

# Biology and Mathematics of the Retina

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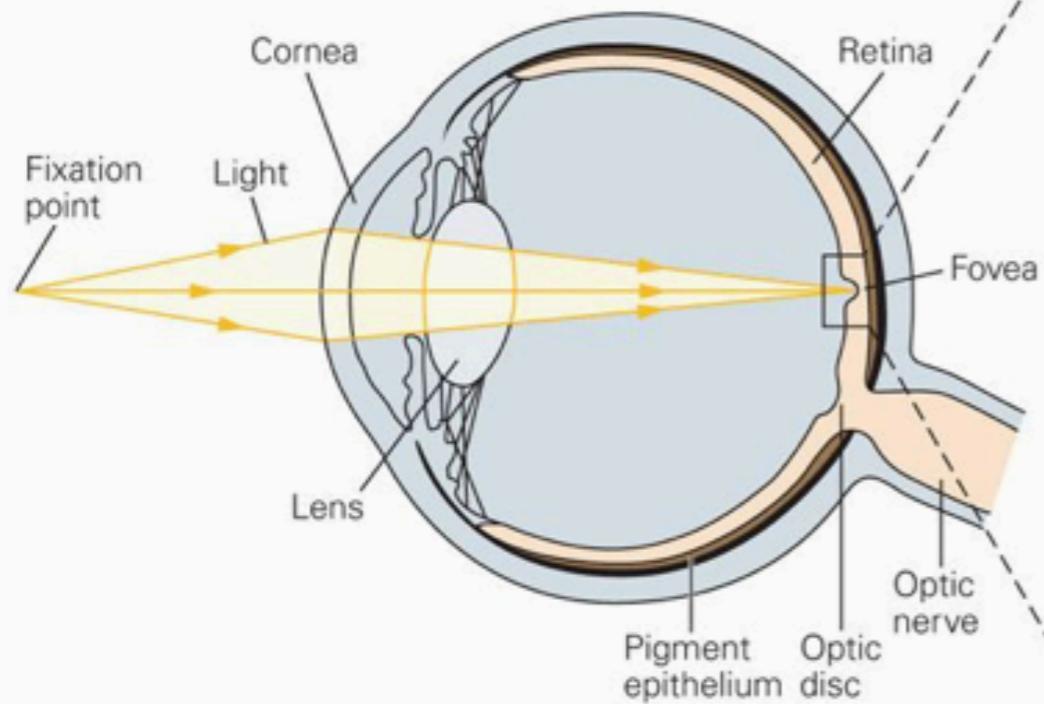
# Plan

- Biology of retina
- RC circuit, convolution and dynamical system
- Dynamic adaptation of the retina
- Vesicle kinetic model for short term learning
- Motion sensing cell and the idea of objects at the retina

