On the Relationship Between the Force JND and the Stiffness JND in Haptic Perception

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ABSTRACT

A large variation of the haptic Just Noticeable difference (JND) in stiffness is found in literature. But no underlying model that explains this variation was found, limiting the practical use of the stiffness JND in the evaluation work of control loading system (CLS). To this end, we investigated the cause of this variation from humans' strategy for stiffness discrimination, by two experiments in which a configurable manipulator was used to generate an elastic force proportional to its angular displacement (deflection). In a first experiment, the stiffness JND was measured for three stiffness levels, and an invariant Weber fraction was obtained. We found that for stiffness discrimination, subjects reproduced the same amount of the manipulator deflection and used the difference in the terminal forces as the indication of the stiffness difference. We demonstrated that the stiffness Weber fraction and the force Weber fraction could be related by a systematic bias in the deflection reproduction, which was caused by the difference in the manipulator stiffness. A second experiment with two conditions was done to verify this model. In one condition, we measured the stiffness JND while asking subjects to move the manipulator to a target angular displacement. Thus the bias in the deflection reproduction was eliminated, and this resulted a stiffness Weber fraction that equaled the force Weber fraction. In the other condition, the stiffness JND was measured without the deflection target, and a bias in deflection reproduction was again observed. This bias related the measurements for the two conditions by the formulation obtained from the first experiment. This suggests that the accuracy of reproducing the manipulator position for stiffness discrimination, which may be susceptible to experimental setting, can be used to explain the variation of stiffness JND in literature. Suggestions are given for CLS evaluation and applications requiring precise manipulator motion control.

CCS CONCEPTS

• **Human-centered computing** → *HCI design and evaluation methods*;

KEYWORDS

Stiffness JND, Control loading system, Force JND, Systematic bias, Weber fraction

1 INTRODUCTION

Nowadays, pilots conduct a major part of their flight training in ground-based simulators (Fig. 1a). A real-world manipulator (such an aircraft sidestick, or a control column) is simulated in a simulator by a control inceptor (see Fig. 1b for an example) attached to a control loading system (CLS). By replicating the physical appearance of a manipulator and reproducing the corresponding feel of control, the control inceptor provides pilots trainees with realism of a flight.

In general cases, the basic dynamics (the feel of control) of a manipulator are defined by mechanical properties such as stiffness, damping and mass. Thus moving an aircraft manipulator resembles moving a mass that is connected with a stiff wall by a spring and a damper. Of these, stiffness, generating force proportionally to the position, contributes an important part to the control feel since most flight control inputs only have energy in the low frequency range. Inevitable problems such as the limitations from the digital control system and actuator, as well as the modeling inaccuracy of the aerodynamic forces, induce differences in stiffness between the simulator's inceptor and the aircraft manipulator. Standard evaluation procedures are required by certifying authorities, e.g., the sweep test required by the European Aviation Safety Agency (EASA) [EASA 2012]. The ultimate goal of such evaluation tests is to ensure that the inceptor stiffness is not distinguishable from the stiffness of the desired aircraft manipulator. However, this goal is not necessarily guaranteed since the evaluation is not based on the haptic perception of humans.

We intend to propose an alternative evaluation standard based on the limitation of humans' continuous haptic perception, i.e., the Just Noticeable Difference (JND) [Feyzabadi et al. 2013; Jones 1989, 2000; Pang et al. 1991]. By comparing the change in stiffness with the JND in the perception of stiffness, the inceptor's fidelity can be known. The JND in stiffness in most cases follows Weber's law [Beauregard et al. 1995; Jones and Hunter 1990, 1993; Tan et al. 1995], i.e., the JND is a invariant fraction of the stiffness level. However, this fraction, usually being referred to as Weber fraction, seems to
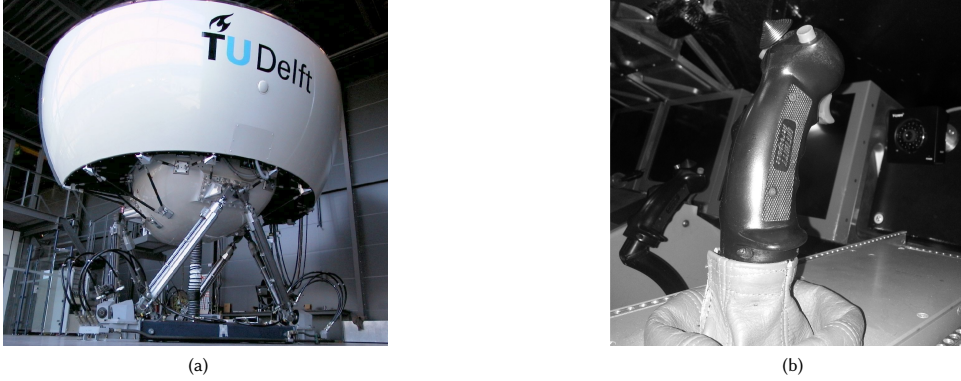
(a)

(b)

Figure 1: (a): Example of a modern flight simulator: the SIMONA Research Simulator (SRS) at the faculty of Aerospace Engineering, Delft University of Technology. (b): the control inceptor in SRS which simulates a typical aircraft side-stick manipulator.

