23.2

(a) Begin with the definition

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt$$

then substitute $s = 1-t$; $t = 1-s$

$$dt = -ds$$

$$= \int_1^0 -(1-s)^{x-1} s^{y-1}\,ds$$

$$= \int_0^1 (1-s)^{x-1} s^{y-1} = B(y,x)$$

(b) Begin with $B(x,y) = \dfrac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$

$$\frac{d}{dx} B(x,y) = \cancel{\frac{\Gamma'(x)\Gamma(y)}{\Gamma(x+y)^2} \cdot \frac{\Gamma(x)}{\Gamma(x)}}$$

$$\cancel{= \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \cdot \frac{\Gamma'(x)}{\Gamma(x)} \cdot \frac{1}{\Gamma(x+y)}}$$

$$\cancel{= B(x,y) \cdot \psi(x+y)}$$

$$\cancel{= \frac{\psi(x)\Gamma(x)\Gamma(y)}{\Gamma(x+y)^2}}$$

$$\cancel{= B(x,y) \cdot \frac{\psi(x)}{\Gamma(x+y)}}$$

$$\frac{\psi(x)\Gamma(x)\Gamma(y) - \Gamma(x)\Gamma(y) \cdots \psi(x+y)}{\Gamma(x+y)} = \frac{\Gamma'(x)\Gamma(y)\Gamma(x+y) - \Gamma(x)\Gamma(y)\Gamma'(x+y)}{\Gamma(x+y)^2}$$

$$B(x,y)\cdot(\psi(x) - \psi(x+y)) = \frac{\Gamma(x)\Gamma(y)(\psi(x) - \psi(x+y))}{\Gamma(x+y)}$$