# Pricing Basket Credit Default Swaps by Copula

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Bob Flagg

Email: bob@calcworks.net

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**Abstract**

We discuss the implementation of Gaussian and Student's t copula models for pricing $k$-th to default basket credit default swap contracts. We investigate the sensitivity of the implementation to several key factors, including the credit quality of the reference entities, the default correlation among the reference entities, the recovery rate, and the discount factors. Callibration procedures are also provided to contruct a credit curve from CDS primiums for a reference entity and to optimize parameters of the copula based on historical pricing data for the reference entities of a contract.

# Contents

# Chapter 1

# Copulas

Copulas have become an important tool in fields where multivariate dependence plays a role and the usual Gaussian assumption is in doubt. In this chapter will review a few elementary properties of copulas that are particularly important in applications to pricing basket credit default swaps and discuss calibrating Gaussian and Student's t copulas.

## 1.1 Elementary Properties

**Definition 1** *An $N$-dimensional* copula *is a function $C$ of $N$ variables on the unit $N$-cube $[0,1]^N$ with the following properties:*

- *the range of $C$ is the unit interval $[0,1]$;*
- *$C(u_1,\ldots,u_N)=0$ if $u_n=0$ for any $n=1,\ldots,N$;*
- *$C(u_1,\ldots,u_N)=u_n$ if $u_m=1$ for all $m\neq n$;*
- *$C$ is increasing in the sense that for any $(a_1,\ldots,a_N),(b_1,\ldots,b_N)\in[0,1]^N$ with $a_n\le b_n$ for $n=1,\ldots,N$ the volume assigned by $C$ to the hypercube $[a_1,b_1]\times\cdots\times[a_N,b_N]$ is nonnegative.*

The complete specification of the default correlation will be given by the joint distribution of default times. Copulas allow us to separate individual default probabilities from the credit risk dependence structure. Formally, this fact is expressed in Sklar's theorem:

**Theorem 1** *If $F$ is an $N$-dimensional distribution function with marginals $F_1,\ldots,F_N$, then there exists an $N$-dimensional copula $C$ such that for all $(t_1,\ldots,t_N)\in\mathbf{R}^N$*

$$F(t_1,\ldots,t_N)=C(F_1(t_1),\ldots,F_N(t_N)). \tag{1.1}$$

*Moreover, if $F_1,\ldots,F_N$ are continuous, then $C$ is unique.*

**Corollary 1** *Let $F$ be an $N$-dimensional distribution function with continuous marginals $F_1, \ldots, F_N$ and define $C : [0,1]^N \to [0,1]$ by*

$$C(u_1, \ldots, u_N) = F(F_1^{-1}(u_1), \ldots, F_N^{-1}(u_N)). \tag{1.2}$$

*Then $C$ is a copula and for all $(t_1, \ldots, t_N) \in \mathbf{R}^N$*

$$F(t_1, \ldots, t_N) = C(F_1(t_1), \ldots, F_N(t_N)).$$

If $F$ has a density function $f$, then Sklar's theorem allows us to write

$$c(F_1(t_1), \ldots, F_N(t_N)) = \frac{f(t_1, \ldots, t_N)}{\prod_{n=1}^N f_n(t_n)}, \tag{1.3}$$

where $f_n$ is the density of the $n$th marginal of $F$ and

$$c(u_1, \ldots, u_N) = \frac{\partial^N C(u_1, \ldots, u_N)}{\partial u_1 \cdots \partial u_N}$$

is the copula density.

## 1.2 Two Important Examples

The applications we make of copulas will be restricted to two specific types: Gaussian and Student's t. To define a Gaussian copula, we need a symmetric, positive definite matrix, $\Sigma$, with unit diagonal entries. Then the *multivariate Gaussian copula* with covariance matrix $\Sigma$ is defined by

$$C(u_1, \ldots, u_N; \Sigma) = \mathcal{N}(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_N); \mathbf{0}, \Sigma), \tag{1.4}$$

where $\mathcal{N}(x_1, \ldots, x_N; \mathbf{0}, \Sigma)$ denotes the multivariate Normal cumulative distribution for mean $\mathbf{0}$ and covariance matrix $\Sigma$ and $\Phi$ is the standard normal cumulative distribution. The density for this copula is obtained from (1.3):

$$c(\Phi(t_1), \ldots, \Phi(t_N); \Sigma) = \frac{\frac{1}{(2\pi)^{\frac{n}{2}}\sqrt{|\Sigma|}} e^{-\frac{1}{2}\mathbf{t}^T \Sigma^{-1} \mathbf{t}}}{\prod_{n=1}^N \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t_n^2}}.$$

