

28.7

Apply the limit comparison test: evaluate  $\lim_{n \rightarrow \infty} \frac{a(n)}{b(n)}$  with  $b(n) = \frac{1}{n}$ ,  $a(n) = \frac{1}{n^{n+1/2}}$ :

$$\lim_{n \rightarrow \infty} \frac{n^{n+1/2}}{n} = \lim_{n \rightarrow \infty} \frac{n \cdot n^{1/2}}{n} = \lim_{n \rightarrow \infty} n^{1/2}$$

Let some  $y = n^{1/2}$ ; then

$$\log(y) = \frac{\log(n)}{2} \quad \text{and} \quad \lim_{n \rightarrow \infty} \frac{\log(n)}{n} = 0 = \lim_{n \rightarrow \infty} (\log(y))$$

Thus,

$$\lim_{n \rightarrow \infty} (\log(y)) = 0 \Rightarrow \lim_{n \rightarrow \infty} (y) = 1 \text{ by power intuition}$$

Since the ratio  $r \neq 0$  and  $\frac{1}{n}$  diverges, then

$$\sum_{n=1}^{\infty} \frac{1}{n^{n+1/2}} \text{ diverges.}$$