7/16/2019

35.1

(a): The written solution first proposes we make the educated ~~guess~~ guess

$$x = A\cos(5t)$$

$$\dot{x} = -A5\sin(5t).$$

$$\ddot{x} = -25A\cos(5t),$$

then by plugging in,

$$-25A \cancel{+9} \cos(5t) + 9A\cos(5t) = 80\cos(5t)$$

$$-25A + 9A = 80$$

$$A = -5,$$

thus we have ~~general~~ particular solution

$$x = -5\cos(5t)$$

To find the homogenous solutions, ~~sets part~~ set parts dependent on t without x to zero;

$$\ddot{x}_h + 9x_h = 0$$

make the exponential ansatz

$$x_h = e^{rt};$$

$$r^2e^{rt} + 9e^{rt} = 0$$

by the quadratic equation,

$$r = \frac{\pm\sqrt{-4\cdot1\cdot9}}{2} = \pm i3$$

and thus we have homogenous solution

$$x_h t = C_1e^{3it} + C_2e^{-3it}$$

And thus by summing particular and homogenous solutions (and applying Euler's formula), we get general solution

$$x(t) = C_1 \cos(3t) + C_2 \sin(3t) - 5\cos(5t)$$

We see that

$$x(0) = 0 \Rightarrow 0 = C_1 - 5 \Rightarrow C_1 = 5,$$

and

$$\dot{x}(0) = 0 \Rightarrow C_2 = 0;$$

thus

$$x(t) = 5\cos(3t) - 5\cos(5t).$$

(5)i Make the ansatz

$$x = At^2 + Bt + C$$

$$\dot{x} = 2At + B$$

$$\ddot{x} = 2A$$

Then,

$$2A + 2 \cdot 2At + 2B + At^2 + Bt + C = t$$

implies $A = 0$ since there are no t^2 terms on the right, thus

$$2B + Bt + C = t,$$

then, $B = 1$ and thus $C = -2$. We then have particular solution

$$x_p = t - 2.$$

For the homogenous solution, make the exponential ansatz

$$x_h = e^{rt},$$

then,

$$r^2 e^{rt} + 2r e^{rt} + e^{rt} = 0$$

$$r^2 + 2r + 1 = 0$$

implies

$$r = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 1}}{2} = -1, -1.$$

Since there is a duplicate solution, it must be true that

$$x_h = te^{rt}$$

is also a homogenous solution. We can then write the general solution

$$x(t) = t - 2 + C_1 e^{-t} + C_2 t e^{-t}.$$

$$x(0) = 1 = -2 + C_1 1 \cancel{+ C_2 te^{-t}}$$

$$C_1 = 3,$$

$$\cancel{\dot{x}(0) = 1 = 1 - 2}$$

Then, $$\dot{x}(t) = 1 + -C_1 e^{-t} + C_2 e^{-t} + -C_2 te^{-t}$$

$$1 = \dot{x}(0) = 1 - 3 + C_2$$

$$C_2 = 3$$

Thus we have general solution

$$x(t) = t - 2 + 3e^{-t} + 3te^{-t}$$

$$= t - 2 + 3e^{-t}(1 + t)$$