24.6

For the third time;

$$(f * f)(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp\left(\frac{-x^2}{2}\right) \exp\left(-\frac{(x-z)^2}{2}\right) dx$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp\left(\frac{-x^2 + (-x^2 + 2zx - z^2)}{2}\right) dx$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp\left(\frac{-2x^2 + 2zx - z^2}{2}\right) dx$$

$$= \text{''} \quad \exp\left(-x^2 + zx - \frac{z^2}{4} - \frac{z^2}{4}\right) dx$$

$$= \text{''} \quad \exp\left(-\left(x^2 - zx + \frac{z^2}{4}\right)\right) \exp\left(\frac{-z^2}{4}\right) dx$$

$$= \text{''} \quad \exp\left(-\left(x - \frac{z}{2}\right)^2\right) \exp\left(\frac{-z^2}{4}\right) dx$$

$$= \frac{1}{2\pi} \exp\left(-\frac{z^2}{4}\right) \int_{-\infty}^{\infty} \exp\left(-\left(x - \frac{z}{2}\right)^2\right) dx$$

We then u-sub $u = x - \frac{z}{2} \Rightarrow du = dx$

$$= \frac{1}{2\pi} \exp\left(-\frac{z^2}{4}\right) \int_{-\infty}^{\infty} \exp(-u^2)\, dx$$

$$= \frac{1}{2\pi} \cdot \sqrt{\pi} \exp\left(\frac{-z^2}{4}\right)$$

$$= \frac{1}{2\sqrt{\pi}} \exp\left(\frac{-z^2}{4}\right)$$