We wish to learn the distribution of:

$$p(Z, \alpha, \theta | X) \propto p(X|\alpha, \theta, Z)P(Z)p(\alpha, \theta)$$

Where $Z = \{z_0, z_1, ...z_n | z_i = 0 \text{ or } 1\}$. $X = \{x_0, x_1, ..., x_n | x_i = \{t_i, k_i\}\}$. Each x_i represents k arrivals in t time for conversation i. We want to favor big priors for the friends hypothesis, and small priors for the not friends hypothesis. So I defined the prior on priors to be:

$$p(\alpha, \theta) \propto \frac{1}{1 - \frac{\alpha_0 \theta_0}{\alpha_1 \theta_1}}$$

so

$$p(X|\alpha, \theta, Z) \propto \prod_i p(x_i|\alpha_0, \theta_0)p(z_i = 0) + p(x_i|\alpha_1, \theta_1)p(z_i = 1)$$

where

$$p(x_i|\alpha, \theta) = \int_0^\infty \frac{(\theta + t)^{\alpha+k}}{\Gamma(\alpha + k)} \lambda^{\alpha+k-1} e^{-\lambda(\theta+t)} d\lambda$$

(A poisson likelihood with a gamma prior, integrating over all possible rates)

$$= \frac{\Gamma(k + \alpha)}{\Gamma(\alpha)} \frac{\theta^\alpha}{(t + \theta)^{k+\alpha}}$$