4/9/2019

~~We~~ 26.4

We see by splitting the integral that

$$F(tri) = \int_{-1}^{0} e^{-ikx}(x+1)\,dx + \int_{0}^{1} e^{-ikx}(1-x)$$

$$= \left(\frac{e^{-ik}-e^{+ik}}{-ik}\right) + \int_{-1}^{0} x e^{-ikx}\,dx - \int_{0}^{1} x e^{-ikx}\,dx$$

We see by integration by parts (see page 2) that

$$\int_a^b x e^{-ikx} = \left[\frac{xe^{-ikx}}{-ik}\right]_a^b - \frac{1}{-ik}\cdot\left[\frac{e^{-ikx}}{-ik}\right]_a^b,$$

Thus,

$$F(tri) = \left(\frac{e^{-ik}-e^{+ik}}{-ik}\right) + \left(\frac{e^{ik}}{-ik} - \frac{1}{-ik}\left(\frac{1}{-ik} - \frac{e^{ik}}{-ik}\right)\right)$$

$$- \left(\frac{e^{-ik}}{-ik} - \frac{1}{-ik}\left(\frac{e^{-ik}}{-ik} - \frac{1}{-ik}\right)\right)$$

We see that the ~~first~~ second term

$$\left(\frac{e^{+ik}}{-ik} - \frac{1}{-ik}\left(\frac{1}{-ik} - \frac{e^{ik}}{-ik}\right)\right) = \cancel{\left(\frac{e^{-ik}}{-ik} + \left(\frac{-1}{-k^2} + \frac{e^{ik}}{-k^2}\right)\right)} = \left(\frac{e^{+ik}}{-ik} - \left(\frac{1}{-k^2} - \frac{e^{ik}}{-k^2}\right.\right.$$

~~Then,~~

$$\cancel{\frac{e^{-ik}}{-ik}\cdot\frac{-ik}{-ik} = \frac{-ike^{-ik}}{-k^2}}$$

$$\cancel{\frac{-ike^{-ik}}{-k^2} - \frac{1}{-k^2} + \frac{e^{ik}}{-k^2}}$$

And the third term

$$\left(\frac{e^{-ik}}{-ik} - \frac{1}{-ik}\left(\frac{e^{-ik}}{-ik} - \frac{1}{-ik}\right)\right) = \left(\frac{e^{-ik}}{-ik} - \frac{e^{-ik}}{-k^2} + \frac{1}{-k^2}\right)$$

The first terms of the second and third terms cancel with the

$$\left(\frac{e^{-ik}-e^{ik}}{-ik}\right),$$

then

$$F(tri) = \frac{1}{k^2} - \frac{e^{ik}}{k^2} - \frac{e^{-ik}}{k^2} + \frac{1}{k^2} = \frac{2-(e^{ik}+e^{-ik})}{k^2}$$

$$= \frac{2-2\cos(k)}{k^2} = \frac{2-(2-4\sin^2(k/2))}{k^2} = \frac{4\sin^2(k/2)}{k^2}$$

$$= \operatorname{sinc}^2\left(\frac{k}{2}\right)$$

26.4 Integration by parts

$\int_a^b x e^{-ikx} \, dx$

Let

$a' = e^{-ikx}$

$a = e^{-ikx} / -ik,$

$b = x,$

$b' = 1.$

Then,

$$\dots = \left[\frac{x e^{-ikx}}{-ik} \right]_a^b - \int_a^b \frac{e^{-ikx}}{-ik} \, dx$$

$$= \quad \text{''} \quad - \frac{1}{-ik} \int_a^b e^{-ikx} \, dx$$

$$= \quad \text{''} \quad - \frac{1}{-ik} \cdot \left[\frac{e^{-ikx}}{-ik} \right]_a^b$$