

```
In [ ]: import numpy as np
import matplotlib.pyplot as plt
from matplotlib.ticker import AutoMinorLocator
import pandas as pd
import matplotlib.ticker as ticker
import scipy
# import the lib we need later
```

```
In [ ]: # homework 9
# a)

spectra = pd.read_csv('data/spectra.csv')
# load the spectra data into notebook

spectra_wavelength = np.array(spectra['wavelength'])
spectra_k_lam_1 = np.array(spectra['k_lam_1'])
spectra_k_lam_2 = np.array(spectra['k_lam_2'])
spectra_k_lam_3 = np.array(spectra['k_lam_3'])
# get all the data from the csv to a way which is easier to work with

"""
for the first spectra, when lam is 1200 A, we can know that N_lam is very close
form the origin datat, we can know that k_lam_1 is 9, and all the data of k_lam_
I think it is reasonalbe to say the N_count should be 3
The total line looks like decreasing
it is reasonalbe to say that the alpha is a negative number,
it decese from around 15 to 0, and the wavelength over 1200 go from 0.9 to 1.16
Thus, it is reasonable to set alpha to be large, I take -5 as a good guess
from the formular of gaussain distribution, we can know that the frequecny of me
I can get a guess of N_line with k_lam at 1216 is 9
since wavelength over 1200 is still samll, I will consider the total formula to
Therefore, it is reasonalbe to guess N_line to be 200
"""

guess_N_count_1 = 3
guess_alpha_1 = -5
guess_N_line_1 = 200
guess_N_lam_1 = np.array(guess_N_count_1*(spectra_wavelength/1200)**guess_alpha_1)

"""
for the second spectra, when lam is 1200 A, we can know that N_lam is very close
form the origin datat, we can know that k_lam_2 is 2, and all the data of k_lam_
I think it is reasonalbe to say the N_count should be very small, 0
The total line looks like increase a little, since the larger part is silightly
it is reasonalbe to say that the alpha is a positive number,
it basically does not increase
Thus, it is reasonable to set alpha to be very small, I take 1 as a good guess
from the formular of gaussain distribution, we can know that the frequecny of me
I can get a guess of N_line with k_lam at 1216 is 1
since wavelength over 1200 is still samll, I will consider the total formula to
Therefore, it is reasonalbe to guess N_line to be 50
"""

guess_N_count_2 = 0
guess_alpha_2 = 1
guess_N_line_2 = 50
guess_N_lam_2 = np.array(guess_N_count_2*(spectra_wavelength/1200)**guess_alpha_2)
```

```

"""
for the third spectra, when lam is 1200 A, we can know that N_lam is very close
form the origin datat, we can know that k_lam_3 is 1, and all the data of k_lam_
I think it is reasonalbe to say the N_count should be small, 1
The total line looks like increase a little, since the larger part is silightly
it is reasonalbe to say that the alpha is a positive number,
it increase a bit, from around 0 to 2.5
Thus, it is reasonable to set alpha to be larger, I take 6 as a good guess
from the formular of gaussain distribution, we can know that the frequecny of me
I can get a guess of N_line with k_lam at 1216 is 2, and the guassain is not so
since wavelength over 1200 is still samll, I will consider the total formula to
Therefore, it is reasonalbe to guess N_line to be 10
"""

guess_N_count_3 = 1
guess_alpha_3 = 6
guess_N_line_3 = 10
guess_N_lam_3 = np.array(guess_N_count_3*(spectra_wavelength/1200)**guess_alpha_3)

# get the function of drawing graph from the problem
def plot_spectrum(ax, wave, k_lam):
    """
    Utility function to plot a spectrum in the low photon regime.

    Parameters
    -----
    ax: matplotlib axis object
        The axis to plot the spectrum on. This is a matplotlib axes object c
        the object oriented API of matplotlib.
    wave: array-like
        The wavelength array of the spectrum.
    k_lam: array-like
        The observed integer number of photons detected per wavelength bin.

    """

    # Calculate bin midpoints
    bin_widths = np.diff(wave) / 2.0
    bin_widths = np.append(bin_widths[0], bin_widths)*0.97
    k_lam_lower = k_lam + 0.5 - np.sqrt(k_lam + 0.25)
    k_lam_upper = k_lam + 0.5 + np.sqrt(k_lam + 0.25)

    for ww, bin_width, kval, kval_lo, kval_hi in zip(wave, bin_widths, k_lam, k_
        # Draw rectangles for the error bars
        ax.fill_betweenx([kval_lo, kval_hi], ww-bin_width, ww+bin_width, color='
        # Draw horizontal lines for the measured value
        ax.hlines(kval, ww-bin_width, ww+bin_width, color='black', linestyle='-
    return

fig, ax = plt.subplots(3,1,figsize = (10,5))
# get the general configuration of diagram
plot_spectrum(ax[0],spectra_wavelength,spectra_k_lam_1)
ax[0].plot(spectra_wavelength,guess_N_lam_1,color='red',drawstyle='steps-mid',la
ax[0].axvline(1216,color='magenta', linestyle=':',label='Emission Line Wavelength
ax[0].legend()
# plot the first graph

plot_spectrum(ax[1],spectra_wavelength,spectra_k_lam_2)
ax[1].plot(spectra_wavelength,guess_N_lam_2,color='red',drawstyle='steps-mid',la

```

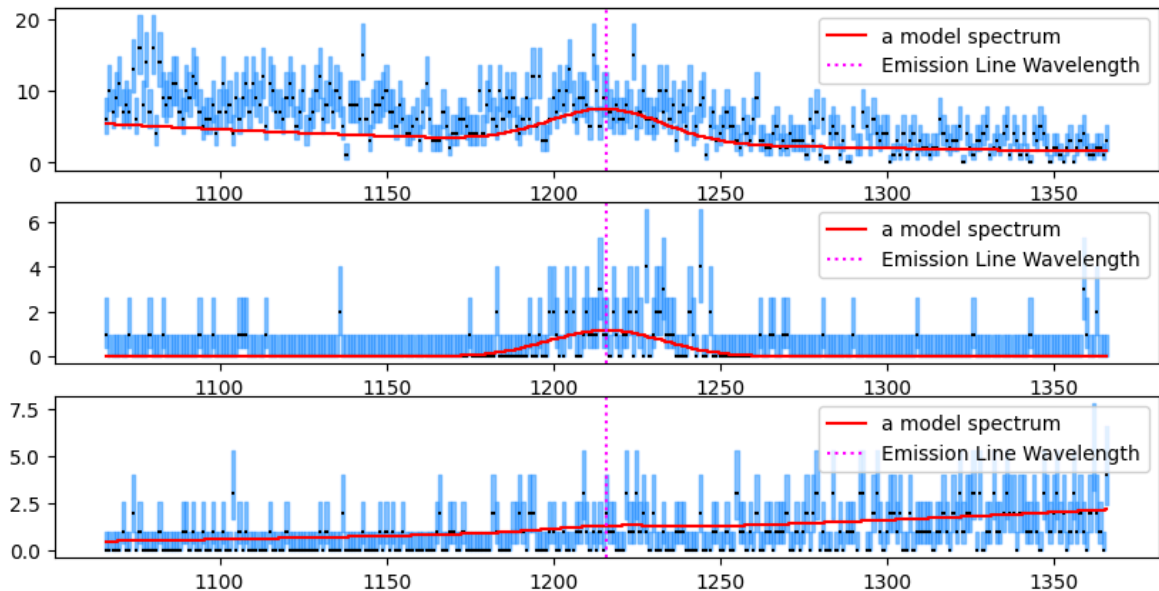
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ax[1].axvline(1216,color='magenta', linestyle=':',label='Emission Line Wavelength')
ax[1].legend()
# plot the second graph

