Biology and Mathematics of the Retina

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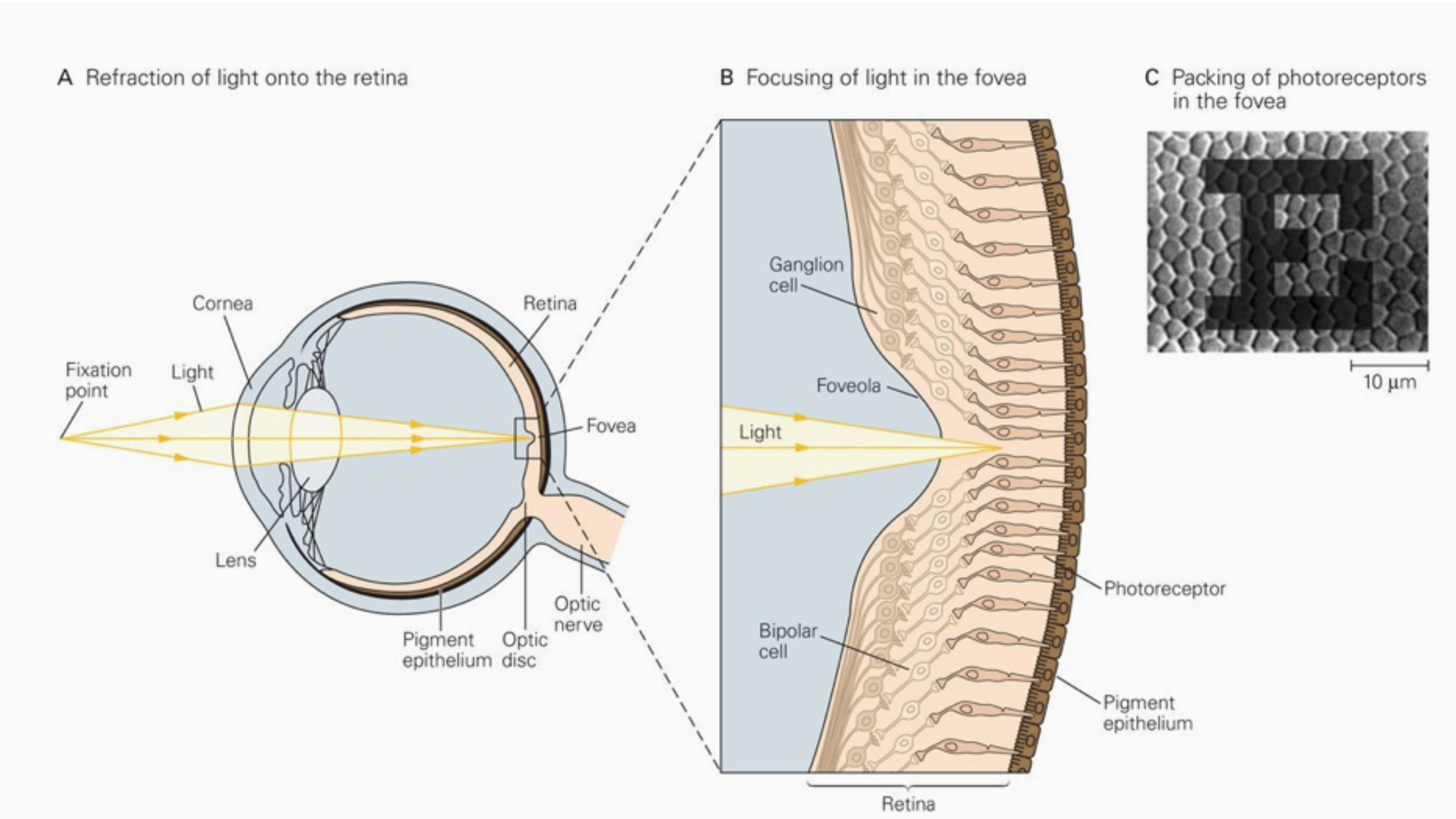
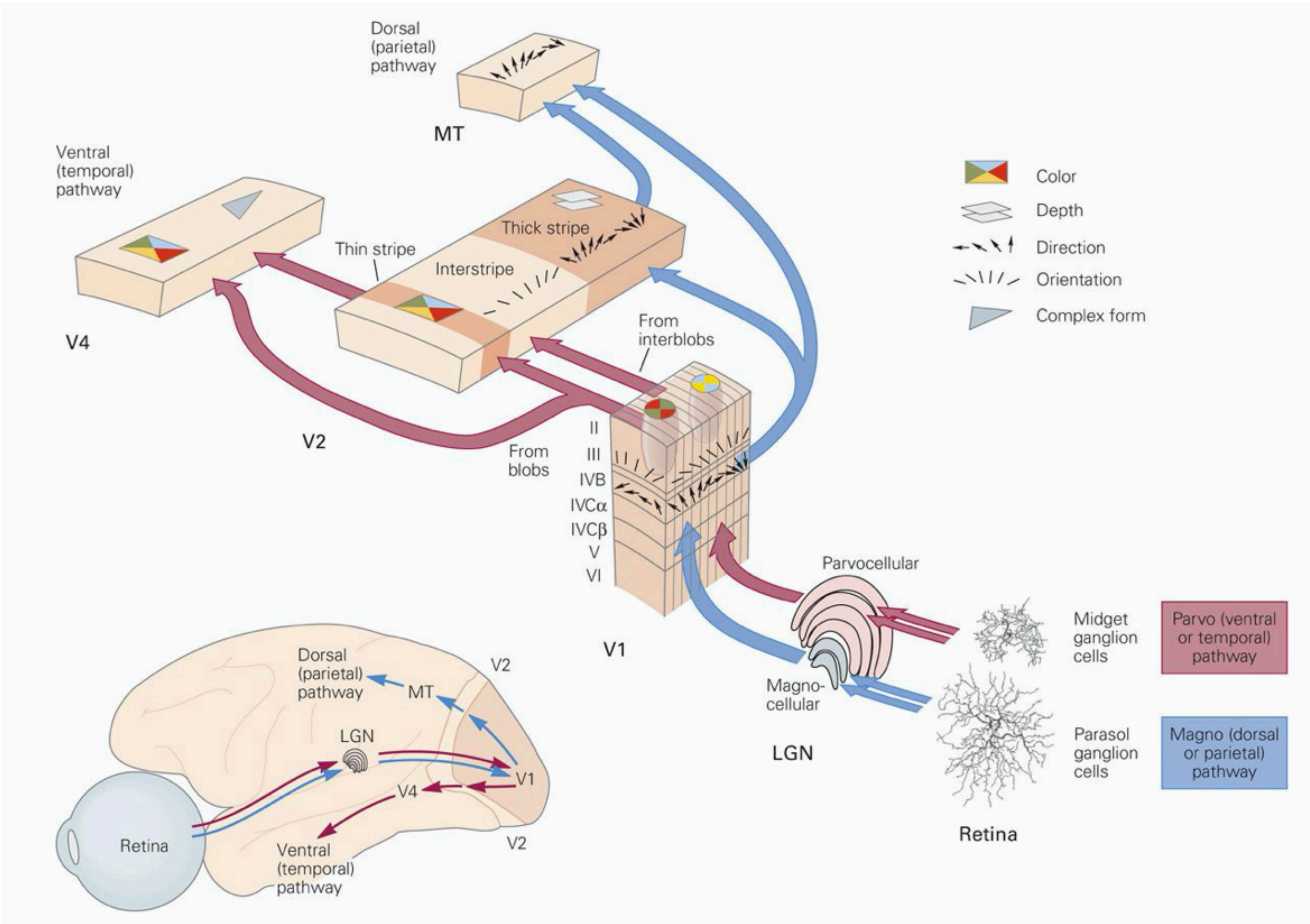
Plan

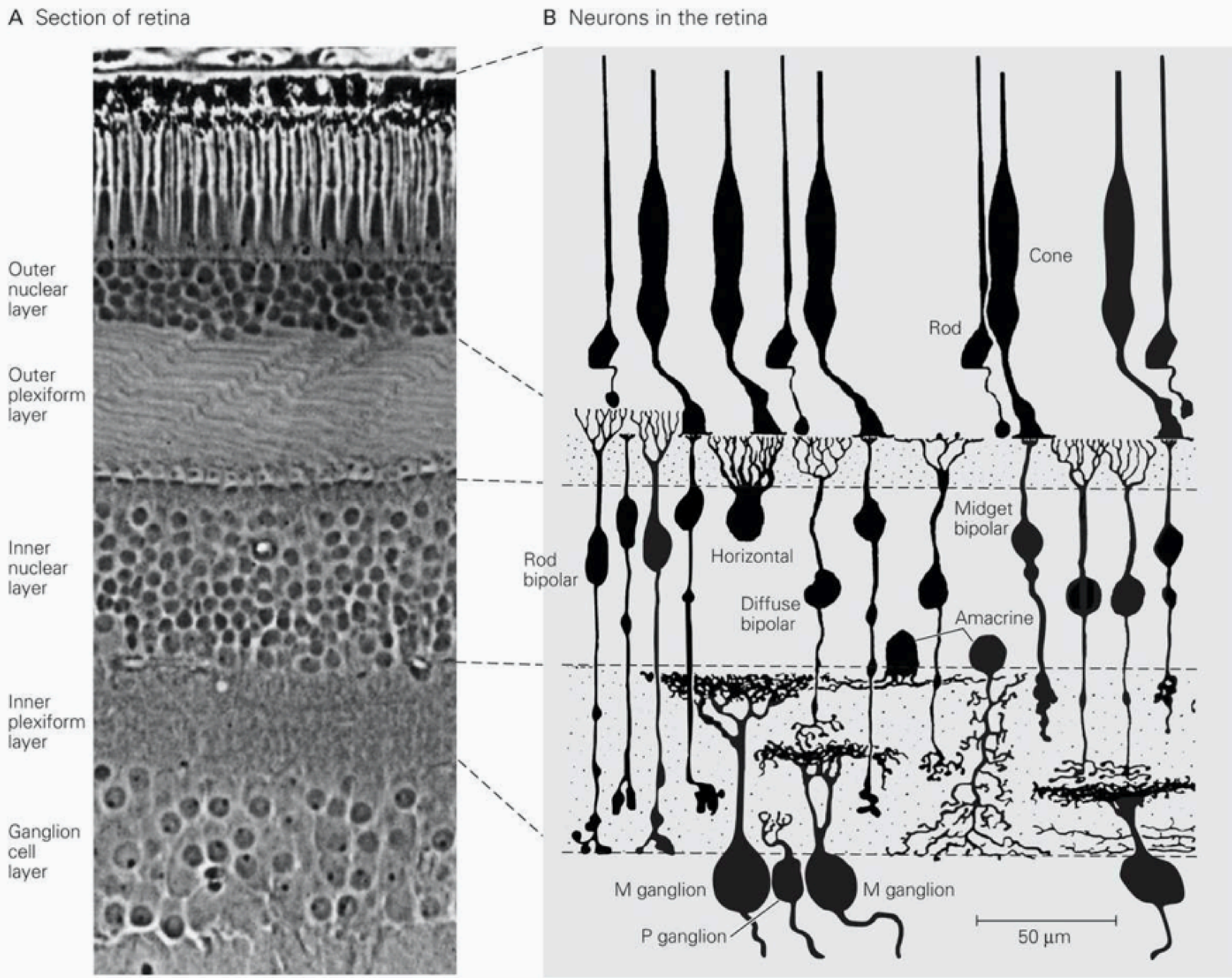
- Biology of retina
- RC circuit, convolution and dynamical system
- Dynamic adaptation of the retina
- Vesicle kinetic model for short term learning
- Motion sensing cell and the idea of objects at the retina

