Reference Information: The following information is for your reference in answering some of the questions in this test.

Volume of a right circular cone with radius r and height h: $V = \frac{1}{3}\pi r^2 h$

Volume of a sphere with radius r: $V = \frac{4}{3}\pi r^3$

Volume of a pyramid with base area B and height h: $V = \frac{1}{3}Bh$

Surface Area of a sphere with radius r: $S = 4\pi r^2$

Number and Operations

1

From a group of 6 juniors and 8 seniors on the student council, 2 juniors and 4 seniors will be chosen to make up a 6-person committee. How many different 6-person committees are possible?

A) 84

B) 85

C) 1,050

D) 1,710

E) 1,890

Choice (C) is the correct answer. The 2 juniors on the committee can be chosen from the 6 juniors in $\binom{6}{2} = 15$ ways. The 4 seniors on the committee can be chosen from the 8 seniors in $\binom{8}{4} = 70$ ways. Therefore, there are $(15)(70) = 1{,}050$ possibilities for the 6-person committee.

Algebra and Functions

$$\text{If } x^4 < |x^3|, \text{ then } 0 < x < 1.$$

2

Which of the following values for x is a counterexample to the statement above?

A) $-\frac{4}{3}$

B) $-\frac{1}{2}$

C) 0

D) $\frac{1}{2}$

E) 1

Choice (B) is the correct answer. A counterexample to the given statement would be a number x such that the hypothesis $x^4 < |x^3|$ is true, but the conclusion $0 < x < 1$ does not hold. Choice (B), $-\frac{1}{2}$, is a counterexample.

It is true that $\left(-\frac{1}{2}\right)^4 < \left|\left(-\frac{1}{2}\right)^3\right|$, since $\frac{1}{16} < \frac{1}{8}$; but it is not true that $-\frac{1}{2}$ lies between 0 and 1. Choice (D) is incorrect. It is true that $\left(\frac{1}{2}\right)^4 < \left|\left(\frac{1}{2}\right)^3\right|$, but the conclusion $0 < \frac{1}{2} < 1$ is also true, so $x = \frac{1}{2}$ is not a counterexample. Choices (A), (C), and (E) are incorrect. None of these values of x satisfies the hypothesis $x^4 < |x^3|$.

3

If $\ln(x) = 1.25$, then $\ln(3x) =$

A) 1.10

B) 1.37

C) 1.73

D) 2.35

E) 3.75

Choice (D) is the correct answer. By the properties of logarithms, $\ln(3x) = \ln(3) + \ln(x) \approx 1.10 + 1.25 = 2.35$.

4

During a thunderstorm, the distance between a person and the storm varies directly as the time interval between the person seeing a flash of lightning and hearing the sound of thunder. When a storm is 4,000 feet away, the time interval between the person seeing the lightning flash and hearing the sound of the thunder is 3.7 seconds. How far away from the person is the storm when this time interval is 5 seconds?

A) 2,960 ft

B) 4,650 ft

C) 5,405 ft

D) 6,284 ft

E) 7,304 ft

Choice (C) is the correct answer. Since the person's distance from the storm varies directly with the time interval between the flash of lightning and the sound of thunder, the distance can be written as $d = kt$, where d is the distance in feet, t is the time in seconds, and k is a constant. This distance is 4,000 feet when the time interval is 3.7 seconds; therefore, $4{,}000 = k(3.7)$, and $k = \frac{4{,}000}{3.7}$. Thus, when the interval between the lightning flash and the sound of the thunder is 5 seconds, the storm is $\frac{4{,}000}{3.7}(5) \approx 5{,}405$ feet away.

5

If $x = \frac{3}{2}$ is a solution to the equation $5(4x - k)(x - 1) = 0$, what is the value of k?

A) $\frac{2}{3}$
B) 1
C) 4
D) 5
E) 6

Choice (E) is the correct answer. The expression $5(4x - k)(x - 1)$ is equal to 0 if and only if $x = 1$ or $4x = k$. If $x = \frac{3}{2}$, it cannot be true that $x = 1$. Thus, if $x = \frac{3}{2}$ is a solution to the equation, it must be true that $4x = k$. It follows that $(4)\left(\frac{3}{2}\right) = 6 = k$.