Table 1: Spring Stiffness JND in literature

Paper	Experimental Conditions	Weber Fraction
[Jones and Hunter 1990]	Bilateral Matching	23%
[Tan et al. 1995]	Fixed Displacement	8%
	Roving Displacement	22%
[Varadharajan et al. 2008]	Visual Information	14.2%
	No Visual Information	17.2%

be susceptible to experimental settings since a large variation can be found among results. Several representative results are shown in Table 1.

Due to this, individual measurements of the stiffness JND lack application value. An appropriate value and an underlying model that explains the variation, are essential for our practical application, but have yet to be found. In our view, addressing this problem will not only give better guidelines for simulator certification, but also be beneficial to the design of haptic support systems for vehicle control and the evaluation of transparency of tele-operation systems. To this end, we investigated the stiffness JND and the strategy that humans use for stiffness discrimination in two experiments performed by human subjects. The main tasks in the two experiments were both discriminating between different levels of manipulator stiffness, but were conducted for different reference levels of stiffness or different instructions for manipulator motion. More details will be given in Section 2. Based on the observed strategy for stiffness discrimination during self exploration, we demonstrated that the stiffness JND can be related to the force JND, by a systematic bias in the reproduced manipulator deflection in the discrimination task. This bias, caused by the stiffness differences that was to be identified in the discrimination tasks, can be used to explain the variation of stiffness JND measurements in literature.

This paper is organized as follows. The experiment setup and procedure, as well as the method for measuring the JNDs, are elaborated in Section 2. Section 3 describes the conditions and results

of the first experiment, as well as subjects' strategy for stiffness discrimination. The stiffness JND was then formulated as a function of the force JND and the accuracy of displacement control. In Section 4 the conditions and results of the second experiment are given. The formulation obtained from the first experiment is validated by the measurements. We discuss the results, conclusion on the strategy and causes of variations of the stiffness JND in Section 5. Conclusions on the contribution of this work are given in Section 6.

2 EXPERIMENTAL SETUP AND METHODOLOGY

2.1 Apparatus

Experiments were conducted in the Human-Machine Interaction Laboratory at the faculty of Aerospace Engineering, TU Delft. An illustration of the devices can be seen in Fig. 2a. A hydraulic side-stick manipulator was used in the experiment. It could be moved around the roll axis (left/right) like a joystick, as can be seen in Fig. 2b. The deflection of the manipulator in the pitch axis (forth/back) was fixed in the neutral point. The manipulator was configured to use minimum mass and damping settings. The stiffness setting K of the manipulator was configured according to different experimental conditions. So the manipulator resembled a spring generating torque ($\tau(t)$) to its deflection (angular displacement, $x(t)$):

$$\tau(t) = K \cdot x(t) \tag{1}$$

The center of the grip point on the manipulator was $0.09\ m$ above the rotation origin, which can be used to calculate the corresponding force. An LCD screen (marked by the rectangle in Fig. 2a) was placed in front of the subject, to show the timing of experimental runs for both experiments, and the visual presentation of the manipulator deflection (shown in Fig. 2c) for the second experiment only. Subjects were asked to wear an active noise suppression headphone (David Clark H10-66XL), to cancel possible auditory cues.