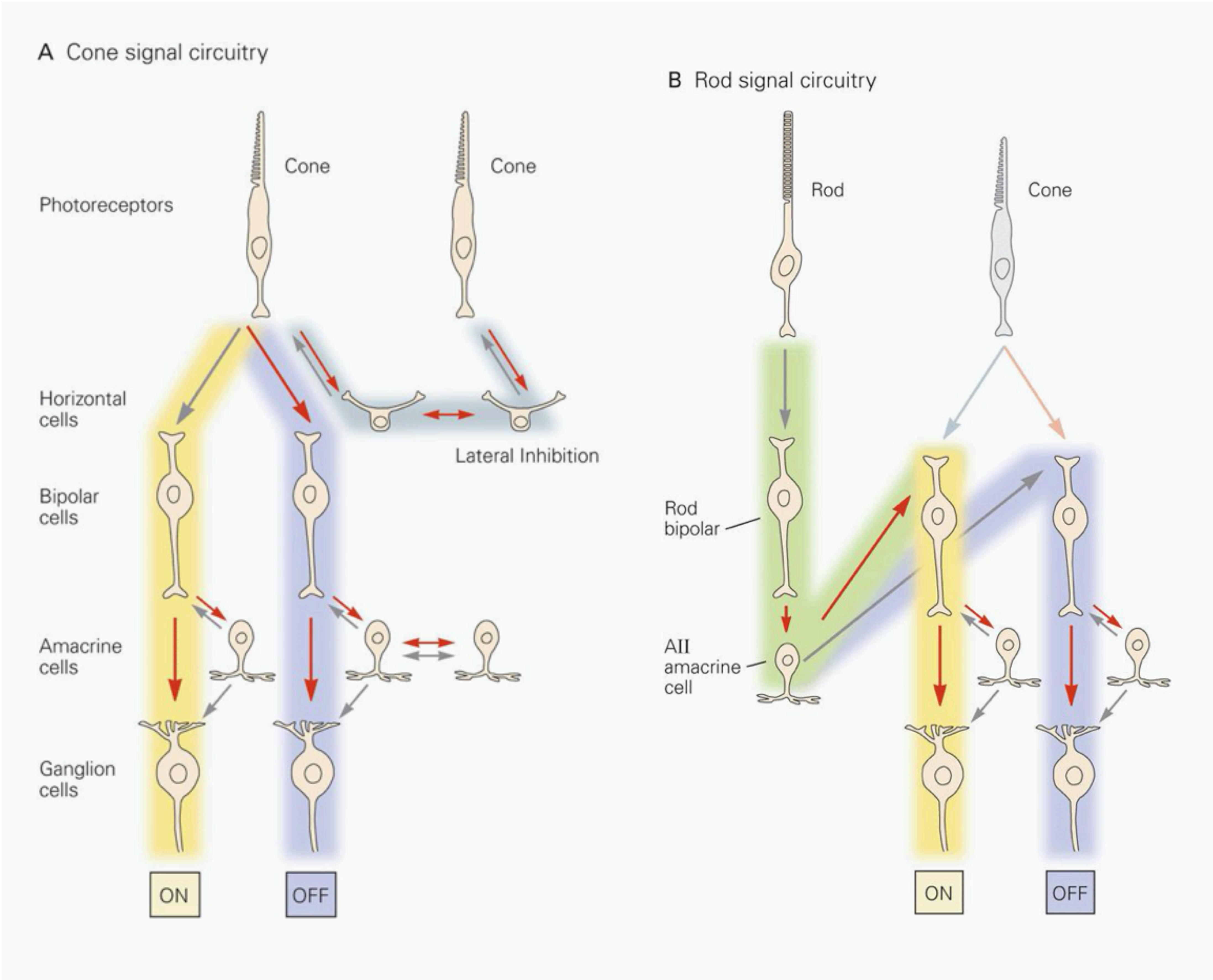
Biology of the Retina

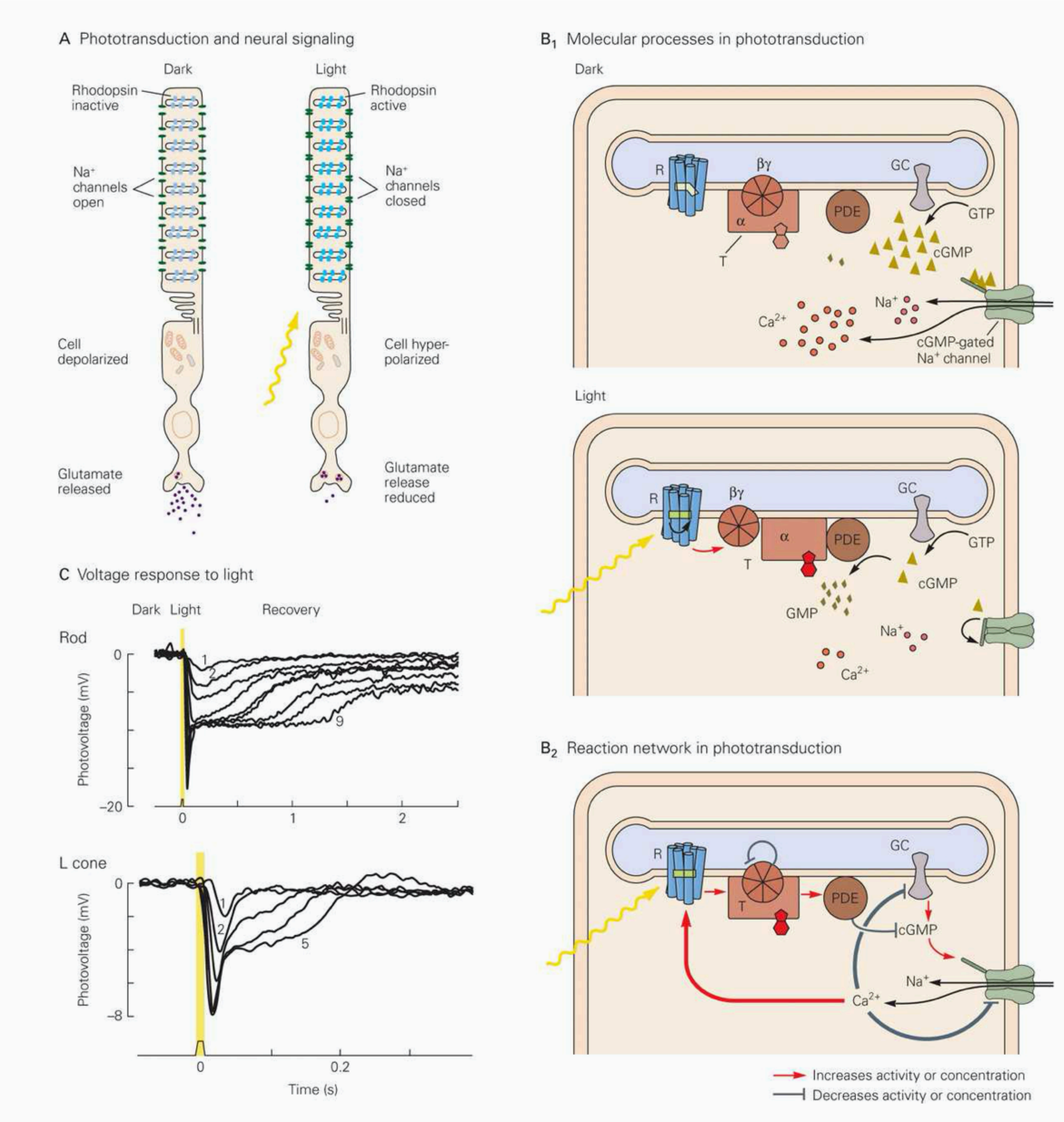
- The structure and cell types of the retina
- The neural circuits of the retina
- Phototransduction
- Gap junctions and ribbon synapses
- Bipolar cells
- Center surround receptive field


A Refraction of light onto the retina
B Focusing of light in the fovea
C Packing of photoreceptors in the fovea
Cornea
Retina
Fixation point
Light
Fovea
Lens
Pigment epithelium
Optic disc
Optic nerve
Ganglion cell
Foveola
Light
Bipolar cell
Photoreceptor
Pigment epithelium
Retina
10 µm


Dorsal (parietal) pathway
MT
Ventral (temporal) pathway
V4
Thin stripe
Interstripe
Thick stripe
V2
From interblobs
From blobs
II
III
IVB
IVCα
IVCβ
V
VI
V1
Parvocellular
Magno-cellular
LGN
Midget ganglion cells
Parasol ganglion cells
Retina
Parvo (ventral or temporal) pathway
Magno (dorsal or parietal) pathway
Color
Depth
Direction
Orientation
Complex form
Dorsal (parietal) pathway
MT
V2
LGN
V1
V4
Retina
Ventral (temporal) pathway
V2


A Section of retina
Outer nuclear layer
Outer plexiform layer
Inner nuclear layer
Inner plexiform layer
Ganglion cell layer
B Neurons in the retina
Cone
Rod
Midget bipolar
Rod bipolar
Horizontal
Diffuse bipolar
Amacrine
M ganglion
M ganglion
P ganglion
50 µm

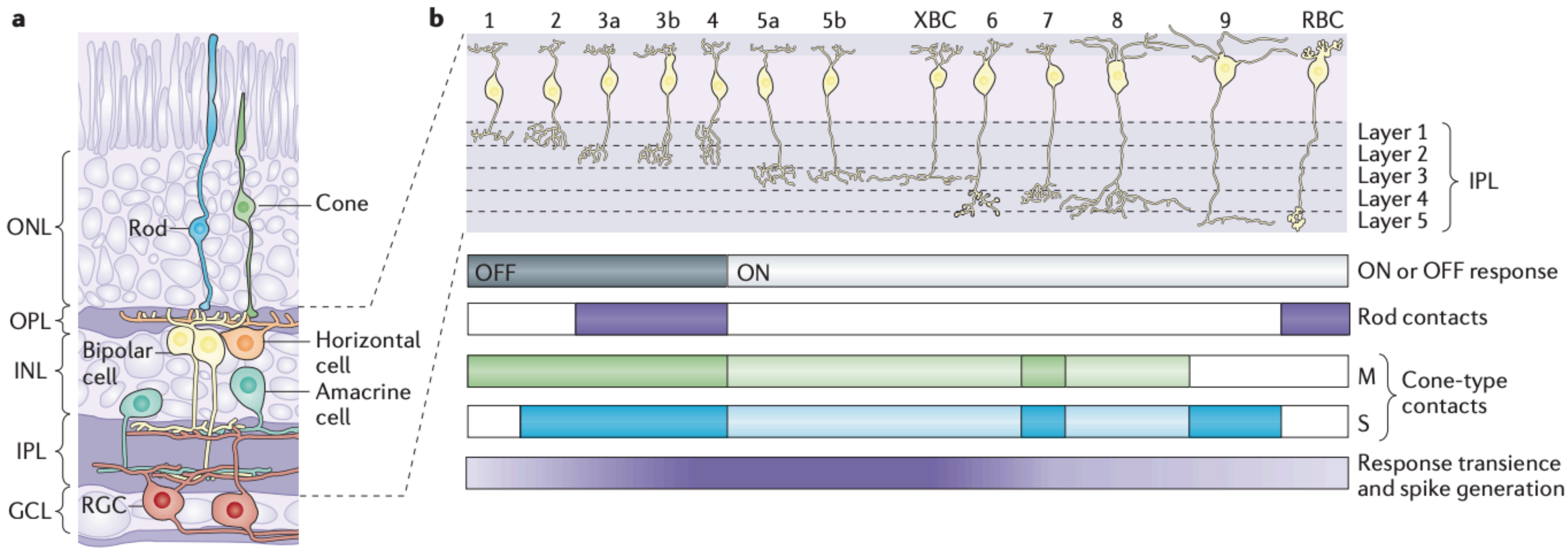

A Cone signal circuitry
Photoreceptors
Cone
Cone
Horizontal cells
Lateral Inhibition
Bipolar cells
Amacrine cells
Ganglion cells
ON
OFF
B Rod signal circuitry
Rod
Cone
Rod bipolar
AII amacrine cell
ON
OFF

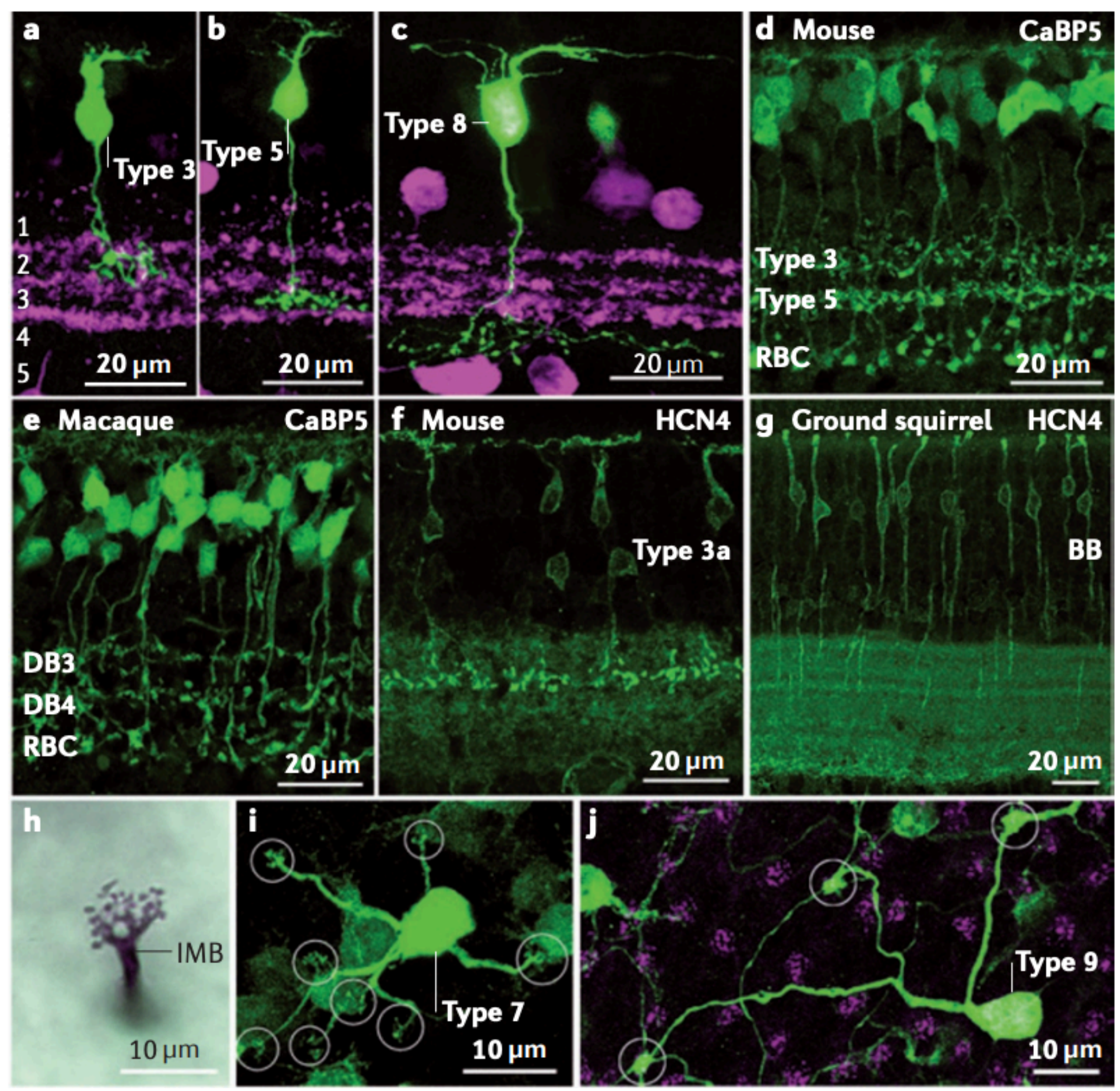


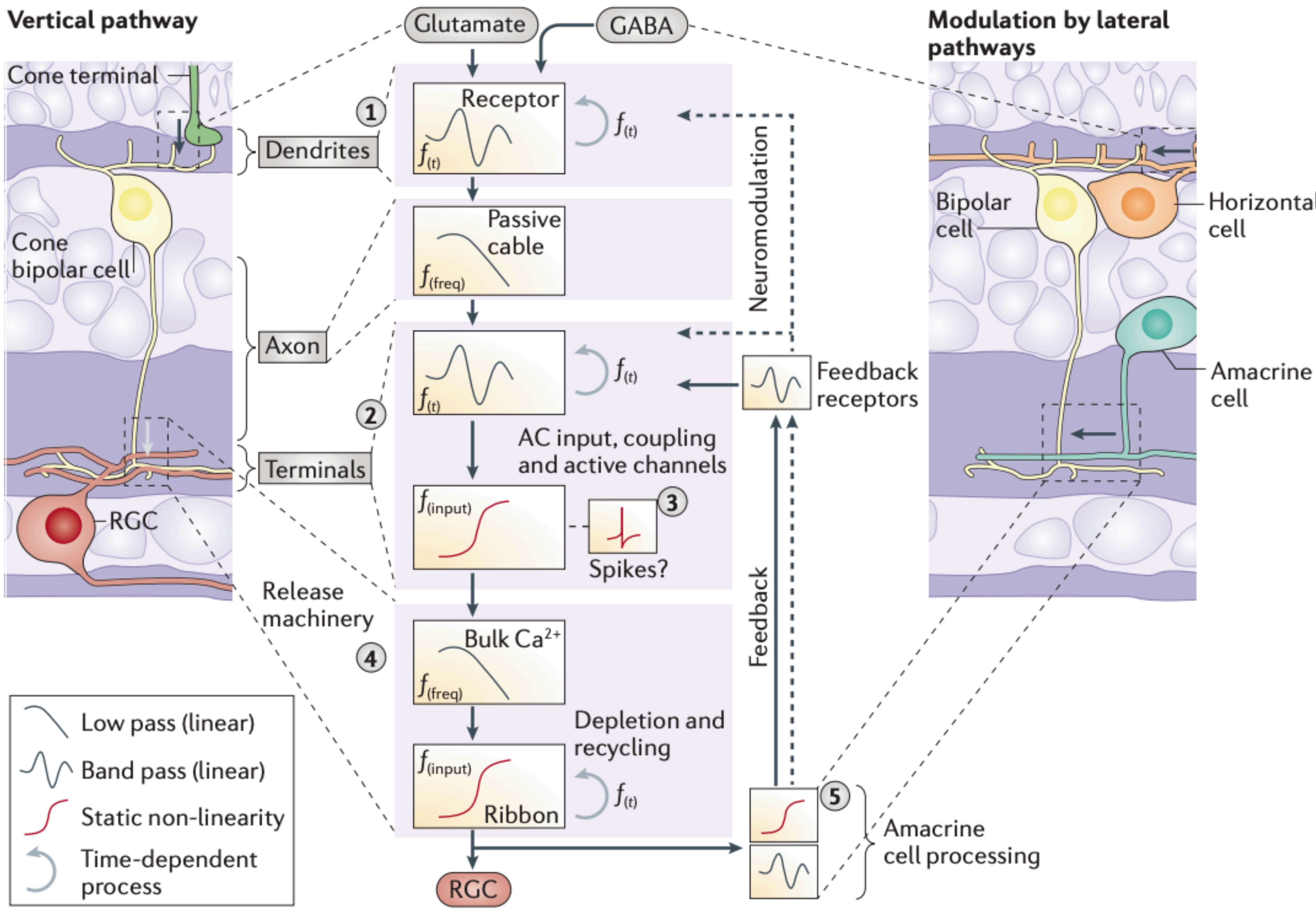
A Phototransduction and neural signaling
Dark
Light
Rhodopsin inactive
Rhodopsin active
Na+ channels open
Na+ channels closed
Cell depolarized
Cell hyper-polarized
Glutamate released
Glutamate release reduced
B1 Molecular processes in phototransduction
Dark
R
βγ
GC
α
PDE
GTP
T
cGMP
Na+
Ca2+
cGMP-gated Na+ channel
Light
GMP
B2 Reaction network in phototransduction
Increases activity or concentration
Decreases activity or concentration
C Voltage response to light
Dark
Light
Recovery
Rod
L cone
Photovoltage (mV)
Time (s)

