LASSO Regression as a Quadratic Program

MATH 5593 - Linear Programming

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November 29, 2025

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Outline

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2. Background information
3. Implementation
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Introduction

Project Motivation

- We wanted to explore the statistical technique known as L_1, or, LASSO regression from a linear programming perspective.
- Vanderbei claims LASSO works as an LP, so surely it can be done in AMPL.
- If so, does this approach reach the same conclusions as standard statistical libraries?
- We thought these questions would be a fun way to bridge the gap between statistics and linear programming.

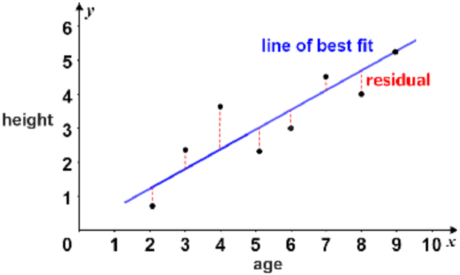
Project Goals

- To convince ourselves, we perfomed this regression in two ways.
- First, using the python library “statsmodels” on a simple dataset.
- Second, using our own AMPL model on the same dataset.
- Lastly, we compare the model coefficients. Are they the same?

Background information

What is regression?

- Regreesion fits a line, plane or hyperplane through a set of points in n dimensional space.
- This fitting is done by treating the "response" variable as a function of other "predictor" variables.
- The fit is improved by minimizing the lines overall distance from the points, otherwise known as the residuals.
- This is known as minimizing the "residual sum of squares", or, RSS.



Penalized Regression

- Having more predictors can often improve model performance, but at the cost of increased complexity.
- There are many consequences to overly complex models, so penalized regression models attempt to help with this.
- LASSO regression is surprise, a type of penalized regression.

Penalized Regression Structure

Penalized regression is made up of two chunks. The RSS part shared with linear regression, and a new penalty chunk.

$$\text{Linear Regression:} \quad \min \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$\text{LASSO Regression:} \quad \min \underbrace{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}_{\text{RSS}} + \underbrace{\lambda \sum_{j=1}^{p} |b_j|}_{\text{Penalty}}$$

$$\hat{y}_i = \sum_{j=1}^{p} b_j x_{ij}$$

LASSO Regression Features

- The penalty is very useful because it can drag coefficients all the way down to zero.
- What this means is LASSO regression can completely remove unimportant variables from a model!
- This makes it a convenient variable selection tool.

LASSO as a Quadratic Program (QP) - Setup

LASSO as a Quadratic Program (QP) - Execution

Implementation

The Data - Overview

- For this project we'll be using the classic ‘mtcars‘ toy dataset.
- “The data was extracted from the 1974 Motor Trend US magazine, and comprises fuel consumption and 10 aspects of automobile design and performance for 32 automobiles (1973–74 models).”
- This is perfect for as the relatively high number of variables to the low number of rows gives us a perfect environment to show the penalty in action.

The Data - Example Rows

model	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
Mazda RX4	21	6	160	110	3.9	2.62	16.46	0	1	4	4
Mazda RX4 Wag	21	6	160	110	3.9	2.875	17.02	0	1	4	4
Datsun 710	22.8	4	108	93	3.85	2.32	18.61	1	1	4	1
Hornet 4 Drive	21.4	6	258	110	3.08	3.215	19.44	1	0	3	1

The Regression Model

Our model uses the following variables from the full dataset:

- mpg: Miles per gallon. (Response variable)
- cyl: Number of cylinders
- disp: Cubic inches of displacement
- hp: Horsepower
- wt: Weight
- qsec: 1/4 mile time; a measure of acceleration

The AMPL Model - Parameters and Variables

```
set Car;
set Variables;

param y {i in Car};
param x {i in Car, j in Variables};
param t;

var bplus{j in Variables}  >= 0;
var bminus{j in Variables} >= 0;
```

The AMPL Model - Obective Function and Constraints

```
minimize SSE:
    sum {i in Car}
        ( y[i] - sum {j in Variables} (bplus[j] - bminus[j]) * x[i,j] )^2;

subject to L1_budget:
    sum {j in Variables: j != 'intercept'} (bplus[j] + bminus[j]) <= t;
```

Results

Python Model Results

AMPL Model Results

Thank You

Questions?

Backup slides go here