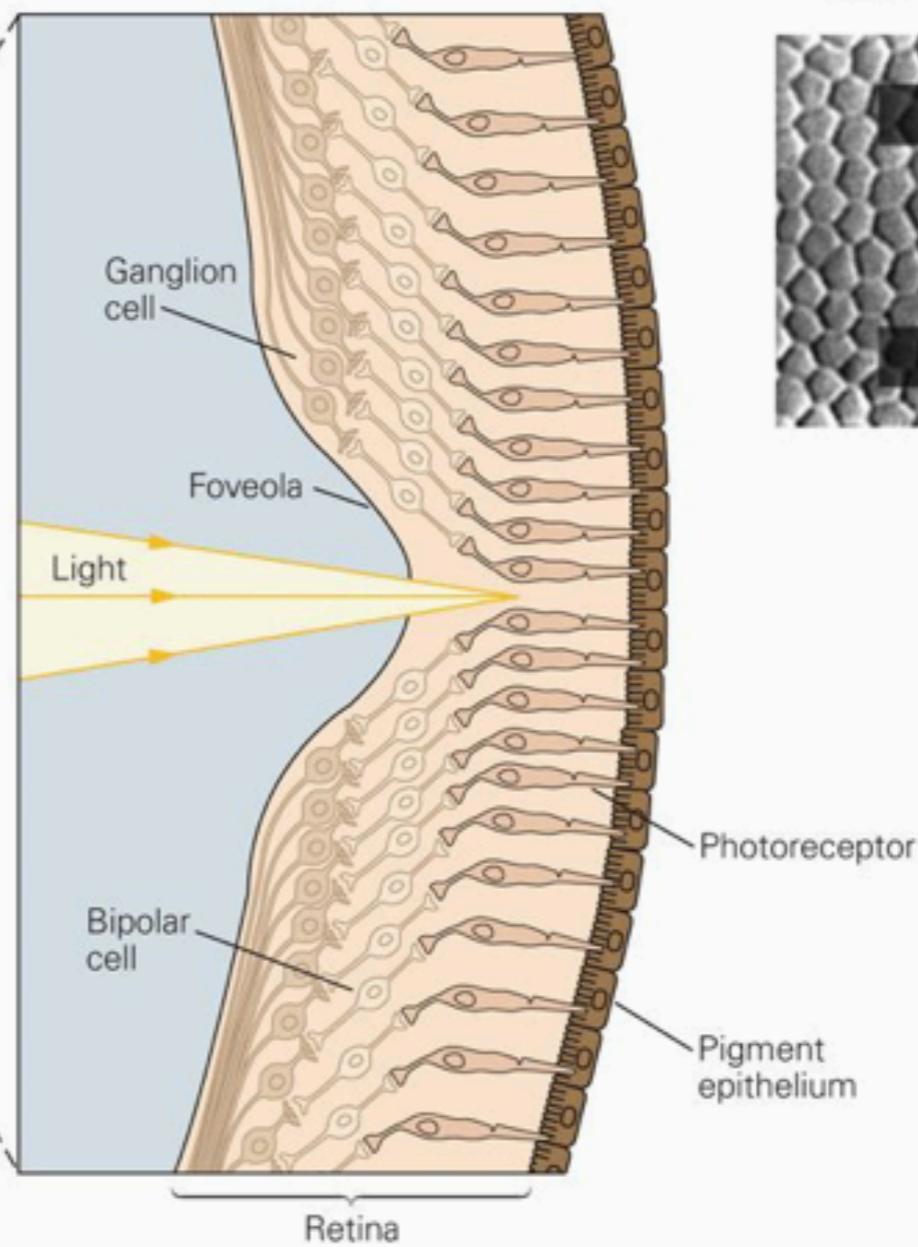
# Biology of the Retina

- The structure and cell types of the retina
- The neural circuits of the retina
- Phototransduction
- Gap junctions and ribbon synapses
- Bipolar cells
- Center surround receptive field

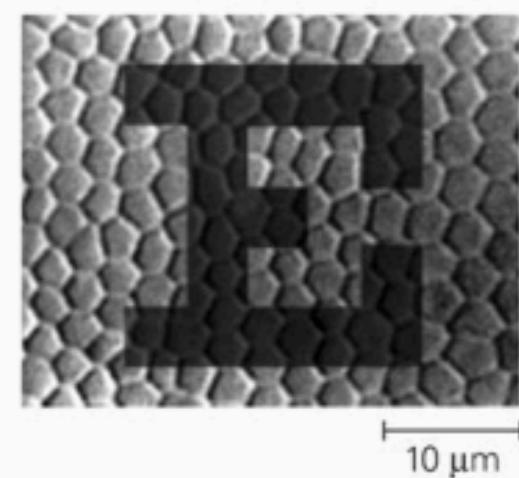
**A** Refraction of light onto the retina

