3.a.

Friday, March 15, 2019 5:30 PM

Derive γ_k, $k \geq 2$

$\gamma_k = Cov(\Delta p_{t-k}, \Delta p_t)$

let $P(k) = \Delta p_{t-k} = (q_{t-k} - q_{t-k-1})c + u_{t-k}$

$\Rightarrow E[(P(k) - E[P(k)])(P(0) - E[P(0)])]$

By the identity: $Cov(X, Y) = E[XY] - E[X]E[Y]$

$= E[P(2)P(0)] - E[P(2)]E[P(0)]$

$P(2) = \Delta p_{t-2} \Rightarrow E[P(2)] = E[\Delta p_{t-2}] = E[(q_{t-2} - q_{t-3})c + u_{t-2}]$

$= (q_{t-2} - q_{t-3})c + u_{t-2}$ $\quad = c(E[q_{t-2}] - E[q_{t-3}]) + E[u_{t-2}]$

$= c(0 - 0) + 0 = 0$

$P(0) = \Delta p_t \Rightarrow E[P(0)] = E[\Delta p_t] = E[(q_t - q_{t-1})c + u_t]$

$= (q_t - q_{t-1})c + u_t$ $\quad = c(0 - 0) + 0 = 0$

$$q_t = \begin{cases} +1 & \mathbb{P}(q_t = 1) = \frac{1}{2} \\ -1 & \mathbb{P}(q_t = -1) = \frac{1}{2} \end{cases} \quad \forall t$$

$E[q_t] = 0$

$E[P(2) \cdot P(0)] = E[[(q_{t-2} - q_{t-3})c + u_{t-2}] \cdot [(q_t - q_{t-1})c + u_t]]$

$= E[c^2(q_t - q_{t-1})(q_{t-2} - q_{t-3}) + u_t(q_{t-2} - q_{t-3})c + u_{t-2}(q_t - q_{t-1})c + \underbrace{u_t u_{t-2}}_{\text{indep by defn. } E[\ldots] = 0}]$

$= c^2 \cdot E[q_t q_{t-2} - q_t q_{t-3} - q_{t-1} q_{t-2} + q_{t-1} q_{t-3}]$

$q_t, q_{t-k} \sim$ Indep. by defn $\forall k \neq 0$

$\Rightarrow 0$

$= E[u_t (q_{t-2} - q_{t-3})c] + E[u_{t-2}(q_t - q_{t-1})c]$

$= c(E[u_t q_{t-2} - u_t \cdot q_{t-3}] + E[u_{t-2} q_t - u_{t-2} q_{t-1}])$

$u_t, q_t \sim$ Independant by defn

$= c(0 - 0) = 0$

$$\boxed{\Rightarrow Cov(\Delta p_{t-k}, \Delta p_t) = 0 \quad \forall k \geq 2}$$