30.6

Begin with what the problem statement gives:

$$\log(1+x) = \int_0^x \frac{1}{1+u}\,du$$

We see that

$$\frac{1}{1+u} = \sum_{n=1}^{\infty} (-u)^n \quad \text{for } u<1 \qquad (1)$$

Then, assuming we are in the radius of convergence $(R=1, \text{by } (1))$ at all times,

$$\ldots = \int_0^x \sum_{n=1}^{\infty} (-u)^n$$

$$= \sum_{n=1}^{\infty} \frac{(-x)^{n+1}}{n}$$

We then see that, for $x=1$,

$$\cancel{\log(1+1) = \sum_{n=1}^{\infty} \frac{1^{1+1}}{1} + \frac{1}{2} + \frac{1}{3} \ldots}$$

$$\log(2) = \cancel{1 + \frac{1}{2} + \frac{1}{3}} = 1 + \frac{-1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$$