question 2 views

Daily Challenge 13.5

(**Due: Tuesday 9/4 at 12:00 noon Eastern**)

Last problem on CD 3 (question 5), and it's fairly short! Copy over when you're done.

(1) Problem: splitting the identity.

Prove that it is impossible to write $x = f(x)g(x)$ for two functions f and g which are differentiable and satisfy $f(0) = g(0) = 0$.

Hint: Differentiate.

daily_challenge

Updated 7 months ago by Christian Ferko

the instructors' answer, *where instructors collectively construct a single answer*

Proof. Suppose by way of contradiction that we could write $x = f(x)g(x)$ for two differentiable functions f and g with $f(0) = 0 = g(0)$. Then we could differentiate both sides to find

$$1 = f'(x)g(x) + f(x)g'(x).$$

But plugging in $x = 0$ to this equation yields

$$1 = f'(0)g(0) + f(0)g'(0) = 0,$$

since $f(0) = 0 = g(0)$. Thus we have arrived at the contradiction $1 = 0$, so it must have been impossible to write the identity function x as a product of two functions f and g in this way. □

Updated 7 months ago by Christian Ferko

followup discussions *for lingering questions and comments*