2.2 Subjects

Eight subjects participated in the first experiment, and 11 subjects participated in the second. All were PhD students or academic staff from Delft University of Technology. They were all right handed and
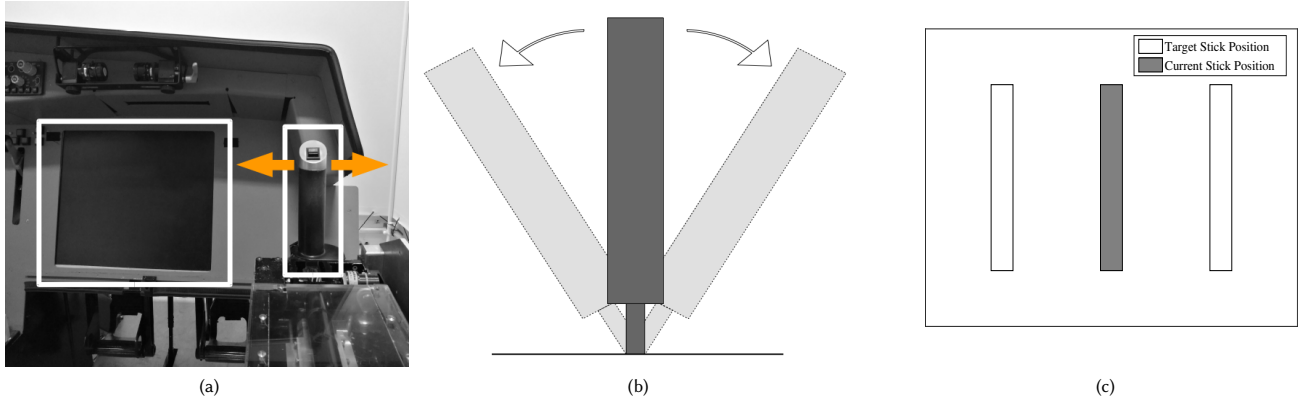
(a)　　　　　　　　　　　　　　　　　　(b)　　　　　　　　　　　　　　　　　　(c)

Figure 2: (a): The devices used in the JND experiment. The side-stick manipulator and the LCD screen are marked by white rectangles. The headphone used in the experiment is not shown in this figure. (b): The motion pattern of the side-stick manipulator. The manipulator could be moved about the roll axis (left/right) like a joystick. (c): The visual presentation of the angle of manipulator deflection shown on the LCD screen in the second experiment. Two target deflections of the manipulator on the two sides represent different directions (left/right) of motion.

reported no abnormality of the neuromuscular system or hand/arm impairment. An informed consent was signed by subjects before experiments.

2.3 Procedure

The upper JND in stiffness accounting for the difference threshold for stiffness increments was investigated in this study. We used an one up/two down weighted adaptive staircase procedure [GarcıÌĄa-Pérez 1998; Kingdom and Prins 2016] to measure the stiffness JND for all the conditions in both experiments. This procedure converges to a JND level corresponding to 80.35% correct performance, an example is given in Fig. 3.

For each condition, a complete staircase procedure, which generally contained about 20 trials, was performed by the subject. Each trial consisted of two 5-second runs. In one of the two runs, a fixed reference stiffness setting (K_r) was simulated by the manipulator, and in the other run an adjustable controlled setting, a stiffer manipulator ($K_c = K_r + \delta K$, $\delta k > 0$) was simulated. The order in which the two settings were simulated in each trial was random. In this paper, we define the run simulating the reference stiffness setting as the reference run and the other run as the controlled run.

During each trial, we asked the subject to identify in which run the manipulator was felt to be stiffer to move. The controlled stiffness was adjusted according to a subject's response. δK was reduced in the following trial when the subject correctly identified the controlled run in two consecutive trials, and was increased by a wrong identification. We define a reversal as a point where the staircase curve changes its direction, as shown by solid circles in Fig. 3. A staircase procedure ended when the 7th reversal occurs, or when the total trial number reached 40. The JND measurement was taken as the average of δK in the last four reversals.

Sufficient training was performed preceding the formal experiment to familiarize our subjects with the procedure and requirements. Note that we define the stiffness by a rotational convention
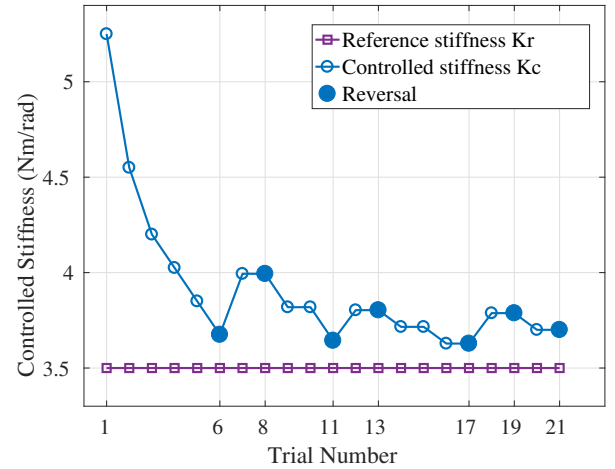

Figure 3: An example of the staircase procedure that obtained during the experiment. One-up one-down procedure was used before the first reversal to achieve a quick convergence.

