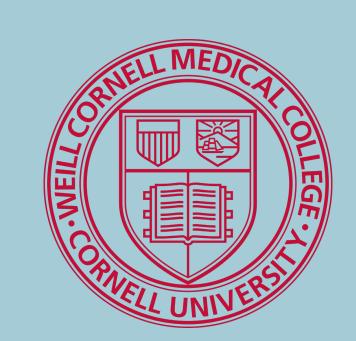
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TITRATION CALORIMETRY

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Introduction

Among the most fundamental molecular interactions in biology are those of small molecules with their proteins. However, deficiencies in the estimation of uncertainty create large challenges in the ability of these ITC experiments to be used in a quantitative way, holding back their use in probing function and aiding design. For instance, most existing analysis procedures fail to incorporate errors in reagent concentrations, which is commonly kept a fixed parameter, whereas previous studies indicate possible errors of 10 % that are not propagated.

A typical ITC experiment

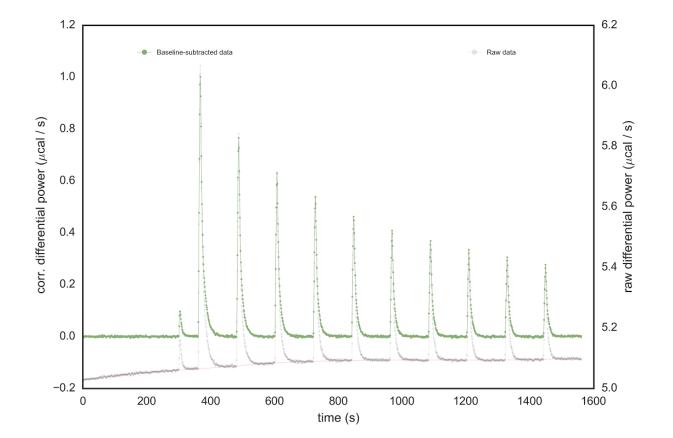


Fig. 1: In an ITC experiment, we inject from a syringe into a sample cell several times, measuring a differential power, and then integrating over that to obtain the heat of the injection, $q_n^{\rm obs}$.

Bayesian ITC

The posterior distribution is defined as

$$\mathcal{P}\left(\theta|\mathcal{D}\right)\propto\mathcal{P}(\mathcal{D}|\theta)\mathcal{P}\left(\theta
ight)$$
 (1)

Here, $\mathcal{P}\left(\theta\right)$ is a prior density of our parameters:

$$\theta = \{\Delta G_{\mathsf{bind}}, \Delta H_{\mathsf{bind}}, \Delta H_0, [\mathsf{X}_{\mathsf{syr}}], [\mathsf{M}_{\mathsf{cell}}], \sigma\}$$
 (2)

which we use to propagate instrumental errors.

Reliable baseline estimates

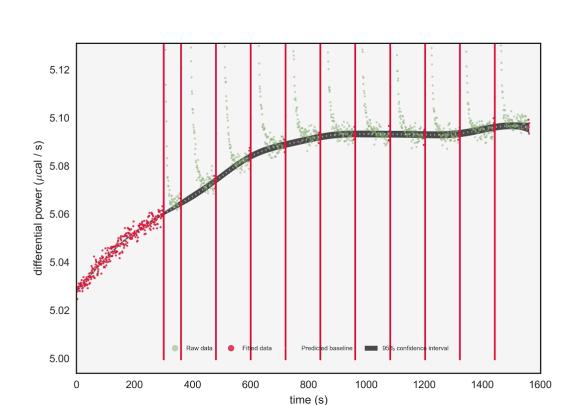


Fig. 2: We apply Gaussian process regression to increase the reliability of our baseline estimates, using scikit learn [1].