plot_spectrum(ax[2],spectra_wavelength,spectra_k_lam_3)
ax[2].plot(spectra_wavelength,guess_N_lam_3,color='red',drawstyle='steps-mid',label='a model spectrum')
ax[2].axvline(1216,color='magenta', linestyle=':',label='Emission Line Wavelength')
ax[2].legend()
# plot the third graph

```

Out[]: <matplotlib.legend.Legend at 0x1dabc6ed510>



In []: # b)

```

import scipy.special
import scipy.stats

def postive_lnl(theta, wave, k_lam):
    """
    The log-likelihood function for the spectrum.

    Parameters
    -----
    theta: array-like
        The model parameters.
    wave: array-like
        The wavelength array of the spectrum.
    k_lam: array-like
        The observed number of photons detected per wavelength bin.

    Returns
    -----
    lnL: float
        The log-likelihood of the model given the data.
    """
    N_count = theta[0]
    alpha = theta[1]
    N_line = theta[2]
    # get the value of model parameters
    N_lam = np.array(N_count*(wave/1200)**alpha+N_line*np.exp(-0.5 * ((wave - 12

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# create the model value from the model parameters
lnL = np.sum(scipy.stats.poisson.logpmf(k_lam,N_lam))
# calculate ln likelihood
return lnL

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In [ ]: # c)

import scipy.optimize

def lnL(theta,wave,k_lam):
    return -postive_lnL(theta,wave,k_lam)

theta_guess = np.array([3,-5,200])
# get the first guess
theta_min_1 = scipy.optimize.minimize(lnL,theta_guess,args=(spectra_wavelength,s
# get the min value
theta_min_1 = theta_min_1['x']
# get the theta for the first guess
maxL_lam_1 = np.array(theta_min_1[0]*(spectra_wavelength/1200)**theta_min_1[1]+t
# get the line of max L

theta_guess = np.array([0,1,50])
# get the first guess
theta_min_2 = scipy.optimize.minimize(lnL,theta_guess,args=(spectra_wavelength,s
# get the min value
theta_min_2 = theta_min_2['x']
# get the theta for the first guess
maxL_lam_2 = np.array(theta_min_2[0]*(spectra_wavelength/1200)**theta_min_2[1]+t
# get the line of max L

theta_guess = np.array([1,6,10])
# get the first guess
theta_min_3 = scipy.optimize.minimize(lnL,theta_guess,args=(spectra_wavelength,s
# get the min value
theta_min_3 = theta_min_3['x']
# get the theta for the first guess
maxL_lam_3 = np.array(theta_min_3[0]*(spectra_wavelength/1200)**theta_min_3[1]+t
# get the line of max L

true_params = pd.read_csv('data/true_params.csv',comment='#')
# get the true values to python

true_params_N_count = np.array(true_params['N_count'])
true_params_alpha = np.array(true_params['alpha'])
true_params_N_line = np.array(true_params['N_line'])
# assign with good name

true_params_N_lam_1 = np.array(true_params_N_count[0]*(spectra_wavelength/1200)*
true_params_N_lam_2 = np.array(true_params_N_count[1]*(spectra_wavelength/1200)*
true_params_N_lam_3 = np.array(true_params_N_count[2]*(spectra_wavelength/1200)*
# generate the value of true value

fig, ax = plt.subplots(3,1,figsize = (10,5))
# get the general configuration of diagram
plot_spectrum(ax[0],spectra_wavelength,spectra_k_lam_1)
ax[0].plot(spectra_wavelength,maxL_lam_1,color='green',drawstyle='steps-mid',lab
ax[0].plot(spectra_wavelength,true_params_N_lam_1,color='red',drawstyle='steps-m
ax[0].legend()
# plot the first graph

```

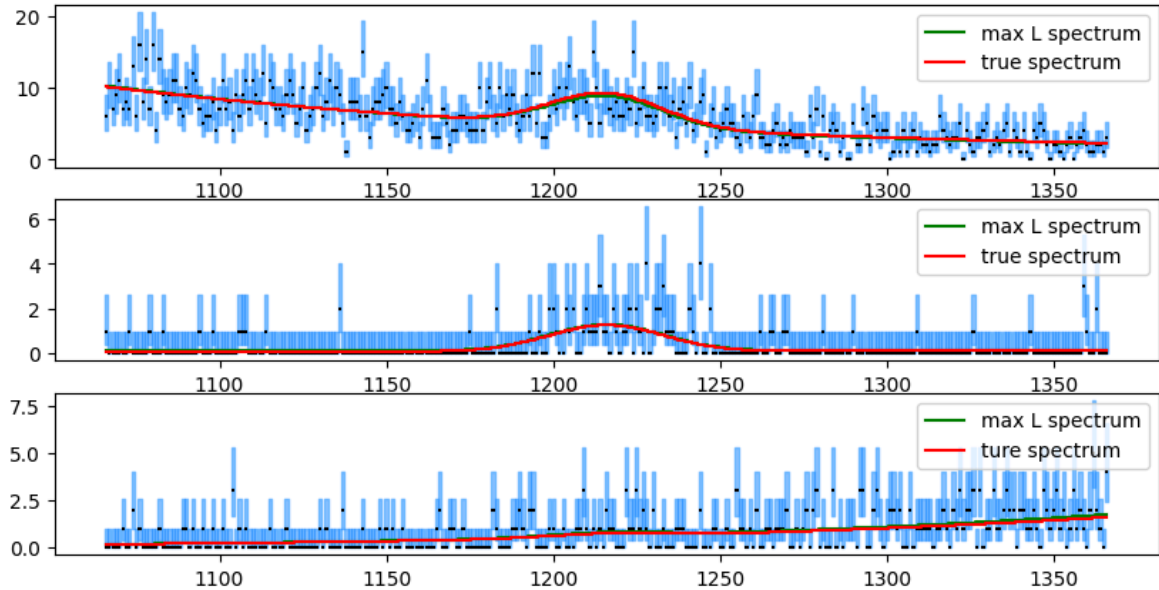
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plot_spectrum(ax[1],spectra_wavelength,spectra_k_lam_2)
ax[1].plot(spectra_wavelength,maxL_lam_2,color='green',drawstyle='steps-mid',label='max L spectrum')
ax[1].plot(spectra_wavelength,true_params_N_lam_2,color='red',drawstyle='steps-mid',label='true spectrum')
ax[1].legend()
# plot the second graph

plot_spectrum(ax[2],spectra_wavelength,spectra_k_lam_3)
ax[2].plot(spectra_wavelength,maxL_lam_3,color='green',drawstyle='steps-mid',label='max L spectrum')
ax[2].plot(spectra_wavelength,true_params_N_lam_3,color='red',drawstyle='steps-mid',label='true spectrum')
ax[2].legend()
# plot the third graph