By setting $\zeta = (\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_N))^T$, we see that

$$c(u_1, \ldots, u_N; \Sigma) = \frac{1}{\sqrt{|\Sigma|}} e^{-\frac{1}{2}\zeta^T(\Sigma^{-1} - \mathbf{I})\zeta} \tag{1.5}$$

The *multivariate Student's t copula* with covariance matrix $\Sigma$ and $\nu$ degrees of freedom is defined by

$$C(u_1, \ldots, u_N; \Sigma, \nu) = \mathcal{T}(t_\nu^{-1}(u_1), \ldots, t_\nu^{-1}(u_N); \Sigma, \nu), \tag{1.6}$$

where $\mathcal{T}(x_1, \ldots, x_N; \Sigma, \nu)$ denotes the multivariate Student's t cumulative distribution for covariance matrix $\Sigma$ and $\nu$ degrees of freedom and $t_\nu$ is the cumulative distribution for a univariate Student's t with $\nu$ degrees of freedom. The density for this copula is again obtained from (1.3):

$$c(u_1, \ldots, u_N; \Sigma, \nu) = \frac{1}{\sqrt{|\Sigma|}} \frac{\Gamma(\frac{\nu+N}{2})}{\Gamma(\frac{\nu}{2})} \Big( \frac{\Gamma(\frac{\nu}{2})}{\Gamma(\frac{\nu+1}{2})} \Big)^N \frac{(1 + \frac{\zeta^T \Sigma^{-1} \zeta}{\nu})^{-\frac{\nu+N}{2}}}{\prod_{n=1}^N (1 + \frac{\zeta_n^2}{\nu})^{-\frac{\nu+1}{2}}}, \quad (1.7)$$

where $\zeta = (t_\nu^{-1}(u_1), \ldots, t_\nu^{-1}(u_N))^T$.

## 1.3 Calibrating Copulas

Calibrating a Gaussian copula to market data is a simple application of maximum likelihood estimation. Suppose $\mathbf{X}_t = (X_1^{(t)}, \ldots, X_N^{(t)})$ for $t = 1, \ldots, T+1$ is a time series of adjusted price data for the reference entities of a basket credit default swap contract. We first transform the price data to returns data by setting

$$Y_n^{(t)} = \log \frac{X_n^{(t+1)}}{X_n^{(t)}} \quad (1.8)$$

for $n = 1, \ldots, N$ and $t = 1, \ldots, T$. Using (1.5), for $\Sigma$ a symmetric positive definite $N \times N$ matrix we obtain the *log-likelihood* of the returns data as

$$l(\Sigma) = -\frac{T}{2} \ln|\Sigma| - \frac{1}{2} \sum_{t=1}^{T} \zeta_t^T (\Sigma^{-1} - \mathbf{I}) \zeta_t. \quad (1.9)$$

As is well known, this function is maximized by taking $\Sigma = \hat{\Sigma}$, where

$$\hat{\Sigma} = \frac{1}{T} \sum_{t=1}^{T} \zeta_t \zeta_t^T. \quad (1.10)$$

Calibrating a Student's t copula is considerably more difficult because it requires simultaneaous optimization of the correlation matrix and the degrees of freedom. There are a number of possible approaches, including *maximum likelihood* (see Johnson & Kotz [2]), *inference functions for margins* (see Joe & Xu [1]) and *canonical maximum likelihood* (CML). The CML method approximates the marginals using the corresponding empirical distributions to avoid simultaneaous optimization. Besides being more tractable, CML makes no assumptions on the distributional form of the marginals. For this reason we restrict our attention to the CML method.

The empirical marginal distributions, denoted by $\hat{F}_n$, are determined by the returns data using the formula

$$\hat{F}_n(y) = \frac{1}{T} \sum_{t=1}^{T} 1_{\{Y_n^{(t)} \leq y\}}, \quad (1.11)$$

for $n = 1, \ldots, N$. Using the empirical marginal distributions, we can transform the returns data into uniform variates by setting

$$\mathbf{U}_t = (U_1^{(t)}, \ldots, U_N^{(t)}) = (\hat{F}_1(Y_1^{(t)}), \ldots, \hat{F}_N(Y_N^{(t)})), \tag{1.12}$$

for $t = 1, \ldots, T$.