6

$$p(t) = 110 + 20\sin(160\pi t)$$

A certain person's blood pressure $p(t)$, in millimeters of mercury, is modeled above as a function of time, t, in minutes. According to the model, how many times in the interval $0 \le t \le 1$ does the person's blood pressure reach its maximum of 130?

A) 60
B) 80
C) 100
D) 110
E) 130

Choice (B) is the correct answer. The maximum of 130 millimeters is achieved exactly when $\sin(160\pi t) = 1$. The sine function has a value of 1 exactly for arguments $\frac{\pi}{2} + 2n\pi$, where n is any integer. Over the interval $0 \le t \le 1$, the argument of $\sin(160\pi t)$ ranges from 0 to 160π. Thus, over the interval in question, $\sin(160\pi t) = 1$, and $p(t) = 110 + 20\sin(160\pi t) = 130$, for $160\pi t = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots, \frac{317\pi}{2}$. Thus, the maximum blood pressure is reached exactly 80 times in the interval. You can also use the period of p to answer the question. The period of p is $\frac{2\pi}{|160\pi|} = \frac{1}{80}$. This means that the graph of p has one complete cycle every $\frac{1}{80}$. On the interval $0 \le t \le 1$, p has 80 complete cycles.

7

x	$f(x)$
−3	87
0	−15
1	15
3	171
5	455

The table above gives selected values for the function f. Which of the following could be the definition of f?

A) $f(x) = 30x - 15$

B) $f(x) = 30x + 15$

C) $f(x) = 30x^2 + 15$

D) $f(x) = 16x^2 - 14x + 15$

E) $f(x) = 16x^2 + 14x - 15$

Choice (E) is the correct answer. The function in choice (A) takes on the correct values at $x = 0$ and $x = 1$, but the value at $x = 3$ is 75, not 171, so choice (A) cannot be correct. The function in choice (B) does not take on the correct value at $x = 1$, so choice (B) cannot be correct. (Another way to eliminate choices (A) and (B) is to note that $f(x)$ decreases and then increases, so that f cannot be linear.) The function in choice (C) takes on only positive values, so choice (C) cannot be correct. The function in choice (D) does not take on the correct value at $x = 0$, so choice (D) cannot be correct. The values of the function in choice (E) do agree with all the values in the table, so this could be the definition of the function. You can also find a quadratic regression for the values using the graphing calculator, which is $y = 16x^2 + 14x - 15$.

8

If $f(x) = \frac{1}{x-5}$ and $g(x) = \sqrt{x+4}$, what is the domain of $f - g$?

A) All x such that $x \neq 5$ and $x \leq 4$

B) All x such that $x \neq -5$ and $x \leq 4$

C) All x such that $x \neq 5$ and $x \geq -4$

D) All x such that $x \neq -4$ and $x \geq -5$

E) All real numbers x

Choice (C) is the correct answer. The function $f - g$ will be defined at exactly those points where f and g are both defined. In other words, the domain of $f - g$ is the intersection of the domain of f and the domain of g. Since $f(x) = \frac{1}{x-5}$ is defined for all $x \neq 5$ and $g(x) = \sqrt{x+4}$ is defined for all $x \geq -4$, the domain of $f - g$ is all x such that $x \neq 5$ and $x \geq -4$. You can also examine the graph of $f - g$. The graph is defined for all real numbers $x \geq -4$ except for $x = 5$, where the graph has a vertical asymptote.

9

A sum of \$10,000 is invested at a rate of 10 percent, with interest compounded semiannually. The value, in dollars, of this investment after t years is given by $V(t) = 10{,}000(1.05)^{2t}$. Approximately how much greater is the value of this investment at the end of 2 years than the same amount invested at the rate of 10 percent compounded annually?

A) \$55

B) \$200

C) \$500

D) \$1,075

E) \$1,155

Choice (A) is the correct answer. Applying the given function with $t = 2$ shows that the value of the investment compounded semiannually after 2 years would be approximately \$12,155. If \$10,000 were invested at 10 percent interest compounded annually, then at the end of 2 years the value of this investment would be $\$10{,}000(1.10)^2 = \$12{,}100$. Thus, difference in the amounts of the investments is approximately $\$12{,}155 - \$12{,}100 = \$55$.

10

For which of the following functions does $f(x, y) = -f(-x, -y)$ for all values of x and y?