```

Out[]: <matplotlib.legend.Legend at 0x1dadfa99490>



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In [ ]: # d)

N_line_min_1 = scipy.optimize.minimize(lnL,[true_params_N_count[0],true_params_a])
# get the value to max L N_line
N_line_min_1 = N_line_min_1['x'][2]
# make it to be a value
maxL_N_line_lam_1 = np.array(true_params_N_count[0]*(spectra_wavelength/1200)**t)
# get the line of max L for N_line

N_line_min_2 = scipy.optimize.minimize(lnL,[true_params_N_count[1],true_params_a])
# get the value to max L N_line
N_line_min_2 = N_line_min_2['x'][2]
# make it to be a value
maxL_N_line_lam_2 = np.array(true_params_N_count[1]*(spectra_wavelength/1200)**t)
# get the line of max L for N_line

N_line_min_3 = scipy.optimize.minimize(lnL,[true_params_N_count[2],true_params_a])
# get the value to max L N_line
N_line_min_3 = N_line_min_3['x'][2]
# make it to be a value
maxL_N_line_lam_3 = np.array(true_params_N_count[2]*(spectra_wavelength/1200)**t)
# get the line of max L for N_line

fig, ax = plt.subplots(3,1,figsize = (10,5))
# get the general configuration of diagram
plot_spectrum(ax[0],spectra_wavelength,spectra_k_lam_1)
ax[0].plot(spectra_wavelength,maxL_N_line_lam_1,color='green',drawstyle='steps-mid',label='max L spectrum')

```

```

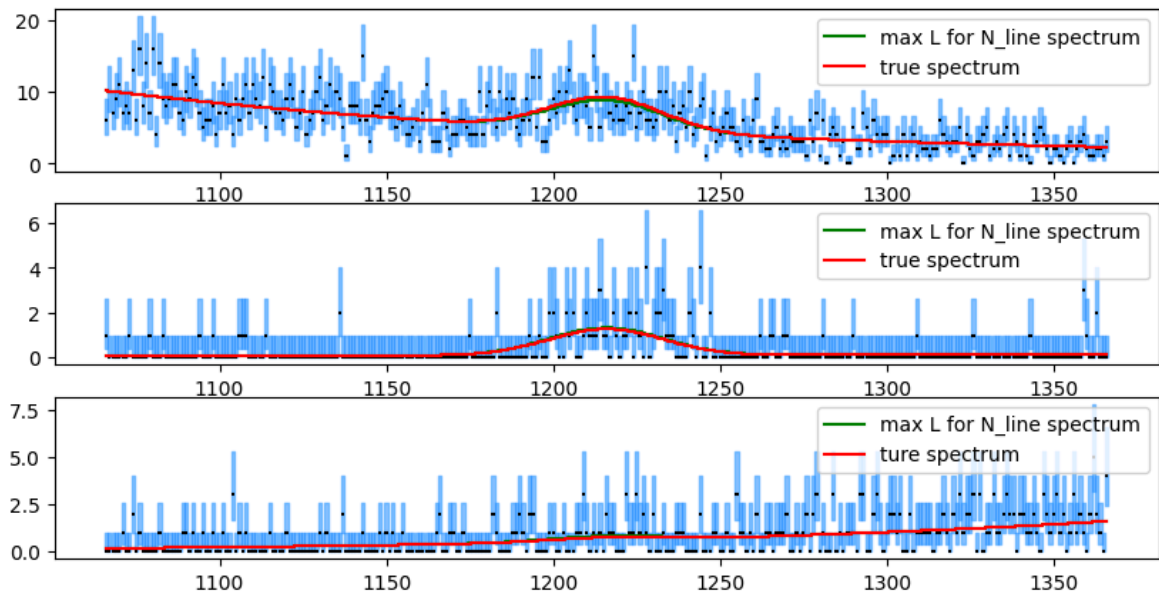
ax[0].plot(spectra_wavelength,true_params_N_lam_1,color='red',drawstyle='steps-m
ax[0].legend()
# plot the first graph

plot_spectrum(ax[1],spectra_wavelength,spectra_k_lam_2)
ax[1].plot(spectra_wavelength,maxL_N_line_lam_2,color='green',drawstyle='steps-m
ax[1].plot(spectra_wavelength,true_params_N_lam_2,color='red',drawstyle='steps-m
ax[1].legend()
# plot the second graph

plot_spectrum(ax[2],spectra_wavelength,spectra_k_lam_3)
ax[2].plot(spectra_wavelength,maxL_N_line_lam_3,color='green',drawstyle='steps-m
ax[2].plot(spectra_wavelength,true_params_N_lam_3,color='red',drawstyle='steps-m
ax[2].legend()
# plot the third graph

```

Out[]: <matplotlib.legend.Legend at 0x1dae13d3ad0>



In []: # e)

```

import scipy.integrate
import scipy.interpolate
import scipy.optimize
import scipy.stats

Pr_N_line = np.linspace(350,0,1000,False)
# get the Pr(N_line) as a unifrom distribution without 0
unif_prior = (Pr_N_line[0] - Pr_N_line[-1])**(-1)
# get the distribution

lnLvalues_1 = [-lnL([true_params_N_count[0],true_params_alpha[0],N_line],spectra
# get ln(L)
P_unnorm_1 = np.exp(lnLvalues_1)*unif_prior
P_norm_1 = -scipy.integrate.trapezoid(P_unnorm_1,Pr_N_line)
# add a negative sign due to the
posterior_1 = P_unnorm_1/P_norm_1
# calculate the posterior
cdf_1 = 1+scipy.integrate.cumulative_trapezoid(posterior_1,Pr_N_line,initial=0)
# get the cdf
cdf_inverse = scipy.interpolate.interp1d(cdf_1,Pr_N_line)
Pr_N_16 = cdf_inverse(0.16)

```

