Question Bank

ACADEMY OF TECHNOLOGY

Subject: MATHEMATICS   Subject Code: BSC-301

Semester: 3RD   Stream: CSE1/CSE2/CSE3

MODULE 1: SEQUENCE AND SERIES

Sl. No.	Question	Marks
1.	State D'Alembert's ratio test and Raabe's test	5
2.	State Leibnitz's Theorem of infinite series with an example	5
3.	Define absolute and conditional convergence of an infinite series.	5
4.	Define convergence of an infinite series with examples.	5

Sl. No.	Question	Marks
5.	For what values of x the following series is convergent: $\sum_{n=1}^{\infty} \frac{(n+1)^n x^n}{n^{(n+1)}}, x > 0$.	5

Sl. No.	Question	Marks
6.	Show that, $e^x = e^2 + (x-2)e^2 + \frac{(x-2)^2}{2!}e^2 + \cdots$	5
7.	Examine the convergence of the series: $1 + 1/2^2 + 2^2/3^3 + 3^3/4^4 + 4^4/5^5 + \text{------------------}$	5
8.	Examine the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$.	5

9.	Examine the convergence of the series $\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}+\ldots\ldots\ldots\ldots\ldots\ldots$	5
10.	Examine the convergence of the series whose n-th term is $\frac{1.3.5.\ldots\ldots..(2n-1)}{2.4.6.\ldots\ldots.(2n-2)}.\frac{x^n}{2n-1}$.	5
11.	Examine the convergence of the series $\sum_{n=1}^{\infty}\frac{n!2^n}{n^n}$.	5
12.	Examine the convergence of the series: $1^2.2^2/1! + 2^2.3^2/2! + 3^2.4^2/3! +$ ------------------.	5
13.	Examine the series: $1/(1+2) + 2/(1+2^2) + 3/(1+2^3) +$ --------	5
14.	Examine the convergence of the series $1 - 1/2^{1/2} + 1/3^{1/2} - 1/4^{1/2} +$ ----------------------.	5
15.	Examine the convergence of the series: $1/2^3 - (1+2)/3^3 + (1+2+3)/4^3 - (1+2+3+4)/5^3 +$ --------------.	5
16.	Examine the convergence of the series: $(1/3)^2 + \{(1.2)/(3.5)\}^2 + \{\{1.2.3)/(3.5.7)\}^2 +$ ----------------	5
17.	Check whether the following series is convergent or divergent: $\sum_{n=1}^{\infty}\frac{n^2+1}{n^2-1}x^n,\ x>0$.	5
18.	Examine the convergence of the following series $1/x - 1/(x+3) + 1/(x+6) -1/(x+9) + \ldots$ $(x> 0)$	5
19.	Solve the series: $1+ \{2^2/3^2\}x + \{(2^2.4^2)/(3^2.5^2)\}x^2 + \{(2^2.4^2.6^2)/(3^2.5^2.7^2)\}x^3 +$ -----------$(x\neq 1)$.	5
20.	Show that the series $1 - 1/2 + 1/3 - 1/4 +$ ---------------- converges conditionally.	5
21.	Determine cosx in powers of x in infinite series stating the condition under which the expansion is valid.	5

MODULE 2: MULTIVARIABLE CALCULUS (DIFFERENTIATION)

Sl. No.	Question	Marks
1.	If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, show that $\partial(x,y,z)/\partial(r,\theta,\phi) = r^2 \sin\theta$.	5
2.	If $y = f(x+ct) + \phi(x-ct)$, show that $\partial^2y/\partial t^2 = c^2\, \partial^2y/\partial x^2$.	5
3.	Let $f(x,y) = \dfrac{xy}{x^2+y^2}$, if $(x,y) \neq (0,0)$. Show that lim f(x,y) as $x\to 0$, $y\to 0$ does not exist.	5
4.	If $u = f(y-z, z-x, x-y)$, show that $(\partial u/\partial x) + (\partial u/\partial y) + (\partial u/\partial z) =0$.	5
5..	If $u = \log(x^3+y^3+z^3-3xyz)$, show that $(\partial u/\partial x) + (\partial u/\partial y) + (\partial u/\partial z)=3/(x+y+z)$	5
6.	If $u = \log(x^3+y^3+z^3-3xyz)$, show that $\partial^2u/\partial x^2 + \partial^2u/\partial y^2 + \partial^2u/\partial z^2 = -3/(x+y+z)^2$	5
7.	If $u = \log(x^3+y^3+z^3-3xyz)$, show that $(\partial/\partial x+\partial /\partial y+\partial /\partial z)2u = -9/(x+y+z)^2$.	5
8.	Given $f(x,y) = \begin{matrix} xy\frac{x^2-y^2}{x^2+y^2} & if \quad x^2+y^2 \neq 0 \\ 0 & if \quad x^2+y^2 = 0 \end{matrix}$, show that $\frac{\partial^2 f}{\partial x\partial y} \neq \frac{\partial^2 f}{\partial y\partial x}$ at the origin.	5
9.	If $u = f(x^2+2yz, y^2+2zx)$, show that $(y^2-zx)\frac{\partial u}{\partial x} +$ $(x^2-yz)\frac{\partial u}{\partial y} + (z^2-xy)\frac{\partial u}{\partial z} = 0$.	5
10.	If $\vec{v} = \vec{\omega}\times\vec{r}$, then show that $\vec{\omega} = \frac{1}{2} Curl\,\vec{v}$, where $\vec{\omega}$ is a constant vector.	5
11.	Calculate the maxima and minima of the function $x^3+y^3-3x-12y+20$ and also the saddle points.	4+2
12.	Calculate the maximum value of x^3y^2 subject to the constraint $x+y=1$, using the method of Lagrange's multiplier.	7