($N \cdot m/rad$), which can be transformed to the linear convention (N/m) by the rotational radius (0.09 m) given in Section 2.1.

3 THE FIRST EXPERIMENT

The aim of the first experiment is to investigate how humans estimate differences in manipulator stiffness during self exploration. The stiffness JND was measured on three reference stiffness levels, as shown in Table. 2.

The stiffness JND is found to follow Weber's law in literature, these three conditions are sufficient to verify such characteristics.

Table 2: Stiffness Settings in the First Experiment

		C1	C2	C3
K_r	$(N \cdot m/rad)$	2.0	3.5	5.0
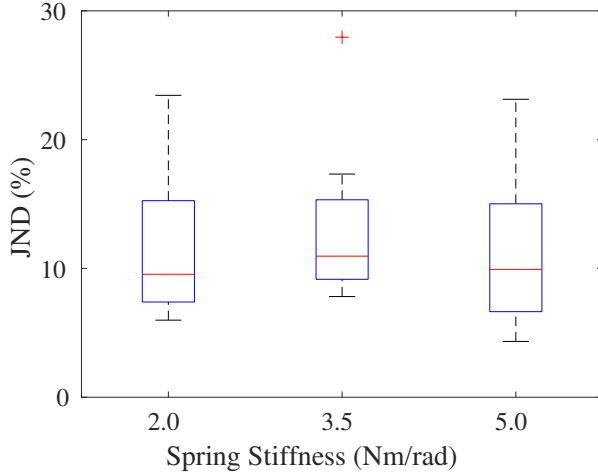
Figure 4: Boxplots of the stiffness JND (plotted as the Weber fraction: $\frac{\Delta K}{K_r}$) for three stiffness settings; the "+" symbol represents the outliers.

We expect to obtain an invariant Weber fraction from the result (define ΔK as the stiffness JND, $\Delta K/K_r$ is constant).

During the experiment, subjects were encouraged to develop their own strategies to identify the stiffness difference between the two runs in each trial. They were also suggested to apply any motion to the manipulator as they would like to (however, extremely fast movement making the system respond at the eigenfrequency was not allowed). The visual display (the LCD screen shown in Fig. 2a) only indicates the time of the starting and ending of each run. So subjects had no additional visual feedback on the manipulator motion from the LCD screen. Such experimental instruction allowed subjects to distinguish between different manipulator stiffness during self exploration, and allowed us to investigate subject's strategies for stiffness discrimination in a more general way.

3.1 Result

Measurements of the stiffness JND for the three conditions are shown as Weber fractions in Fig. 4. As expected, different stiffness levels have no significant effects on Weber fraction for stiffness (one-way ANOVA, $F(2, 21 = 0.23), p = 0.7969$). Thus Weber's law is indeed observed for the stiffness JND. The average of the Weber fractions is $12.13\% \pm 1.06\%$. The remaining questions is how humans estimate a stiffness change.

3.2 Strategy

After the experiment, we verbally questioned subjects on strategies they used for the assessment of stiffness differences. We found that all subjects used the terminal (maximum) force as an indication
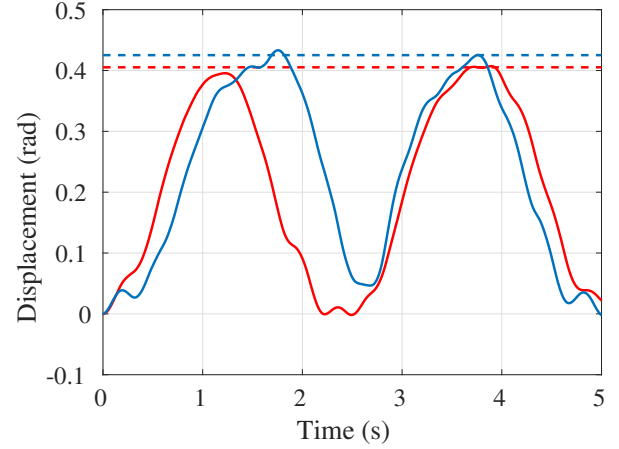
Figure 5: An example of the side-stick motion trajectories in one trial. The blue solid line represents the manipulator deflection in the reference run, and the red one is for the controlled run. The dashed lines show the average amounts of terminal displacements (peaks of the deflection) applied in the two runs (blue and red ones ones for the reference and controlled runs respectively).

of a change in manipulator stiffness. This finding is in line with the conclusion given by [Tan et al. 1995], in which the author also suggested the contribution of the terminal force.

