question 2 views

Daily Challenge 17.6

(**Due: Saturday 11/10 at 12:00 noon Eastern**)

Today I've chosen a short problem so we don't fall behind. If this takes you more than 5 minutes, ask for help.

(1) Problem: an x under the integral.

Let $F(x)$ be given by

$$F(x) = \int_0^x x \cdot f(t)\, dt,$$

where f is some integrable function. Find $F'(x)$.

[Hint: the answer is *not* $xf(x)$. You should perform an obvious manipulation on the integral before trying to find F'. It is crucial to recognize a constant when you see it.]

daily_challenge

Updated 5 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

The described integral is interested in some changing t, and therefore x is a scalar/constant within this integral and we can "pull" it out;
$F(x) = \int_0^x \ x \cdot f(t) = x \int_0^x \ f(t)$.
We then take the derivative of each side and apply the FTC to see that
$F'(x) = \int_0^x f(t)\, dt + x \cdot f(x)$.

Updated 5 months ago by Christian Ferko and Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

The integral runs over the variable t, so x is a constant with respect to the integral. Thus

$$F(x) = x \int_0^x f(t)\, dt.$$

Now take the derivative of each side and apply the product rule,

$$F'(x) = \left(\int_0^x f(t)\, dt \right) + x \frac{d}{dx} \left(\int_0^x f(t)\, dt \right).$$

By the fundamental theorem of calculus, we conclude

$$F'(x) = \left(\int_0^x f(t)\, dt \right) + x f(x).$$

Updated 5 months ago by Christian Ferko

followup discussions *for lingering questions and comments*