A) $f(x, y) = x + y^2$

B) $f(x, y) = x - y^2$

C) $f(x, y) = x^2 - y$

D) $f(x, y) = x + y^3$

E) $f(x, y) = x - y^4$

Choice (D) is the correct answer. If $f(x, y)$ is a polynomial in x and y, then $f(x, y) = -f(-x, -y)$ if and only if every nonzero term of f is of odd degree. Of the given choices, this is true only for $f(x, y) = x + y^3$. In this case, $f(-x, -y) = (-x) + (-y)^3 = -x - y^3 = -(x + y^3) = -f(x, y)$. So, $f(x, y) = -f(-x, -y)$.

Geometry and Measurement: Coordinate Geometry

11

Which of the following describes the set of points (a, b) for which $|a| + |b| = 5$ in the xy-plane?

A) A circle with radius 5

B) A circle with radius $5\sqrt{2}$

C) A square with sides of length $5\sqrt{2}$

D) A square with sides of length 10

E) A regular hexagon with sides of length 5

If $|a| + |b| = 5$, consider the four cases:

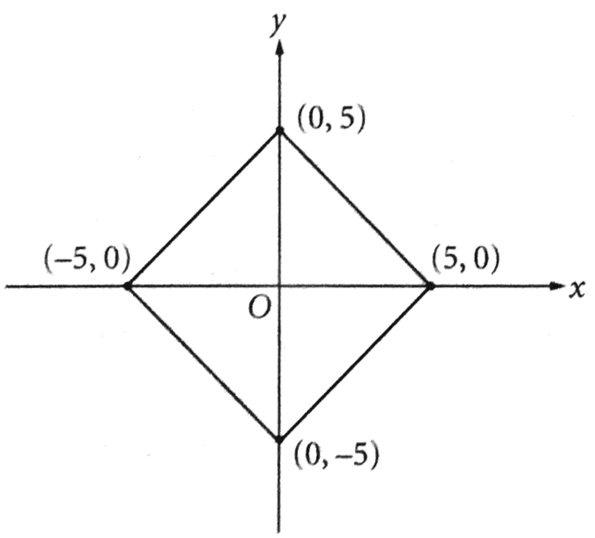
$a > 0, b > 0$: $a + b = 5$ so $b = 5 - a$ is a line with slope -1 and y-intercept 5

$a > 0, b < 0$: $a - b = 5$ so $b = -5 + a$ is a line with slope 1 and y-intercept -5

$a < 0, b > 0$: $-a + b = 5$ so $b = 5 + a$ is a line with slope 1 and y-intercept 5

$a < 0, b < 0$: $-a - b = 5$ so $b = -5 - a$ is a line with slope -1 and y-intercept -5

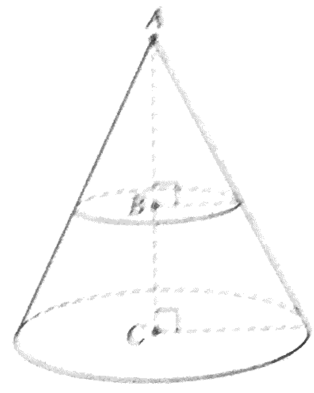
Sketch the graphs of the 4 lines. The 4 lines intersect to form a square with vertices on the coordinate axes located 5 units from the origin.



Choice (C) is the correct answer. As shown in the figure above, the set of points (a, b) for which $|a| + |b| = 5$ is the square with vertices $(0, 5)$, $(5, 0)$, $(0, -5)$, and $(-5, 0)$. By the Pythagorean theorem, the sides of this square are of length $5\sqrt{2}$.

Geometry and Measurement: Three-Dimensional Geometry

12



In the figure above, points B and C are the centers of the bases of two right circular cones, each with vertex A. If $AB = 1$ and $AC = 2$, what is the ratio of the volume of the smaller cone to the volume of the larger cone?

A) $\frac{1}{8}$

B) $\frac{1}{4}$

C) $\frac{3}{8}$

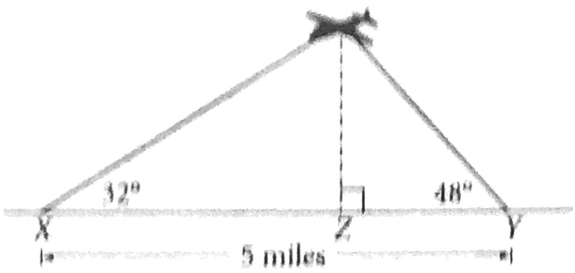
D) $\frac{1}{2}$

E) It cannot be determined from the information given.