Three types of synapse

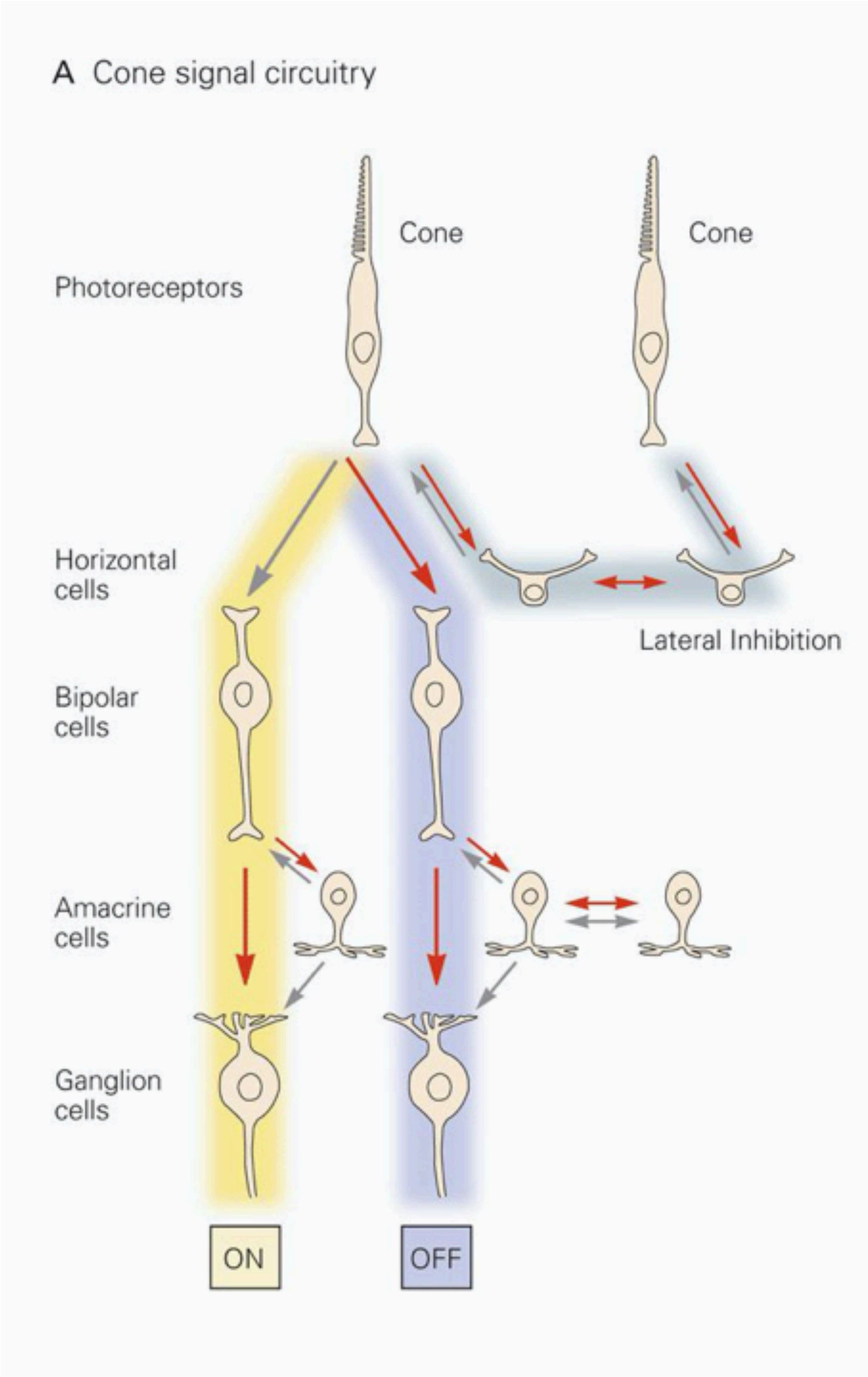
- Chemical synapse, digital output
- Ribbon synapse, analog output
- Gap junction, bidirectional analog output

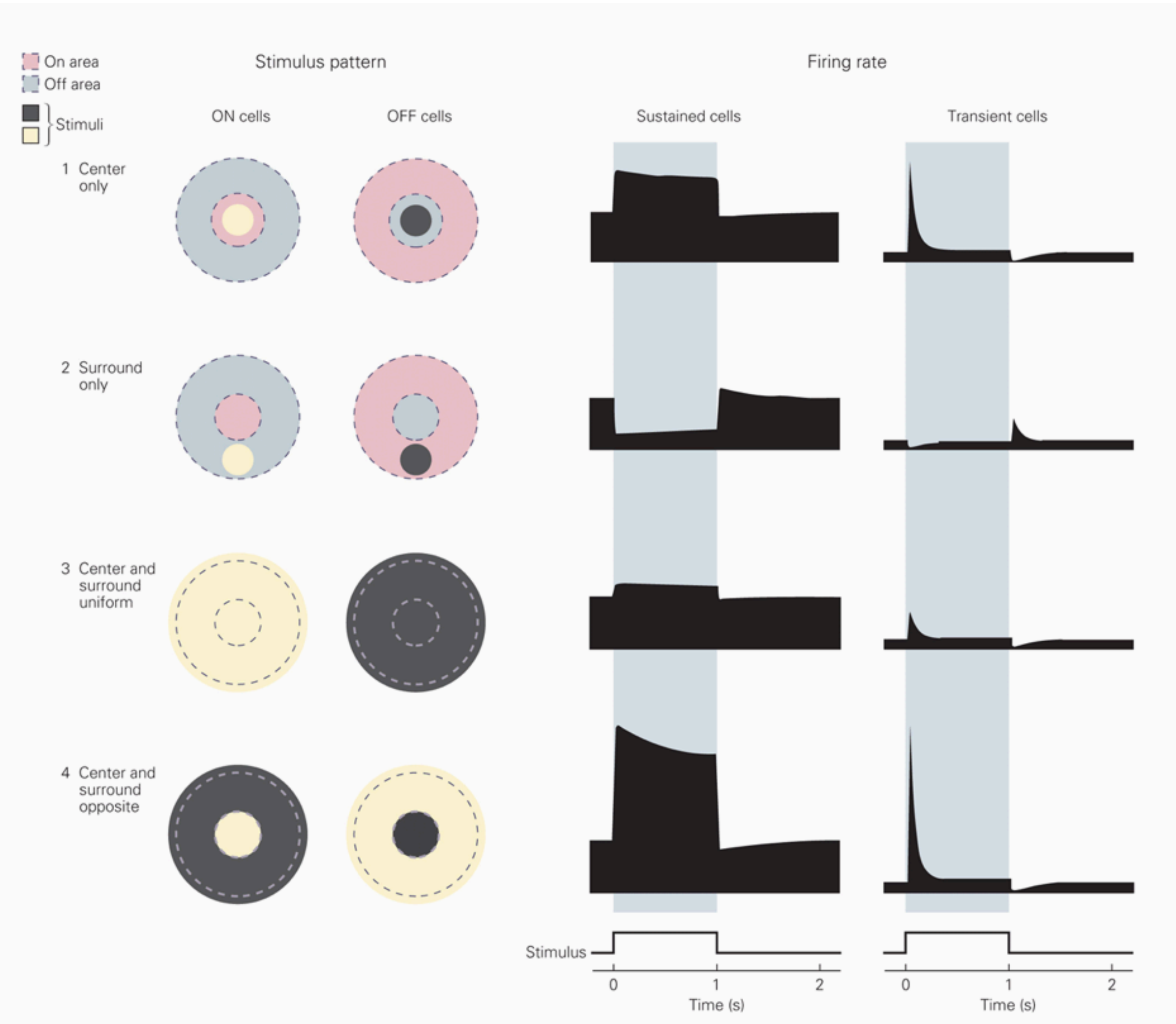

a
ONL
OPL
INL
IPL
GCL
Rod
Cone
Bipolar cell
Horizontal cell
Amacrine cell
RGC
b
1
2
3a
3b
4
5a
5b
XBC
6
7
8
9
RBC
Layer 1
Layer 2
Layer 3
Layer 4
Layer 5
IPL
OFF
ON
ON or OFF response
Rod contacts
M
S
Cone-type contacts
Response transience and spike generation

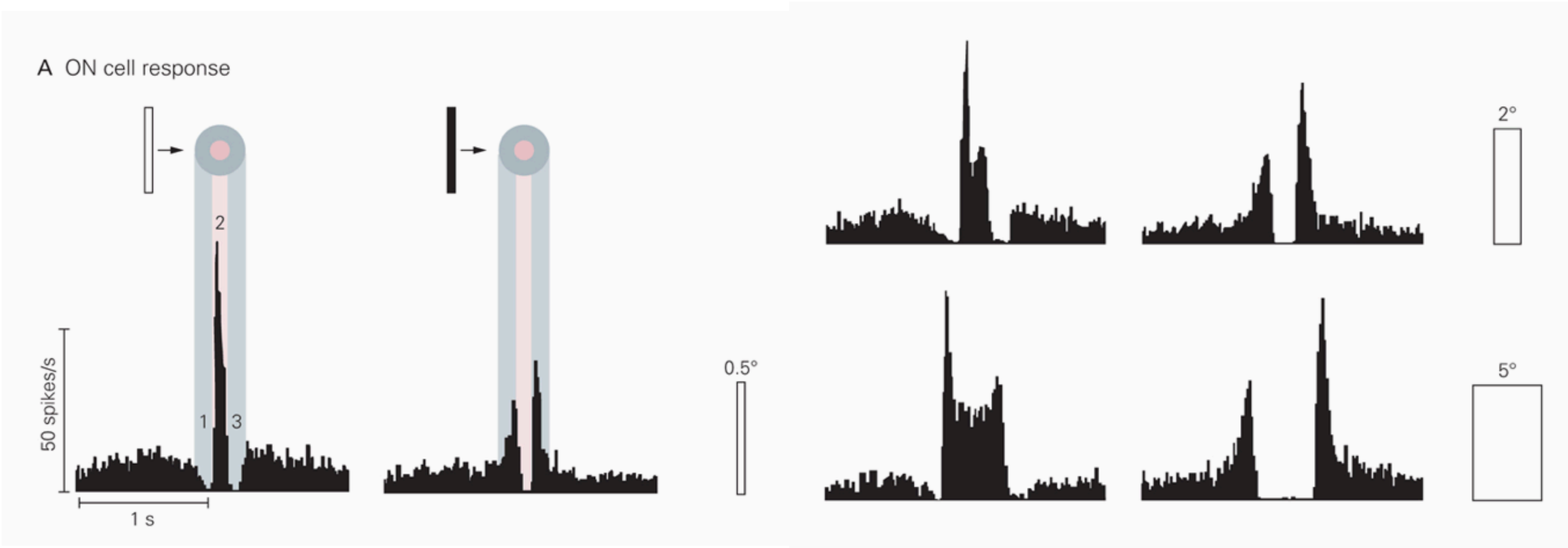

a
Type 3
1
2
3
4
5
20 μm
b
Type 5
20 μm
c
Type 8
20 μm
d Mouse
CaBP5
Type 3
Type 5
RBC
20 μm
e Macaque
CaBP5
DB3
DB4
RBC
20 μm
f Mouse
HCN4
Type 3a
20 μm
g Ground squirrel
HCN4
BB
20 μm
h
IMB
10 μm
i
Type 7
10 μm
j
Type 9
10 μm

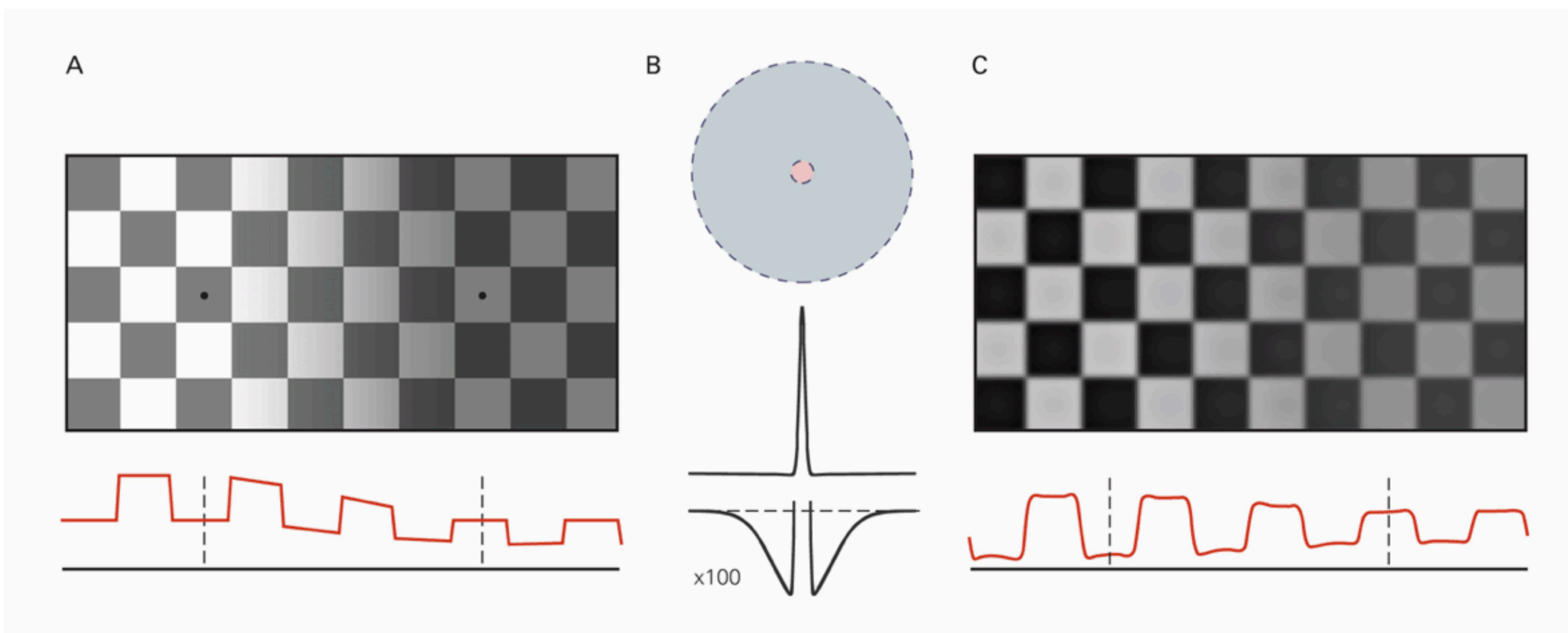


Vertical pathway
Cone terminal
Dendrites
Cone bipolar cell
Axon
Terminals
RGC
Glutamate
GABA
Receptor
$f_{(t)}$
Passive cable
$f_{(freq)}$
AC input, coupling and active channels
$f_{(input)}$
Spikes?
Release machinery
Bulk Ca^{2+}
Depletion and recycling
Ribbon
RGC
Neuromodulation
Feedback receptors
Feedback
Amacrine cell processing
1
2
3
4
5
Modulation by lateral pathways
Bipolar cell
Horizontal cell
Amacrine cell
Low pass (linear)
Band pass (linear)
Static non-linearity
Time-dependent process


A Cone signal circuitry
Cone
Cone
Photoreceptors
Horizontal cells
Lateral Inhibition
Bipolar cells
Amacrine cells
Ganglion cells
ON
OFF