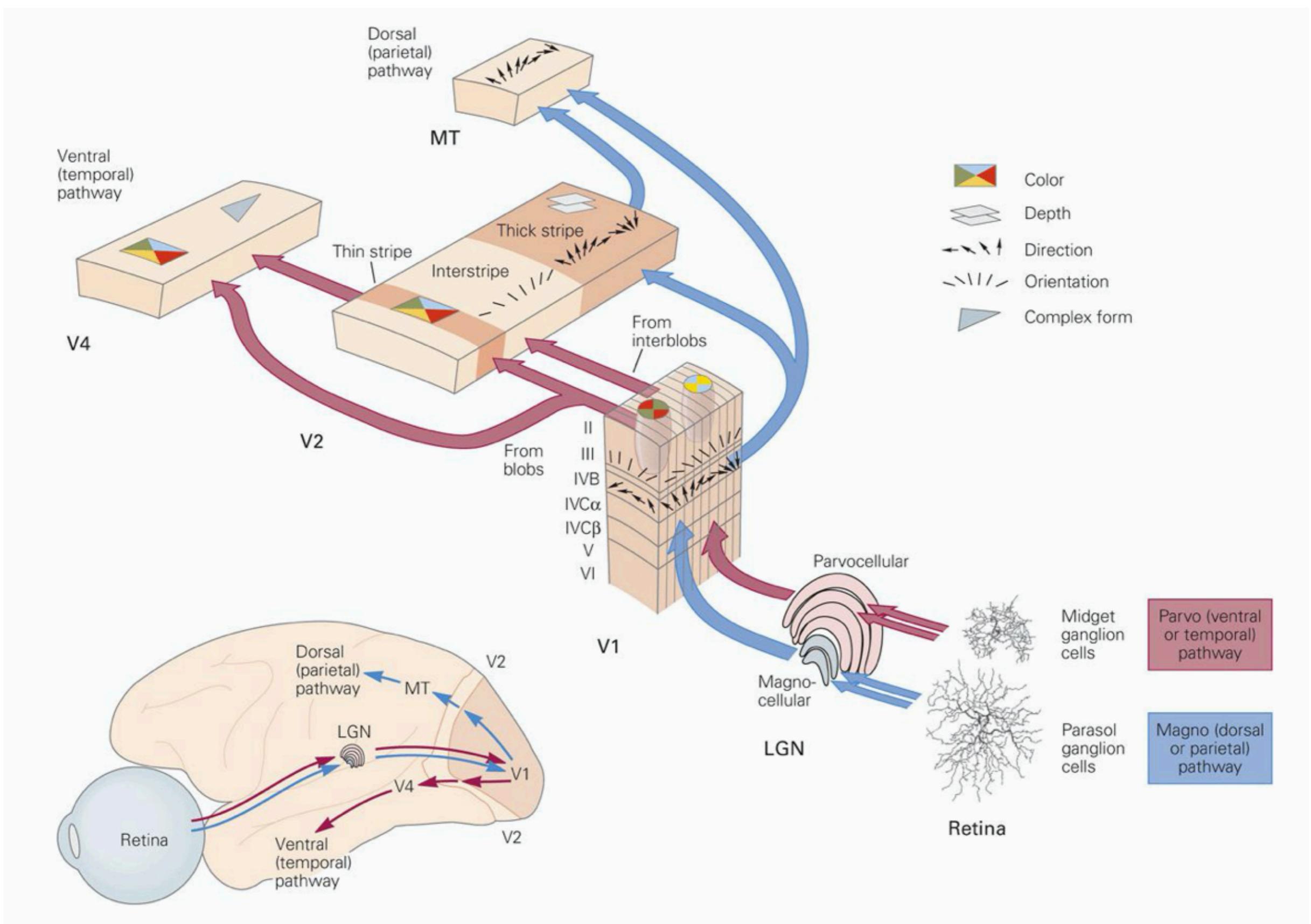


**B** Focusing of light in the fovea

