35.2

(a): Recall the definition of the second order Taylor series:

$$f(x) \text{ near } x_0: \approx f(x_0) + \frac{f'(x_0)}{1}(x-x_0) + \frac{f''(x_0)}{2}(x-x_0)^2$$

Then, since x_0 is a minimum of $V(x)$,

$$V(x) \text{ near } x_0 \approx \cancel{V}(x_0) + \frac{V''(x_0)}{2}(x-x_0)^2$$

Plugging into given F,

$$F = -\frac{dV}{dx} = -V''(x_0)(x-x_0) = m\frac{d^2x}{dt^2}$$

then,

$$-m\frac{d^2x}{dt^2} - V''(x_0)x + V''(x_0)x_0 = 0$$

$$\frac{d^2x}{dt^2} + \frac{V''(x_0)x}{m} - \frac{V''(x_0)x_0}{m} = 0$$

not sure why I distributed before,

$$\frac{d^2x}{dt^2} + \frac{V''(x_0)}{m}(x-x_0) = 0$$

(b): Redefining $y = x - x_0$ and thus $\frac{d^2x}{dt^2} = \frac{d^2y}{dt^2}$,

$$\frac{d^2y}{dt^2} + \frac{V''(x_0)}{m}y = 0$$

In session 71, we saw that an equation of the form

$$\ddot{x} + w^2x = 0$$

has general solution

$$x = x_0\cos(wt) + \frac{v_0}{w}\sin(wt)$$

Thus, we have

$$y = A\cos(wt) + B\sin(wt) \text{ where } w = \sqrt{\frac{V''(x_0)}{m}}$$

and since $y = x - x_0$,

$$x = x_0 + A\cos(wt) + B\sin(wt)$$