With more detailed verbal survey we found that the two variables: the manipulator deflection and force, played different roles for stiffness discrimination. During each trial, the subject reproduced a same angular displacement (deflection) in the two runs, then reported the decision on the stiffer manipulator in response to the stronger terminal force. In each run, all subjects generally resampled the force maximums (the terminal forces) for several times through several individual motions, in order to confirm their perceptions of the terminal force. An example of the manipulator deflection data obtained in one trial is given in Fig. 5. In this case, an angular displacement was reproduced in the two runs, and each terminal force (the forces at the displacement peaks) was sampled twice.

We use a diagram to illustrate the stiffness discrimination procedure in each trial, as shown in Fig. 6a. In each trial, a desired angular displacement of the manipulator (X_d) is reproduced for several times in both runs. We define X_r and X_c as the averages of realizations of the angular displacement in the corresponding runs (subscripts r and c correspond to the reference and controlled runs respectively), an example is shown in Fig. 5, in which X_r and X_c are represented by the blue and red dashed lines respectively. F_r and F_c denote the terminal forces corresponding to these displacements. These two force maximums are compared as the indication of stiffness changes. The actual force discrimination procedure undertaken in the central nervous system is assumed to be probability based. We assume a fixed value to account for the converged threshold effect, as shown by a deadband in Fig. 6b. Since the force difference threshold can be formulated by the Weber's law, in addition it was found that threshold sizes for elastic forces and constant forces are

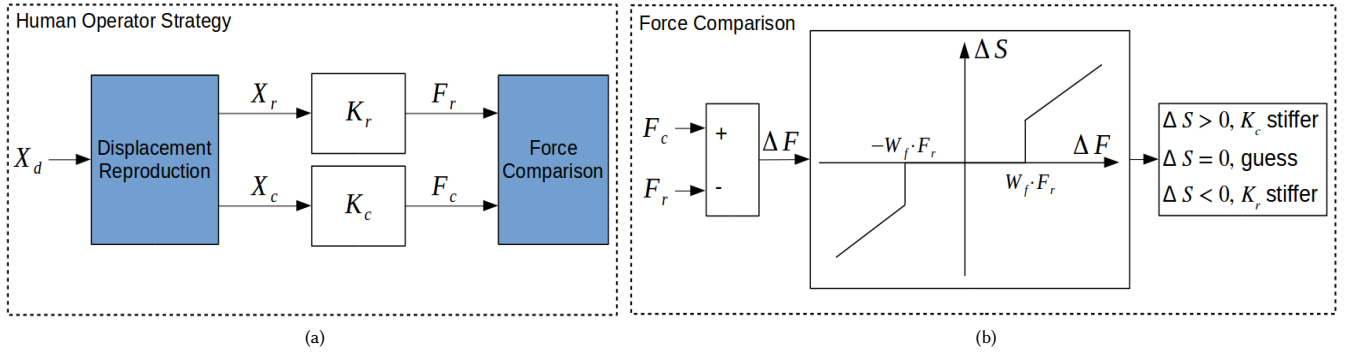(a)   (b)

Figure 6: Spring stiffness comparison procedure in each trial (a) and details of the force comparison process (b).

roughly the same [Tan et al. 1995]. We quantify the deadband's threshold size as a constant fraction W_f of the terminal force in the reference run F_r. The output of this deadband ΔS accounts for the perceived force change. Subjects identify the stiffer manipulator according to a criterion based on ΔS:

$$\Delta S \begin{cases} > 0, \text{select } K_c \text{ as the stiffer simulation} \\ = 0, \text{a random selection (guess)} \\ < 0, \text{select } K_r \text{ as the stiffer simulation} \end{cases} \quad (2)$$

A decision regarding to the stiffer manipulator is made in response to the larger force. When no force difference is perceived, e.g., the force difference is smaller than the force JND, a subject has to guess.

Normalizing ΔF to F_r, we get:

$$r = \frac{\Delta F}{F_r} = \frac{X_c \cdot K_c - X_r \cdot K_r}{X_r \cdot K_r} \quad (3)$$

In each trial, if the absolute value of r is larger than the Weber fraction for the force W_f, a decision will be made accordingly. A guess answer occurs when r falls into the range of $[-W_f, W_f]$. Simplifying this equation by defining α as the ratio between X_c and X_r:

$$r = (\alpha - 1) + \alpha \cdot \frac{\delta K}{K}$$
$$\text{where} \quad \alpha = \frac{X_c}{X_r} \quad , \quad K_c = K_r + \delta K \quad (4)$$

As shown by the example in Fig. 5, subjects seldom made identical deflections in the two runs in each trial ($\alpha \neq 1$). The variation of α could result a fluctuation in the level of detectable stiffness difference from trial to trial. For a staircase procedure, the converged level of δK, i.e., the stiffness JND measurement ΔK, is determined by the average of α. By equating r to W_f, the Weber fraction for stiffness can be formulated:

$$W_k = \frac{\Delta K}{K_r}$$
$$= \frac{W_f - (E(\alpha) - 1)}{E(\alpha)} \quad (5)$$

Here $E(\alpha)$ denotes the average of α.