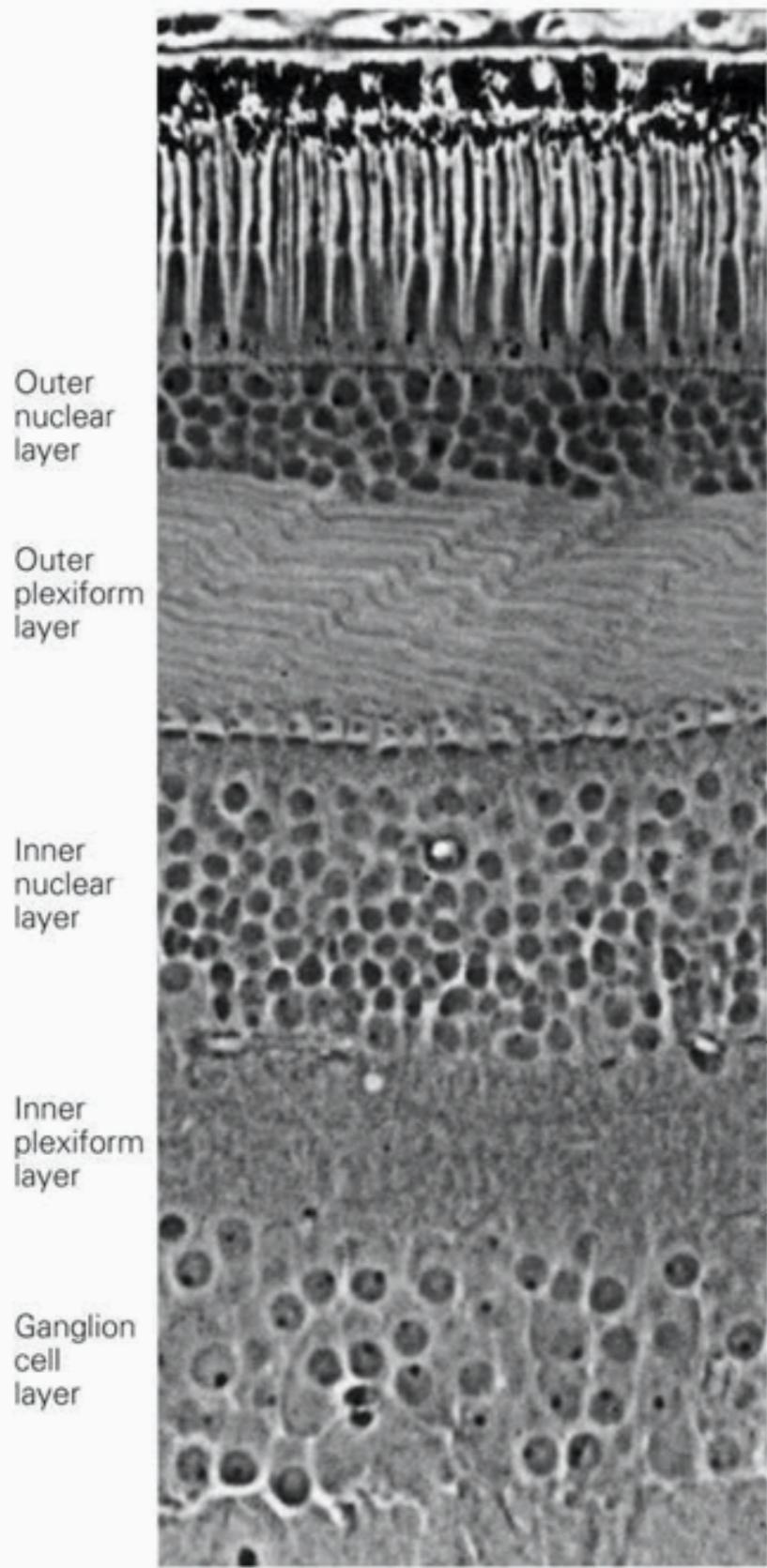


**C** Packing of photoreceptors in the fovea

