question 2 views

Daily Challenge 26.6

(**Due: Thursday 4/11 at 12:00 noon Eastern**)

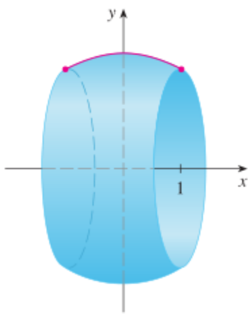
In session 59 we showed that integrals can be used to find the surface areas of surfaces of revolution by slicing into tiny cylindrical strips, just as we found volumes earlier by slicing into discs or shells.

(**Part a**) Re-read slides 12-15 in session 59 and quickly skim this Wikipedia section to refresh your memory.

Write down the two formulas for the surface areas of shapes obtained by rotating a function $f(x)$ between a and b about (1) the x axis, and (2) the y-axis.

[Hint: both involve an integral from a to b of an expression multiplying the arc length element $\sqrt{1+(f'(x))^2}$, which is roughly the "height" of the tiny cylinders. In one formula we multiply by $f(x)$ and in the other we multiply by x, depending on whether the radius of the tiny cylinders is y or x.]

(**Part b**) The curve $y=\sqrt{4-x^2}$, $-1 \leq x \leq 1$, is an arc of the circle $x^2+y^2=4$. Find the surface area of the shape obtained by rotating this arc about the x-axis. (The surface is a portion of a sphere of radius 2.)



Make sure you find a surface area of 8π.

(**Part c**) Suppose you want to find the function $f(x)$ which gives the *smallest possible surface area* when you rotate it to form a surface of revolution as above. The shape that you obtain by rotating this special curve $f(x)$ is called a minimal surface.

For simplicity, say we rotate $f(x)$ about the y-axis between $-\ell$ and ℓ. By the formula you wrote down in part (a), the surface area is

$$A[f]=2\pi\int_{-\ell}^{\ell} f(x)\sqrt{1+(f'(x))^2}\,dx.$$

Without doing any work at all, write down a function $f(x)$ which gives the smallest possible surface area $A[f]$.

[Hint/Solution: Compare this functional $A[f]$ to the functional you minimized in the catenary problem yesterday. You've solved this exact problem before so you can simply quote the result. The answer is that the minimal surface of revolution is achieved for $f(x)=a\cosh\left(\frac{x-b}{a}\right)$, where a and b are constants.]

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Updated 5 days ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

yeet

Updated 2 days ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

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