Uncertainty estimation

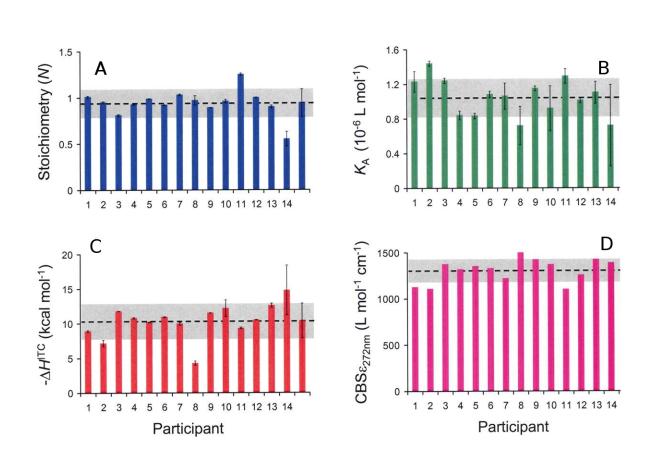


Fig. 3: Binding measurements of CBS to bovine carbonic anhydrase II from the ABRF-MIRG'2 study.

A: Stoichiometry. B: Association constant. C:

Binding enthalpy. D: Extinction coefficient of CBS, as reported by 14 participants [2].

MCMC sampling

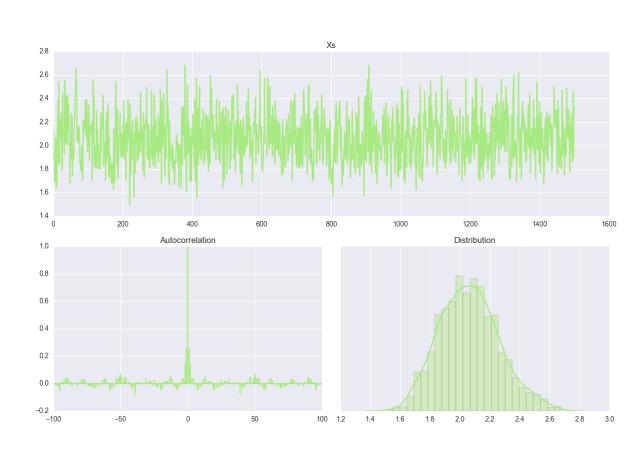


Fig. 4: An example distribution sampled for the syringe concentration using pymc [3].

Observation model

We model the integrated heats as being samples from a normal distribution \mathcal{N} ,

$$q_n^{\mathsf{obs}} \sim \mathcal{N}(q_n^{\mathsf{true}}, \sigma^2)$$
 , (3)

with the true heats $q_n^{\rm true}$ as a mean, with a variance of σ^2 .

Posterior predictions

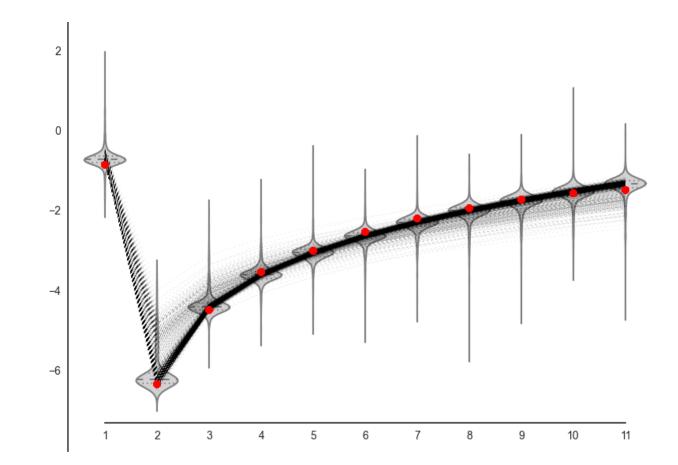


Fig. 5: Our observations (red dots) and our sampled posterior heats (violins) provide us with new estimates plus credible intervals. Model traces are shown as dotted lines.

Conclusions

- Not propagating errors in concentrations leads to large underestimation of uncertainty.
- Using Bayesian inference, we can incorporate prior information into our modeling
- MCMC will then give us posterior distributions with more accurate uncertainty estimates
- This allows us to use ITC experiments as a means of validating free energy calculations

References

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