To further simplify the optimization, we follow Mashal & Zeevi's recommendation of estimating the correlation matrix of the Student's t copula with Kendall's tau. This approach depends on the following special case of a result from Lindskog, McNeil, & Shmock [4].

**Theorem 2** *Let $\mathbf{X}$ be an $N$-dimensional Student's t distribution. Then for $n, m \in \{1, \ldots, N\}$*

$$\tau(X_n, X_m) = \frac{2}{\pi} \arcsin R_{nm}, \tag{1.13}$$

*where $\tau(X_n, X_m)$ denotes Kendall's tau for the random variables $(X_n, X_m)$ and $R_{nm}$ denotes Pearson's linear correlation coefficient for the random variables $(X_n, X_m)$.*

Given a time series of adjusted price data $\mathbf{X}_t = (X_1^{(t)}, \ldots, X_N^{(t)})$ for $t = 1, \ldots, T+1$, our calibration procedure is made up of the following steps:

1. Transform the price data to returns data, $\{\mathbf{Y}_t\}_t$, using (1.8);
2. Transform the returns data to uniform variates, $\{\mathbf{U}_t\}_t$, using (1.12);
3. Estimate the correlation matrix, $\hat{\Sigma}$, using (1.13): $\hat{\Sigma}_{nm} = \sin\left(\frac{\pi}{2}\tau(U_n, U_m)\right)$;
4. Estimate the degrees of freedom, $\hat{\nu}$, by maximizing the log-likelihood function of the Student's t copula density (1.7) with correlation matrix $\hat{\Sigma}$ over a grid.

# Chapter 2

# Building a Hazard Rate Term Structure

A key component in our approach to pricing basket credit default swaps is estimating the instantaneous default probabilities of each reference entity in the underlying portfolio. In this section we describe a method of constructing a term structure of instantaneous default probabilities by bootstraping from credit default swap (CDS) spread data.

In brief, a CDS is used to transfer the credit risk of a reference entity from one party to another. In a standard CDS contract one party purchases credit protection from another party, to cover the loss of the face value of an asset following a credit event. To pay for this protection, the protection buyer makes a regular stream of payments, known as the *premium leg*, to the protection seller. The size of these premium payments is calculated from a quoted default swap spread which is paid on the face value of the protection. These payments are made until a credit event occurs or until maturity, whichever occurs first. If a credit event does occur before the maturity date of the contract, there is a payment by the protection seller, known as the *protection leg*. This payment equals the difference between par and the price of the cheapest to deliver (CTD) asset of the reference entity on the face value of the protection and compensates the protection buyer for the loss. It can be made in cash or physically settled format.

## 2.1 Pricing a CDS

To price a CDS we use a simplified method known as the *JP Morgan Approach*. In this approach there are $N$ periods, indexed by $n = 1, \dots, N$ and each period is of length $\Delta t$, expressed in years. The end of period maturities are $T_n = n \cdot \Delta t$. The risk-free forward interest rate that can be locked in at time 0 for investing over the period from $T_{n-1}$ to $T_n$ is denoted by $r_n = r(T_{n-1}, T_n)$. The discount factors can then be written as functions of the forward rates:

$$D_n = D(0, T_n) = \exp\left( -\sum_{k=1}^{n} r_k \Delta t \right).$$

For a given obligor, we assume that the hazard rate is constant over forward period $n$ with value $\lambda_n$. The survival probability of the obligor at the end of period maturities is then given by

$$P_n = \mathbf{P}(\tau > T_n) = e^{-\sum_{k=1}^{n} \lambda_k \Delta t}.$$

We denote the $N$-period CDS spread as $S_N$, stated as an annualized percentage of the nominal value of the contract, which, without loss of generality, we can take to be 1.00. We assume that defaults occur only at the end of the period so the premiums will be paid until the end of the period. Since the premium payments are made as long as the reference entity survives, the expected present value of the premium leg ($PL_N$) is

$$PL_N = S_N \sum_{n=1}^{N} D(0, T_n) P_n \Delta t.$$

The expected loss payment in period $n$ is based on the probability of default in period $n$, conditional on no default in a prior period. This is given by the probability of surviving until period $n-1$ and then defaulting in period $n$. It follows that the expected present value of the default leg ($DL_N$) is

$$DL_N = (1 - R) \sum_{n=1}^{N} D(0, T_n)(P_{n-1} - P_n).$$

The fair pricing of the $N$-period CDS, i.e. the fair quote of the spread $S_N$, must be such that the expected present value of payments made by buyer and seller are equal, i.e. $PL_N = DL_N$. From the above discussion, we see that

$$S_N = \frac{(1 - R) \sum_{n=1}^{N} D(0, T_n)(P_{n-1} - P_n)}{\sum_{n=1}^{N} D(0, T_n) P_n \Delta t}. \tag{2.1}$$

