Poisson Likelihood, Gamma Prior

$p(\lambda \mid d) = p(d \mid \lambda)p(\lambda)$ Poisson likelihood

$\frac{t\lambda^k e^{-\lambda t}}{k!} \propto t\lambda^k e^{-\lambda t}$

Prior

$\frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda} \propto \lambda^{\alpha-1}e^{-\beta\lambda}$

Prior * Likelihood

$\propto t\lambda^{\alpha+k-1}e^{-\lambda(\beta+t)}$

$=$ t * Gamma$(\lambda; \alpha + k, \beta + t)$

$= \frac{(\beta+t)^{\alpha+k}}{\Gamma(\alpha+k)}\lambda^{\alpha+k-1}e^{-\lambda(\beta+t)}$

If we derive WRT λ, there is a root at $\lambda = \frac{\alpha+k-1}{\beta+t}$