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MATHEMATICS LEVEL 2 TEST

For each of the following problems, decide which is the BEST of the choices given. If the exact numerical value is not one of the choices, select the choice that best approximates this value. Then fill in the corresponding circle on the answer sheet.

Notes: (1) A scientific or graphing calculator will be necessary for answering some (but not all) of the questions in this test. For each question you will have to decide whether or not you should use a calculator.

(2) For some questions in this test you may have to decide whether your calculator should be in the radian mode or the degree mode.

(3) Figures that accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

(4) Unless otherwise specified, the domain of any function f is assumed to be the set of all real numbers for which $f(x)$ is a real number. The range of f is assumed to be the set of all real numbers $f(x)$, where x is in the domain of f .

(5) Reference information that may be useful in answering the questions in this test can be found on the page preceding Question 1.

USE THIS SPACE FOR SCRATCH WORK

1. $x^m x^{-2m} =$

(A) x^m

(B) $\frac{1}{x^m}$

(C) $\frac{1}{x^{-m}}$

(D) x^{-3m}

(E) x^{-2m^2}

MATHEMATICS LEVEL 2 TEST—Continued

2. All of the following numbers satisfy the inequality $(2x + 1)(x - 5) < 0$ EXCEPT
(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

USE THIS SPACE FOR SCRATCH WORK.

3. For all real numbers m , the equation $y = mx + 3$ represents which of the following in the xy -plane?

- (A) Lines whose x -intercept is 3
(B) Lines whose y -intercept is 3
(C) Lines whose slope is 3
(D) Vertical lines through $(3, 0)$
(E) Horizontal lines through $(0, 3)$

4. If $2^a = 4^b = 64$, what is the value of $a + b$?

- (A) 3 (B) 8 (C) 9 (D) 18 (E) 48

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MATHEMATICS LEVEL 2 TEST—Continued

5. What is the domain of the function f defined by

$$f(x) = \frac{x^2}{x^2 + 1}?$$

- (A) $-1 < x \leq 1$
- (B) $0 \leq x < 1$
- (C) $x \geq 0$
- (D) All real numbers except -1
- (E) All real numbers

USE THIS SPACE FOR SCRATCH

6. If $y = 2x^3 + x^2$, what is the value of $|y|$ when $x = -2$?

- (A) -20
- (B) 8
- (C) 12
- (D) 20
- (E) 60

7. If $f(x) = (x - 3)^2$, what is the greatest value of x for which $f(x) = 5$?

- (A) -0.76
- (B) 0.76
- (C) 3.74
- (D) 4.00
- (E) 5.24

TCH WORK.

MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

2	6		
3	2	7	
4	5	5	
5	6	8	
6	1	3	6

2 | 6 represents 26.

8. The stem-and-leaf plot above shows the mathematics scores on a national test for a group of juniors at Pacific High School. What is the median score for this group?

(A) 45
(B) 49.7
(C) 50.5
(D) 56
(E) 58

9. If $f(x) = 2x + 1$ and $g(x) = \frac{1}{x} - 2$, for what value of x is $g(f(x))$ equal to 0?

(A) -1
(B) $-\frac{1}{4}$
(C) $\frac{1}{4}$
(D) $\frac{1}{2}$
(E) $\frac{2}{3}$

MATHEMATICS LEVEL 2 TEST—Continued

10. Let a be a nonzero constant. If $2x^2 - 4 = a$,
then $x^2 - 2 =$

USE THIS SPACE FOR SCRATCH WORK.

- (A) $\frac{1}{2}$ (B) $\frac{a}{2}$ (C) $\frac{2}{a}$ (D) 2 (E) $2a$

x	Number of months in business	0	1	2	3	4	5
$P(x)$	Profit (in thousands of dollars)	0	1	4.2	9.1	15.8	25.3

11. The table above shows the profit made by a new company. Of the following functions, which best models the relationship between the company's profit and the number of months in business?

