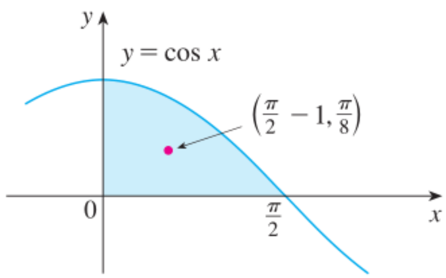
question 2 views

Daily Challenge 24.1

(**Due: Sunday 3/10 at 12:00 noon Eastern**)

(a) A thin mass is cut out in the shape of the region bounded by the curve $y = \cos(x)$, $y = 0$, $x = 0$, and $x = \frac{\pi}{2}$, as shown below.



Assume the region has **constant** density σ in units of mass per area. Thus the total mass of the region is

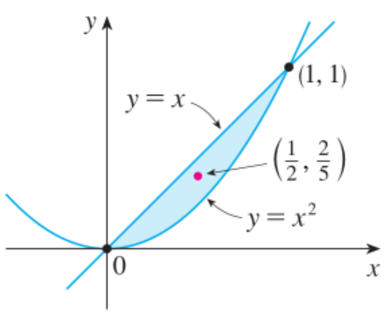
$$M = \sigma A$$

where A is the area under the curve. Likewise, the x coordinate of the center of mass is

$$\begin{aligned} x_{\text{com}} &= \frac{1}{M}\int_0^{\pi/2} x\,\sigma\, f(x)\, dx \\ &= \frac{1}{A}\int_0^{\pi/2} x\cos(x)\, dx \end{aligned}$$

Do the two integrals (first the integral definining the area A under the curve, and second the integral appearing in x_{com}). Show that the x-coordinate of the center of mass occurs at $x_{\text{com}} = \frac{\pi}{2} - 1$.

(b) Find only the x coordinate of the center of mass for a region bounded by the line $y = x$ and the parabola $y = x^2$, as shown below.



Again assume constant density σ. Use the same strategy: find the total mass $M = \sigma A$ by doing the integral that defines A, then compute

$$\begin{aligned} x_{\text{com}} &= \frac{1}{M}\int_0^{1} x(f(x) - g(x))\, dx \\ &= \frac{1}{A}\int_0^{1} x(x - x^2)\, dx \end{aligned}$$

where $f(x)$ is the upper curve $y = x$ and $g(x)$ is the lower curve $y = x^2$.

Show that $x_{\text{com}} = \frac{1}{2}$, as claimed in the figure.

daily_challenge

Updated 1 month ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

green boi

~ An instructor (Christian Ferko) endorsed this answer ~

Updated 1 month ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

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