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An Adaptive Grid Algorithm For Computational Shock Hydrodynamics

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Abstract

During the development of computational methods that solve time dependent shock hydrodynamic problems, two underlying strategies have emerged that enable flow features to be resolved clearly. One, employ a numerical scheme of inherently high resolution, usually a second-order Godunov-type method. Two, locally refine the computational mesh in regions of interest. It has been demonstrated by Berger & Collela that a combination of both strategies is necessary if a solution of very high resolution is sought. The present study combines Roe's flux-difference splitting scheme with an adaptive mesh refinement algorithm developed from the ideas of Berger. The result being a general purpose scheme that can fully resolve complicated flows but which requires only modest computing power.

The material in this thesis reflects three broad aims. First, to explain the methodology and intricacies of our scheme. Compared to non-adaptive methods our scheme is undeniably complicated, for it contains many elements which must be carefully co-ordinated. Second, to vindicate this complexity. To this end, computational results are presented which are comparable in resolution to Schlieren photographs, yet the calculations were performed on a small desktop workstation. Third, to give sufficient details of our implementation so as to allay the apprehensions of any person who might wish to code up the scheme.

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Nomenclature

Hierarchical Grid System

G	Computational grid
l	Grid level l
l_{max}	Highest grid level
G_l	Grid at level l
nG_l	Number of meshes at level l
$G_{l,k}$	k^{th} mesh at level l
Gp_l	Index for 1^{st} mesh at level l
$\mathbf{C}_l^2$	Logical co-ordinate system for level l
$< IW, JS, IE, JN >$	Mesh extent
$\Box_{l,k}$	Mesh extent for $G_{l,k}$ using $\mathbf{C}_l^2$ co-ordinate system
$\Box_{l,k}^c$	Mesh extent for $G_{l,k}$ using $\mathbf{C}_{l+1}^2$ co-ordinate system
$IM_{l,k}$	Width of $G_{l,k}$
$JM_{l,k}$	Height of $G_{l,k}$
$G_{l,k:i,j}$	The ij^{th} cell contained by $G_{l,k}$
$G_{l,k:N;i}$	The i^{th} interface along the Northern boundary of $G_{l,k}$
$G_{l,k:S;i}$	The i^{th} interface along the Southern boundary of $G_{l,k}$
$G_{l,k:E;j}$	The j^{th} interface along the Eastern boundary of $G_{l,k}$
$G_{l,k:W;j}$	The j^{th} interface along the Western boundary of $G_{l,k}$
$\mathbf{W}$	Field solution contained by G
$\mathbf{W}_l$	Field solution contained by G_l
$\mathbf{W}_{l,k}$	Field solution contained by $G_{l,k}$
$\mathbf{W}_{l,k:i,j}$	Solution vector contained by $G_{l,k:i,j}$
rI_l	Number of sub-divisions made along I co-ordinate lines
rJ_l	Number of sub-divisions made along J co-ordinate lines
Δt_l	Time step used to integrate $\mathbf{W}_l$

Automatic Adaption Process

$\widetilde{G}$	Newly adapted computational grid
$\widetilde{W}$	Newly adapted field solution
F_{tol}	Factor used to control flagging for refinement process
P_{tol}	Factor used to control clustering process

Interface Fluxes and Riemann Solvers

$(\mathbf{W_L}, \mathbf{W_R})$	Riemann problem with left state $\mathbf{W_L}$ and right state $\mathbf{W_R}$
$(\mathbf{W^*_L}, \mathbf{W^*_R})$	Intermediate states for the solution to $(\mathbf{W_L}, \mathbf{W_R})$
$\mathbf{F}$	Conservative flux vector
$\mathbf{A}$	Jacobian matrix, $\frac{\partial \mathbf{F}}{\partial \mathbf{W}}$
(x, y)	Cartesian co-ordinate system
(n, t)	Local co-ordinate system normal and tangential to a cell interface
V_x, V_y	Velocity components using (x, y) co-ordinate system
V_n, V_t	Velocity components using (n, t) co-ordinate system
τ	Time
$\mathbf{F}_{i+\frac{1}{2}}$	Numerical flux across the interface between the i^{th} and $i^{th}+1$ cells
$\widetilde{\mathbf{A}}$	Linearized Jacobian matrix
α_k	Strength of k^{th} wave
$\widetilde{\lambda}_k$	Velocity of k^{th} wave
ν_k	Courant number for k^{th} wave
$\widetilde{\mathbf{e}}_k$	k^{th} eigenvector of $\widetilde{\mathbf{A}}$
δ_k	Spreading rate for k^{th} wave
B_k	Limiter function for k^{th} wave
A_k	Amplification factor for k^{th} wave
w_k	k^{th} weight for Weighted Average Flux method
$\Delta()$	Difference between right and left values, $()_{\mathbf{R}} - ()_{\mathbf{L}}$
$\Delta^{(k)}$	Difference across the k^{th} wave
$\widetilde{()}$	Roe averaged quantity

Miscellaneous

$()_\infty$	Freestream reference conditions
a	Speed of sound
P	Pressure
ρ	Density
T	Temperature
γ	Ratio of specific heats
E_t	Total Energy
H	Total Enthalpy
(u, v)	Cartesian velocity components
Re	Reynolds number
Pr	Prandtl number
μ	Coefficient of viscosity
λ	Coefficient of heat conduction

$\tau_{xx}, \tau_{yy}, \tau_{xy}, \tau_{yx}$	Components of shear stress tensor
q_x, q_y	Components of heat flux vector

Moving Shock Relationships

$()_1$	Quiescent fluid
$()_2$	Post-shock fluid
M_s	Shock Mach number
U_s	Shock speed

Acronyms

AMR	Adaptive Mesh Refinement
CFD	Computational Fluid Dynamics
CFL	Courant – Friedrichs – Lewy condition
MR	Mach Reflection
RR	Regular Reflection
CMR	Complex Mach Reflection
DMR	Double Mach Reflection
SMR	Simple Mach Reflection
SIMD	Single Instruction Multiple Data
MIMD	Multiple Instruction Multiple Data

Some notation which is used infrequently is labelled within the main body of the text.

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Finally, I wish to thank all my associates for putting up with my bad temper over the period of writing up this work. Hopefully this behavioural trait is a mere transient brought about by an attack of thesis blues, but doubtless many people would proffer alternative, less palatable diagnoses.