Test Prep Day 3

(3): (a): We see ~~by~~ the FTC that

$$f(-5) = f(1) - \int_{-5}^{1} g(x)\,dx$$

$$= 3 - \left(\left(-3x\right)\Big|_{-5}^{-2} + \left(\int 3x\right)\Big|_{-2}^{-1} + \left(\int 0\right)\Big|_{-1}^{0} + \left(\int 2x\right)\Big|_{0}^{1} \right)$$

$$= 3 + \frac{19}{2} = \frac{25}{2}$$

(b): The area under [1,3] is 4, and

$$\cancel{\int_3^6 2(x^2 - 8x + 16) = 2\left(\frac{x^3}{3} - \frac{x^2}{4} + 16x\right)\Big|_3^6}$$

$$\cancel{= 2((}$$

$$\int_3^6 2(x-4)^2 = \left(\frac{2}{3}(x-4)^3\right)\Big|_3^6 = \frac{16}{3} + \frac{2}{3} = 6$$

Thus,

$$\int_1^6 g(x) = 6 + 4 = 10.$$

(c): For both [0,1] and (4,6), we see that the first derivative and second are both positive; thus, on these ranges the function is concave up.

(d): There is one point of inflection at 4, since $g''(x)$ goes from ~~zero~~ negative to positive there.