

question

2 views

Daily Challenge 5.3

(Due: Thursday 5/24 at 12:00 noon Eastern)

As we build skill reading proofs, it will be useful to grade some supposed "proofs", by which I mean we will read a potential proof (which may or may not be correct) and determine whether it is logically valid.

Review

I have recorded two videos grading example "proofs", and a third video which introduces another "proof" which you will grade in the problem.

The first video:



The second video:



Problem

Watch the [third video](#), which introduces a "proof" for you to grade.



Then answer the following questions:

1. Which step of the "proof" in the first video was wrong, and why?
2. The second video was a proof by contrapositive. Write the contrapositive of the statement "if t is irrational, then $5t$ is irrational."
3. Grade the proof in the third video. If it is totally correct, choose a couple of deductions in the proof and explain why they are true. If it is partially correct but has some errors or missing steps, explain what is wrong. If it is totally incorrect, point out the logical flaws.

daily_challenge

Updated 10 months ago by Christian Ferko

the students' answer, where students collectively construct a single answer

Logan Pachulski:

1. The incorrect step of this first proof would be: "Since $x \in B$, $x \in A$ by definition of subset. This is incorrect because it was previously implied that $A \subseteq B$, and it is not guaranteed that all elements of B are present in A .
2. "If $5t$ is rational, then t is rational."
3. To my knowledge, this proof is worthy of an A for correct logic and deductions. There is very little to go depth into about, but examples include the logical conclusion of "If $x \in A \cap B$, then $x \in A$ and $x \in B$ " due concept of the "and" statement in intersection; this correct thought process also applies to the inverse seen later in the problem. In a related manner, the author correctly used the concept of **universal quantification**. (lol, submitted at 9:21 on the nose)
4. lol
5. even more lol

Updated 10 months ago by Logan Pachulski

the instructors' answer, where instructors collectively construct a single answer

My answers follow.

1. Which step of the "proof" in the first video was wrong, and why?

Answer. The error occurs in the sentence "Since $x \in B$, $x \in A$, by the definition of subset." This is not true! We have assumed $A \subseteq B$, which means every $x \in A$ also satisfies $x \in B$, but in this sentence the implication is reversed -- the author incorrectly claims that, for every $x \in B$, we have $x \in A$.

2. The second video was a proof by contrapositive. Write the contrapositive of the statement "if t is irrational, then $5t$ is irrational."

Answer. The original statement, "if t is irrational, then $5t$ is irrational", is of the form $p \implies q$ where p is the statement " t is irrational" and q is the statement " $5t$ is irrational". The contrapositive is $(\text{not } p) \implies (\text{not } q)$, or "if $5t$ is rational, then t is rational".

3. Grade the proof in the third video. If it is totally correct, choose a couple of deductions in the proof and explain why they are true. If it is partially correct but has some errors or missing steps, explain what is wrong. If it is totally incorrect, point out the logical flaws.

Answer. I would give the proof partial credit. It is mostly correct, but the sentence "This shows that $A \cap B = A$ " is incorrect. In the previous sentence, the author showed that $A \cap B \subseteq A$. To show that the two sets are equal, one must *also* show that $A \subseteq (A \cap B)$. This part of the proof is missing, so the proof is not logically valid. If an extra sentence establishing $A \subseteq (A \cap B)$ were added, the proof would be totally correct.

Updated 10 months ago by Christian Ferko

followup discussions for lingering questions and comments