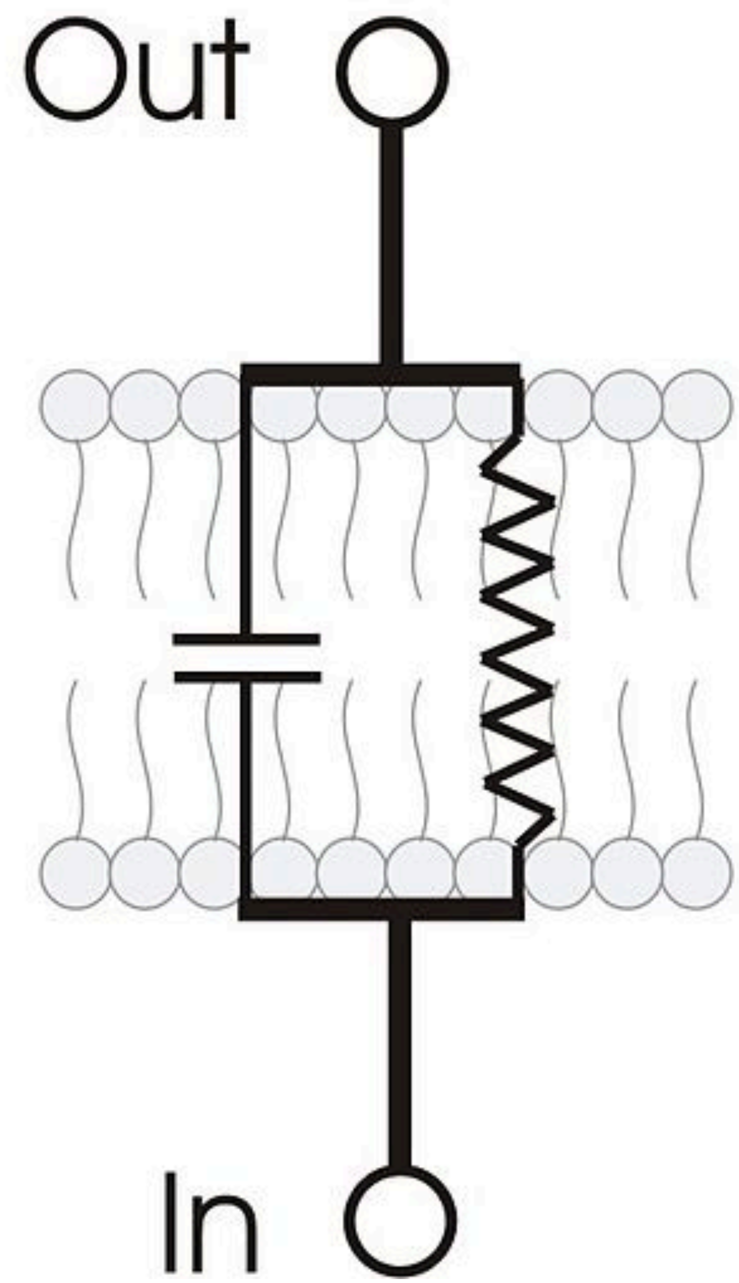

On area
Off area
Stimuli
Stimulus pattern
ON cells
OFF cells
Firing rate
Sustained cells
Transient cells
1 Center only
2 Surround only
3 Center and surround uniform
4 Center and surround opposite
Stimulus
0
1
2
Time (s)
0
1
2
Time (s)


A ON cell response
2
1
3
50 spikes/s
1 s
0.5°
2°
5°


A
B
C
x100

RC Circuit

- We can treat membrane as a RC circuit
- Ex: Hogkin-Huxley model with voltage-depedent resistance
- $CV = Q \Rightarrow C\frac{dV}{dt} = I_C$
- $I_C + I_R - I = 0$
- $C\frac{dV}{dt} + \frac{V - V_m}{R} - I = 0$



Solution

- $\tau \frac{dV}{dt} = -V + V_m + RI(t),\ \tau = RC$
- $V(t) = e^{-\frac{t}{\tau}} \left(V(0) + \int_{s=0}^{t} e^{\frac{s}{\tau}} \frac{V_m + RI(s)}{\tau} ds \right)$
- When the input current is a constant we have
- $V(t) = e^{-\frac{t}{\tau}} V(0) + (1 - e^{-\frac{t}{\tau}})(V_m + RI)$

Convolution View

- In the above equation, set the membrane potential and initial voltage as 0. We get
- $V(t) = \int_0^t e^{\frac{s-t}{\tau}} \frac{RI}{\tau} ds$, which we can rewrite as
- $V(t) = \frac{R}{\tau} e^t * I(t)$

Adaptation of the retina

- Recall that by efficient coding theory, we derive that the connection of the retina should be PCA, which induces center surround receptive field.

- This is working under the assumption that the correlation of natural image statistics are high for nearby pixels and inverse proportion to the square of the distance.

- By subtracting the nearby values, we only transmit what is different from expectation and hence can more efficiently communicate. Ex: edge detection

What happens if the environment is different?

Different adaptation

- Contrast adaptation
- Spatial frequency adaptation
- Orientation adaptation
- Temporal sequence adaptation
- Proposal of the underlying mechanism: Short term learning with vesicles and desensitization with AMPA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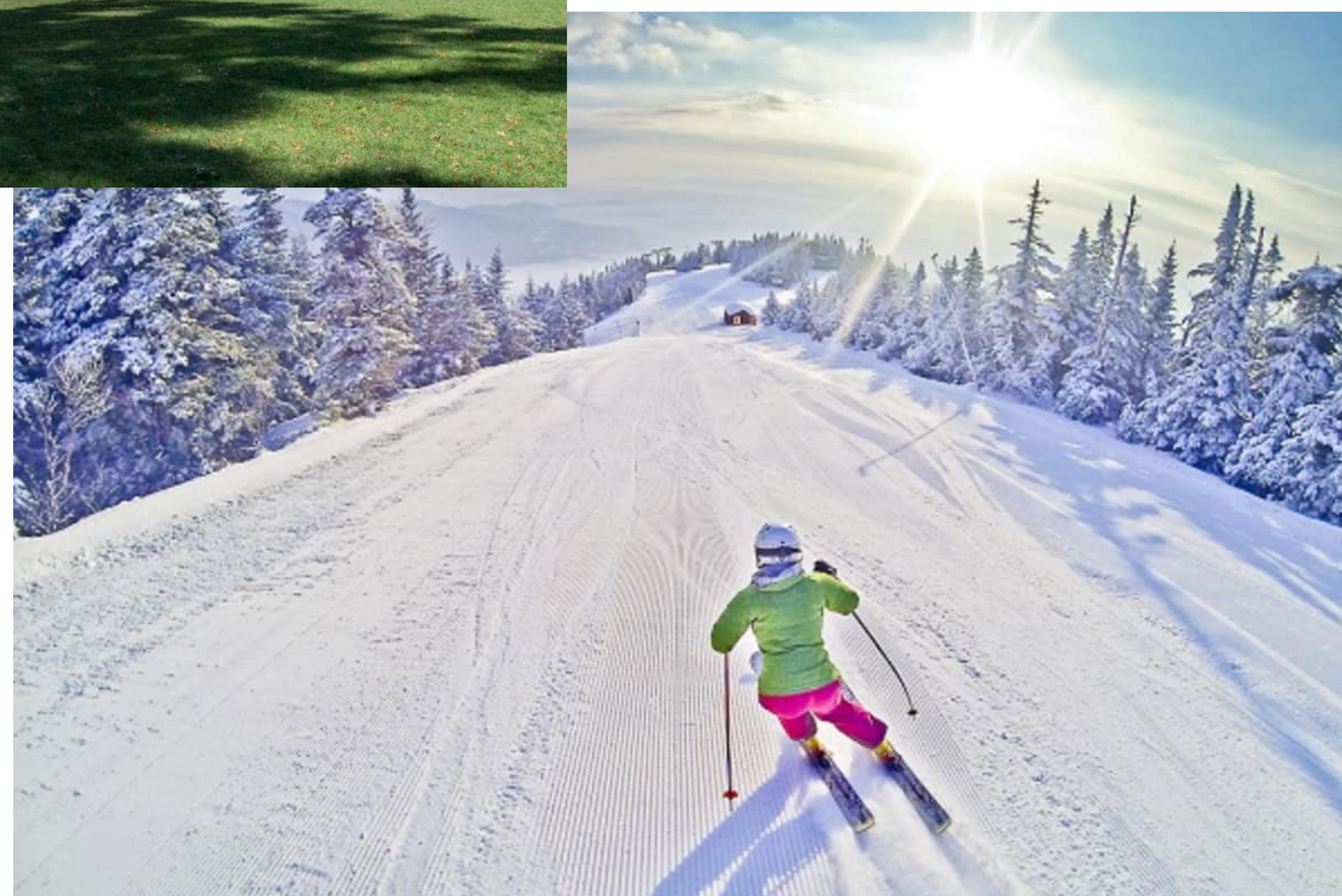
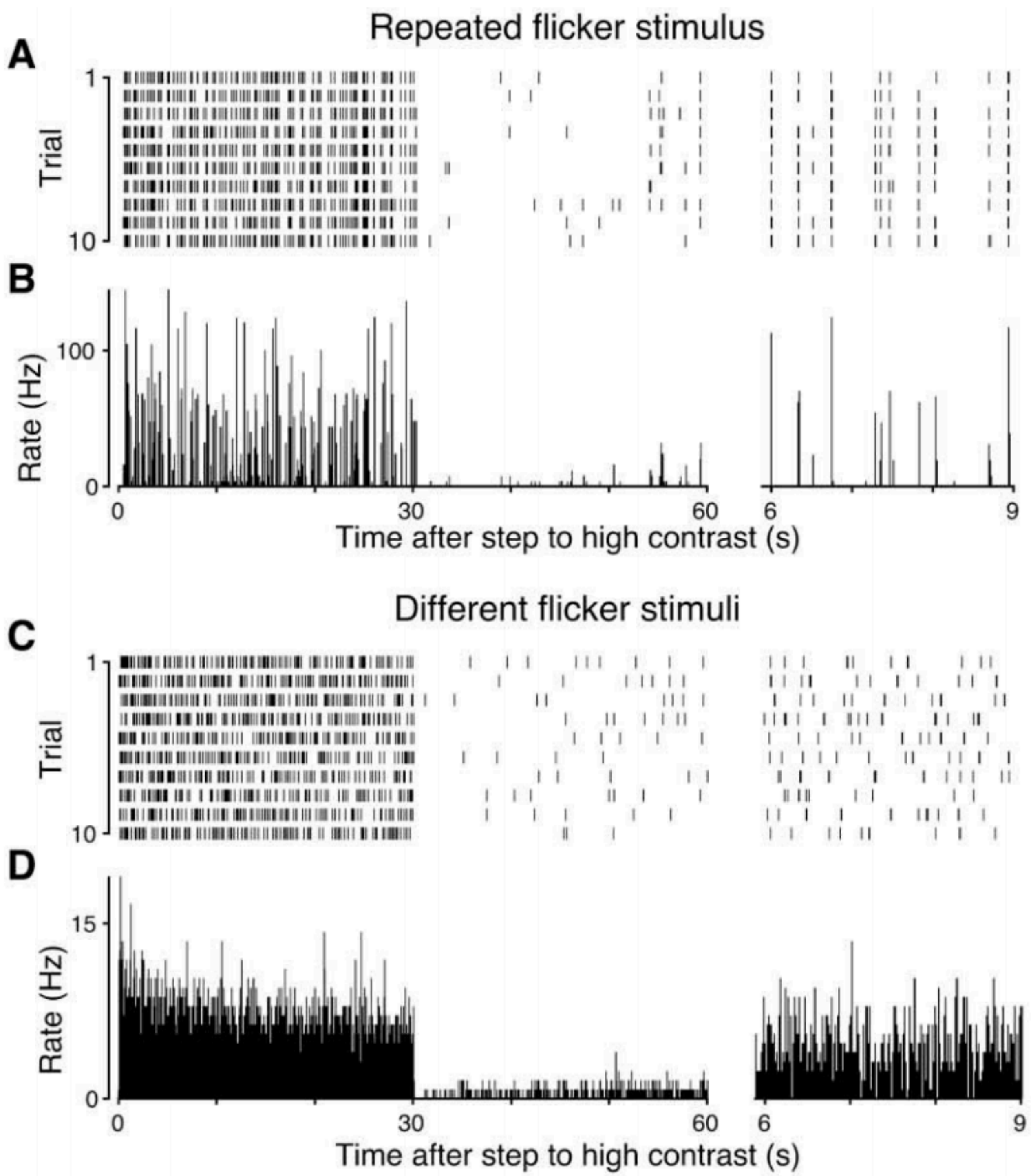
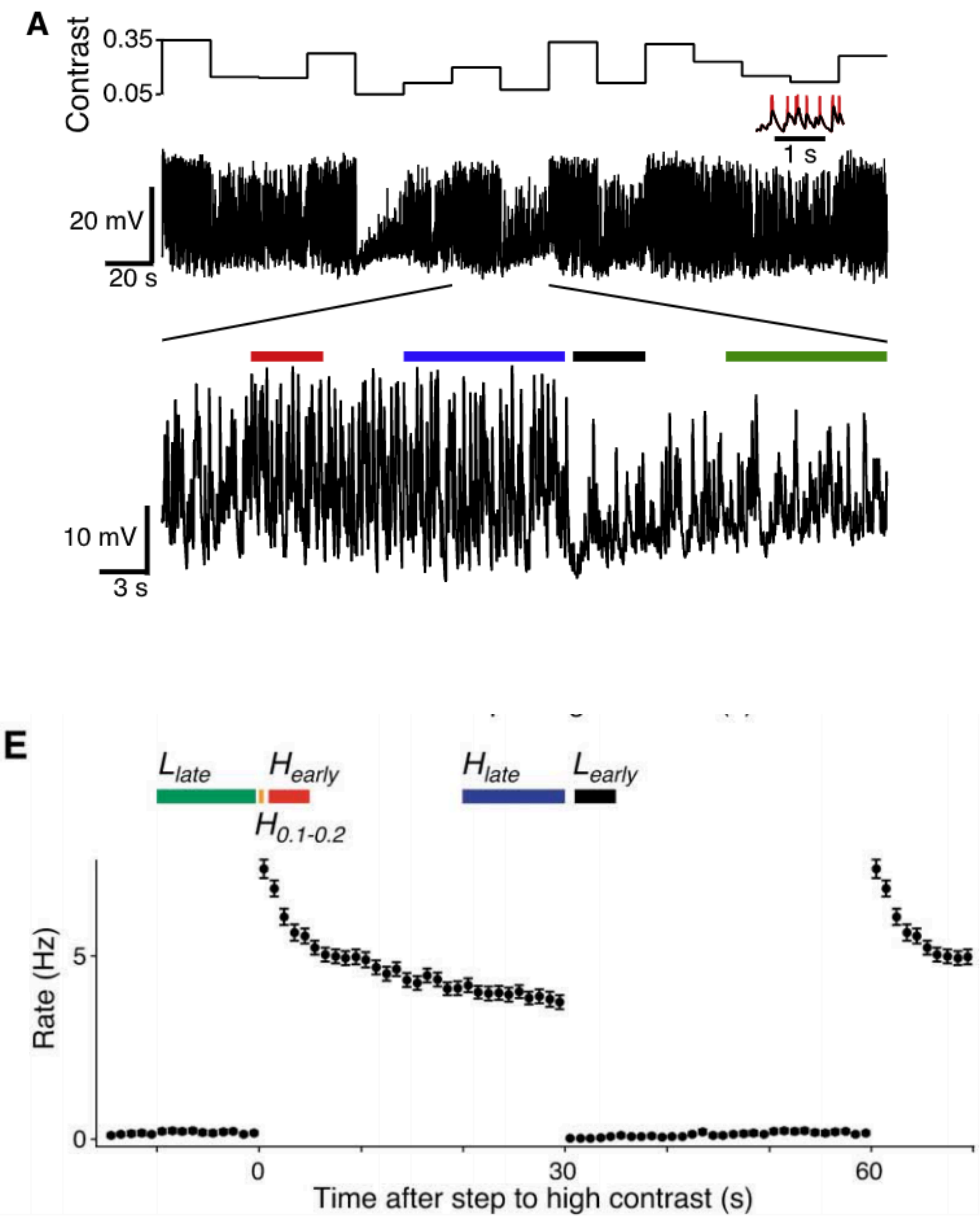