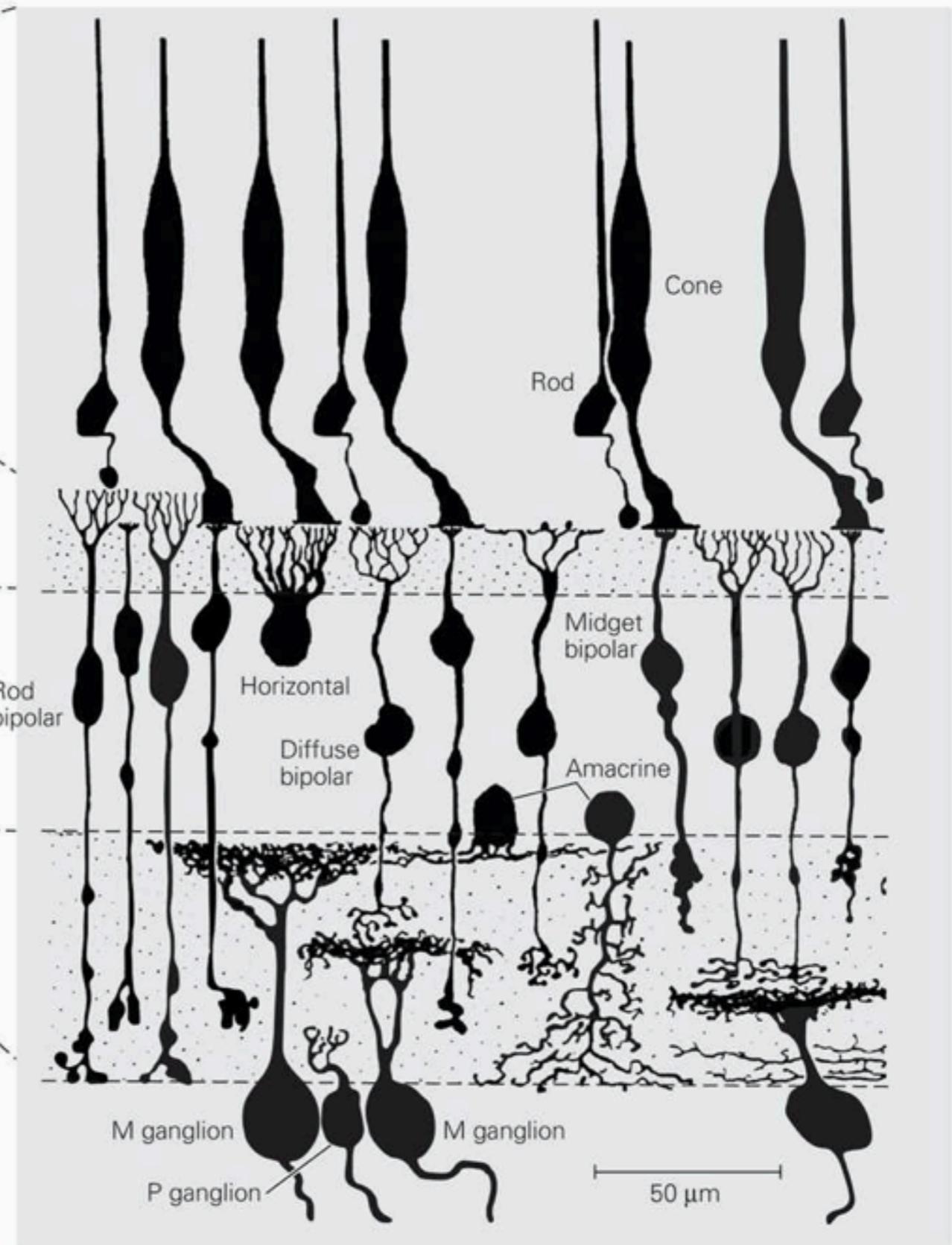




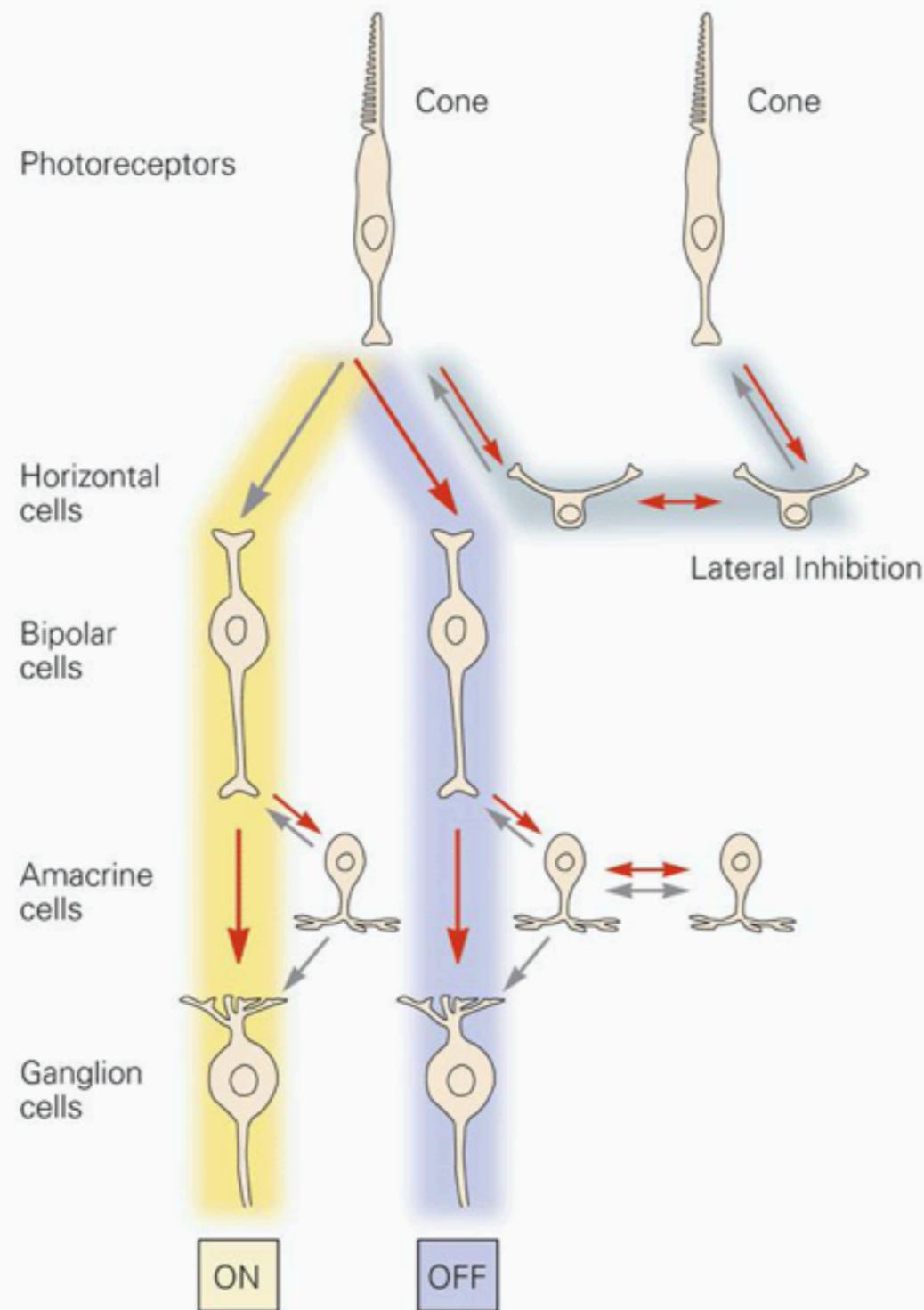