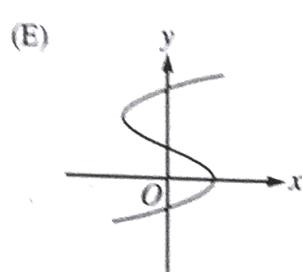
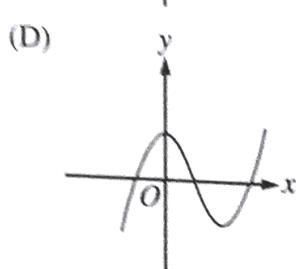
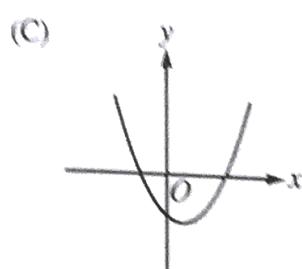
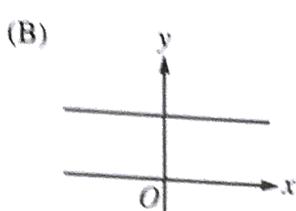
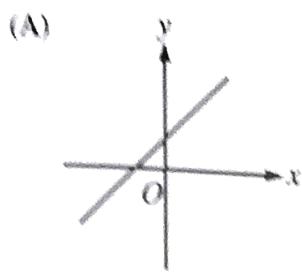
- (A) $P(x) = 2x - 1$
 (B) $P(x) = 5x - 3$
 (C) $P(x) = x^2$
 (D) $P(x) = 2x^2 - 1$
 (E) $P(x) = x^3$

12. If $y_n = 1 - (-1)^n$, where $n = 1, 2, 3, \dots$, which of the following statements is true?

- (A) For all n , $y_n = 0$ only.
 (B) For all n , $y_n = 0$ or $y_n = 2$.
 (C) For all n , $y_n = 0$ or $y_n = 1$.
 (D) For $n \geq 1,000$, $y_n > 0$.
 (E) For $n \geq 1,000$, $y_n < 0$.

MATHEMATICS LEVEL 2 TEST—Continued

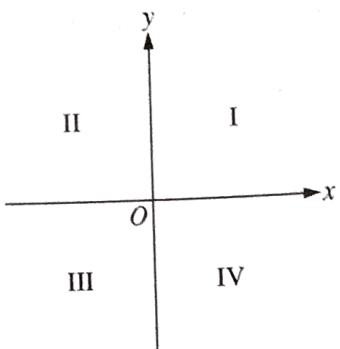
13. Which of the following is NOT a graph of y as a function of x ?



USE THIS SPACE FOR SCRATCH WORK.

MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK



14. The figure above shows the xy -plane. If $f(x) = cx + 3$ and $g(x) = dx + 1$ for $0 < c < d$, which of the following is true about the graphs of f and g ?

- (A) The graphs intersect in quadrant I.
- (B) The graphs intersect in quadrant II.
- (C) The graphs intersect in quadrant III.
- (D) The graphs intersect in quadrant IV.
- (E) The graphs do not intersect.

15. Eight cars, each of a different color (red, blue, black, gray, white, green, tan, gold), travel one behind the other to a campground. The red car must lead and the green car must be last. How many different orderings of the cars are there?

- (A) $6!$
- (B) $8!$
- (C) $2 \cdot 6!$
- (D) $2 \cdot 8!$
- (E) $\frac{8!}{2}$

16. If $\ln($

- (A) 1
- (B) 2
- (C) 2
- (D) 3
- (E) 3

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- (A)
- (B)
- (C)
- (D)
- (E)

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- (A)
- (B)
- (C)
- (D)
- (E)

MATHEMATICS LEVEL 2 TEST—Continued

16. If $\ln(x) = 1.58$, then $\ln(2x) =$

- (A) 1.15
- (B) 2.27
- (C) 2.49
- (D) 3.16
- (E) 3.58

USE THIS SPACE FOR SCRATCH WORK.

