question 2 views

Daily Challenge 5.5

(**Due: Saturday 5/26 at 12:00 noon**)

I will post a short daily challenge (really just an exercise) to leave time for make-up work.

Review

It will be useful for us to have a few adjectives for describing functions with certain properties. Today I will introduce three.

Definition. Let f be a function.

- We say that f is **periodic** if there exists a positive real number k such that $f(x) = f(x+k)$ for all $x \in \mathrm{Dom}(f)$. The smallest such k, if it exists, is called the **period** of f.
- We say that f is **strictly increasing** if, whenever $x > y$, we have $f(x) > f(y)$.
- We say that f is **strictly decreasing** if, whenever $x > y$, we have $f(x) < f(y)$.

Problem

Answer the following questions:

1. Let $f(x) = \sin(2\pi x)$. Is f periodic, by our above definition? What is its period?
2. Let $g(x) = 1$. Is g periodic, by our above definition (be careful!)? Why or why not? If so, does it have a period?
3. Let $h(x) = 3x^3$. Is h periodic? Strictly increasing? Strictly decreasing?

daily_challenge

Updated 10 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

Logan Pachulski:

1. Yes, it has a period of 1
2. No, because the distance between the "same x values" as I understand it is 0.
3. No, strictly increasing.

Submitted at 9:27.

And damnit, just saw the instructor answer

Updated 10 months ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

My answers follow.

1. Let $f(x) = \sin(2\pi x)$. Is f periodic, by our above definition? What is its period?

   Answer. Yes, $f(x)$ is periodic. We recall that $\sin(\theta + 2\pi) = \sin(\theta)$ for all $\theta \in \mathbb{R}$, since adding an angle of 2π just wraps around the circle by one complete revolution. In this function $f(x) = \sin(2\pi x)$, the argument comes multiplied by 2π. Therefore, the period will be $k = 1$, since

   $$\begin{aligned} f(x+1) &= \sin(2\pi(x+1)) = \sin(2\pi x + 2\pi) \\ &= \sin(2\pi x) \\ &= f(x) \end{aligned}$$

   for all $x \in \mathbb{R}$.

2. Let $g(x) = 1$. Is g periodic, by our above definition (be careful!)? Why or why not? If so, does it have a period?

   Answer. Yes! This function $g(x)$ is periodic since, for *any* real number k, it is true that $g(x+k) = g(x)$, since both the left side and the right side equal 1. This means that it satisfies our definition of "periodic".

   However, g does not have a period because there is no *smallest* number k for which this is true. Any value of k at all will do.

3. Let $h(x) = 3x^3$. Is h periodic? Strictly increasing? Strictly decreasing?

   Answer. This function $h(x)$ is *not* periodic (in fact, it passes the horizontal line test, so it cannot be periodic), nor is it strictly decreasing. It *is* strictly increasing. To see this, note that if $a > b$, then $3a^3 > 3b^3$, so $h(a) > h(b)$. One can also see this by thinking about the graph of $h(x) = 3x^3$, which always goes upward as we move to the right.

Updated 10 months ago by Christian Ferko

followup discussions *for lingering questions and comments*