A Section of retina



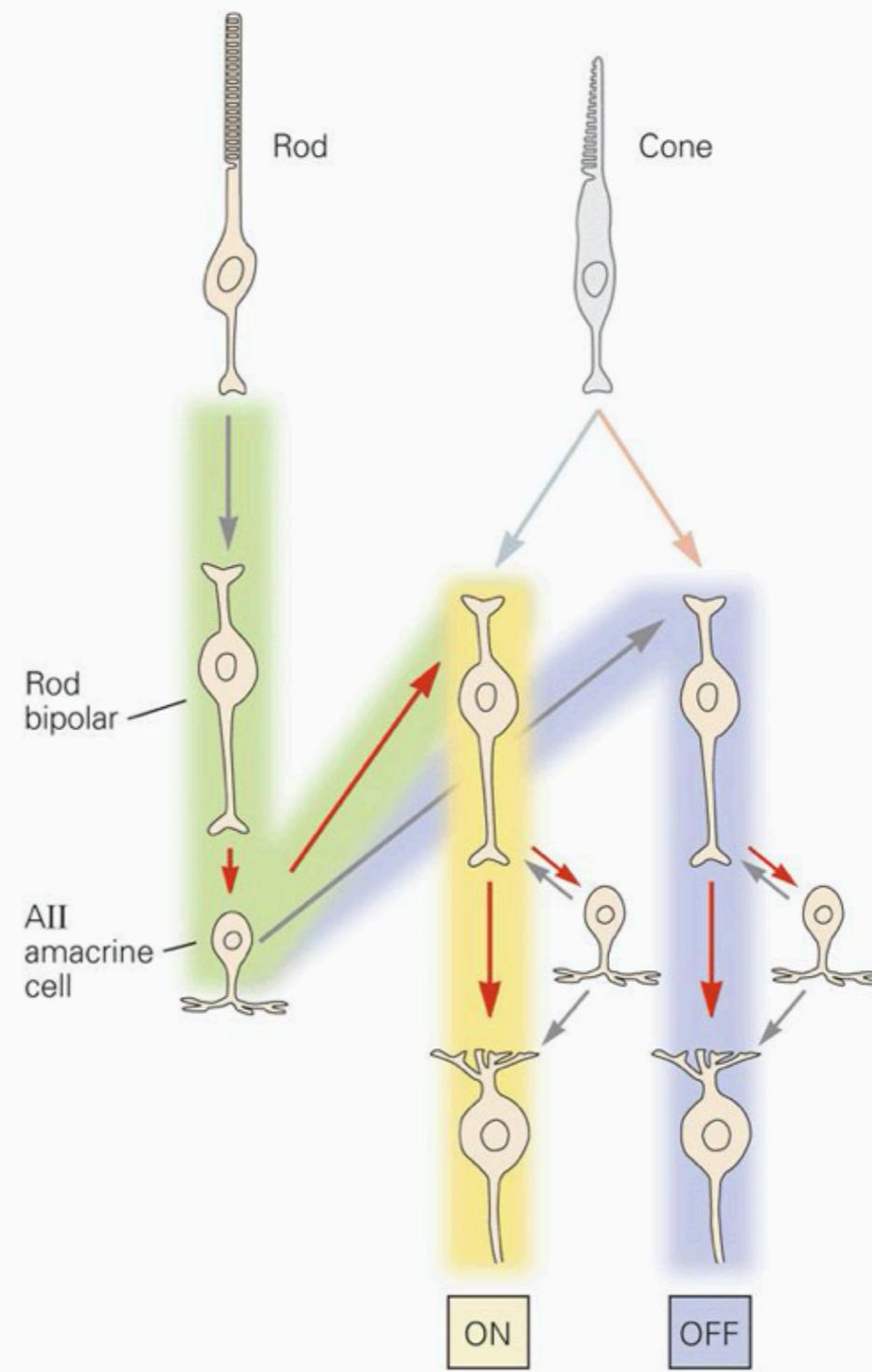
B Neurons in the retina