## 2.2 Bootstrapping Hazard Rates

If we know the discount factors $D_n$ and credit default swap fair spreads for the maturities $T_1, \cdots, T_N$, we can use (2.1) to "bootstrap" the hazard rate values. Precisely,

$$P_1 = \frac{L}{L + S_1 \Delta t}, \tag{2.2}$$

where $L = 1 - R$; and

$$\begin{aligned} P_{n+1} &= \frac{\sum_{k=1}^{n} D_k[LP_{k-1} - (L + \Delta t S_{n+1})P_k]}{d_{n+1}(L + \Delta t S_{n+1})} + \frac{P_n L}{L + \Delta t S_{n+1}} \\ &= \frac{L \sum_{k=1}^{n} D_k[P_{k-1} - P_k] - \Delta t S_{n+1} \sum_{k=1}^{n} d_k P_k}{d_{n+1}(L + \Delta t S_{n+1})} + \frac{P_n L}{L + \Delta t S_{n+1}} \\ &= \frac{L X_n - \Delta t S_{n+1} Y_n}{d_{n+1}(L + \Delta t S_{n+1})} + \frac{P_n L}{L + \Delta t S_{n+1}}, \end{aligned}$$

where

$$X_n = \sum_{k=1}^{n} D_k[P_{k-1} - P_k] \quad \text{and} \quad Y_n = \sum_{k=1}^{n} d_k P_k.$$

Pseudocode implementing this procedure is given below. In that code, $D$ is an array of discount factors, $T$ is an array of maturity times, $S$ is an array of CDS market spreads, $m$ is the number of periods and $r$ is the recovery rate. The algorithm returns an array of survival probabilities at the maturity times.

**Algorithm 2.2.1:** DEFAULTPROBABILITIES$(D, T, S, m, r)$

$x \leftarrow 0; y \leftarrow 0; p \leftarrow 1; l \leftarrow 1 - r; t \leftarrow 0;$
**for** $i \leftarrow 1$ **to** $m$

$$\textbf{do} \begin{cases} \delta \leftarrow T_i - t; \\ d = D_i * (l + \delta * S_i); \\ P_i \leftarrow (l * x - \delta * S_i * y)/d + p * l/(l + \delta * S_i); \\ x \leftarrow x + D_i * (p - P_i); \\ y \leftarrow y + D_i * P_i; \\ p \leftarrow P_i; \\ t \leftarrow T_i; \end{cases}$$

**return** $(P)$

Using the procedure just described we obtain survival probabilities at the maturity times. Now we would like to find a corresponding stepwise hazard function; that is, we would like values $\lambda_1, \ldots, \lambda_m$ so that

$$\mathbf{P}(\tau > T_n) = e^{-\sum_{i=1}^{n} \lambda_i (T_i - T_{i-1})},$$

(where we set $T_0 = 0$) for $n = 1, \ldots, m$. A procedure for finding these values is given below. In that code, $T$ is an array of maturity times and $P$ is an array of survival probabilities at the maturity times. The algorithm returns an array of constants defining the hazard function.

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**Algorithm 2.2.2:** $\textsc{hazardRates}(T, P)$

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$t \leftarrow 0; p \leftarrow 1;$
**for** $i \leftarrow 1$ **to** $m$

$$\textbf{do} \begin{cases} \delta \leftarrow T_i - t; \\ \lambda_i \leftarrow \frac{\log P_i - \log p}{\delta}; \\ p \leftarrow P_i; \\ t \leftarrow T_i; \end{cases}$$

**return** $(\lambda)$

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# Chapter 3

# Basket Default Swap Pricing

## 3.1 Introduction

In this chapter we discuss the pricing of basket credit default swaps using the copula framework developed above to model dependencies between defaults. A basket credit default swap is a credit derivative contract where the protection seller agrees to take on some credit risk in a portfolio of reference entities. In a $k$-th to default basket credit default swap there is protection payment in the event (and at the time) of the $k$-th default. In exchange for this kind of protection, the protection buyer makes periodic payments to the seller – the *premium* – until the maturity date of the contract.