17. An insect population is growing in such a way that the number in each generation is approximately 1.5 times that of the previous generation. If there are 100 insects in the first generation, approximately how many insects will there be in the fourth generation?

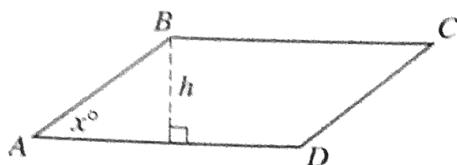
- (A) 338
- (B) 475
- (C) 506
- (D) 813
- (E) 1,319

18. Which of the following are true?

- I. If $x \neq 2$, then $x^2 + 4 \neq 8$.
- II. If $x^2 + 4 \neq 8$, then $x \neq 2$.
- III. If $x^2 + 4 = 8$, then $x = 2$.

- (A) I only
- (B) II only
- (C) I and II only
- (D) I and III only
- (E) I, II, and III

MATHEMATICS LEVEL 2 TEST—Continued



USE THIS SPACE FOR SCRATCHING.

Note: Figure not drawn to scale.

19. In parallelogram $ABCD$ above, $AB = 8$ and $AD = 13$. If $x = 42$, what is the value of h ?

- (A) 5.35
- (B) 5.95
- (C) 7.20
- (D) 8.70
- (E) 9.66

20. If $f(1) = -3$, $f(3) = 2$, and for all real numbers x , $f(x) = ax + b$, then $(a, b) =$

- (A) $\left(\frac{5}{2}, \frac{11}{2}\right)$
- (B) $\left(\frac{5}{2}, -\frac{11}{2}\right)$
- (C) $\left(\frac{5}{2}, -\frac{13}{2}\right)$
- (D) $\left(-\frac{5}{2}, \frac{11}{2}\right)$
- (E) $\left(-\frac{5}{2}, -\frac{13}{2}\right)$

MATHEMATICS LEVEL 2 TEST—*Continued*

21. Which value of x in the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$ satisfies the equation $\cos x = \sec x$?

USE THIS SPACE FOR SCRATCH WORK.

- (A) $-\frac{\pi}{3}$
- (B) $-\frac{\pi}{4}$
- (C) 0
- (D) $\frac{\pi}{4}$
- (E) $\frac{\pi}{3}$

22. The function g is defined by

$$g(x) = 3\sin(2x + 1) - 1.$$

What is the range of g ?

- (A) $-4 \leq g(x) \leq 2$
- (B) $-2 \leq g(x) \leq 4$
- (C) $-1 \leq g(x) \leq 3$
- (D) $-\frac{1}{2} \leq g(x) \leq 0$
- (E) $2 \leq g(x) \leq 3$

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MATHEMATICS LEVEL 2 TEST—*Continued*

23. In the first quadrant of the xy -plane, the point of

USE THIS SPACE FOR SCRATCHING.

intersection of the graphs of the line $y = x$ and
the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ is which of the following?

- (A) (3.12, 3.12)
- (B) (4.47, 4.47)
- (C) (4.50, 4.50)
- (D) (6.67, 6.67)
- (E) (9.76, 9.76)

24. The probability of randomly drawing a piece of red candy from a bag containing only red and purple candies is $\frac{2}{3}$. Which of the following could be the number of red and the number of purple candies in the bag?

