Theoretische Infomatik: Selbststudium - Blatt 1

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Aufgabe 1

- G_1
  The pure application of the derivation rules leads to the following:
  $$XaYaZ \to a^iXaYaZ \to a^ibaYaZ \to a^ibab^jYaZ \to a^ibab^jaaZ \to a^ibab^jaac^kZ \to a^ibab^jaac^k$$
  Hence, let us define the language we would expect from this grammar:
  $$L_1 := \{a^ibab^jaac^k \mid i,j,k \in \mathbb{N}\}$$
  Let's prove $L_1 = L(G_1)$ by proving mutual inclusions.
  - $L_1 \subseteq L(G_1)$

    $$w \in L_1 \Rightarrow \exists i,j,k \in \mathbb{N} \mid w = a^ibab^jaac^k$$
    Let's see if we can generate such a word in the scope of grammar G_1.
    $S \to XaYaZ$ is applied anyway
    We can apply $X \to aX$ i times, followed by $X \to b$
    $$XaYaZ \to a^ibaYaZ$$
    We can apply $Y \to bY$ j times, followed by $Y \to a$
    $$a^ibaYaZ \to a^ibab^jaaZ$$
    We can apply $Z \to cZ$ k times, followed by $Z \to c$
    $$a^ibab^jaaZ \to a^ibab^jaac^k$$
    As we strictly applied the derivation rules of G_1 for any word of L_1, the inclusion is proven.
  - $L(G_1) \subseteq L_1$

    Assume $w \in L(G_1)$
    We know that w has been generated by a finite number of applications of derivations rules, starting with S.
    There is only one possible derivation with S as left-hand side.
    $$\to XaYaZ$$
    X can only be replaced by either aX or b.
    It is obvious that any finite sequence of applications of those rules leads to something of the pattern a^ib
    Y can only be replaced by either bY or a.
    It is obvious that any finite sequence of applications of those rules leads to something of the pattern b^ja
    Z can only be replaced by either cZ or λ .
    It is obvious that any finite sequence of applications of those rules leads to something of the pattern c^k
    Putting it all together, this results in a word of the pattern $a^ibab^jaac^k$, which is the exact definition of L_1

- G_2
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    If $S \to 0$ is applied, $w \in L(G_2) \Rightarrow w = 0$

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$$\text{If not, } S \to X1X1X1XY \text{ is applied}$$
$$X1X1X1XY \to 0^i10^j10^k10^lY$$
$$0^i10^j10^k10^lY \to 0^i10^j10^k10^l[X1X1X1X1XY]$$
$$0^i10^j10^k10^l[X1X1X1X1XY] \to 0^i10^j10^k10^l[0^m10^n10^o10^p10^qY]$$
$$\text{Y can be recursively replaced finitely many times.}$$
$$A_1 := \{0\}^*, w_i \in A$$
$$A_2 := \{w_01w_11w_21w_31w_4\}^*, A_3 \in A_2$$
$$\Rightarrow w \in A \cdot \{1\} \cdot A \cdot \{1\} \cdot A \cdot \{1\} \cdot A \cdot A_3$$
$$\Rightarrow L(G_2) =: L_2 = A \cdot \{1\} \cdot A \cdot \{1\} \cdot A \cdot \{1\} \cdot A \cdot A_3$$

Aufgabe 2

(a) Let's treat both conditions seperately.

- $(|x|_a - |x|_b) \bmod 3 = 1$
  Let X,Y,Z be non-terminals.
  $X \to (|x|_a - |x|_b) \bmod 3 = 1; Y \to (|x|_a - |x|_b) \bmod 3 = 2; Z \to (|x|_a - |x|_b) \bmod 3 = 0$
  Let's define the derivation rules.
  $$S \to aX; S \to bY$$
  $$X \to \lambda; X \to aY; X \to bZ$$
  $$Y \to aZ; Y \to bX$$
  $$Z \to aX; Z \to bY$$
- $bbb \in x$
  $$S \to aA; S \to bB_1$$
  $$A \to aA; A \to bB_1$$
  $$B_1 \to aA; B_1 \to bB_2$$
  $$B_2 \to aA; B_2 \to bB_3$$
  $$B_3 \to aB_3; B_3 \to bB_3; B_3 \to \lambda$$

P is the set containing all those rules.

(b) Let's define the first rule.
$S \to 001A101D011$
Let's treat both conditions seperately.

- $|x| \bmod 3 = 0$
  $$A \to 0A; A \to 1B; A \to \lambda$$
  $$B \to 0B; B \to 1C$$
  $$C \to 0C; C \to 1A$$

- $|y|_0 \bmod 2 = 1$

$$D \to 1D; D \to 0E$$
$$E \to 1E; E \to 0D; E \to \lambda$$

P is the set containing all those rules.

Aufgabe 3

In order to get to grammar regular and normed, the following derivation rules have to be adapted:

(1) $S \to bS$

(2) $S \to aaaX$

(3) $Y \to bbZ$

(4) $Y \to X$

(5) $Y \to \lambda$

They become

(1): $S \to bH_0; H_0 \to bS; H_0 \to aH_1; H_0 \to bY$

(2): $S \to aH_1; H_1 \to aH_2; H_2 \to aX$

(3): $Y \to bH_3; H_3 \to bZ$

(4): X only derivates to b, hence we can replace (4) by: $Y \to b$

(5): There is only one rule with Y as right-hand side. Hence substituting Y by λ is that rule is sufficient: $S \to b$

We also need $Z \to b$, since we had $Z \to bY \to \lambda$.