Problem 1

a)

since we have A is a constant, $\langle A\rangle = \int AP(x,y)dxdy = A\int P(x,y)dxdy$. Since P(x,y) is the distribution function, we have $\int P(x,y)dxdy = 1$. Thus we have $\langle A\rangle = A\int P(x,y)dxdy = A*1 = A$

b)

$\mathrm{Var}(A) = \int[A-\langle A\rangle]^2P(x,y)dxdy = \int[A-A]^2P(x,y)dxdy = \int 0P(x,y)dxdy$

c)

$$\langle Ax\rangle = \int AxP(x,y)dxdy = A\int xP(x,y)dxdy = A\langle x\rangle$$

$$\mathrm{Var}(Ax) = \int[Ax-\langle Ax\rangle]^2P(x,y)dxdy = \int[Ax-A\langle x\rangle]^2P(x,y)dxdy = \int[A($$

Similarly, all these can be applied to $\mathrm{Var}(By)$, and we can get $\mathrm{Var}(By) = B^2\mathrm{Var}(y)$

d)

$$\langle Ax+By\rangle = \int(Ax+By)P(x,y)dxdy = \int(Ax)P(x,y)+(By)P(x,y)dxdy =$$

e)

$$\langle x\rangle = \int xP(x,y)dxdy = \int xP(x)P(y)dxdy = \int xP(x)dx*\int P(y)dy = \int x1$$

Similarly we can probe $\langle y\rangle = \int yP(y)dy$

f)

$$\mathrm{Var}(x) = \int[x-\langle x\rangle]^2P(x,y)dxdy = \int[x-\langle x\rangle]^2P(x)P(y)dxdy = \int[x-\langle x\rangle]^2$$

Similarly, we can prove $\mathrm{Var}(y) = \int[y-\langle y\rangle]^2P(y)dy$

g)

$$
\begin{aligned}
{\rm Var}(Ax+By) = \int [(Ax+By) - \langle Ax+By\rangle]^2 P(x,y) dxdy = \int [Ax + By - A \\
+ [B(y-\langle y\rangle)]^2 P(x,y) \\
= \int A^2[x-\langle x\rangle]^2 P(x,y)dxdy + \int B^2 [y - \\
= A^2 \int [x-\langle x\rangle]^2 P(x) * \int P(y)dy + B^2 \int [y-\langle y\rangle]^2 \\
= A^2 {\rm Var}(x) * 1 + B^2 {\rm Var}(y) * 1
\end{aligned}
$$

since x and y is independed, $\int (x - \langle x\rangle)P(x)dx * \int (y - \langle y\rangle)P(y)dy$ is 0.

Thus, we can get ${\rm Var}(Ax+By) = A^2{\rm Var}(x) + B^2{\rm Var}(y)$

Problem 2

a)

we have probobility

$$
U(x|a,b) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x < b, \\ 0 & \text{otherwise.} \end{cases}
$$

, the $\langle x\rangle$ is $\int xU(x)dx = \int_a^b \frac{x}{b-a}dx = \frac{b^2-a^2}{2(b-a)} = \frac{a+b}{2}$

b)

$$

{\rm Var}(x) = \int [x - \langle x\rangle]^2 U(x) dx = \int_{a}^{b} \frac{[x - \frac{a+b}{2}]^2}{b-a} dx = \frac{1}{b-a} \int_{a}^{b} x^2 - (a+b)x + \frac{a^2+2ab+b^2}{4} dx \

= \frac{1}{b-a} (\frac{b^3-a^3}{3} - \frac{(a+b)(b^2-a^2)}{2} + \frac{a^2+2ab+b^2}{4}(b-a)) = \frac{1}{b-a} (\frac{b^3-a^3}{3} - \frac{b^3+ab^2-a^2b-a^3}{4}) \

= \frac{1}{b-a} \frac{(b-a)^3}{12} = \frac{(b-a)^2}{12} \

\sigma = \sqrt{{\rm Var}(x)} = \sqrt{\frac{(b-a)^2}{12}} = \frac{b-a}{\sqrt{12}}

In [ ]:

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.ticker import AutoMinorLocator
import pandas as pd
import matplotlib.ticker as ticker
import scipy
# import the lib we need later
```

In [ ]:
```
# c)
a = 0
b = 12
# provide a and be as asked

randomnumber = np.random.uniform(a,b,1000000)
# generate the random numbers as asked

print(randomnumber.mean(),np.std(randomnumber))
# print the mean and standard divation from the random number generated