13.	Calculate the directional derivative of $f = xyz$ at (1,1,1) in the direction $2\hat{i} - \hat{j} - 2\hat{k}$.	5

Sl. No.	Question	Marks
14.	Evaluate div $\vec{F}$ and curl $\vec{F}$, where $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$.	5
15.	Evaluate the directional derivative of $f = xyz$ at (1,1,1) in the direction $2\hat{i} - \hat{j} - 2\hat{k}$.	5
16.	Justify Euler's theorem for the function f(x,y) = $(x^{1/4} + y^{1/4})/(x^{1/5} + y^{1/5})$.	5
17.	If $z = e^{xy^2}$, x = tcost, y = tsint, evaluate the value of (dz/dt) at t = π/2.	5

MODULE 3: MULTIVARIABLE CALCULUS (INTEGRATION)

Sl. No.	Question	Marks
1.	State Green's theorem in plane. State Divergence Theorem of Gauss.	5

Sl. No.	Question	Marks
2.	Evaluate ∬ $(4x^2-y^2)^{1/2}$dxdy over the triangle formed by the straight lines y =0, x =1 and y =x.	5
3.	Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\ dzdxdy$	5
4.	Evaluate $\int_0^a \int_0^{\sqrt{(a^2-y^2)}} (x^2 + y^2) dxdy$ by changing to polar	5

	coordinates.	
5.	Evaluate by Green's theorem $\int_C [(xy+x^2)dx+(x^2+y^2)dy]$, where C is the square formed by the lines $y=\pm 1,\ x=\pm 1$, described in the positive sense.	5
6.	Verify Green's theorem in the plane for $\int_C [(xy+y^2)dx+x^2dy]$, where C is the closed curve of the region bounded by $y=x$ and $y=x^2$.	5
7.	Evaluate by Green's theorem $\int_C [(xy+x^2)dx+(x^2+y^2)dy]$, where C is the square formed by the lines $y=\pm 1,\ x=\pm 1$, described in the positive sense.	5
8.	Evaluate by Green's theorem $\oint_C \{(\cos x \sin y - xy)dx + \sin x \cos y dy\}$, where C is the circle $x^2+y^2=1$.	5
9.	Evaluate $\int_0^{\frac{\pi}{2}}\int_0^{\pi} \sin(x+y)dxdy$	5
10.	Verify divergence theorem for $\vec{F}=4xz\hat{i}-y^2\hat{j}+yz\hat{k}$ taken over the cube bounded by x=0, x=1; y=0, y=1 and z =0, z =1.	5

11.	Evaluate by Green's theorem $\oint_C \{(\cos x \sin y - xy)dx + \sin x \cos y dy\}$, where C is the circle $x^2+y^2=1$.	5
12.	Evaluate ∬ $(4x^2-y^2)^{1/2}$dxdy over the triangle formed by the straight lines y =0, x =1 and y =x.	5
13.	Evaluate by Green's theorem $\int_C [(xy+x^2)dx+(x^2+y^2)dy]$, where C is the square formed by the lines $y=\pm 1,\ x=\pm 1$, described in the positive sense.	5

14.	Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} \, dz dx dy$	5

MODULE 4: ORDINARY DIFFERENTIAL EQUATIONS

Sl. No.	Question	Marks
1.	Find the general and singular solution for the equation $y = px + \sqrt{a^2 p^2 + b^2}$, where p $\equiv \frac{dy}{dx}$.	5
2.	Find the general and singular solution for the equation y = px + p – p^2 where p $\equiv \frac{dy}{dx}$.	5
3.	Find the general and singular solution of $(y - px)(p - 1) = e^{2p}$.	5