If the variation of α is only caused by random errors, i.e., $E(\alpha) = 1$, the staircase would converge to a stiffness Weber fraction identical
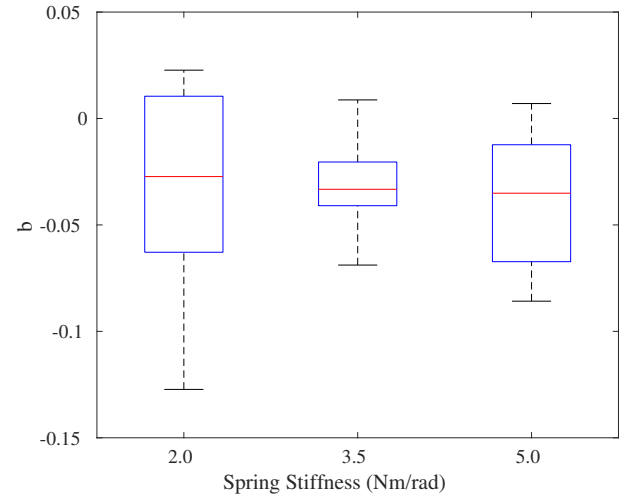

Figure 7: The bias ratios $b = E(\alpha) - 1$ of subjects for the three conditions.

to the force Weber fraction ($W_k = W_f$). Whereas if the errors also contain systematic components (bias), differences between the two fractions would be observed. $E(\alpha) < 1$ indicates a stiffness Weber fraction larger than the force Weber fraction. For $E(\alpha) > 1$ the opposite happens.

3.3 Effects of the Deflection Reproduction

In order to investigate whether a systematic bias exists, we examined the ratio of bias (b) of each subject's data obtained in trials after the third reversal, according to:

$$b = E(\alpha) - 1 = \frac{1}{N} \sum_{i=1}^{N} \frac{X_{c,i} - X_{r,i}}{X_{r,i}} \quad , \quad (6)$$

in which N denotes the total number of trials after the third reversal of a staircase, $i = 1$ denotes the first trial after the third reversal.

The distribution of the bias (b) of subjects is shown by a boxplot in Fig. 7. No significant difference is found among different condi-

tions (one-way ANOVA, $F(2, 21) = 0.08, p = 0.9204$). The average of b is -0.032, the result from T-test shows this average differs from zero with significance. This implies that a systematic bias indeed exists in the angular displacement reproduction, and the angular displacements of the stiffer manipulator are 3.2% smaller on average for all three stiffness levels, $E(\alpha) = 0.968$.

Taking the averages of both b and the stiffness Weber fraction measurement W_k into Eq. (5), we get 8.2% for the force Weber fraction W_f. This value is in line with the literature results (5% - 10%,[Jones 1989; Pang et al. 1991]), indicating that the force JND and the stiffness JND can indeed be related by Eq. (5).

From this, it seems that the level of humans' haptic JND in stiffness is determined by two factors, i.e., the accuracy of the position reproduction and the force JND. The model in Eq. (5) states that humans' sensitivity to stiffness variations can be improved by increasing their accuracy in the position reproduction. When the bias is removed, the stiffness Weber fraction is equal to the force Weber fraction. To verify this, a second experiment was designed.

4 THE SECOND EXPERIMENT

4.1 Experiment Settings

In the second experiment, we measured the stiffness JND for a reference stiffness of 3.5 $N \cdot m/rad$ with two conditions. In one condition, on the LCD screen (marked by the rectangle in Fig. 2a) a presentation of the manipulator angular displacement (see Fig. 2c) was shown. As can be seen, the presentation provides the target and the current manipulator defections. The current manipulator deflection is shown by the solid bar. The targets are shown by the empty bars on the two sides, they are both 0.37 rad but denote different directions. Deflecting the manipulator to the right by an angle of 0.37 rad will reach the target on the right side, and a deflection to the left will reach the left target.