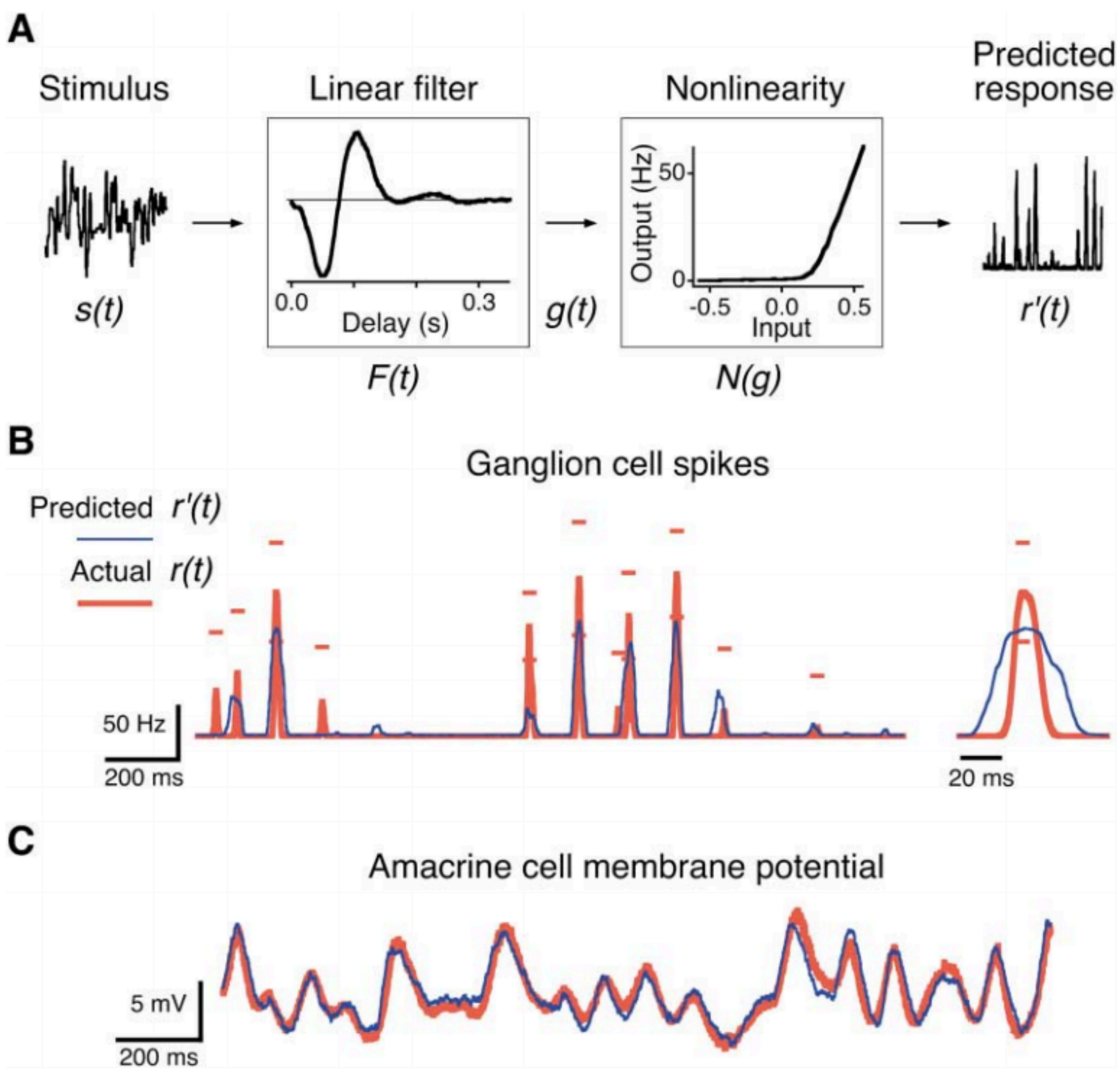
Contrast adaptation

- When you are walking in MIT, because the scene contains wide range of illumination, the retina decreases the sensitivity to cover the whole range. Therefore it is much harder to notice a random person.

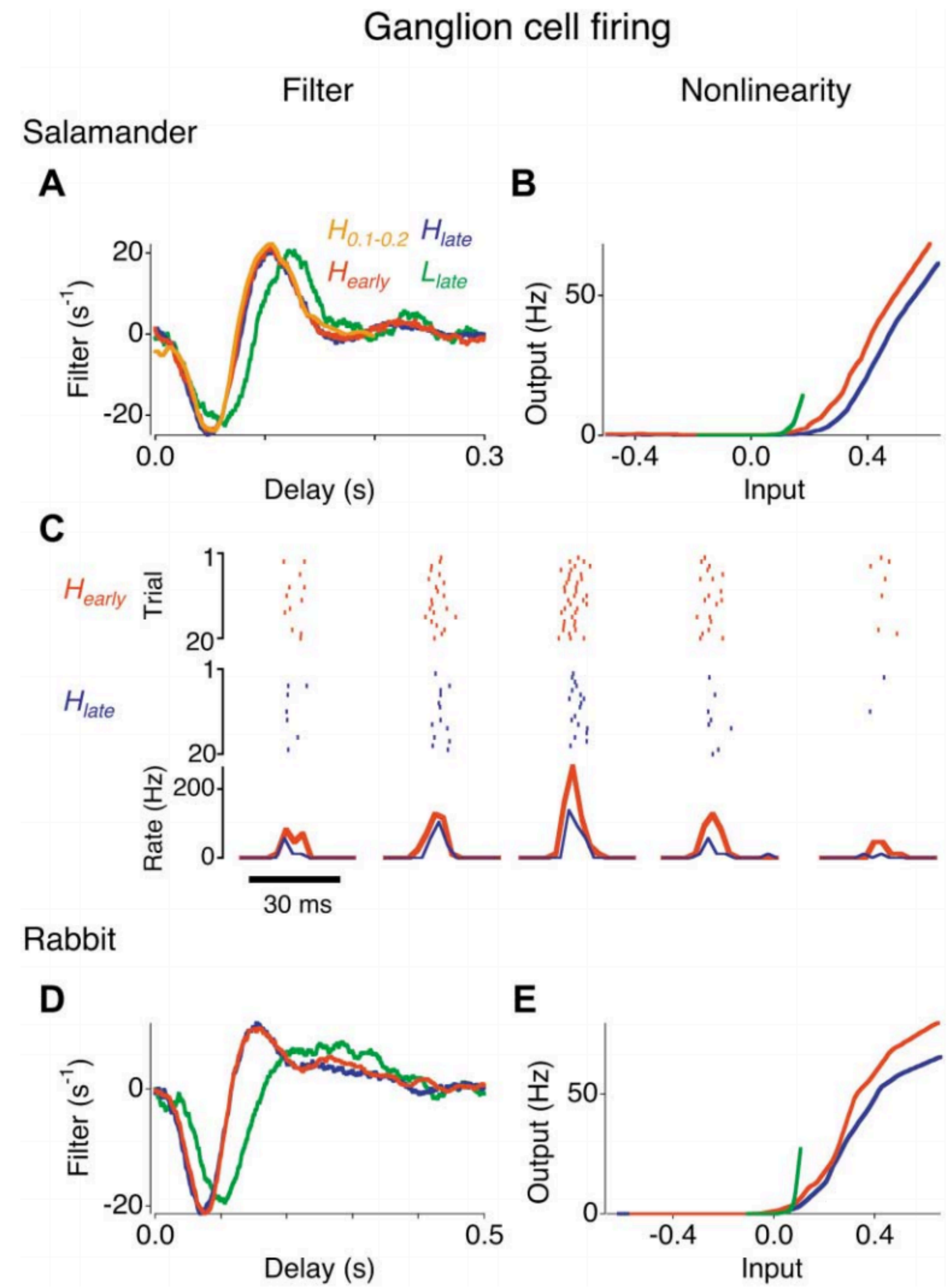
- When you are skiing in Killington, because the scene has very low contrast, the retina increases its sensitivity to distinguish different shades of the white. Therefore it is much easier to notice other skiers.


Repeated flicker stimulus
A
Trial
1
10
B
Rate (Hz)
100
0
0
30
60
6
9
Time after step to high contrast (s)
Different flicker stimuli
C
Trial
1
10
D
Rate (Hz)
15
0
0
30
60
6
9
Time after step to high contrast (s)

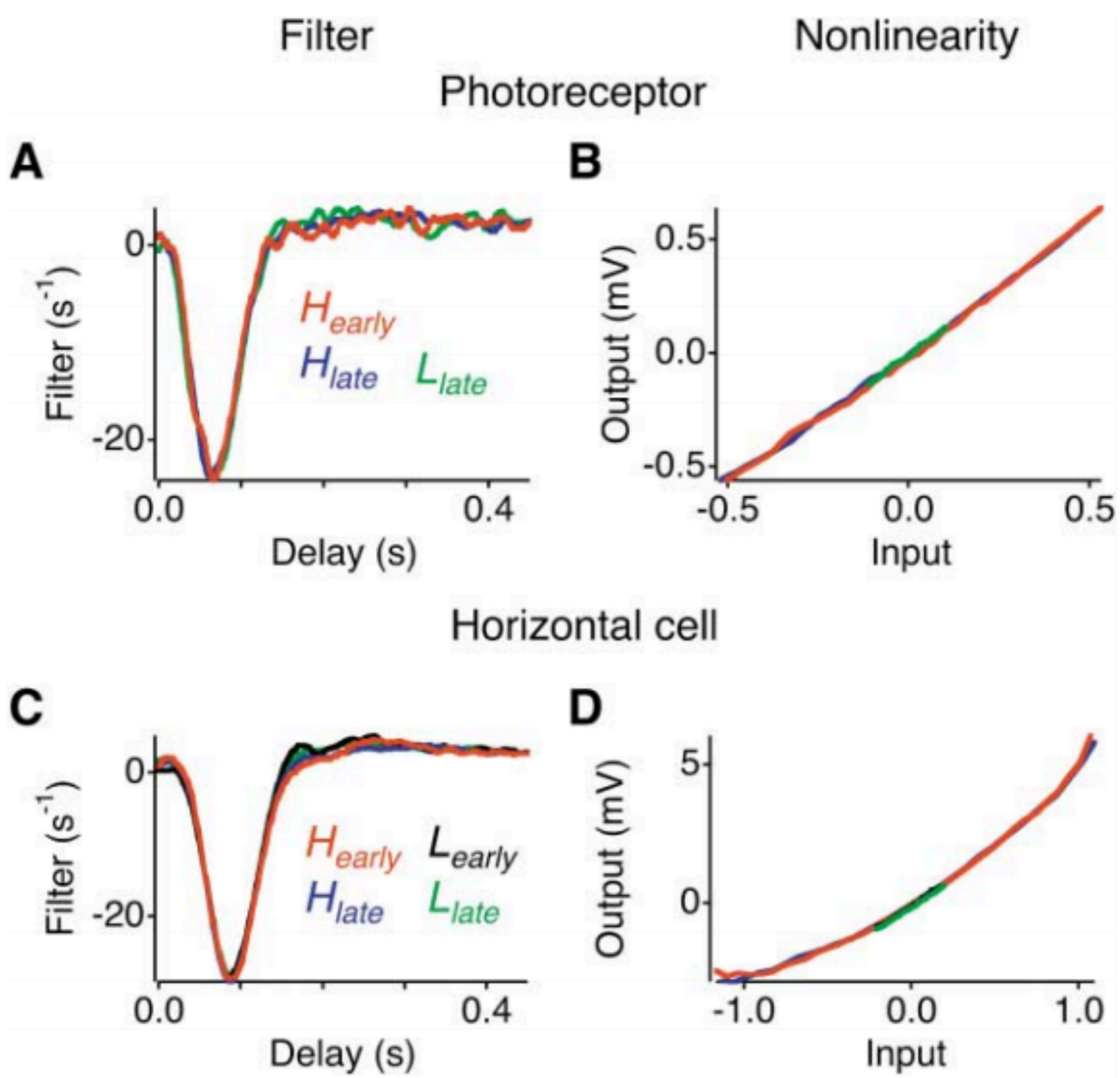

A
Contrast
0.35
0.05
1 s
20 mV
20 s
10 mV
3 s
E
L_late
H_early
H_0.1-0.2
H_late
L_early
Rate (Hz)
5
0
0
30
60
Time after step to high contrast (s)