```

Pr_N_50 = cdf_inverse(0.50)
Pr_N_84 = cdf_inverse(0.84)

# Create a figure with two subplots
fig, (ax1, ax2) = plt.subplots(2, 1, figsize=(10, 8), sharex=True, gridspec_kw={

# Plot the CDF in the upper subplot
ax1.plot(Pr_N_line, cdf_1, color='g', label=r'\rm CDF(\leq g)$ for $P(g|\math
ax1.fill_between(Pr_N_line, -5, cdf_1, where=((Pr_N_line >= Pr_N_16) & (Pr_N_lin
# vertical lines at the true, maxL, and median values
ax1.axvline(true_params_N_line[0], color='r', linestyle='--', linewidth=2.0)
ax1.axvline(N_line_min_1, color='b', linestyle=':', alpha=0.5, linewidth=2.0)
ax1.axvline(Pr_N_50, color='magenta', linestyle='-', alpha=0.5)

ax1.set_ylabel(r'\rm CDF$', fontsize=15)
ax1.legend(loc='lower right', fontsize=12)
ax1.set_ylim(-0.001, 1.001)

# Plot the posterior distribution in the lower subplot
ax2.plot(Pr_N_line, posterior_1, color='k', label=r'Posterior')
# vertical lines at the true, maxL, and median values
ax2.axvline(true_params_N_line[0], color='r', linestyle='--', alpha=0.9, linewidth=2.0)
ax2.axvline(N_line_min_1, color='b', linestyle=':', linewidth=2.0, alpha=0.7, label='N_min')
ax2.axvline(Pr_N_50, color='magenta', linestyle='-', linewidth=2.0, alpha=0.7, label='Pr_N_50')
ax2.fill_between(Pr_N_line, -10.0, posterior_1, where=((Pr_N_line >= Pr_N_16) &
ax2.set_ylabel('P', fontsize=15)
ax2.set_ylim(-1e-4, 0.02)

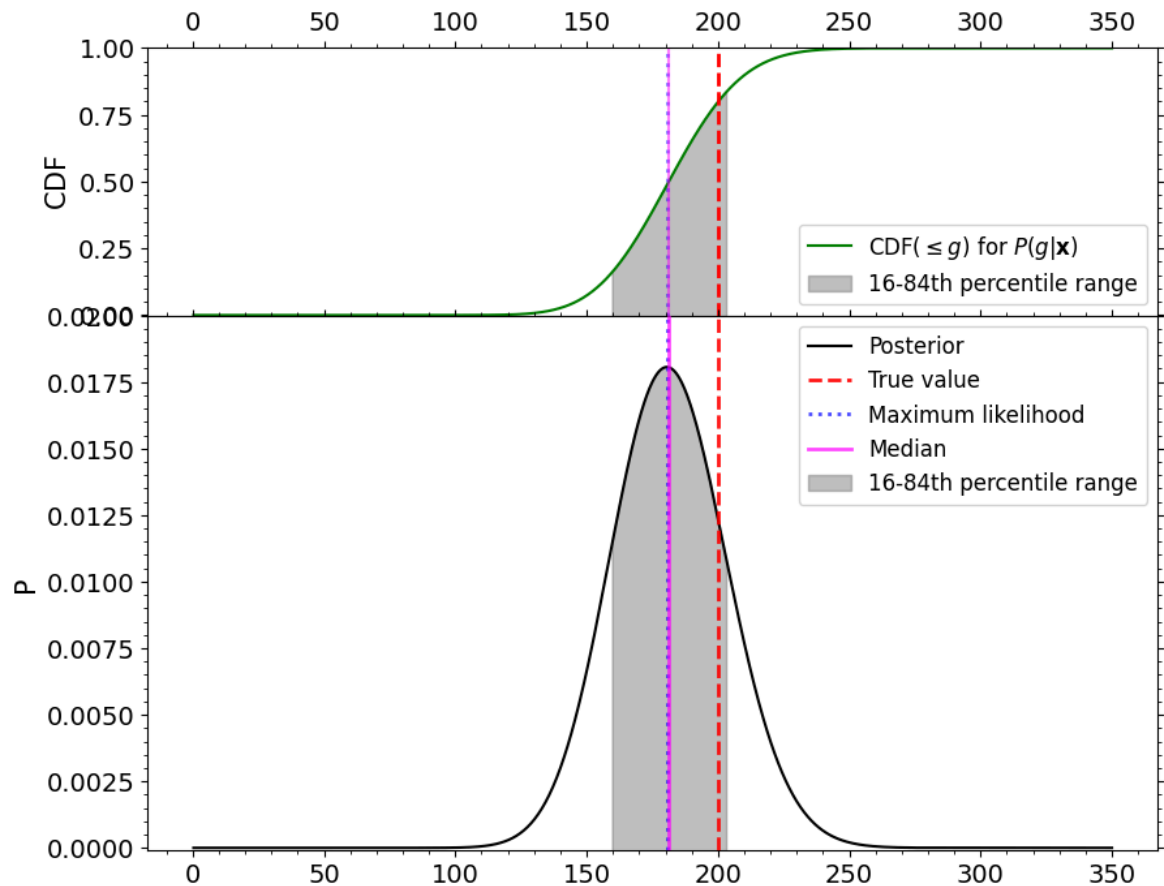
# Some tick mark things
ax1.tick_params(which='both', bottom=True, top=True, left=True, right=True, labelsize=10)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.yaxis.set_minor_locator(AutoMinorLocator())

# Some tick mark things
ax2.tick_params(which='both', bottom=True, top=True, left=True, right=True, labelsize=10)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.yaxis.set_minor_locator(AutoMinorLocator())

# Add the Legend from the upper plot to the Lower plot
ax2.legend(loc='upper right', fontsize=12)

plt.show()

```



```
In [ ]: Pr_N_line = np.linspace(100,0,1000,False)
# get the Pr(N_line) as a unifrom distribution without 0
unif_prior = (Pr_N_line[0] - Pr_N_line[-1])**(-1)
# get the distribution

lnLvalues_2 = [-lnL([true_params_N_count[1],true_params_alpha[1],N_line],spectra
# get ln(L)
P_unnorm_2 = np.exp(lnLvalues_2)*unif_prior
P_norm_2 = -scipy.integrate.trapezoid(P_unnorm_2,Pr_N_line)
# add a negative sign due to the
posterior_2 = P_unnorm_2/P_norm_2
# calculate the posterior
cdf_2 = 1+scipy.integrate.cumulative_trapezoid(posterior_2,Pr_N_line,initial=0)
# get the cdf

cdf_inverse = scipy.interpolate.interp1d(cdf_2,Pr_N_line)
Pr_N_16 = cdf_inverse(0.16)
Pr_N_50 = cdf_inverse(0.50)
Pr_N_84 = cdf_inverse(0.84)