During each trial of the staircase procedure, in addition to the stiffness discrimination task, we asked our subjects to move the manipulator to either of the targets for two or three times in each run. Since in this way no errors should exist in the displacement reproduction, this condition would give a stiffness JND that is only determined by the force JND. In the remainder of this paper, we refer to this condition as the visual condition. The other condition was performed without this displacement presentation, subjects were asked to discriminate between different stiffness settings in a way similar to the first experiment. In order to minimize the uncertainty resulting from different manipulator movement frequencies, we suggested subjects to apply a manipulator motion with a similar pattern (two or three moves) as for the visual condition. We refer to this condition as the non-visual condition. Results of the two conditions should be related by Eq. (5), if their difference is mainly determined by the displacement reduction for the stiffer-manipulator simulation.

4.2 Result

The obtained Weber fractions for the two conditions are shown in Fig. 8. An improvement in the stiffness JND induced by the visual presentation was found to be significant (T-test). The average Weber fraction was 11.08% for the non-visual condition, and this fraction reduced to 7.79% for the visual condition.
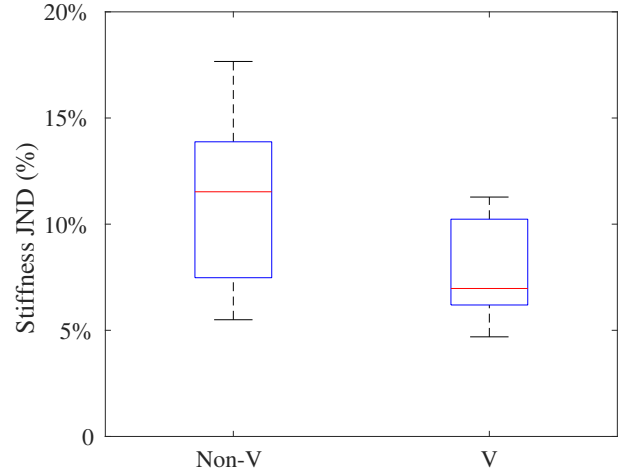

Figure 8: The Weber fractions for stiffness ($\Delta K / K_r$) obtained from the two conditions.

4.3 Analysis of results

As expected, lower stiffness JND was found when subjects were helped in the visual condition to perform two almost identical manipulator deflections. The resulting Weber fraction, 7.79%, as discussed earlier in Section 4.1 is also the force Weber fraction of our subjects. The JND measurement for the non-visual condition (11.08%) is similar to that obtained in the first experiment (12.13%). Moreover, for the non-visual condition a reduction in deflection of the stiffer manipulator is again observed. The average of the bias ratio b is 3.22%, significantly differs from zero (T-test). Thus $E(\alpha)$ for the non-visual condition is 0.9678. By taking this value and the JND measurement (11.08%) into Eq. (5), we get 7.50% for the force Weber fraction W_f. This value is almost identical to the measurement for the visual condition, the model obtained from the first experiment is therefore validated.

5 DISCUSSION

We can conclude that in our experiment the estimate of stiffness a change is dominated by the perceived force difference, and therefore can be affected by a systematic biases existing in the position reproduction. Subjects appear to use a strategy in which they compare the force at its maximum, and in the cases where there is a position bias, a stiffness difference is no longer fully reflected by the force difference. The smaller deflection of a stiffer manipulator would make the force difference less evident, and a larger stiffness difference is thus required to provide a detectable quantity.

In [Pressman et al. 2007], it was shown that humans detect the time delay in elastic force by interpreting the change in the perceived stiffness. There it was suggested that humans use the ratio between maximum force and perceived (not actual) amount of spring deflection as the indication of the amount of stiffness. In our work the perceived manipulator deflections during each discrimination task should be the same, since subjects intended to reproduce this variable. Hence comparing the maximum force is basically in

line with this previous finding, implying that this strategy is the intuitive choice for stiffness discrimination.

The second experiment validated the conclusion obtained from the first experiment. The measurements obtained for the two conditions, i.e., the force Weber fraction obtained in the visual condition and the stiffness Weber fraction obtained in the non-visual condition, can be related by the formulation in Eq. 5. The accurate estimate of the force Weber fraction excludes the significance of other possible causes for the differences between stiffness and force Weber fractions, allowing us to formulate the stiffness JND as a function of the force JND and position reproduction accuracy.