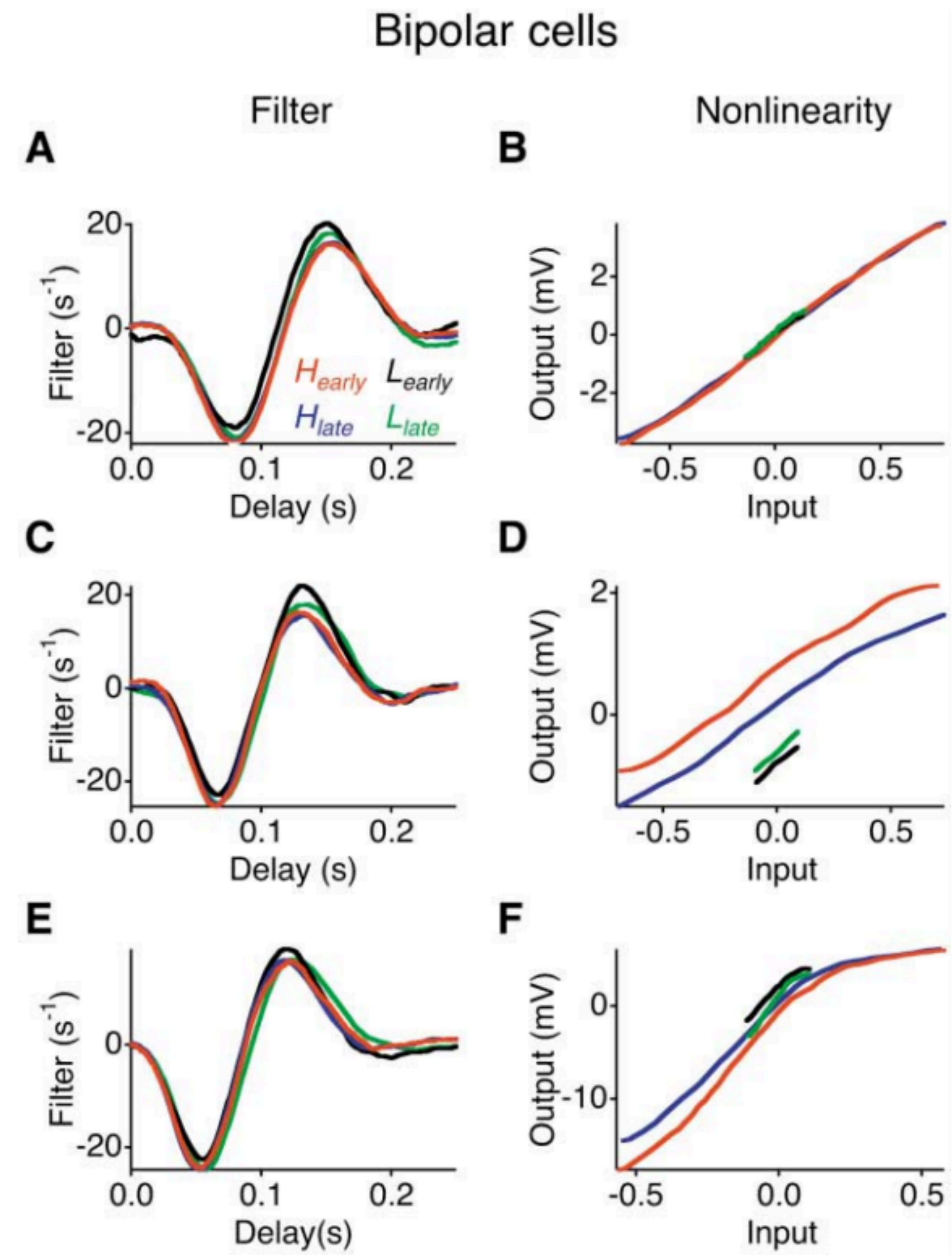


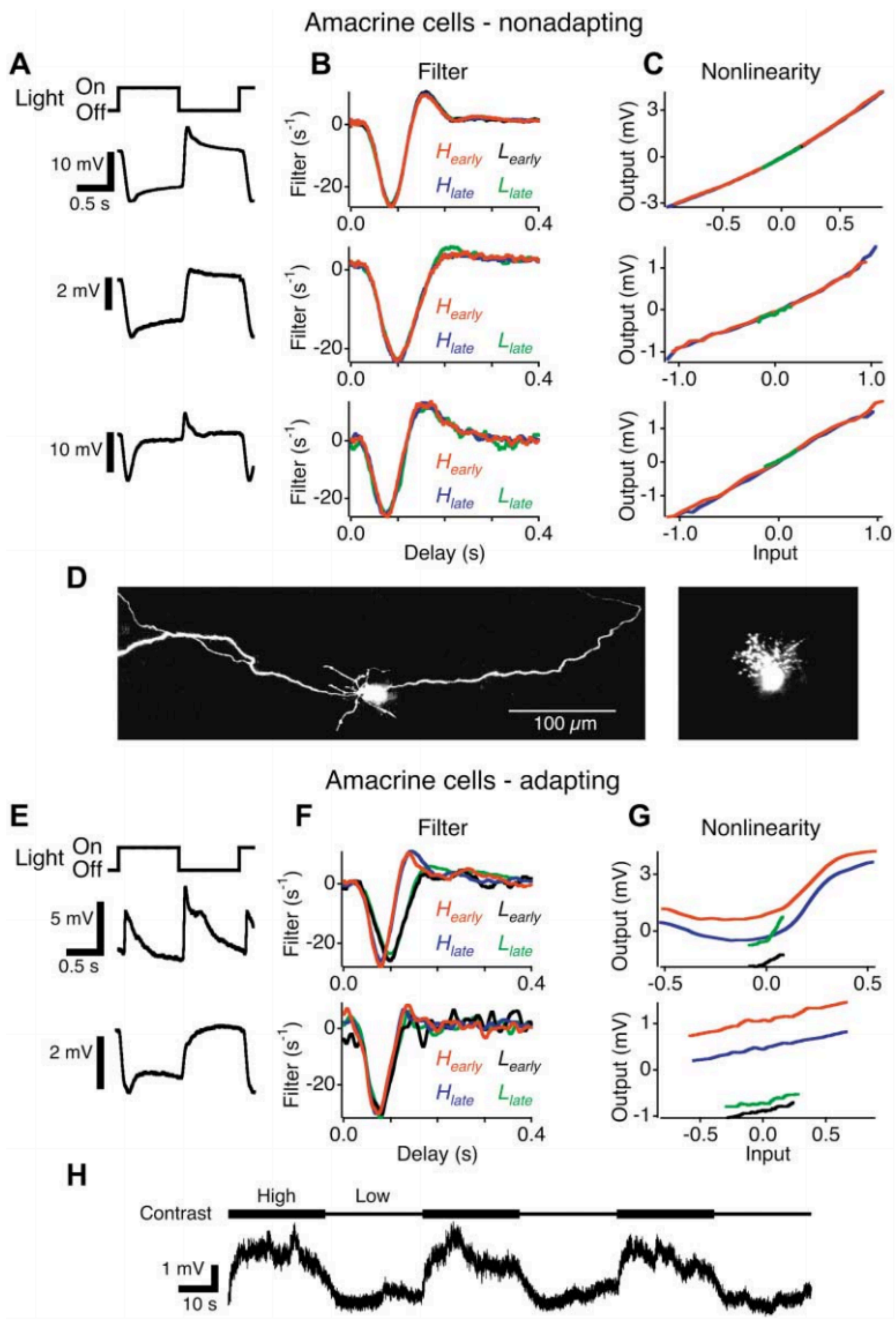
A
Stimulus
Linear filter
Nonlinearity
Predicted response
s(t)
Delay (s)
F(t)
g(t)
Output (Hz)
Input
N(g)
r′(t)
B
Ganglion cell spikes
Predicted r′(t)
Actual r(t)
50 Hz
200 ms
20 ms
C
Amacrine cell membrane potential
5 mV
200 ms

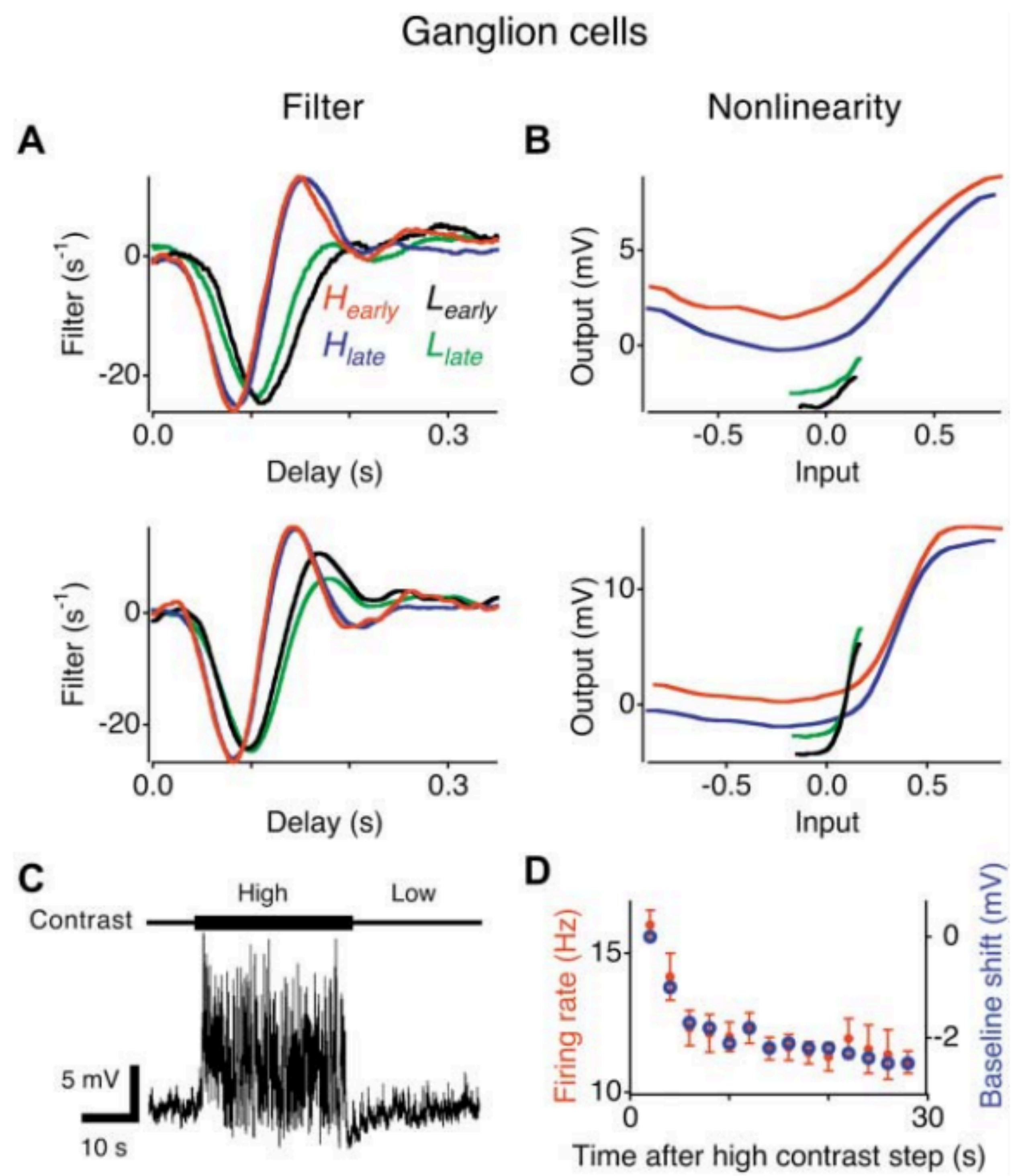


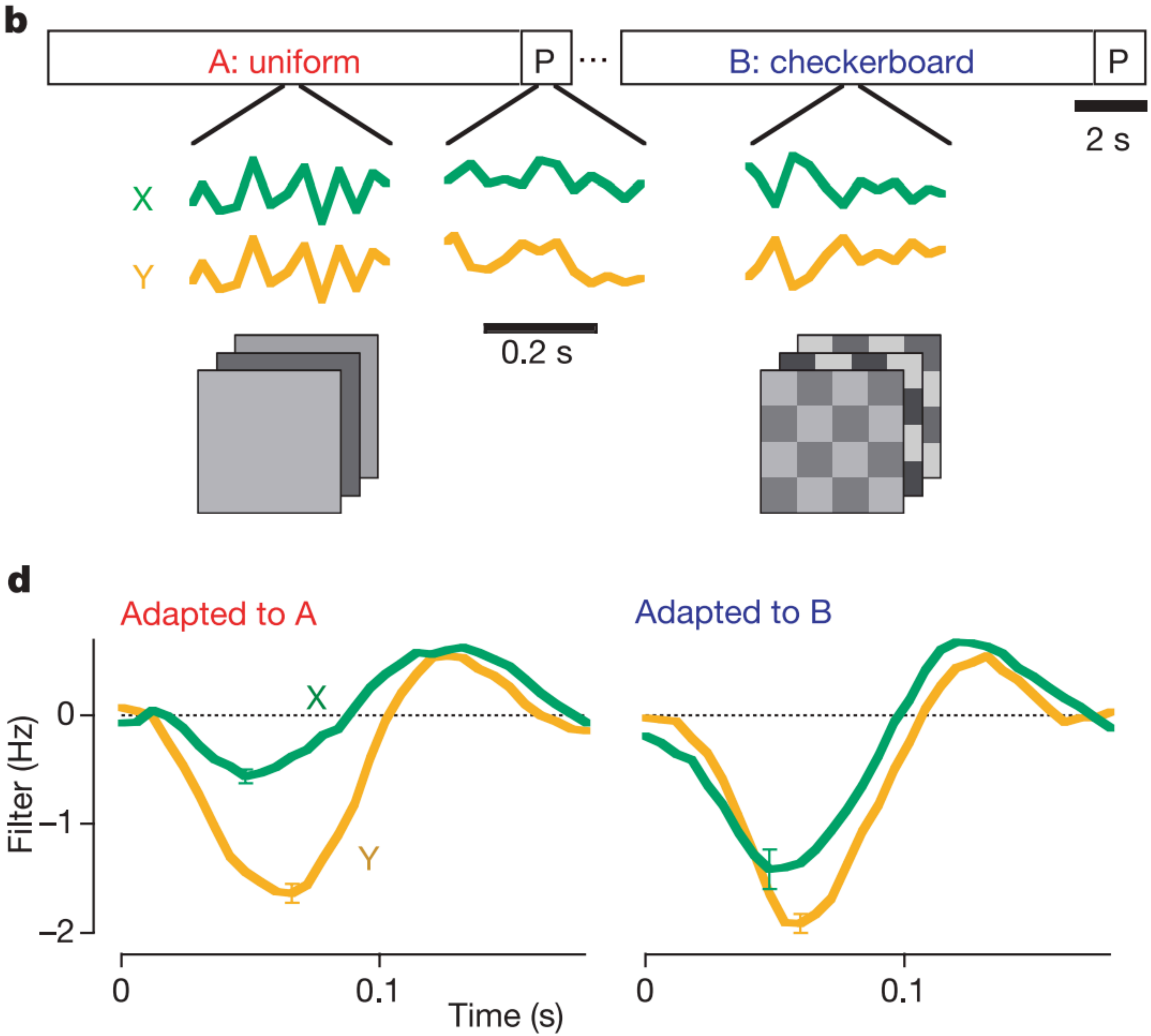
Ganglion cell firing
Filter
Nonlinearity
Salamander
A
B
Filter (s-1)
Delay (s)
Output (Hz)
Input
C
Trial
Rate (Hz)
30 ms
Rabbit
D
E
Filter (s-1)
Delay (s)
Output (Hz)
Input

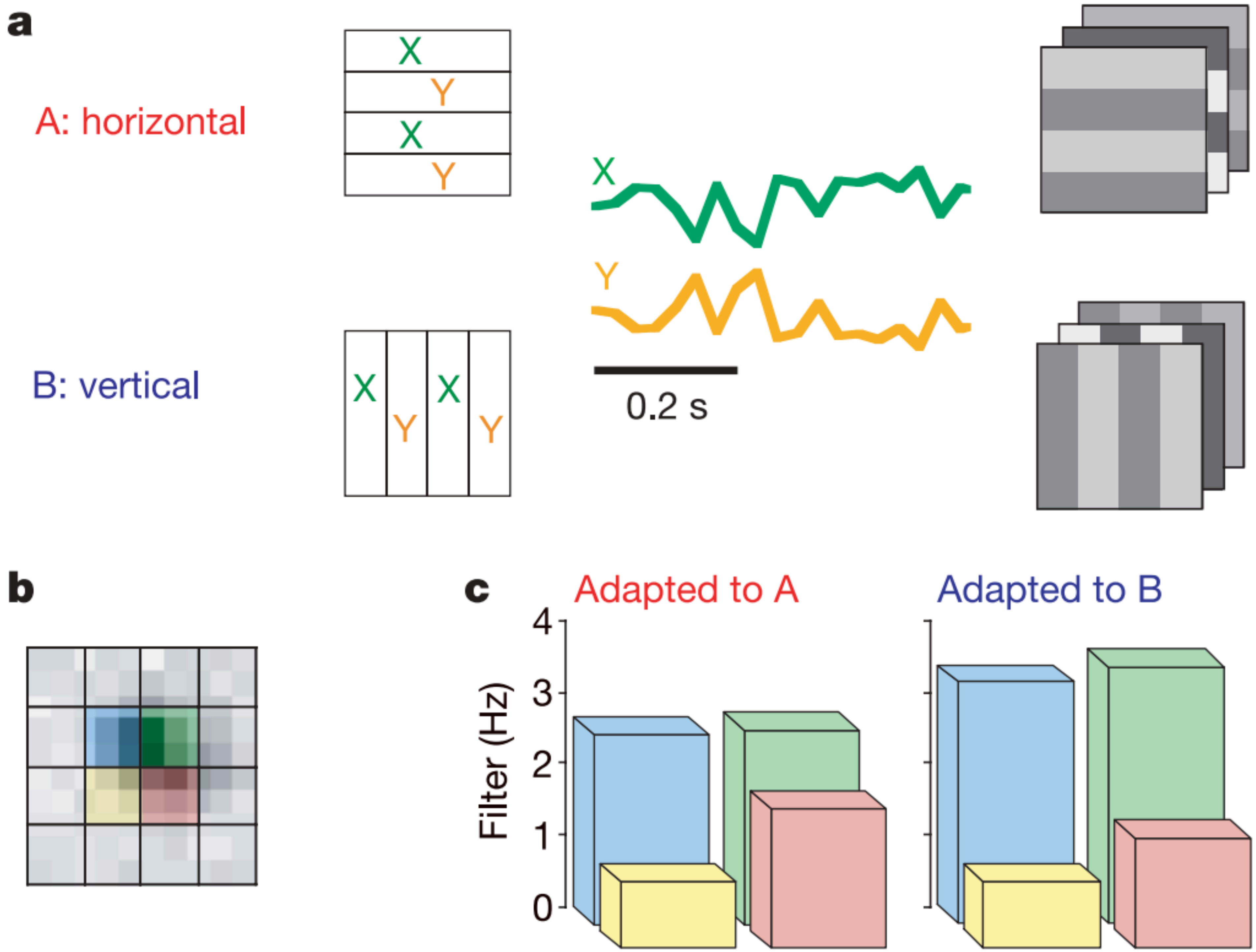

Filter
Nonlinearity
Photoreceptor
A
B
Filter (s-1)
0
-20
0.0
0.4
Delay (s)
Hearly
Hlate
Llate
Output (mV)
0.5
0.0
-0.5
Input
Horizontal cell
C
D
Hearly
Learly
Hlate
Llate
5
-1.0
1.0

