LASSO Regression as a Quadratic Program

MATH 5593 - Linear Programming

Kevin Ruiz Brady Lamson

November 29, 2025

College of Letters Arts and Sciences

Denver

Outline

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3. Implementation
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Introduction

Project Motivation

- We wanted to explore the statistical technique known as L_1, or, LASSO regression from a linear programming perspective.
- Vanderbei claims LASSO works as an LP, so surely it can be done in AMPL.
- If so, does this approach reach the same conclusions as standard statistical libraries?
- We thought these questions would be a fun way to bridge the gap between statistics and linear programming.

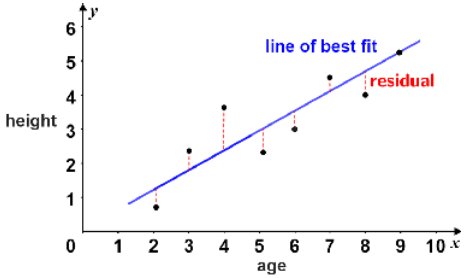
Project Goals

- To convince ourselves, we perfomed this regression in two ways.
- First, using the python library "statsmodels" on a simple dataset.
- Second, using our own AMPL model on the same dataset.
- Lastly, we compare the model coefficients. Are they the same?

Background information

What is regression?

- Regreesion fits a line, plane or hyperplane through a set of points in n dimensional space.
- This fitting is done by treating the "response" variable as a function of other "predictor" variables.
- The fit is improved by minimizing the lines overall distance from the points, otherwise known as the residuals.
- This is known as minimizing the "residual sum of squares", or, RSS.



Penalized Regression

- Having more predictors can often improve model performance, but at the cost of increased complexity.
- There are many consequences to overly complex models, so penalized regression models attempt to help with this.
- LASSO regression is surprise, a type of penalized regression.

Penalized Regression Structure

Penalized regression is made up of two chunks. The RSS part shared with linear regression, and a new penalty chunk.

$$\text{Linear Regression:} \quad \min \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$\text{LASSO Regression:} \quad \min \underbrace{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}_{\text{RSS}} + \underbrace{\lambda \sum_{j=1}^{p} |b_j|}_{\text{Penalty}}$$

$$\hat{y}_i = \sum_{j=1}^{p} b_j x_{ij}$$

LASSO Regression Features

- The penalty is very useful because it can drag coefficients all the way down to zero.
- What this means is LASSO regression can completely remove unimportant variables from a model!
- This makes it a convenient variable selection tool.

LASSO as a Quadratic Program (QP) - Setup

- Converting this into the form of an LP is pretty straightforward, but some modifications are needed.
- The RSS portion becomes the objective function, but it's squared so this is actually a quadratic program (QP).
- The penalty will become the constraint, but the absolute value is inappropriate for an LP (or QP).
- To fix that, we let $b_j = b_j^+ - b_j^-$ where $b_j^+, b_j^- \geq 0$. The absolute value then is just $|b_j| = b_j^+ + b_j^-$.

LASSO as a Quadratic Program (QP) - Execution

This gives us the following program.

$$\min \sum_i (y_i - \hat{y}_i)^2$$

$$\text{subject to: } \sum_{j=1}^{p} (b_j^+ + b_j^-) \leq t$$

$$\hat{y}_i = \sum_j b_j^+ x_{ij} - \sum_j b_j^- x_{ij}$$

$$b_j^+, b_j^- \geq 0$$

Implementation

The Data - Overview

- For this project we'll be using the classic 'mtcars' toy dataset.
- "The data was extracted from the 1974 Motor Trend US magazine, and comprises fuel consumption and 10 aspects of automobile design and performance for 32 automobiles (1973–74 models)."
- This is perfect for as the relatively high number of variables to the low number of rows gives us a perfect environment to show the penalty in action.

The Data - Example Rows

model	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
Mazda RX4	21	6	160	110	3.9	2.62	16.46	0	1	4	4
Mazda RX4 Wag	21	6	160	110	3.9	2.875	17.02	0	1	4	4
Datsun 710	22.8	4	108	93	3.85	2.32	18.61	1	1	4	1
Hornet 4 Drive	21.4	6	258	110	3.08	3.215	19.44	1	0	3	1

The Regression Model

Our model uses the following variables from the full dataset:

- mpg: Miles per gallon. (Response variable)
- cyl: Number of cylinders
- disp: Cubic inches of displacement
- hp: Horsepower
- wt: Weight
- qsec: 1/4 mile time; a measure of acceleration

The AMPL Model - Parameters and Variables

```
set Car;
set Variables;

param y {i in Car};
param x {i in Car, j in Variables};
param t;

var bplus{j in Variables}  >= 0;
var bminus{j in Variables} >= 0;
```

The AMPL Model - Obective Function and Constraints

```
minimize SSE:
    sum {i in Car}
        ( y[i] - sum {j in Variables} (bplus[j] - bminus[j]) * x[i,j] )^2;

subject to L1_budget:
    sum {j in Variables: j != 'intercept'} (bplus[j] + bminus[j]) <= t;
```

Results and Conclusion

Model Results

Model	Intercept	cyl	disp	hp	wt	qsec
statsmodels	19.99	-1.64	0.0	-1.16	-2.99	0.0
ampl	20.09	-1.62	0.0	-1.08	-2.94	0.02

As can be seen from this table, our implementation reached nearly identical results.

Conclusion

As we've seen, tackling LASSO regression from the perspective of linear programming is very straightforward.

I think this demonstrates just how close many of our seemingly disparate domains truly are!

Thank You

Questions?

Backup Slides

The Python Model - Setup Code

```
mtcars = pd.read_csv("data/mtcars.csv")
X = mtcars[["cyl", "disp", "hp", "wt", "qsec"]]
y = mtcars['mpg']

# Standardize features
scaler = StandardScaler()
X_scaled = scaler.fit_transform(X)
X_scaled_df = pd.DataFrame(X_scaled, columns=X.columns)

# Add constant for intercept
X_scaled_df = sm.add_constant(X_scaled_df)
```

The Python Model - Modeling Code

```
model = sm.OLS(y, X_scaled_df)
results = model.fit_regularized(method='elastic_net', L1_wt=1.0,
↪  alpha=0.1)

for variable, coefficient in results.params.items():
    print(f"{variable}: {round(coefficient, 5)}")
```