- (A) 10 red, 20 purple
- (B) 20 red, 10 purple
- (C) 20 red, 30 purple
- (D) 20 red, 50 purple
- (E) 30 red, 20 purple

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MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.

x	$f(x)$
0	3
1	0
2	1
3	4
4	2

25. The table above defines the function f , which has domain $\{0,1,2,3,4\}$. What is the value of $f(f(3))$?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
26. Functions f and g are defined for all real numbers. The function f has zeros at -2 , 3 , and 7 ; and the function g has zeros at -3 , -1 , 4 , and 7 . How many distinct zeros does the product function $f \cdot g$ have?
- (A) Three
(B) Four
(C) Six
(D) Seven
(E) Twelve

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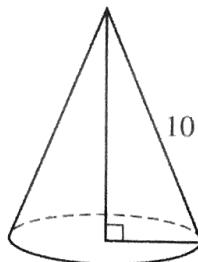
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MATHEMATICS LEVEL 2 TEST—Continued

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27. In the figure above, the diameter of the base of the right circular cone is 8. What is the volume of the cone?

(A) 153.6
(B) 167.6
(C) 351.9
(D) 402.1
(E) 670.2

28. In the xy -plane, what is the distance between the points whose coordinates are $(2\sqrt{3}, 4\sqrt{5})$ and $(-\sqrt{3}, 7\sqrt{5})$?

(A) 3.45 (B) 4.24 (C) 6.93 (D) 8.00
(E) 8.49

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TCH WORK.

MATHEMATICS LEVEL 2 TEST—Continued

29. An isosceles triangle has a base of length 25 centimeters and a vertex angle (the angle opposite the base) of 50° . What is the perimeter of the triangle?

- (A) 42.3 cm
- (B) 57.9 cm
- (C) 67.3 cm
- (D) 84.2 cm
- (E) 101.0 cm

USE THIS SPACE FOR SCRATCH WORK.

30. A taxi charges a base fee of \$1.25 plus \$0.75 for each mile (or part thereof). Which of the following would represent the taxi fare for a trip of length x miles? (Let $\lceil x \rceil$ represent the least integer that is greater than or equal to x .)

- (A) \$2.00 $\lceil x \rceil$
- (B) \$1.25 + \$0.75 $\lceil x \rceil$
- (C) \$0.75 + \$1.25 $\lceil x \rceil$
- (D) \$1.25 + \$0.75 $\lceil x + 1 \rceil$
- (E) \$0.75 + \$1.25 $\lceil x + 1 \rceil$

31. If $f(x, y) = \frac{x^2 + y^2}{x^2 - y^2}$, then $f(x, -y) =$

(A) -1

(B) 1

(C) $\frac{y^2 - x^2}{x^2 + y^2}$

(D) $\frac{x^2 + y^2}{y^2 - x^2}$

(E) $\frac{x^2 + y^2}{x^2 - y^2}$

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MATHEMATICS LEVEL 2 TEST—Continued

32. Twenty-seven identical cubes are arranged to form a larger cube. The diagonal of each face of each of the 27 identical cubes measures 1.415 centimeters. What is the volume of the larger cube?

(A) 1.002 cm³
(B) 2.833 cm³
(C) 9.018 cm³
(D) 27.045 cm³
(E) 76.495 cm³

USE THIS SPACE FOR SCRATCHING.

t , 8, 5, 4, 12, 8

33. In the list above, t is an integer. For the list, if the mean is equal to the median and the range is less than 10, what is the value of t ?

(A) 2 (B) 6.5 (C) 8 (D) 11 (E) 14

34. Which of the following are polar coordinates of a point on the graph of $r = \cos \theta$?

(A) $\left(1, \frac{\pi}{2}\right)$
(B) $\left(0.5, \frac{\pi}{3}\right)$
(C) $(1, \pi)$
(D) $\left(0.5, \frac{2\pi}{3}\right)$
(E) $\left(1, \frac{\pi}{4}\right)$

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SCRATCH WORK

MATHEMATICS LEVEL 2 TEST—Continued

35. The prime factorization of a positive integer n is p^3 . Which of the following is true?

- I. n cannot be even.
- II. n has only one positive prime factor.
- III. n has exactly three distinct factors.

- (A) I only
- (B) II only
- (C) III only
- (D) II and III only
- (E) I, II, and III

USE THIS SPACE FOR SCRATCH WORK.