We use the following notation and terminology in describing our pricing algorithm:

- $N$ is the number of reference names.
- $N_p$ is the notional principle, which we will normalize to 1.00.
- $T$ is the maturity of the basket default swap and we assume there are $M$ periods, $t_1, \ldots, t_M$ with $t_M = T$.
- $k$ is the *seniority* of the basket default swap; that is, the number of defaults required to trigger a default payment by the protection seller.
- $\tau_n$ is the time of default of the $n$th reference entity, for $n = 1, \ldots, N$.
- $\tau_{(k)}$ is the time of the $k$-th default.
- $s$ is the fair spread of the contract, to be paid $\frac{1}{\delta}$ times per annum until $T$ or $\tau_{(k)}$.
- $Z(t, T)$ is the risk free zero coupon bond price as a discount factor.
- $R$ is the constant recovery rate and $L = 1 - R$.

- $F_n$ is the distribution function of $\tau_n$, for $n = 1, \ldots, N$.
- $F_{(k)}$ is the distribution function of $\tau_{(k)}$.

The spread of $k$-th to default basket credit default swap is computed by equating the expected value of the discounted premium leg with the expected value of the discounted default leg under the risk neutral probability measure. The premium leg is

$$\mathrm{PL} = s \times N_p \times \delta \times \sum_{m=1}^{M} Z(0, T_m) \times (1 - F_{(k)}(T_m)). \tag{3.1}$$

Assuming default payment is paid at the end the default period, the default leg is

$$\mathrm{DL} = (1 - R) \times N_p \times \sum_{m=1}^{M} Z(0, T_m) \times (F_{(k)}(T_m) - F_{(k)}(T_{m-1})). \tag{3.2}$$

It follows that the fair spread is

$$s = \frac{(1 - R) \sum_{m=1}^{M} Z(0, T_m)(F_{(k)}(T_m) - F_{(k)}(T_{m-1}))}{\delta \sum_{m=1}^{M} Z(0, T_m)(1 - F_{(k)}(T_m))}. \tag{3.3}$$

We may adjust the default leg by substracting the accrued premium, which, if we assume default can only occur at the middle of each period, is

$$\mathrm{AP} = \frac{1}{2} \times s \times N_p \times \delta \times \sum_{m=1}^{M} Z(0, T_m) \times (F_{(k)}(T_m) - F_{(k)}(T_{m-1})). \tag{3.4}$$

## 3.2 Implementation

Our implementation of $k$-th to default basket credit default swap pricing uses Monte Carlo simulation of a copula model and has the following main steps:

1. For each reference name use CDS data to estimate the term structure of default probabilities.
2. Estimate the parameters of the copula using historical price data for the reference entities of the contract.
3. For each simulation step do the following
   (a) Generate a vector of correlated uniform random variates by the copula.

(b) For each reference entity transform the uniform random variate to a default time by taking the inverse of the variate under the marginal cumulative distribution function.

(c) Sort the default times and choose the $k$-th one, $\tau_{(k)}$.

(d) Based on $\tau_{(k)}$ calculate the discounted value of the premium leg and the default leg.

4. Calculate the average of premium leg and default leg and then use equation (3.3) (or the corresponding formula adjusted for accrued premium) to determine the fair spread.

## 3.3 Generating Correlated Uniform Random Variates

The execution of Step 3(a) of the above outline will depend on the copula used to model default correlation. We consider just two types: Gaussian and Student's t.

### 3.3.1 Sampling from a Multivariate Gaussian Copula

To generate samples from an $N$-dimensional Gaussian copula, $C_{\Sigma}^{G}$, with correlation matrix $\Sigma$, first find a Cholesky factorization, $A$, of $\Sigma$ so that $\Sigma = A \cdot A^T$. Then proceed as follows:

1. Draw an $N$-dimensional vector $\mathbf{z}$ of independent standard normal variates.

2. Set $\mathbf{x} = A \cdot \mathbf{z}$.

3. Transform $\mathbf{x}$ to a vector of uniform variates, $\mathbf{u}$, by setting $u_n = \Phi(x_n)$, for $n = 1, \ldots, N$.

Then $\mathbf{u}$ is distributed according to $C_{\Sigma}^{G}$.