# Create a figure with two subplots
fig, (ax1, ax2) = plt.subplots(2, 1, figsize=(10, 8), sharex=True, gridspec_kw={

# Plot the CDF in the upper subplot
ax1.plot(Pr_N_line, cdf_2, color='g', label=r'\rm CDF(\leq g) for $P(g|\mathbf{x})$')
ax1.fill_between(Pr_N_line, -5, cdf_2, where=((Pr_N_line >= Pr_N_16) & (Pr_N_line <= Pr_N_84)))
# vertical lines at the true, maxL, and median values
ax1.axvline(true_params_N_line[1], color='r', linestyle='--', linewidth=2.0)
ax1.axvline(N_line_min_2, color='b', linestyle=':', alpha=0.5, linewidth=2.0)
ax1.axvline(Pr_N_50, color='magenta', linestyle='-', alpha=0.5)

ax1.set_ylabel(r'\rm CDF', fontsize=15)
```



```

ax1.legend(loc='lower right', fontsize=12)
ax1.set_ylim(-0.001,1.001)

# Plot the posterior distribution in the lower subplot
ax2.plot(Pr_N_line, posterior_2, color='k', label=r'Posterior')
# vertical lines at the true, maxL, and median values
ax2.axvline(true_params_N_line[1], color='r', linestyle='--', alpha=0.9, linewidth=2)
ax2.axvline(N_line_min_2, color='b', linestyle=':', linewidth=2.0, alpha=0.7, label='N_line_min_2')
ax2.axvline(Pr_N_50, color='magenta', linestyle='-', linewidth=2.0, alpha=0.7, label='Pr_N_50')
ax2.fill_between(Pr_N_line, -10.0, posterior_2, where=((Pr_N_line >= Pr_N_16) &
ax2.set_ylabel('P', fontsize=15)
ax2.set_ylim(-1e-4,0.06)

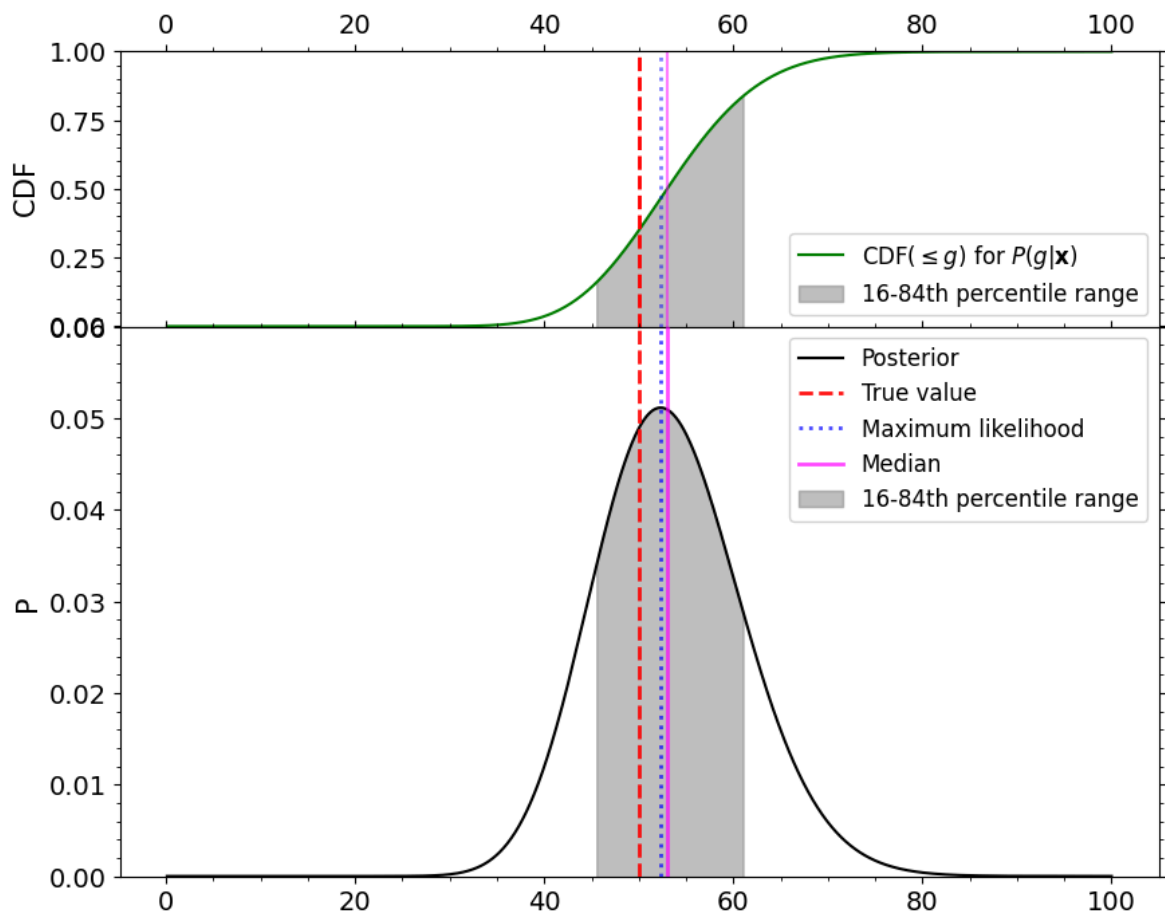
# Some tick mark things
ax1.tick_params(which='both', bottom=True, top=True, left=True, right=True, labelsize=12)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.yaxis.set_minor_locator(AutoMinorLocator())

# Some tick mark things
ax2.tick_params(which='both', bottom=True, top=True, left=True, right=True, labelsize=12)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.yaxis.set_minor_locator(AutoMinorLocator())

# Add the Legend from the upper plot to the lower plot
ax2.legend(loc='upper right', fontsize=12)

plt.show()

```



```

In [ ]: Pr_N_line = np.linspace(50,0,1000,False)
# get the Pr(N_line) as a unifrom distribution without 0

```

