$

where $N' = h' + t'$, by the Bernoulli distribution ~~formula~~, and the denominator, by the law of total probability,

$$P(H=h') = \int_0^1 P(H=h' \mid q)\, f_P(q)\, dq$$

which is the product of a Bernoulli distribution and a beta distribution. Substitute such terms

$$P(H=h') = \int_0^1 \binom{N'}{h'} q^{h'}(1-q)^{t'} \frac{q^{h_0}(1-q)^{t_0}}{B(h_0+1, t_0+1)}\, dq$$

then by substituting in,

$$f_P(p \mid H=h') = \frac{\binom{N'}{h'} p^{h'}(1-p)^{t'} \cdot p^{h_0}(1-p)^{t_0}}{B(h_0+1, t_0+1)\int_0^1 \binom{N'}{h'} q^{h'}(1-q)^{t'} \frac{q^{h_0}(1-q)^{t_0}}{B(h_0+1, t_0+1)}\, dq}$$

cancel terms (and simplify)

$$\ldots = \frac{p^{h'}(1-p)^{t'} \cdot p^{h_0}(1-p)^{t_0}}{\int_0^1 q^{h'}(1-q)^{t'} \cdot q^{h_0}(1-q)^{t_0}} = \frac{p^{h'+h_0}(1-p)^{t'+t_0}}{B(h'+h_0+1, t'+t_0+1)}$$