### 3.3.2 Sampling from a Multivariate Student's t Copula

To generate samples from an $N$-dimensional Student's t copula, $C_{\Sigma,\nu}^{T}$, with correlation matrix $\Sigma$ and $\nu$ degrees of freedom, first find a Cholesky factorization, $A$, of $\Sigma$ so that $\Sigma = A \cdot A^T$. Then proceed as follows:

1. Draw an $N$-dimensional vector $\mathbf{z}$ of independent standard normal variates.

2. Draw an independent $\chi_{\nu}^{2}$ random variate $s$.

3. Set $\mathbf{y} = A \cdot \mathbf{z}$.

4. Set $\mathbf{x} = \sqrt{\frac{\nu}{s}}\mathbf{z}$.
5. Transform $\mathbf{x}$ to a vector of uniform variates, $\mathbf{u}$, by setting $u_n = t_\nu(x_n)$, for $n = 1, \ldots, N$.

Then $\mathbf{u}$ is distributed according to $C^T_{\Sigma,\nu}$.

## 3.4 Sensitivity Analysis

The key factors on which the price of a $k$th to default credit default swap depends are

- the credit quality of the reference entities;
- the default correlation among the reference entities;
- the recovery rate; and
- the discount factors.

In this section we investigate the influence of each of these factors on the simulated fair spread of a basket credit default swap. To simplify this sensitivity analysis, we will assume that the short rate is constant, the underlying credit default swap curve is flat and the same for all reference entities, and that correlation among the reference entities is pairwise constant. We will also use the same number of steps (250,000) in all simulations related to sensitivity analysis and fix other parameters such as the maturity ($T = 5$) and the notional principle ($N_p = 1.00$).

### 3.4.1 Credit Quality

To investigate the influence of credit quality on basket credit default swap premiums, we consider constant shifts in a flat underlying CDS curve for the reference entities with values at $50, 75, 100, 125, 150, 175, 200, 225, 250, 275,$ and 300 bps. In addition to the general simplifying assumptions mentioned above, we also assume a constant recovery rate of 40%, a fixed pairwise default correlation among reference entities of 0.5 and a constant spot interest rate of 4%. As expected and as shown in Figure 3.1 basket credit default premiums increase with increasing premiums for the underlying CDS curve.

### 3.4.2 Default Correlation

As shown in Figure 3.2, the premium for 1st to default basket credit default swap decreases as the pairwise default correlation increases. For 2nd or 3rd to default basket credit default swaps however, premiums appear to increase as the pairwise default correlation increases though the effect is quite small.

### 3.4.3 Recovery Rate

As evident from Figure 3.3, higher recovery rates decrease the fair spread of a basket credit default swap. This is consistent across seniority. Clearly higher recovery rates decrease the expected loss so these results are easily explained.

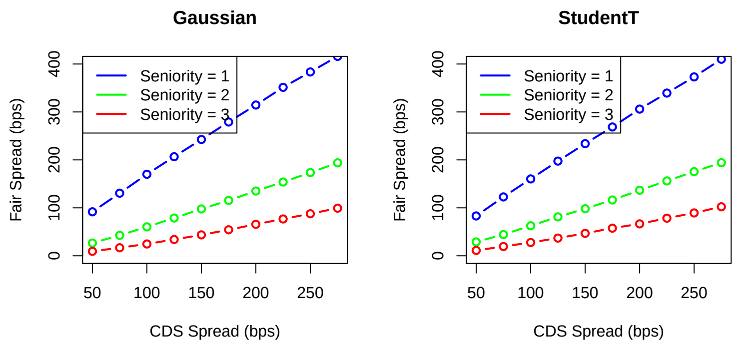



Figure 3.1: Basket Credit Default premiums vs. Credit Quality

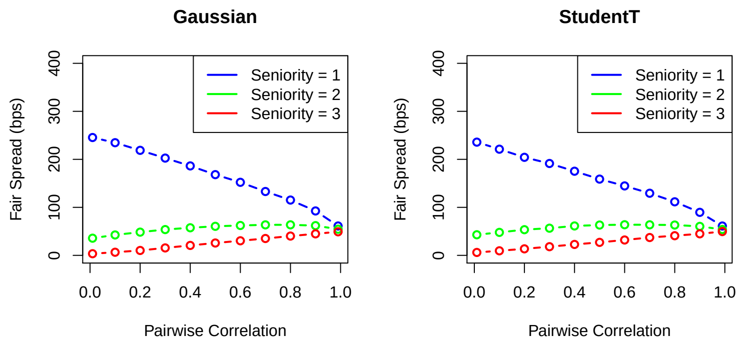



Figure 3.2: Basket Credit Default premiums vs. Correlation

### 3.4.4 Discount Factors

We explore the influence of discount factors by considering constant shifts in the yield curve, letting the short rate vary from 10% to 100% in increments of 10%. Again we assume a constant recovery rate of 40% and a fixed pairwise default correlation among reference entities of 0.5.