```

unif_prior = (Pr_N_line[0] - Pr_N_line[-1])**(-1)
# get the distribution

lnLvalues_3 = [-lnL([true_params_N_count[2],true_params_alpha[2],N_line],spectra
# get ln(L)
P_unnorm_3 = np.exp(lnLvalues_3)*unif_prior
P_norm_3 = -scipy.integrate.trapezoid(P_unnorm_3,Pr_N_line)
# add a negative sign due to the
posterior_3 = P_unnorm_3/P_norm_3
# calculate the posterior
cdf_3 = 1+scipy.integrate.cumulative_trapezoid(posterior_3,Pr_N_line,initial=0)
# get the cdf

cdf_inverse = scipy.interpolate.interp1d(cdf_3,Pr_N_line)
Pr_N_16 = cdf_inverse(0.16)
Pr_N_50 = cdf_inverse(0.50)
Pr_N_84 = cdf_inverse(0.84)

# Create a figure with two subplots
fig, (ax1, ax2) = plt.subplots(2, 1, figsize=(10, 8), sharex=True, gridspec_kw={

# Plot the CDF in the upper subplot
ax1.plot(Pr_N_line, cdf_3, color='g', label=r'$\rm CDF(\leq g)$ for $P(g|\mathbf{g})$')
ax1.fill_between(Pr_N_line, -5, cdf_3, where=((Pr_N_line >= Pr_N_16) & (Pr_N_line <= Pr_N_84)))
# vertical lines at the true, maxL, and median values
ax1.axvline(true_params_N_line[2], color='r', linestyle='--', linewidth=2.0)
ax1.axvline(N_line_min_3, color='b', linestyle=':', alpha=0.5, linewidth=2.0)
ax1.axvline(Pr_N_50, color='magenta', linestyle='-', alpha=0.5)

ax1.set_ylabel(r'$\rm CDF$', fontsize=15)
ax1.legend(loc='lower right', fontsize=12)
ax1.set_ylim(-0.001,1.001)

# Plot the posterior distribution in the lower subplot
ax2.plot(Pr_N_line, posterior_3, color='k', label=r'Posterior')
# vertical lines at the true, maxL, and median values
ax2.axvline(true_params_N_line[2], color='r', linestyle='--', alpha=0.9, linewidth=2.0)
ax2.axvline(N_line_min_3, color='b', linestyle=':', linewidth=2.0, alpha=0.7, label='N_min')
ax2.axvline(Pr_N_50, color='magenta', linestyle='-', linewidth=2.0, alpha=0.7, label='P_50')
ax2.fill_between(Pr_N_line, -10.0, posterior_3, where=((Pr_N_line >= Pr_N_16) & (Pr_N_line <= Pr_N_84)))
ax2.set_ylabel('P', fontsize=15)
ax2.set_ylim(-1e-4,0.07)

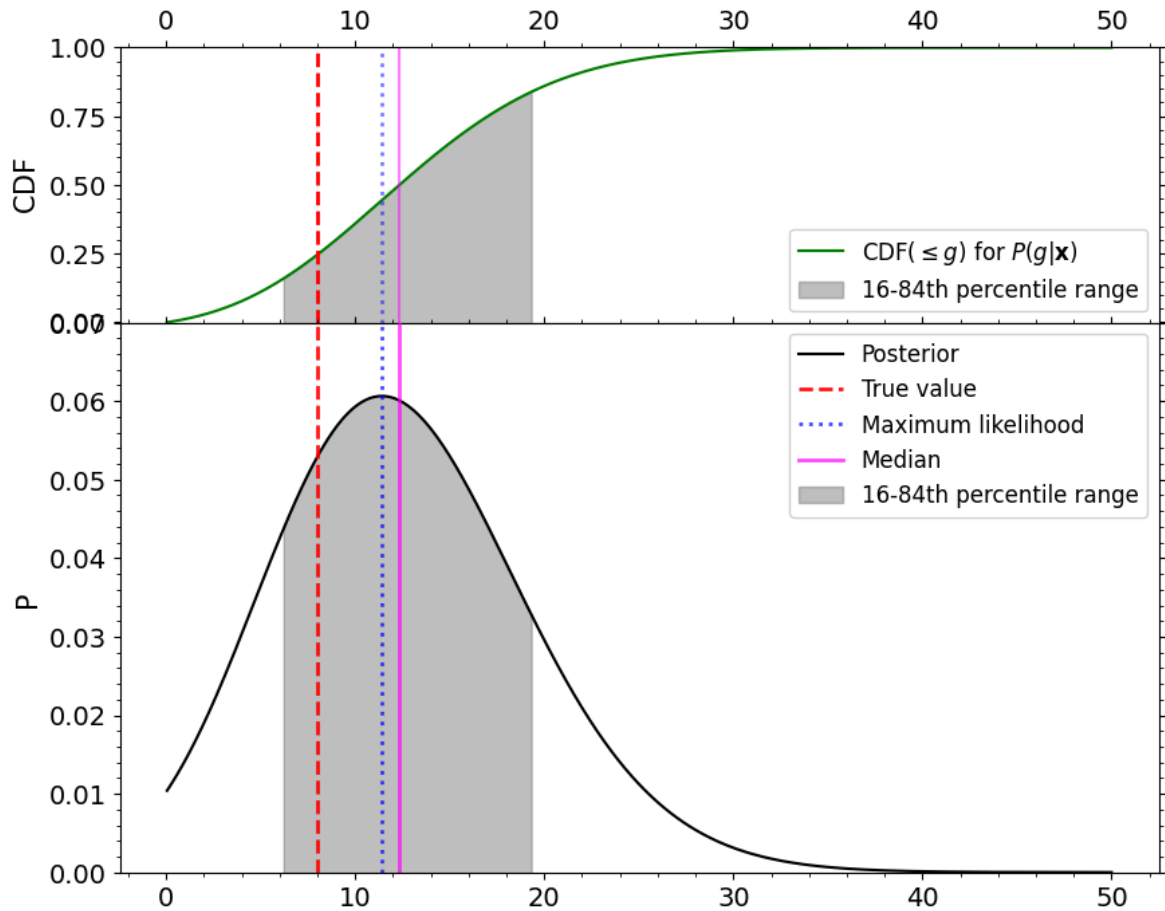
# Some tick mark things
ax1.tick_params(which='both', bottom=True, top=True, left=True, right=True, labelsize=12)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.yaxis.set_minor_locator(AutoMinorLocator())

# Some tick mark things
ax2.tick_params(which='both', bottom=True, top=True, left=True, right=True, labelsize=12)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.yaxis.set_minor_locator(AutoMinorLocator())

# Add the Legend from the upper plot to the Lower plot
ax2.legend(loc='upper right', fontsize=12)

plt.show()