Sl. No.	Question	Marks
4.	Solve: $(\sin x \cos y + e^{2x})dx + (\cos x \sin y + \tan y)dy = 0$.	5
5.	Solve: $x\frac{dy}{dx} + y - x^3 y^6 = 0$.	5
6.	Solve: $y = 2px + yp^2$, where p $\equiv \frac{dy}{dx}$.	5
7.	Solve: $p^2 + 2py\cot x = y^2$, where p $\equiv \frac{dy}{dx}$.	5
8.	Solve: $(D^2 + 1)y = \cos x$, where $D \equiv \frac{d}{dx}$.	5
9.	Solve: $(D^2 + 4)y = x\cos x$, where $D \equiv \frac{d}{dx}$.	5
10.	Solve: $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^x \cos x$	5
11.	Solve: $e^x \sin y dx + (e^x + 1)\cos y dy = 0$.	5
12.	Solve by the method of variation of parameters the equation	5

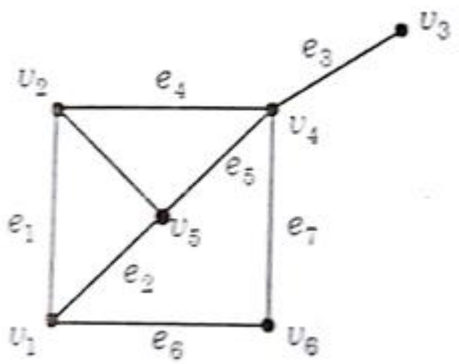
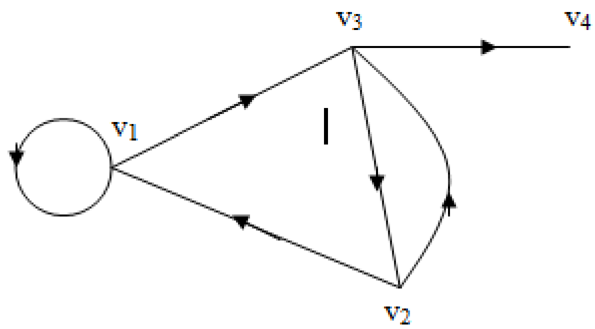
	$\frac{d^2y}{dx^2}+9y=\sec 3x.$	
13.	Solve: $y=-px+x^4p^2$, where p $\equiv \frac{dy}{dx}$.	5
14.	Solve: $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+2y=x\log x$.	5
15.	Solve: $(D^2+4D+3)y=e^{-2x}$, where D $\equiv \frac{d}{dx}$.	5
16.	Solve: $(D^2 - 4D + 4)y = x^2e^{2x}$, where D $\equiv \frac{d}{dx}$.	
17.	Solve: $xy\frac{dy}{dx}=\frac{1+y^2}{1+x^2}(1+x+x^2)$.	5
18.	Solve: $\left(x+2y^3\right)\frac{dy}{dx}=y$	5
19.	Solve: $\frac{d^2y}{dx^2}-y=xe^x\sin x$.	5
20.	Solve: $(D^2-5D+6)y=e^x cosx$ where D$\equiv\frac{d}{dx}$	5
21.	Apply the method of variation of parameters to solve the equation: $\frac{d^2y}{dx^2}+y=\sec^3 x\tan x$	5
22.	Solve: $(x+3)^2\frac{d^2y}{dx^2}-4(x+3)\frac{dy}{dx}+6y=x$.	5
23.	Solve: $(x^2D^2-xD+4)y=x\sin(\log x)$ where D$\equiv\frac{d}{dx}$.	5

MODULE 5: GRAPH THEORY

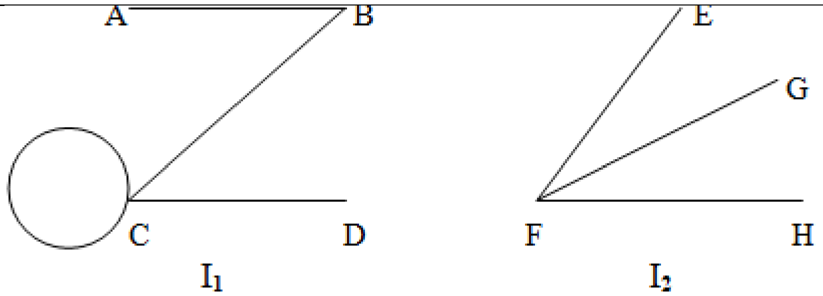
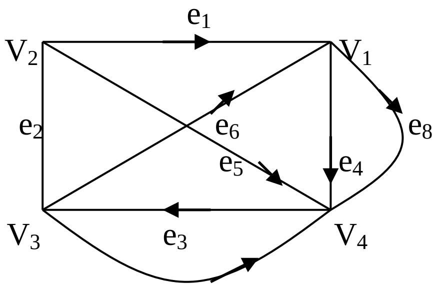
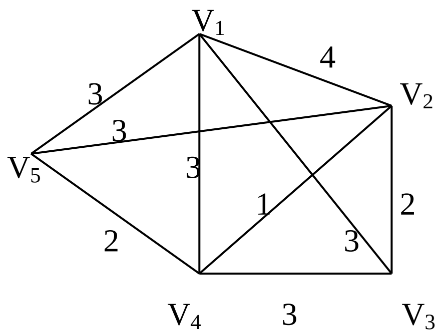
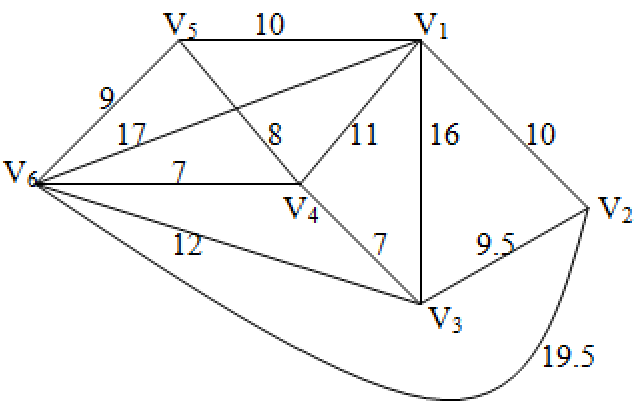
Sl. No.	Question	Marks
1.	Define Hamiltonian graph with an example	5

