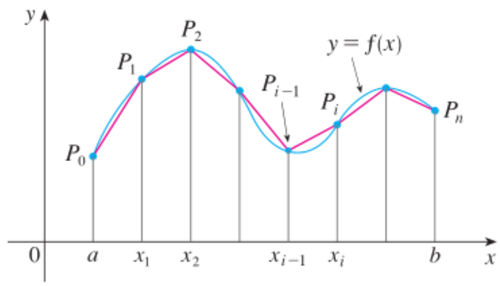
question 2 views

Daily Challenge 22.1

(**Due: Tuesday 2/19 at 12:00 noon Eastern**)

In session 50, we give a definition of the length of a curve by chopping it into piecewise-linear segments and then taking a supremum.



In the case where the curve is differentiable, we proved that the result can be written as

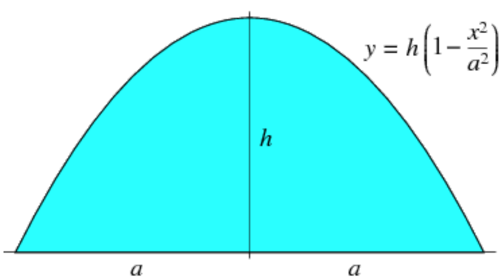
$$L(f, [a, b]) = \int_a^b \sqrt{1 + (f'(x))^2}\, dx.$$

(1) Problem: a parabolic segment.

Using the formula for the arc length of a differentiable curve that we derived in session 50, find the length of the parabolic curve

$$y = h\left(1 - \frac{x^2}{a^2}\right)$$

between $x = -a$ and $x = a$. That is, find the length of the top part of the curve in the figure below:



~~Hint: you will get an integrand involving~~ $\sqrt{1 + Bx^2}$ ~~for some constant~~ B~~. Refer to the second sub here for the desired~~ u~~-substitution. Then you'll end up with something like~~ $\int \cos^2(u)\, du$~~, which you can handle with the double-angle identity~~ $\cos^2(u) = \frac{1}{2}(\cos(2u) + 1)$~~. Remember to substitute back in for~~ x ~~instead of~~ u~~, and use your trig-of-inverse-trig skills to simplify the resulting~~ $\cos(\arcsin(\text{ thing }))$ ~~term.~~

Hint: You may use the following fact without proof:

$$\int \sqrt{x^2 + b^2}\, dx = \frac{x}{2}\sqrt{b^2 + x^2} + \frac{b^2}{2}\log\left(\sqrt{b^2 + x^2} + x\right),$$

which can be derived using the substitution $x = b\tan(u)$, the identity $\tan^2(u) + 1 = \sec^2(u)$, and the integral of secant.

Answer on Overleaf.

daily_challenge

Updated 1 month ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

:logswet:

Updated 1 month ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

Submit your answer in Overleaf: https://www.overleaf.com/1231232126rckrscxfchyf

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followup discussions *for lingering questions and comments*