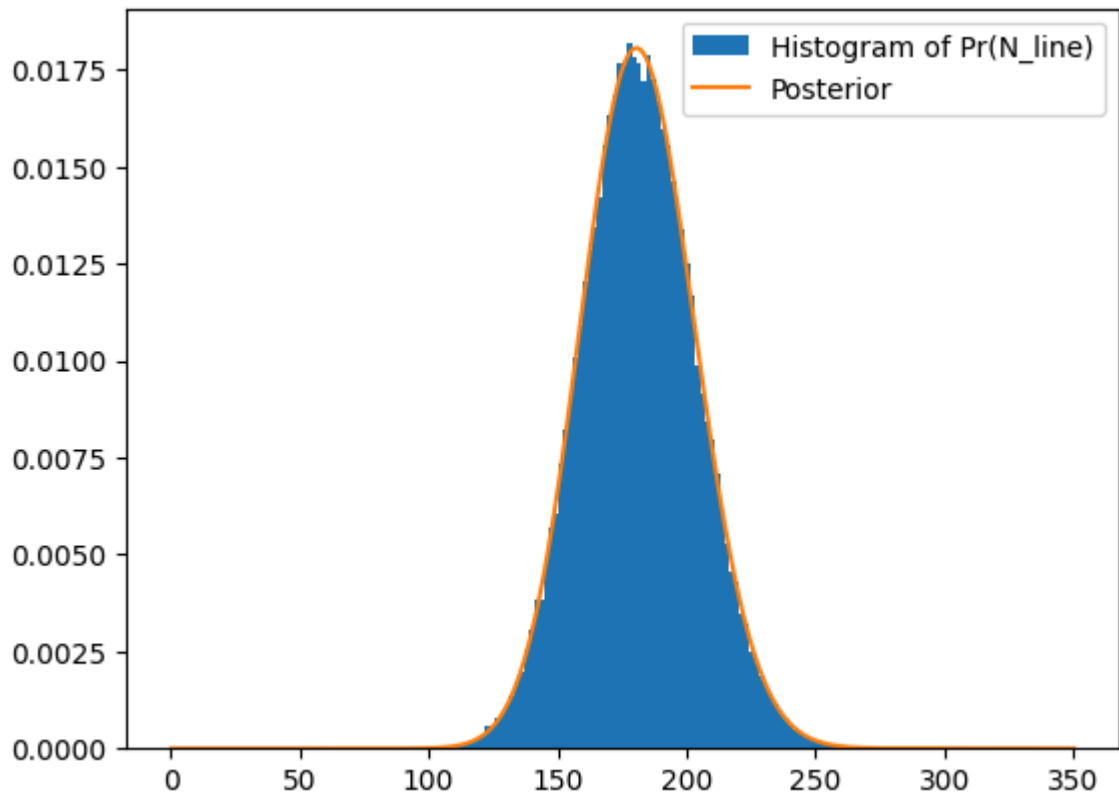
```



```
In [ ]: # f)

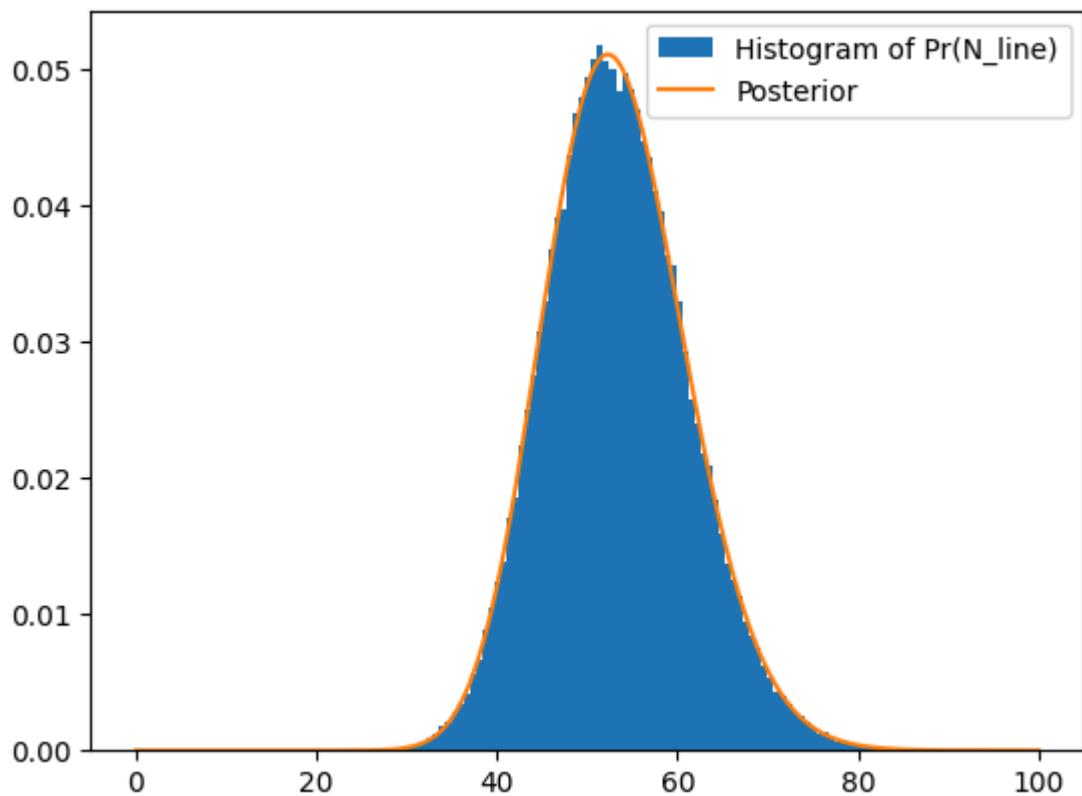
uniform_sample = np.random.uniform(0,1,100000)
# generate a random uniform sample
Pr_N_line = np.linspace(350,0,1000,False)
cdf_inverse = scipy.interpolate.interp1d(cdf_1,Pr_N_line)
Pr_N_samples = cdf_inverse(uniform_sample)
# get the sample
plt.hist(Pr_N_samples,100,density=True,label='Histogram of Pr(N_line)')
plt.plot(Pr_N_line,posterior_1,label='Posterior')
# get the values
plt.legend()
```