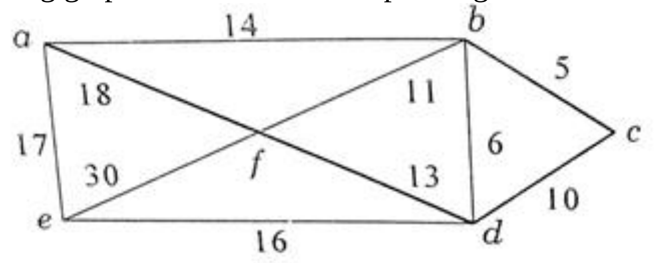
Sl. No.	Question	Marks
2.	If G is a graph with n vertices and m edges then prove that $\sum_{i=1}^{n} d(v_i)$ = 2m, where $d(v_i)$ is the degree of vertex v_i.	5
3.	Show that the number of edges in a complete graph with n vertices is n(n-1)/2.	5
4.	Prove that the maximum degree of any vertex in a simple graph with n vertices is (n-1).	5
5.	Prove that the number of odd vertices in any graph is even.	5
6.	Prove that there exist no graphs with 4 edges and 4 vertices with degree sequence 1,2,3,4.	5
7.	Prove that a tree with n vertices contains exactly (n-1) edges.	5
8.	Prove that the number of pendant vertices in a binary tree is $\frac{n+1}{2}$ where n is the number of vertices in the tree.	5
9.	Prove that if there is one and only path between every pair of vertices in a graph G then G is a tree.	5
10	Prove that a simple graph with n vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ no of edges.	5
11	The maximum number of edges in a connected simple graph with n vertices is $\frac{n(n-1)}{2}$.	5
12.	Prove that a graph G has a spanning tree if and only if G is connected.	5
13.	Let G be a graph with n vertices and e edges. Prove that G has a vertex of degree m such that $m \geq \frac{2e}{n}$.	5

14.	Determine the adjacency matrix of the following graph:	5

		
15.	Construct the Adjacency Matrix of the following di-graph : 	5
16.	Let *e* be an edge of a connected weighted graph G whose weight is smaller than that of other edges in G. Prove that every shortest spanning tree of G contains the edge.	5
17.	Let G be a di-graph with an odd number of vertices. If each vertex of G has an odd out-degree then prove that there is an odd number of vertices of G with odd in-degree.	5
18.	If G be a graph having no cycles, n vertices and k components, then prove that G has (n-k) edges.	5

Sl. No.	**Question**	**Marks**
19.	Determine the minimum and maximum number of edges of a simple graph with 10 vertices and 3 components.	5
20.	Determine the adjacency matrix of the following graph:	5

	 	
21.	Determine the incidence matrix of the following di-graph:  	5
22.	Describe Prim's algorithm to find a minimal spanning tree in a graph G. Use this algorithm to find the minimal spanning tree for the following graph:  	5
23.	Describe Kruskal's algorithm to find the shortest spanning tree in a graph G. Apply this algorithm to find the shortest spanning tree for the following graph:  	5

24.	Apply Prim's algorithm to find the shortest spanning tree of the following graph and find the corresponding minimum weight 	5
25	Construct a graph whose adjacency matrix is given as: $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$	5

Sl. No.	Question	Marks
26.	If a simple regular graph has n vertices and 24 edges, find all possible values of n. Draw a graph against each of such values of n.	5
27.	Explain Konisberg bridge problem. Represent the problem by means of graph. Does the problem have a solution?	5