Choice (A) is the correct answer. The volume V of a cone is given by $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height. The smaller cone and larger cone are similar geometric figures, and the ratio of the height of the smaller cone to the height of the larger cone is $\frac{1}{2}$. It follows that the ratio of the base radius of the smaller cone to the base radius of the larger cone is also $\frac{1}{2}$. Therefore, if r is the base radius of the smaller cone, then $2r$ is the base radius of the larger cone. Thus, the volume of the smaller cone is $V = \frac{1}{3}\pi r^2(1)$, the volume of the larger cone is $V = \frac{1}{3}\pi (2r)^2 (2) = \frac{8}{3}\pi r^2$. Thus, the ratio of the volume of the smaller cone to the volume of the larger cone is $\frac{\frac{1}{3}}{\frac{8}{3}} = \frac{1}{8}$.

Geometry and Measurement: Trigonometry

11



The airplane in the figure above is flying directly over point Z on a straight, level road. The angles of elevation for points X and Y are 32° and 48°, respectively. If points X and Y are 5 miles apart, what is the distance, in miles, from the airplane to point X?

A) 1.60
B) 2.40
C) 2.69
D) 3.77
E) 7.01

Choice (D) is the correct answer. Label the location of the airplane as point W. Then in ΔXYW, the measure of $\angle X$ is 32°, the measure of $\angle Y$ is 48°, and the measure of $\angle W$ is 100°. Let x, y, and w denote the lengths, in miles, of the sides of ΔXYW opposite $\angle X$, $\angle Y$, and $\angle W$, respectively. Then by the law of sines, $\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{w}{\sin W}$. Since $w = 5$ and the distance from the plane to point X is y, it follows that $\frac{5}{\sin 100^\circ} = \frac{y}{\sin 48^\circ}$. This gives $y \approx 3.77$ for the distance, in miles, from the plane to point X.

14

If $\cos\theta = \frac{x}{3}$, where $0 < \theta < \frac{\pi}{2}$ and $0 < x < 3$, then $\sin\theta =$

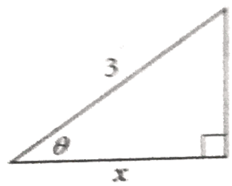
A) $\frac{\sqrt{9-x^2}}{3}$

B) $\frac{\sqrt{x^2-9}}{x}$

C) $\frac{\sqrt{9-x^2}}{x}$

D) $\frac{\sqrt{3-x^2}}{3}$

E) $\frac{\sqrt{3-x^2}}{x}$



Choice (A) is the correct answer. Since $\cos\theta = \frac{x}{3}$, $0 < \theta < \frac{\pi}{2}$, and $0 < x < 3$, the figure above can be drawn. By the Pythagorean theorem, the other leg of the right triangle is $\sqrt{9-x^2}$. Thus, $\sin\theta = \frac{\sqrt{9-x^2}}{3}$. Alternatively, since $\sin^2\theta + \cos^2\theta = 1$,

$$\begin{aligned} \sin^2\theta &= 1 - \cos^2\theta \\ &= 1 - \left(\frac{x}{3}\right)^2 \\ &= \frac{9-x^2}{9} \\ \sin\theta &= \frac{\sqrt{9-x^2}}{3} \end{aligned}$$

since $0 < \theta < \frac{\pi}{2}$ and $0 < x < 3$.

Data Analysis, Statistics, and Probability

15

Revenue for Company X	
Years after merger	**Revenue (in billions of dollars)**
0	\$3
2	\$4
4	\$11
7	\$25

Two companies merged to form Company X, whose revenues are shown in the table above for selected years after the merger. If a least-squares exponential regression is used to model the data above, what revenue, in billions of dollars, would be predicted for the company 13 years after the merger?

A) \$31

B) \$43

C) \$109

D) \$172

E) \$208

Choice (D) is the correct answer. A graphing calculator can be used to find the least-squares exponential regression for the data (0, 3), (2, 4), (4, 11), and (7, 25). This gives a function of the form $y = ab^x$, where $a \approx 2.678$ and $b \approx 1.377$. The exponential regression is $y = (2.678)(1.377)^x$. Evaluating this function at $t = 13$, without rounding the values of a and b, gives approximately 172. Thus, the revenue predicted is \$172 billion.