print((a+b)/2,(b-a)/np.sqrt(12))
# print the mean and standard divation from the forumlar
```

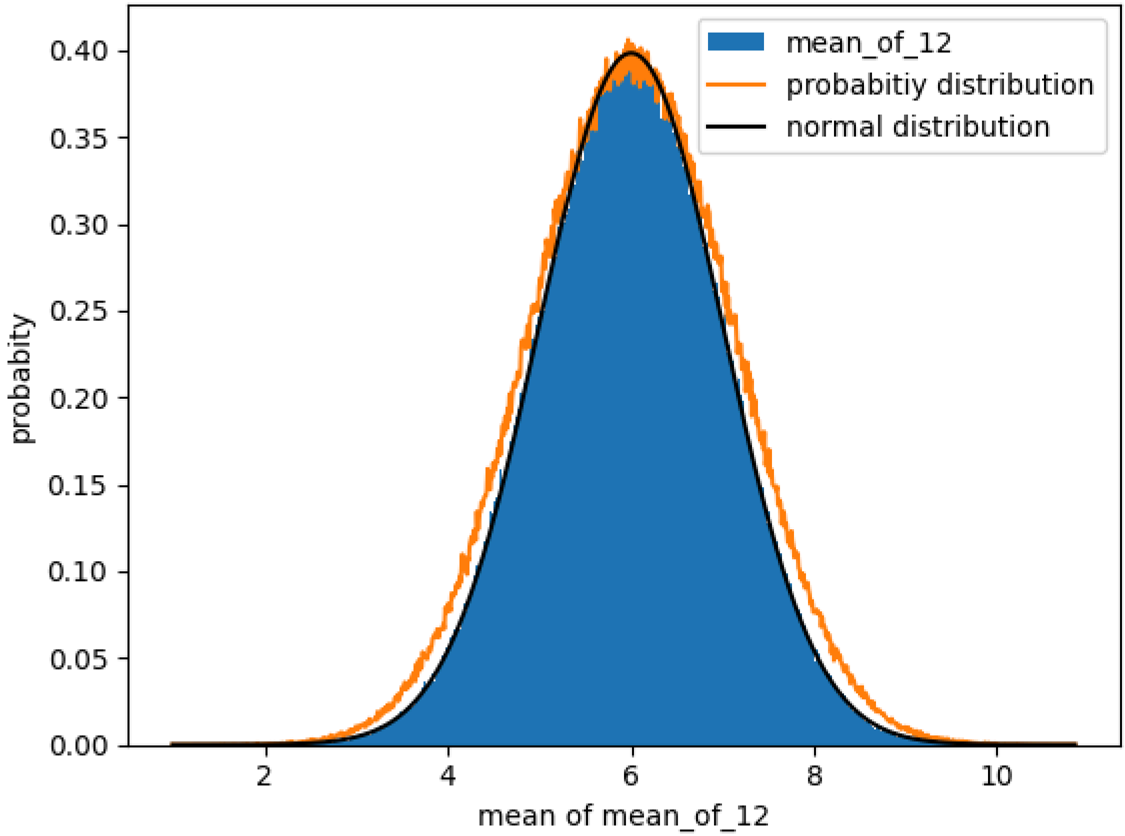
```
5.994752497169571 3.464093868334015
6.0 3.464101615137755
```

In [ ]:
```
# d)

sample = np.random.uniform(a,b,(1000000,12))
# generate a sample as asked

mean_of_12 = sample.mean(1)
# calculate the sample of each 12 numbers from a sample

print(mean_of_12.mean(),mean_of_12.std())
# calculate the mean and standard divation of the mean_of_12
```

```
6.001455623866872 1.001180385215327
```

In [ ]:
```
# e)

import scipy.stats


p = plt.hist(mean_of_12,1000,density=True,label='mean_of_12')
# plot the histogram with 1000 bars showing the probobility of having each value

xmin,xmax=plt.xlim()
# get the max and min of the plot

x = np.linspace(xmin,xmax,1000)
# generate x whcih give very point a x

plt.plot(x,p[0],label='probabitiy distribution')
# plot the probabity distribution we get in the plot

n = scipy.stats.norm.pdf(x,mean_of_12.mean(),mean_of_12.std())
# generate the nromal distribution with mean of mean_of_12 and standarddivation

plt.plot(x,n,label='normal distribution',color = 'black')
# plot the normal distribution

plt.xlabel('mean of mean_of_12')
plt.ylabel('probabity')
plt.legend()
# plot the labels for poeple to read the graph easly
```

Out[ ]: <matplotlib.legend.Legend at 0x1f021ab1fd0>



f)

since we have a great number of values randomly choose uniformly from 0 to 12. it is obvious that since very number have a equal change to show up, the mean will be the middle point to the unifromaly distribution range, since for every number x smaller than 6, there is a equal chance that a nubmer which is x larger than 6 wiil show up. Thus, the 6 will be the mean.

since every element is choose randomly with unifrom possiblity, we know that the mean of every 12 numbers should be a distribution with center at 6, and the chance of a mean which is very far away from the mean will be very less likely to show up. With this feature, it should be a normal distribution. Since ,the show up of mean is randome, it is reanonalbe to say that it follows tha normal distribution with standard divation 1.