```
Out[ ]: <matplotlib.legend.Legend at 0x1dae45f9710>
```



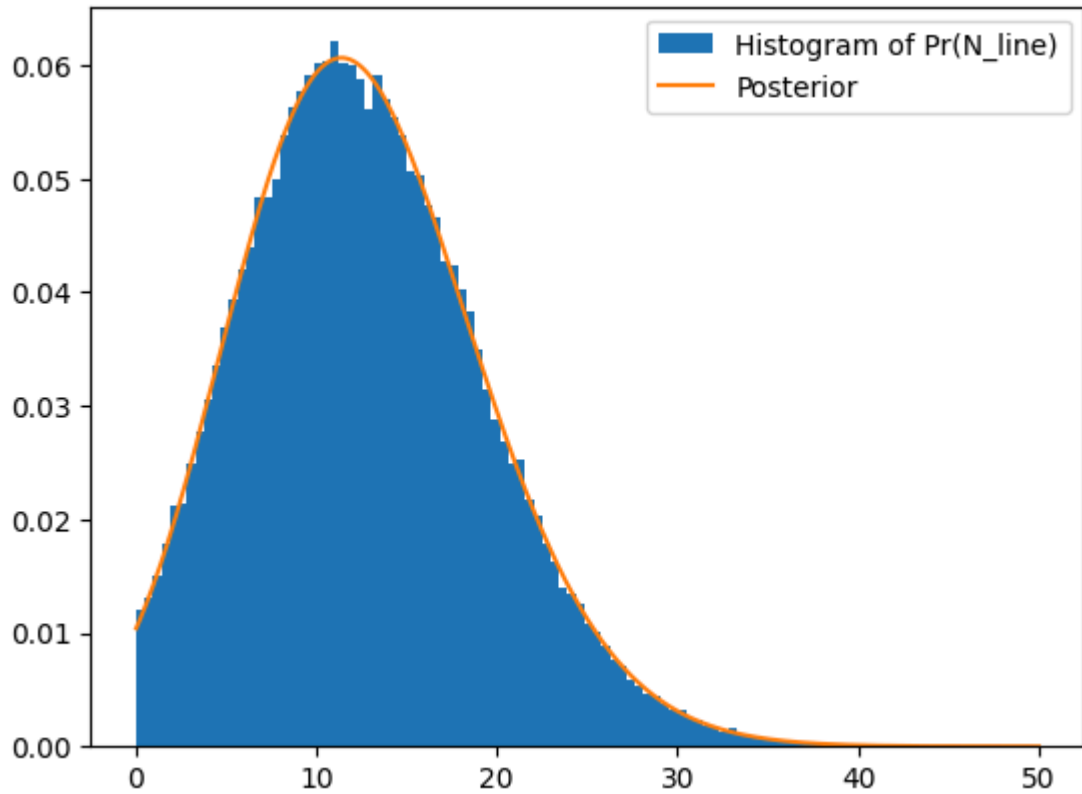
```
In [ ]: Pr_N_line = np.linspace(100,0,1000,False)
cdf_inverse = scipy.interpolate.interp1d(cdf_2,Pr_N_line)
Pr_N_samples = cdf_inverse(uniform_sample)
# get the sample
plt.hist(Pr_N_samples,100,density=True,label='Histogram of Pr(N_line)')
plt.plot(Pr_N_line,posterior_2,label='Posterior')
# get the values
plt.legend()
```

```
Out[ ]: <matplotlib.legend.Legend at 0x1dae47644d0>
```



```
In [ ]: Pr_N_line = np.linspace(50,0,1000,False)
cdf_inverse = scipy.interpolate.interp1d(cdf_3,Pr_N_line)
Pr_N_samples = cdf_inverse(uniform_sample)
# get the sample
plt.hist(Pr_N_samples,100,density=True,label='Histogram of Pr(N_line)')
plt.plot(Pr_N_line,posterior_3,label='Posterior')
# get the values
plt.legend()
```

```
Out[ ]: <matplotlib.legend.Legend at 0x1dae479da10>
```



```
In [ ]: # g)

uniform_sample = np.random.uniform(0,1,100)
# generate a random unifrom sample
fig, ax = plt.subplots(3,1,figsize = (10,10))

Pr_N_line = np.linspace(350,0,1000,False)
cdf_inverse = scipy.interpolate.interp1d(cdf_1,Pr_N_line)
Pr_N_samples = cdf_inverse(uniform_sample)
# get the general configuration of diagram
plot_spectrum(ax[0],spectra_wavelength,spectra_k_lam_1)
for ip,Pr_N in enumerate(Pr_N_samples):
    sample_line = true_params_N_count[0]*(spectra_wavelength/1200)**true_params_
    ax[0].plot(spectra_wavelength,sample_line,'gray', alpha=0.2, linewidth=1, la
ax[0].plot(spectra_wavelength,true_params_N_lam_1,color='red',drawstyle='steps-m
ax[0].plot(spectra_wavelength,maxL_N_line_lam_1,color='green',drawstyle='steps-m
ax[0].set_xlim(1156,1267)
ax[0].legend()
# plot the first graph

Pr_N_line = np.linspace(100,0,1000,False)
cdf_inverse = scipy.interpolate.interp1d(cdf_2,Pr_N_line)
Pr_N_samples = cdf_inverse(uniform_sample)
plot_spectrum(ax[1],spectra_wavelength,spectra_k_lam_2)
for ip,Pr_N in enumerate(Pr_N_samples):
    sample_line = true_params_N_count[1]*(spectra_wavelength/1200)**true_params_
    ax[1].plot(spectra_wavelength,sample_line,'gray', alpha=0.2, linewidth=1, la
ax[1].plot(spectra_wavelength,true_params_N_lam_2,color='red',drawstyle='steps-m
ax[1].plot(spectra_wavelength,maxL_N_line_lam_2,color='green',drawstyle='steps-m
ax[1].set_xlim(1156,1267)
ax[1].legend()
# plot the second graph

Pr_N_line = np.linspace(50,0,1000,False)
cdf_inverse = scipy.interpolate.interp1d(cdf_3,Pr_N_line)
```

```

Pr_N_samples = cdf_inverse(uniform_sample)
plot_spectrum(ax[2],spectra_wavelength,spectra_k_lam_3)
for ip,Pr_N in enumerate(Pr_N_samples):
    sample_line = true_params_N_count[2]*(spectra_wavelength/1200)**true_params_N_lam_3
    ax[2].plot(spectra_wavelength,sample_line,'gray', alpha=0.2, linewidth=1, label=f'Pr_N {ip}')
ax[2].plot(spectra_wavelength,true_params_N_lam_3,color='red',drawstyle='steps-mid',label='true spectrum')
ax[2].plot(spectra_wavelength,maxL_N_line_lam_3,color='green',drawstyle='steps-mid',label='maximum likelihood spectrum')
ax[2].set_xlim(1156,1267)
ax[2].legend()
# plot the third graph

```

Out[]: <matplotlib.legend.Legend at 0x1dafae21490>

