question 2 views

Daily Challenge 20.6

In tonight's meeting we will meet *reduction formulas* for integrals.

Let $I_n = \int x^n e^{ax}\, dx$. Prove the reduction formula

$$I_n = \frac{1}{a}(x^n e^{ax} - nI_{n-1})$$

using integration by parts.

daily_challenge

Updated 2 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

(log): We shall integrate by parts, where $a' = e^{ax}$, $a = \frac{e^{ax}}{a}$, $b = x^n$, $b' = nx^{n-1}$.
For clarity of substitution we shall first say

$$\begin{aligned} \int a'b &= ab - \int ab' \\ \int e^{ax} x^n &= \frac{e^{ax}}{a} x^n - \int \frac{e^{ax}}{a} n x^{n-1} \\ &= \frac{e^{ax}}{a} x^n - \frac{n}{a} \int e^{ax} x^{n-1} \\ &= \frac{1}{a}(e^{ax} x^n - nI_{n-1}) \end{aligned}$$

Good warmup.

Updated 2 months ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

Begin with $I_n = \int x^n e^{ax}\, dx$ and let $u = x^n$, $dv = e^{ax}\, dx$, $du = nx^{n-1}\, dx$, and $v = \frac{1}{a} e^{ax}$. Using the integration by parts formula, one has

$$\begin{aligned} I_n &= \int x^n e^{ax}\, dx \\ &= \int u\, dv \\ &= uv - \int v\, du \\ &= \frac{1}{a} x^n e^{ax} - \frac{1}{a} \int n e^{ax} x^{n-1}\, dx, \end{aligned}$$

but the far right integral in the line above is, by definition, $\frac{n}{a} I_{n-1}$. Thus we have shown that

$$I_n = \frac{1}{a}(x^n e^{ax} - nI_{n-1}),$$

as claimed.

Updated 2 months ago by Christian Ferko

followup discussions *for lingering questions and comments*