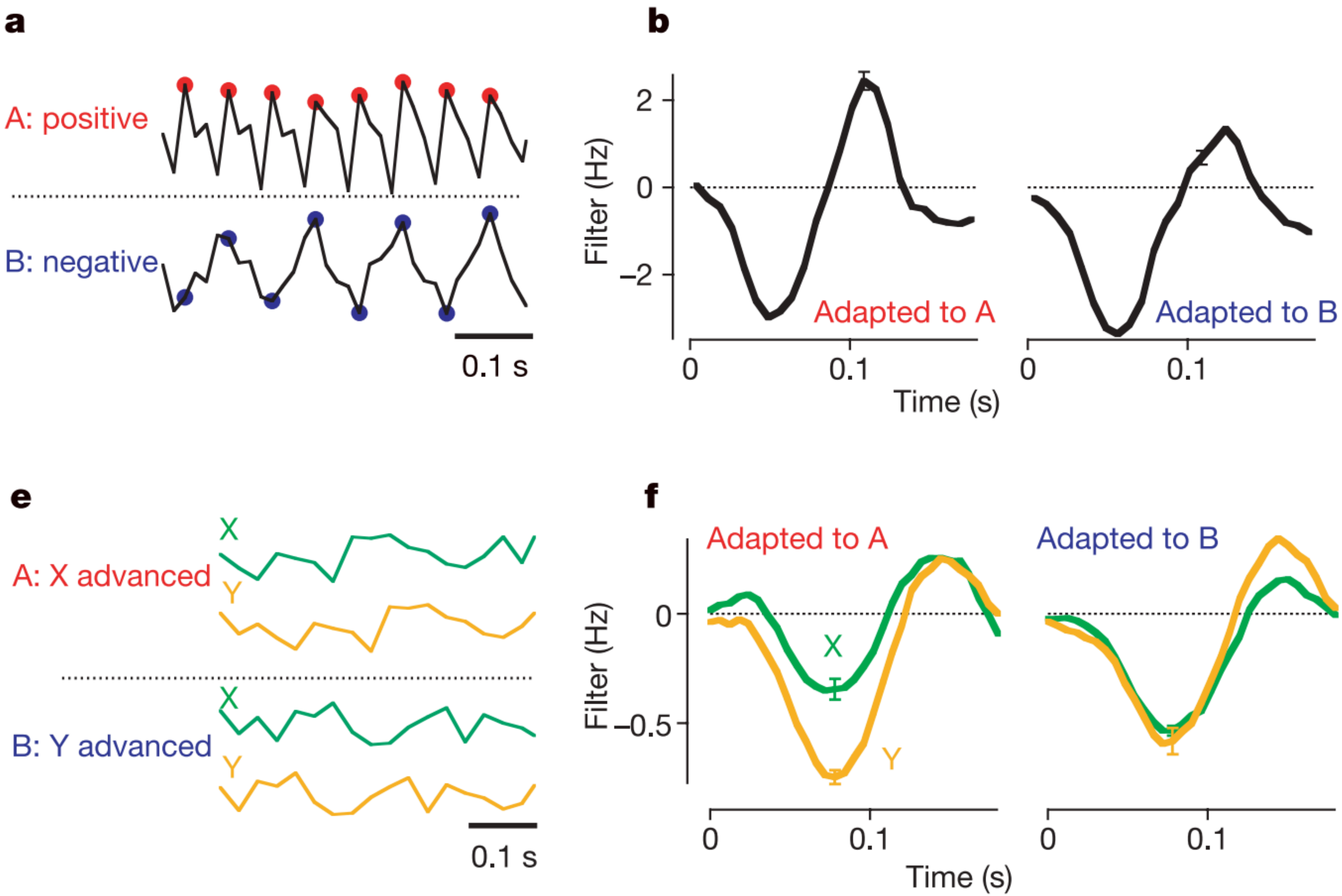

Bipolar cells
Filter
Nonlinearity
A
B
C
D
E
F
Filter (s-1)
20
0
-20
0.0
0.1
0.2
Delay (s)
Delay(s)
Hearly
Learly
Hlate
Llate
Output (mV)
2
-2
-10
-0.5
0.5
Input


Amacrine cells - nonadapting
A
Light On Off
10 mV
0.5 s
2 mV
10 mV
B Filter
Filter (s^{-1})
H_{early} L_{early}
H_{late} L_{late}
Delay (s)
C Nonlinearity
Output (mV)
Input
D
100 µm
Amacrine cells - adapting
E
Light On Off
5 mV
0.5 s
2 mV
F Filter
Filter (s^{-1})
H_{early} L_{early}
H_{late} L_{late}
Delay (s)
G Nonlinearity
Output (mV)
Input
H
Contrast
High
Low
1 mV
10 s


Ganglion cells
Filter
Nonlinearity
A
B
Filter (s^{-1})
H_{early} L_{early}
H_{late} L_{late}
Delay (s)
Output (mV)
Input
C
Contrast
High
Low
5 mV
10 s
D
Firing rate (Hz)
Baseline shift (mV)
Time after high contrast step (s)


b
A: uniform
P
B: checkerboard
P
2 s
X
Y
0.2 s
d
Adapted to A
Adapted to B
X
Y
Filter (Hz)
0
−1
−2
0
0.1
Time (s)
0
0.1

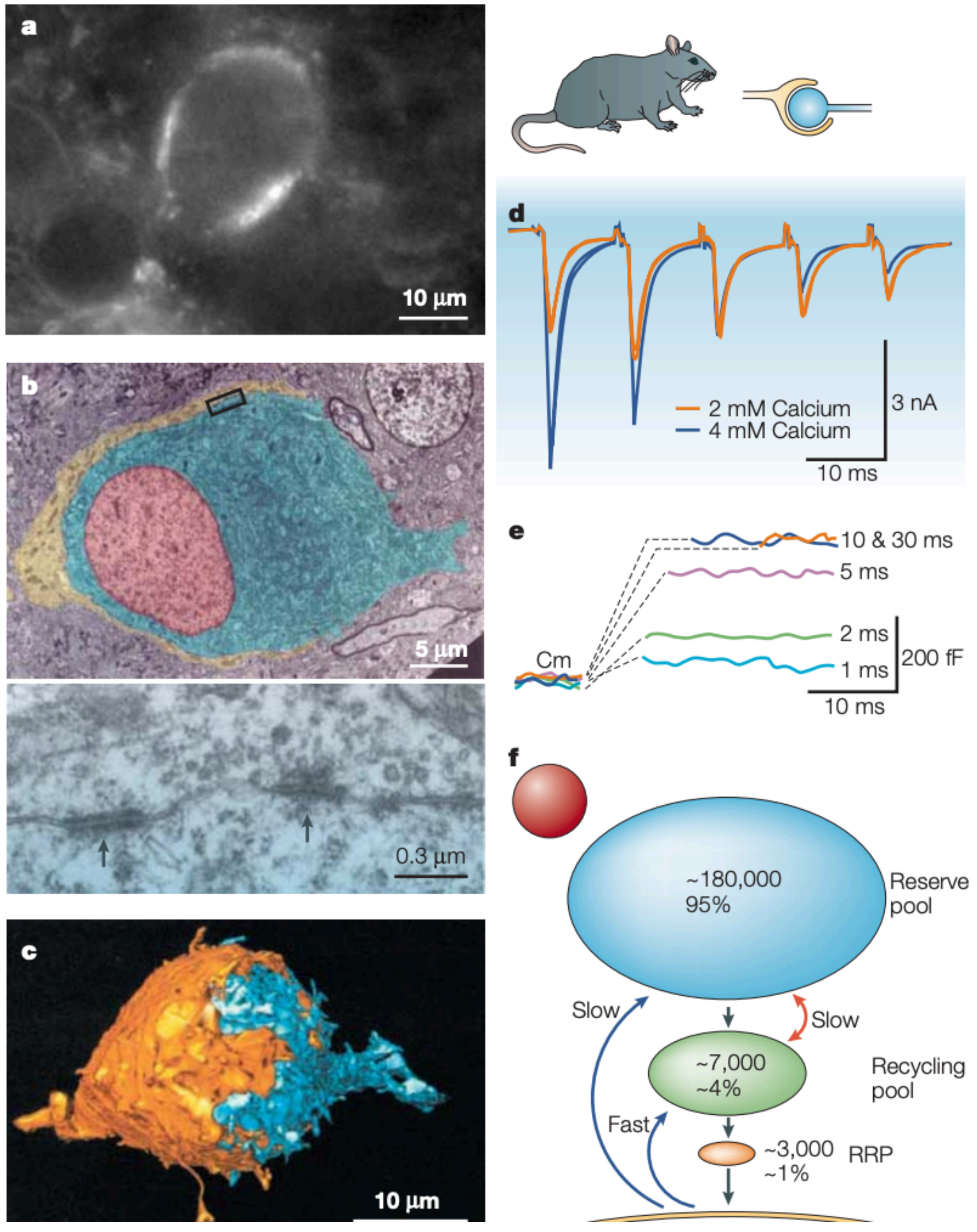

a
A: horizontal
B: vertical
X
Y
X
Y
X
Y
X
Y
X
Y
0.2 s
b
c
Adapted to A
Adapted to B
Filter (Hz)
0
1
2
3
4



a
A: positive
B: negative
0.1 s
b
Filter (Hz)
2
0
−2
Adapted to A
Adapted to B
0
0.1
Time (s)
e
A: X advanced
B: Y advanced
X
Y
0.1 s
f
Adapted to A
Adapted to B
Filter (Hz)
0
−0.5
Time (s)

Vesicle release

- In addition to the long term Hebbian plasticity, there is also short term plasticity which does not change the synaptic weight
- There are three pools: reserve, recycling and ready release pools
- When RRP and recycling pools deplete, the synapse becomes desensitized



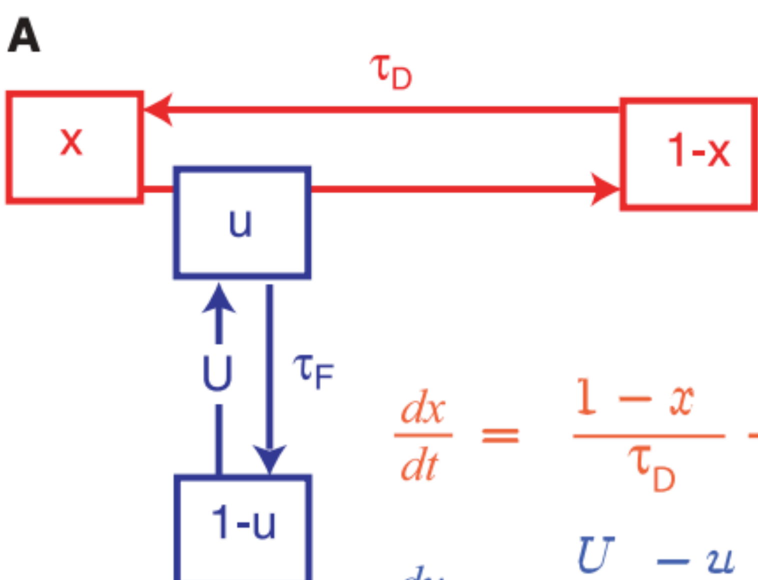
STP vs STD

- There are actually two competing factors here
- The increase release probability from accumulation of calcium
- The depletion of synaptic resources from release of RRP and recycling pools
- So for a synapse with low release probability, we can actually observe short term potentiation

Simplified STL

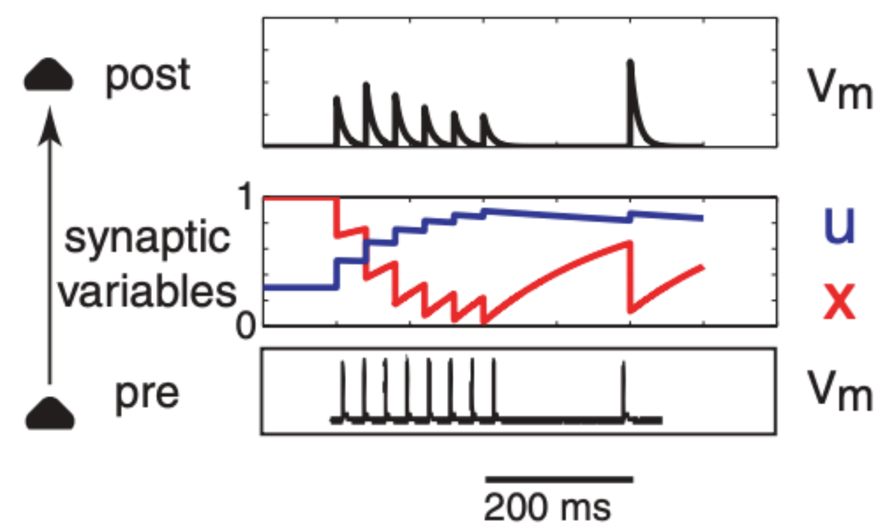
- So instead of modeling three vesicle pools, we can just model synapse resources and synapse utilization

