question 2 views

Daily Challenge 18.5

(**Due: Friday 11/16 at 12:00 noon Eastern**)

We continue with the week of exercises rather than challenges.

(1) Exercise: antiderivatives of polynomials.

(a) Let n be a positive integer. Find $\int x^n \, dx$.

(That is, find a function $F(x)$ such that $F'(x) = x^n$. Don't forget the C.)

(b) Does your formula from (a) work for $n = 0$? Why or why not?

(c) Does your formula from (b) work for negative integers? Why or why not?

daily_challenge

Updated 5 months ago by Christian Ferko

the students' answer, *where students collectively construct a single answer*

(a) We see by "applying the power rule in reverse" that $\int x^n \, dx = \frac{1x^{n+1}}{n+1} + C$.

(b) We see by inserting zero into our given integral that $\int x^0 \, dx = \int 1 \, dx$, where of course the antiderivative of 1 would be an line with slope 1, ie $x + C$ where C is some constant; We apply our formula seen in (a) to see $\int x^0 \, dx = \frac{1x^1}{1} + C = x + C$, identical to our "intuition". Yes.

(c) No, consider the negative integer -1; this immediately does not work because $\frac{1x^0}{0}$ is undefined.

Updated 4 months ago by Logan Pachulski

the instructors' answer, *where instructors collectively construct a single answer*

(a) If $n \in \mathbb{N}_+$, we have

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

since we see by inspection that the power rule for differentiation gives $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1} + C\right) = x^n$.

(b) Yes. If $n = 0$, the indefinite integral of interest is $\int x^0 \, dx = \int 1 \, dx$. Clearly an antiderivative of 1 which works is $x + C$ for any constant C. But this matches the prediction of the formula when $n = 0$, since it produces $\frac{x^{n+1}}{n+1} + C \xrightarrow{n=0} x + C$.

(c) No. In particular, it fails for $n = -1$, since our formula would predict $\frac{x^0}{0} + C$ which is undefined. The true antiderivative in this case is of course given by

$$\int \frac{1}{x} \, dx = \log(x) + C,$$

since we recall that $\frac{d}{dx}\log(x) = \frac{1}{x}$.

The formula works for other negative integers $n < -1$. For instance, if $n = -2$ our formula predicts

$$\int x^{-2} \, dx = \frac{x^{-1}}{-2+1} + C = -\frac{1}{x} + C,$$

which indeed works because $\frac{d}{dx}\left(-\frac{1}{x} + C\right) = \frac{1}{x^2}$.

Updated 4 months ago by Christian Ferko

followup discussions *for lingering questions and comments*