36. In the xy -plane, the asymptotes of a hyperbola are the lines $y = x + 5$ and $y = -x - 5$. What are the coordinates of the center of the hyperbola?

- (A) $(0, 0)$
- (B) $(-5, 0)$
- (C) $(0, -5)$
- (D) $(0, 5)$
- (E) $(5, 0)$

MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK

Test Number	Student's Score	Class Mean	Class Standard Deviation
1	85	76	3
2	87	89	7
3	94	88	4
4	88	80	6
5	82	72	5

37. The table above shows a student's record of performance on five tests. On which test did the student rank the highest in relation to the other students in the class? (Assume that the test scores on each test are normally distributed.)

- (A) Test 1
- (B) Test 2
- (C) Test 3
- (D) Test 4
- (E) Test 5

38. When a positive integer n is divided by 4, the remainder is 3. Which of the following could equal n for some integer t ?

- (A) $4t + 2$
- (B) $4t$
- (C) $4t - 1$
- (D) $4t - 2$
- (E) $4t - 3$



39. In the figure above, a circle has its center at point C , with radius 1, and a secant line intersects the circle at point A . If the measure

- length of \overline{CD} is
- (A) 2.53
 - (B) 2.62
 - (C) 2.93
 - (D) 3.14
 - (E) 3.20

40. Let $f(x) = e^x$

$x = 1.27$ and

of the following

- (A) -4.70
- (B) -4.75
- (C) -4.80
- (D) -4.85
- (E) -4.90

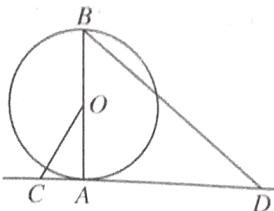
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ATCH WORK.

MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.



39. In the figure above, O is the center of the circle with radius 1, and line CD is tangent to the circle at A . If the measure of $\angle AOC$ is 30° and the length of \overline{CD} is 3, what is the length of \overline{BD} ?

(A) 2.53
(B) 2.62
(C) 2.93
(D) 3.14
(E) 3.20

40. Let $f(x) = e^x + x + k$. If $f(x) < 0$ when $x = 1.27$ and $f(x) > 0$ when $x = 1.28$, which of the following could be a value of k ?

(A) -4.70
(B) -4.75
(C) -4.80
(D) -4.85
(E) -4.90

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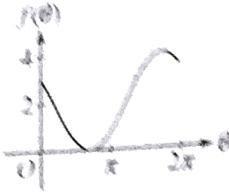
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MATHEMATICS LEVEL 2 TEST—Continued

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Graph I



Graph II

41. In the graphs above, Graph I shows a portion of the graph of $f(\theta) = A[\sin(B\theta + C)] + D$, where A , B , C , and D are constants. Graph II could result from changing which of the constants in $f(\theta)$?

- (A) A only
- (B) B only
- (C) C only
- (D) D only
- (E) A and D

42. The number of birds on each of islands X and Y remains constant from year to year; however, the birds migrate between islands. After one year, 20 percent of the birds on X have migrated to Y , and 15 percent of the birds on Y have migrated to X . If the total number of birds is 14,000, how many birds are on island X ?

- (A) 2,800
- (B) 6,000
- (C) 6,788
- (D) 7,212
- (E) 8,000

43. If $z = 1 -$
above is

- (A) A

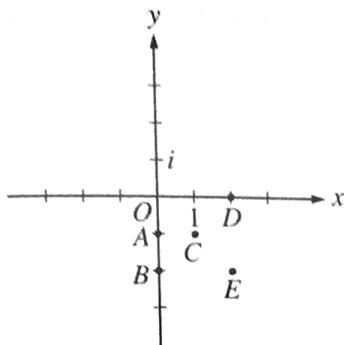
44. What is the volume of a cylinder
and its height?