We were not able to establish a statistically significant pattern (see Figure 3.4) in the fair spread value of a basket credit default swap when varying the discount factors.

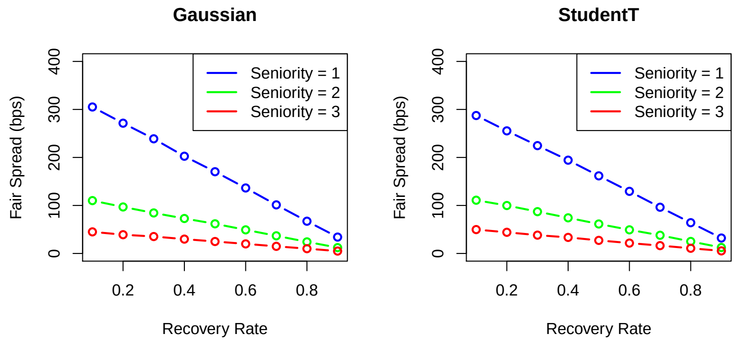



Figure 3.3: Basket Credit Default premiums vs. Recovery Rate

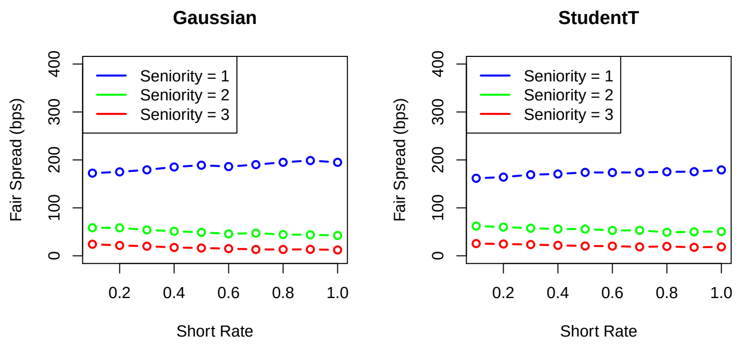



Figure 3.4: Basket Credit Default premiums vs. Short Rate

# Appendix A

# Using the Software

Our implementation of BCD copula pricing is done in Java 6 and distributed as a jar archive named **CQF-BCD.jar**. The jar file is available in the directory **BCD** on the included CD. That directory has a subdirectory **lib** which contains Java libraries required by the application. The main source code is also included on the CD in a subdirectory of **BCD** called **src**. Additional source code used by the BCD application but not specific to that project is included on the CD in a separate folder call **CQF**.

## A.1 Using the Java Application

To start the application, right click on the archive **CQF-BCD.jar** and choose *Open with Java 6 Runtime*.[1]

When the application is launched, the main application frame is displayed as shown in Figure A.1. The panel on the top left and the entity selection dialog (Figure A.2) allow you to choose which reference entities to include in the BCD contract. You can specify the properties of the BCD product you would like to price and properties of the Monte Carlo simulation using the two lower panels on the left of the main frame. After setting the specifications, click the **Calibrate** button at the bottom left of the main frame. When the calibration is complete, the **Price** button is enabled and you can click that to obtain an approximation to the fair spread of the BCD contract. This value will be shown in the top right subpanel. The two panels on the lower right show simple calibration and simulation visualizations and the model parameters. Examples of these are show below. To quit the application, close the window.

[1]This assumes you have Java 6 installed on your machine. If this is not the case, it can be obtained from Oracle at http://www.java.com/en/download/index.jsp.