The systematic bias in the position reproduction caused by the stiffness variation reflects the effect of motion in the stiffness JND, and could be used to explain the variation of the stiffness JND measurements in the past research. Different experimental settings and procedures may affect the accuracy of position reproduction, hence inconsistency of stiffness JND measurement is expected. The large Weber fraction given by [Jones and Hunter 1990] may indicate that the accuracy is lower when one tries to reproduce the position of one arm by the other arm. The force threshold may also be affected by bilateral comparison, so that both a smaller α and a higher W_f result a larger stiffness JND. Similar to our work, in [Tan et al. 1995], when a fixed displacement was imposed the force and stiffness Weber fractions were found to be equal. The involvement of other sensory cues such as visual presentation may, on the upside, provide improvement in motion perceiving accuracy, on the downside, introduce bias or even suppress proprioceptive position cues and induce illusions. Without a fixed visual target as introduced in our work [Varadharajan et al. 2008], the stiffness JND was still improved from 17% to 14% when vision cues were provided. Because of the visual contribution, the bias in perception of stiffness caused by different spatial locations of the object was eliminated, and the JND was also improved from 10% to 5% [Wu et al. 1999]. Visual cues are also found to suppress the proprioceptive position percept, which could largely vary the perception of stiffness so that even an obviously detectable stiffness increment may not be perceived or misperceived as a decrement [Srinivasan et al. 1996]. We conjecture that these changes are due to the effect of vision on the perceiving and reproducing of the arm motion, which results in a different force for comparison. Hence, with different visual cues or other causes affecting the position reproduction process, the stiffness JND measurements in previous research are bound to show variation. In some other cases due to some imposed constraints subjects had to choose different strategies rather than the intuitive one obtained in this work. When the amounts of spring deflection during discrimination were artificially varied, such as by providing roving displacements [Tan et al. 1995], subjects couldn't compare the maximum force leading to a significant increase in the stiffness JND. In addition to the above, the variations in the devices should also be considered. For different devices, the dynamics of actuators in moderate and high frequency range, which could be effectively perceived as mechanical properties such as the mass and damping, are different. If the stiffness discrimination is conducted during non-static motion, for example using a sinusoidal motion profile, the stiffness JND could also be altered by the effective mass and

damping perceptions due to the masking effect among mechanical properties [Rank et al. 2012].

Typical situations in flight control are similar to a tracking process with a compensatory task [McRuer and Jex 1967; Van Paassen et al. 2004]. This leads to a high accuracy of the manipulator position control. The stiffness Weber fraction would in these cases approach the force Weber fraction, indicating that the force Weber fraction is more appropriate to evaluate the stiffness of the simulator inceptor.

The result obtained in this study is from a manipulator that has minimal achievable mass and damping settings, so that the perception of manipulator stiffness is in isolation. The results can be used to examine that whether the stiffness difference between the aircraft manipulator and simulator inceptor cause a changed perception when the pilot maintains a static inceptor motion. The current results will need to be extended for cases where the control input contains energy in higher frequency range. This is because the mass and damping of a real-world manipulator are usually much higher than the settings used in this study. So that the force caused by manipulator mass and damping can not be ignored, which will probably affect the stiffness JND since the perceptions of these three mechanical properties are found to be coupled [Rank et al. 2012].

Nevertheless, the cause of the observed position bias should be addressed. It may be a result of the involvement of the force in humans' haptic position estimates. Evidence [Mugge et al. 2009; Wydoodt et al. 2006] has shown that when a correlation exists between the force and motion, e.g., a spring loaded manipulator, the perception of the position is affected by the force. An unnoticeable stiffness increment causes the blindly reproduced displacement to be overestimated (a negative bias) [Mugge et al. 2009], which is in line with our observation. This phenomenon still needs detailed research, and will be addressed in our future work.

6 CONCLUSION

In this study with two experiments we formulated the stiffness JND as a function of the force JND and position reproduction accuracy. In the first experiment, the stiffness JND was measured at three stiffness settings. Consistent with results in literature, the results followed Weber's law. Subjects used a maximum force strategy based on position reproduction for stiffness discrimination. A negative bias was observed in the reproduced deflection of the stiffer manipulator. The stiffness JND was then formulated as a function of this bias and the force JND. In order to validate this formulation, we used an experiment condition in which a visual presentation eliminated the bias in the deflection reproduction during stiffness discrimination, which resulted a stiffness Weber fraction equal to the force Weber fraction. The result was compared to the stiffness JND obtained without visual cues from the same group of subject. A negative bias along with the stiffer manipulator was again observed when the visual cues were turned off. With this bias, the force JND and stiffness JND measured in the two conditions could be related. The formulation obtained from the first experiment was validated.

Future work includes investigation into the cause of the systematic error observed in this work. Since other mechanical properties

such as the manipulator damping and mass also produce force responses to the motion in ways similar to the stiffness, we expect to characterize the JNDs for these two properties by similar models. These models will probably be based on the control of arm velocity, and the control of arm acceleration (through controlling the arm velocity and accelerating time).

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