- (A) 2π
- (B) π
- (C) 8π
- (D) 6π
- (E) 2π

SCRATCH WORK.

MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK.



43. If $z = 1 - i$, which of the points in the figure above is the graphical representation of z^2 ?

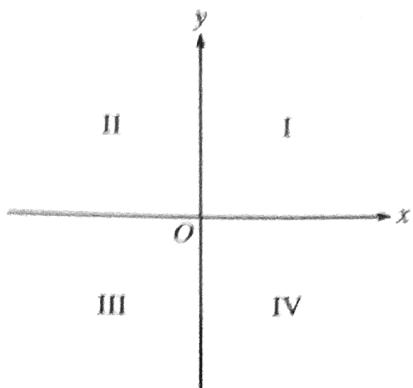
(A) A (B) B (C) C (D) D (E) E

44. What is the total surface area of a right circular cylindrical solid if the diameter of its base is $2r$ and its height is $2r$?

(A) $2\pi r + 2r$
(B) $\pi r^2 + 2r$
(C) $8\pi r^2$
(D) $6\pi r^2$
(E) $2\pi r^2$

MATHEMATICS LEVEL 2 TEST—Continued

USE THIS SPACE FOR SCRATCH WORK



45. If the terminal side of an angle θ , in standard position, lies in quadrant IV of the xy -plane above, which of the following must be true?

- (A) $\csc \theta < 0$ and $\sec \theta > 0$
- (B) $\csc \theta < 0$ and $\sec \theta < 0$
- (C) $\csc \theta < 0$ and $\tan \theta > 0$
- (D) $\csc \theta > 0$ and $\sec \theta < 0$
- (E) $\csc \theta > 0$ and $\tan \theta < 0$

Time (seconds)	0	0.5	1.5	2
Height (feet)	5	17	20.5	11.9

46. A ball is tossed into the air, and the height of the ball above the ground at different times is recorded in the table above. A quadratic regression equation is obtained for these data, with height expressed as a function of time. According to the regression equation, what is the maximum height of the ball?

- (A) 18.8 ft
- (B) 20.5 ft
- (C) 22.2 ft
- (D) 22.4 ft
- (E) 30.9 ft

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MATHEMATICS LEVEL 2 TEST—Continued

47. In right triangle ABC , the measure of $\angle A$ is 90° , $AB = 3$, and $BC = x$. If $\sin C = k$, then, in terms of k , $AC =$

- (A) $\sqrt{9k^2 - 9}$
(B) $\sqrt{9 - \left(\frac{3}{k}\right)^2}$
(C) $\sqrt{9 - \left(\frac{k}{3}\right)^2}$
(D) $\sqrt{\left(\frac{k}{3}\right)^2 - 9}$
(E) $\sqrt{\left(\frac{3}{k}\right)^2 - 9}$

USE THIS SPACE FOR SCRATCH WORK.

48. If $f(x) = \begin{cases} x & \text{when } 0 \leq x < 1 \\ f(x-1) & \text{when } x \geq 1 \end{cases}$,

what is the value of $f(4.7)$?

- (A) 4.7
(B) 3.7
(C) 0.7
(D) 0.3
(E) -0.3

MATHEMATICS LEVEL 2 TEST—Continued

49. How many noncongruent triangles ABC exist such that the measure of $\angle A$ is 42° , $AB = 6$, and $BC = 4$?

(A) None
(B) One
(C) Two
(D) Three
(E) An infinite number

USE THIS SPACE FOR SCRATCH WORK

50. A magazine article described the growth of a computer network as exponential. The article stated that in 10 years, the number of users of this network had risen from 1 million to 20 million. Assuming that this article was correct, in how many years would the number of users increase from 20 million to 200 million?

(A) 20 (B) 18 (C) 15 (D) 10 (E) 8

S T O P

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS TEST ONLY.
DO NOT TURN TO ANY OTHER TEST IN THIS BOOK.**