- Usually utilization is a slower constant while resources is a faster constant



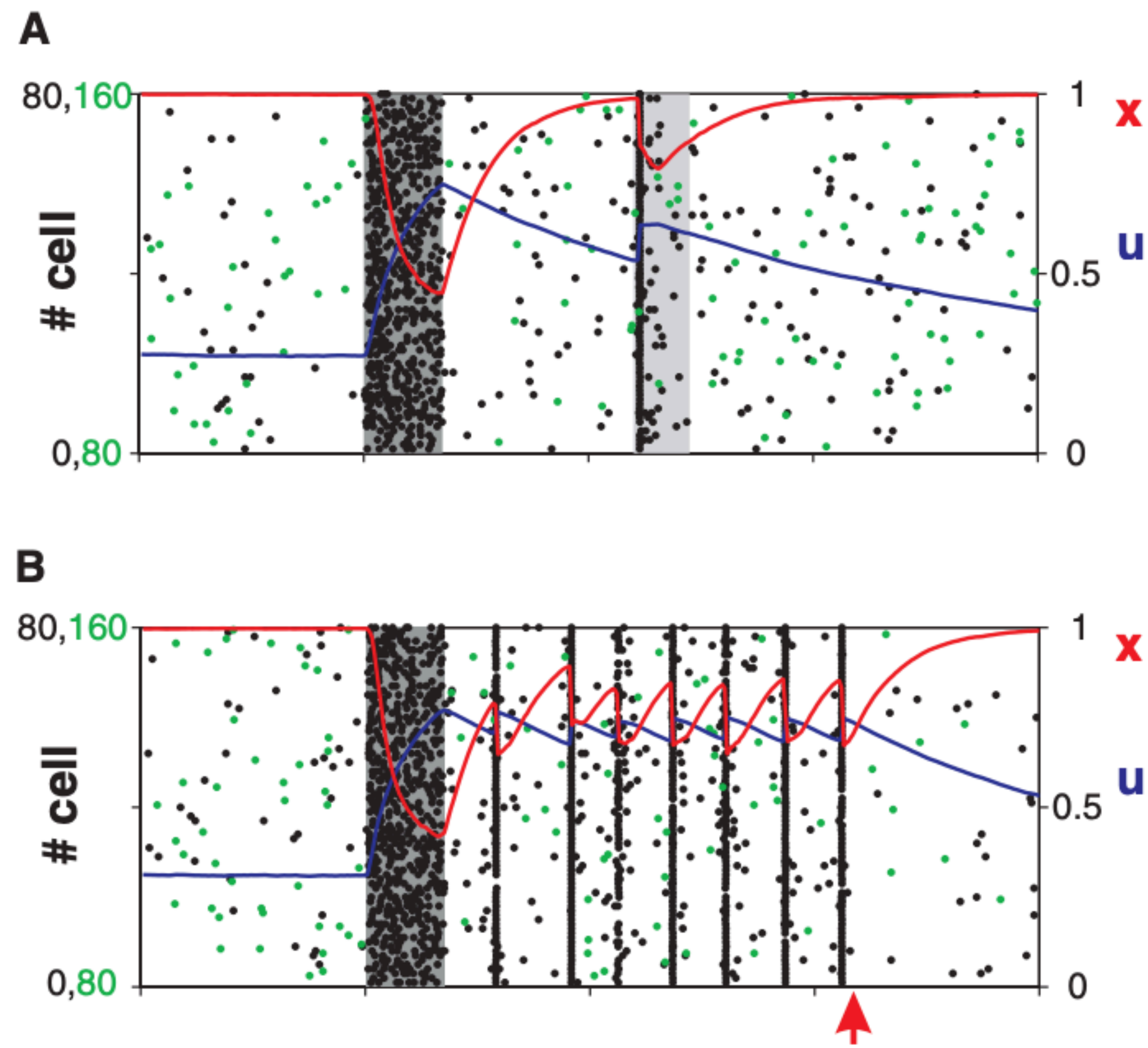
$$\frac{dx}{dt} = \frac{1-x}{\tau_D} - u\ x\ \delta(t-t_{sp})$$

$$\frac{du}{dt} = \frac{U\ -u}{\tau_F} + U\ (1-u\ )\ \delta(t-t_{sp})$$



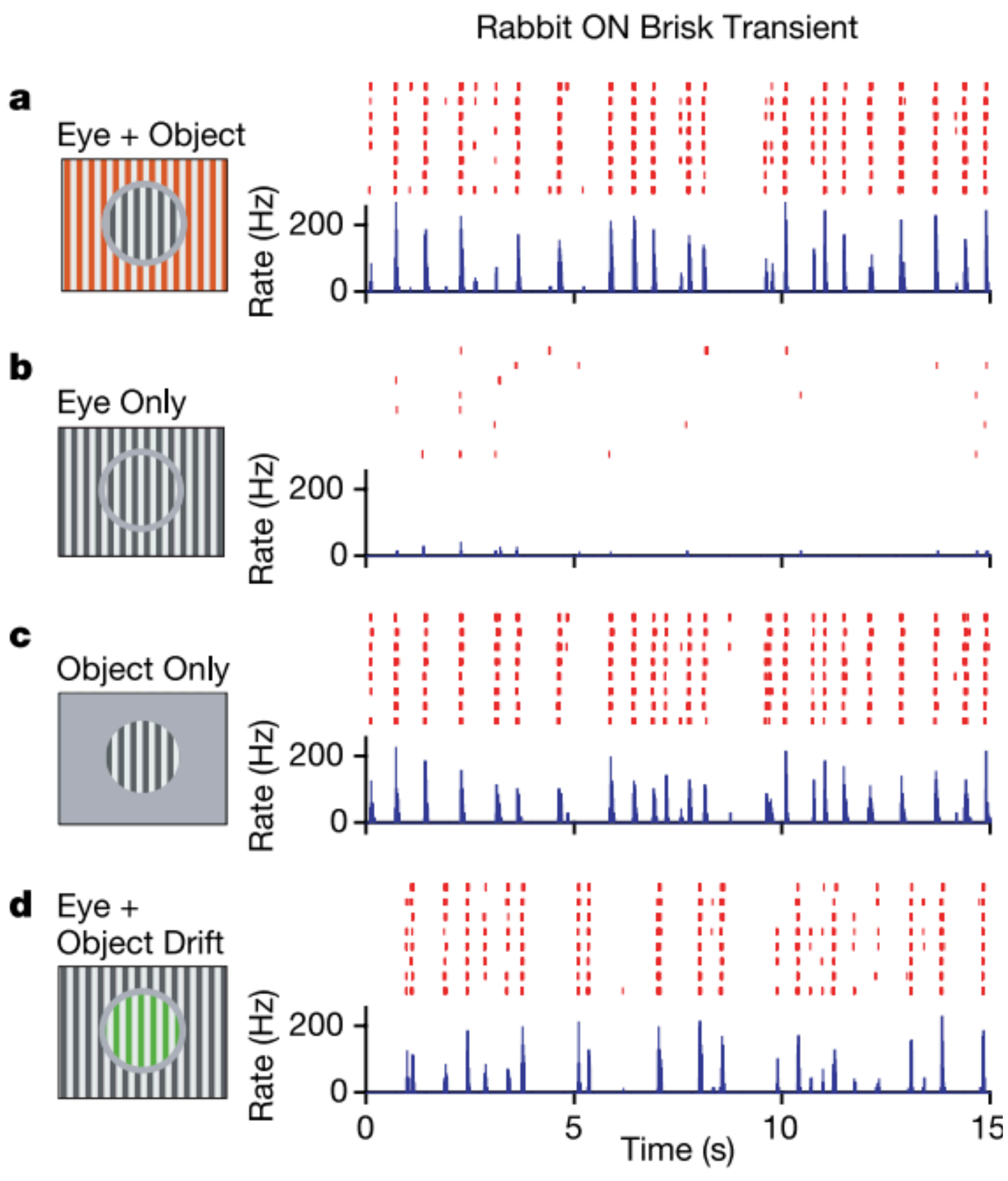
Assembly without LTP

- Even without LTP, we are able to demonstrate cell assembly
- Generally, with STDP along, it is very unstable to form assembly. Only with the aids of STL, STDP can stabilize more robustly
- So maybe STL is the main driving force while LTP is a mechanism to consolidate?

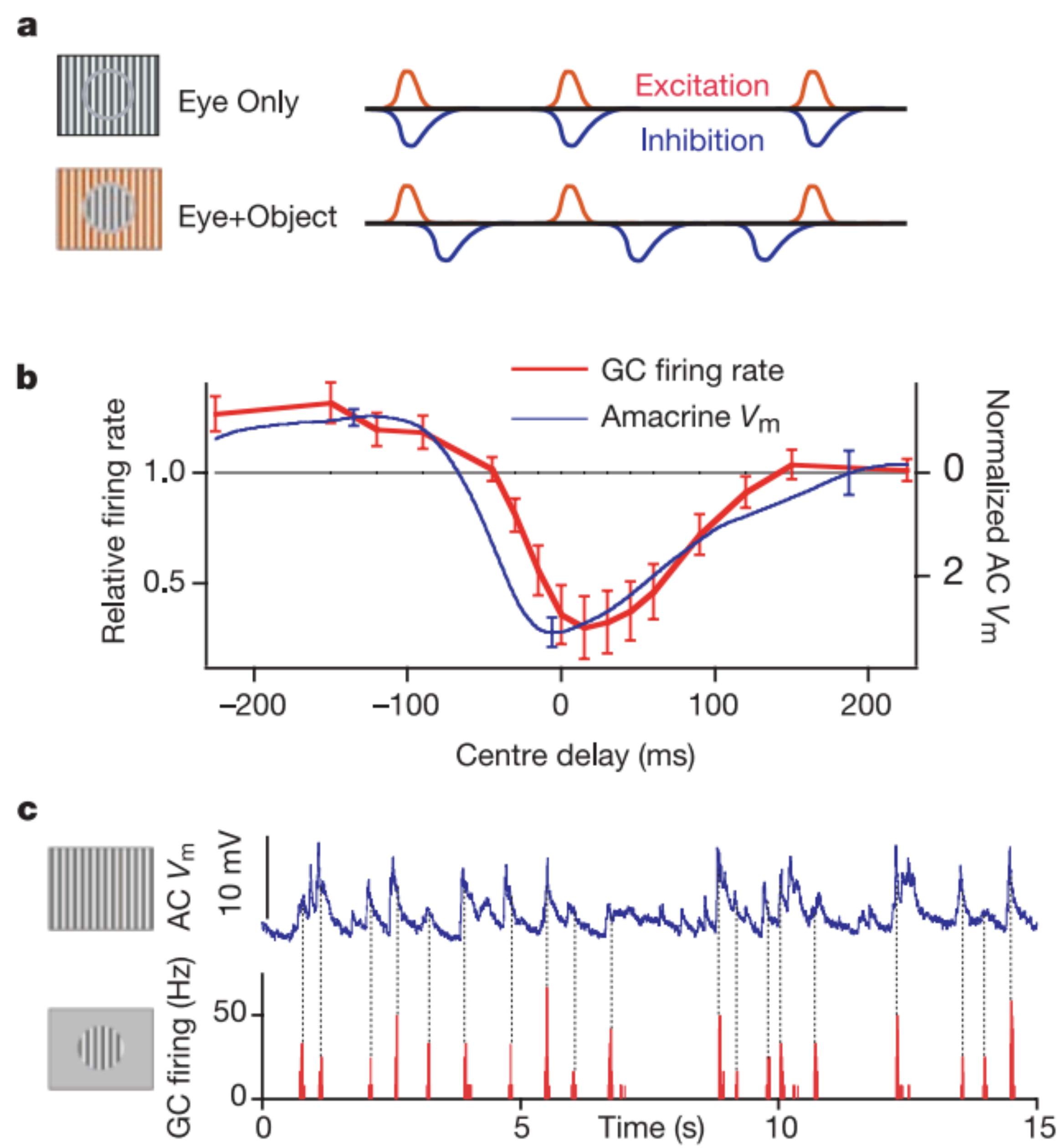


Pixel vs Object

- Object is a higher level visual feature. Does the retina understand object?
- We can define a simple rough definition of the object—matter that moves together.
- We already mentioned that the receptive fields of the retina will emphasize the boundary of the moving objects.
- But what if there are many objects are moving?



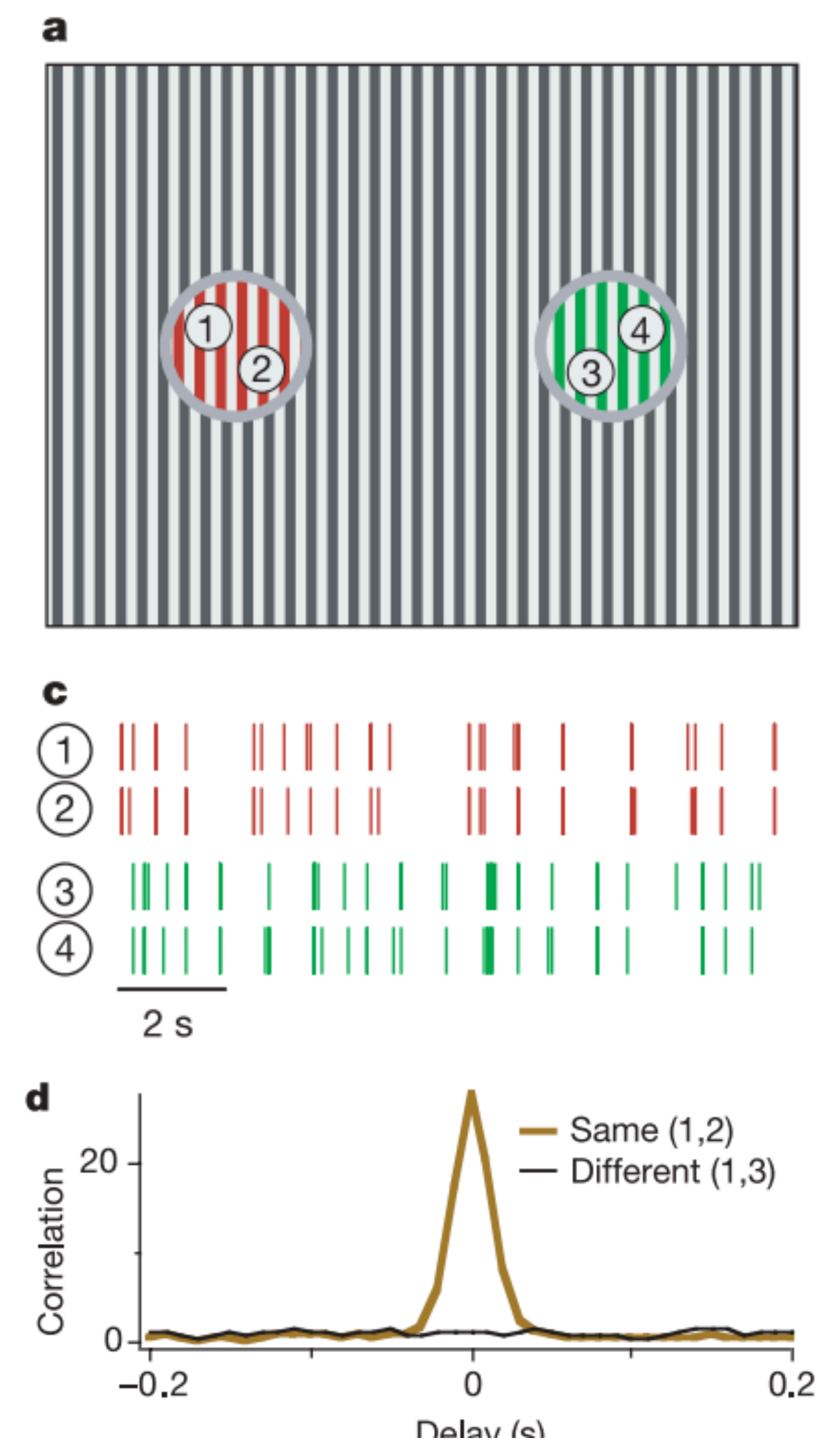
Rabbit ON Brisk Transient
a
Eye + Object
Rate (Hz)
200
0
b
Eye Only
Rate (Hz)
200
0
c
Object Only
Rate (Hz)
200
0
d
Eye +
Object Drift
Rate (Hz)
200
0
0
5
10
15
Time (s)



a
Eye Only
Excitation
Inhibition
Eye+Object
b
GC firing rate
Amacrine V_m
Relative firing rate
1.0
0.5
Normalized AC V_m
0
2
−200
−100
0
100
200
Centre delay (ms)
c
AC V_m
10 mV
GC firing (Hz)
50
0
0
5
Time (s)
10
15

Vision is dynamic

- Different objects move at different directions.
- So different directions generate different spike sequences.
- RGC already understands object.
- Normal ML trains on static images. However, since brain receives dynamic input, it has extra information to work with.



Summary

- Retina develops a center surround receptive field for natural image statistics.
- Retina adapts to new statistics to code more efficiently.
- Short term learning has two competing factors, synaptic resource and synaptic utilization.
- The motion of the scene helps the retina understand what an object is.