30.2 a

(a): First the pointwise argument; we see that

for $0 \leq x < \frac{\pi}{2}$, $\sin(x) < 1 \Rightarrow \lim_{n\to\infty} \sin^n(x) = 0$

thus the pointwise limit is $f(x) = 0$. We can prove uniform convergence by showing that for $n >$ some N, we can make

$\sin^n(x) <$ ~~som~~ given ϵ.

To do this, we let M, a term ~~grea~~ we desire to be greater than $\sin^n(x)$, equal $\sin(a)$; then since ~~$M \in$~~ $0 < m < 1$, then

$$\lim_{n\to\infty} m^n = 0$$

We ~~see that since let~~ choose some N such that

$$n > N \Rightarrow m^n < \epsilon$$

We have that

$\sin(x) < \sin(a)$ by sine being strictly increasing on $[0, \pi/2]$,

then by making N large enough, we can have that

$$\sin^n(x) < m^n = \sin^n(a) < \epsilon.$$

This argument of course does not apply for the range $[0, \pi/2]$, since the pointwise limit jumps up discontinuously at $\pi/2$ to 1, thus $\sin^n(x)$ is not convergent uniformly ~~at $\pi/2$~~ when we include $\pi/2$.