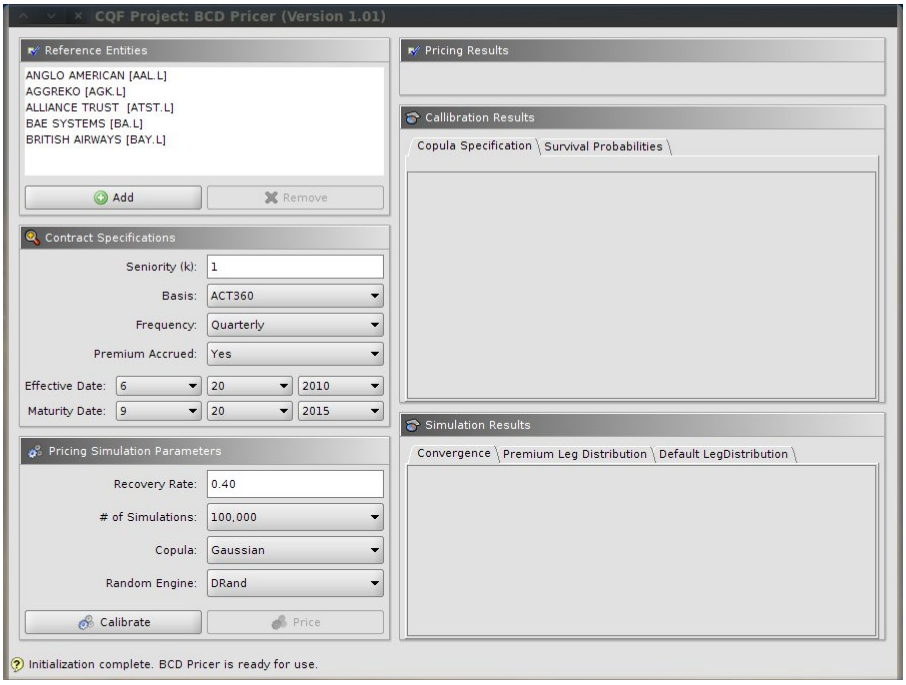



Figure A.1: BCD Main Application Frame

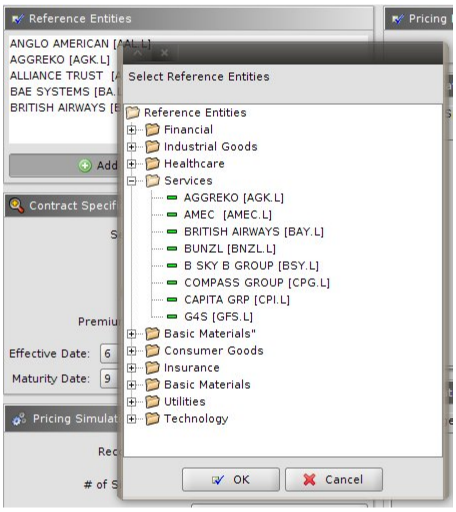



Figure A.2: BCD Entity Selection

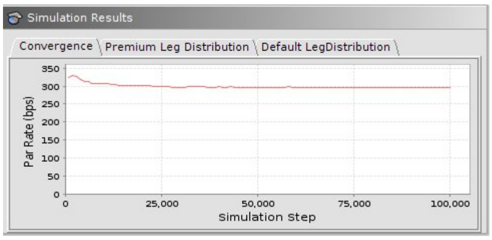



Figure A.3: BCD Convergence Visualization

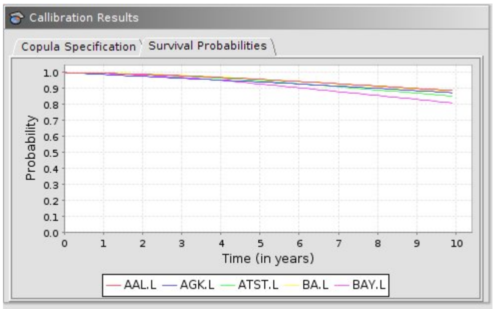
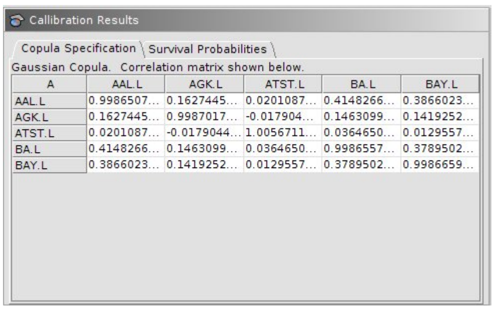



Figure A.4: BCD Survival Probabilities Visualization



Callibration Results

Copula Specification | Survival Probabilities

Gaussian Copula. Correlation matrix shown below.

| A | AAL.L | AGK.L | ATST.L | BA.L | BAY.L |
|---|---|---|---|---|---|
| AAL.L | 0.9986507... | 0.1627445... | 0.0201087... | 0.4148266... | 0.3866023... |
| AGK.L | 0.1627445... | 0.9987017... | -0.017904... | 0.1463099... | 0.1419252... |
| ATST.L | 0.0201087... | -0.0179044... | 1.0056711... | 0.0364650... | 0.0129557... |
| BA.L | 0.4148266... | 0.1463099... | 0.0364650... | 0.9986557... | 0.3789502... |
| BAY.L | 0.3866023... | 0.1419252... | 0.0129557... | 0.3789502... | 0.9986659... |

Figure A.5: BCD Parameters Table

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