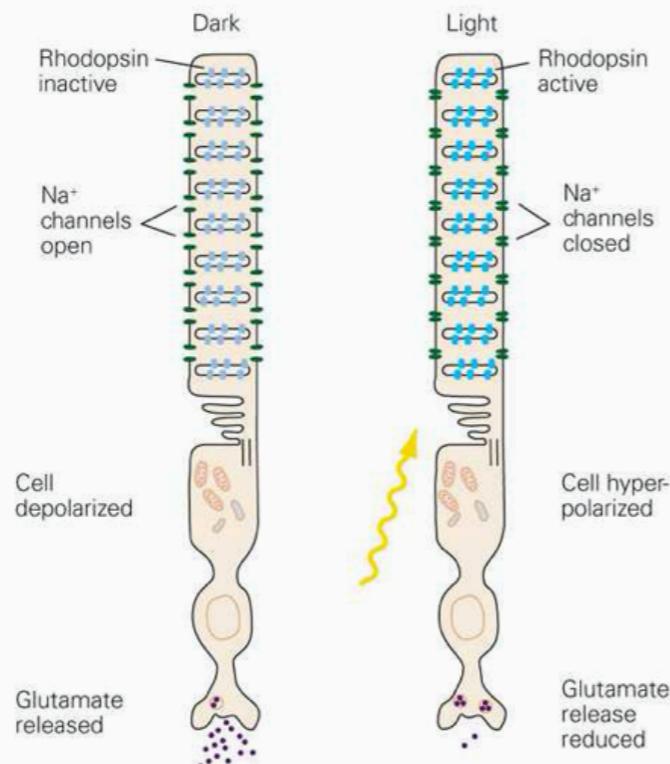
**A Cone signal circuitry**



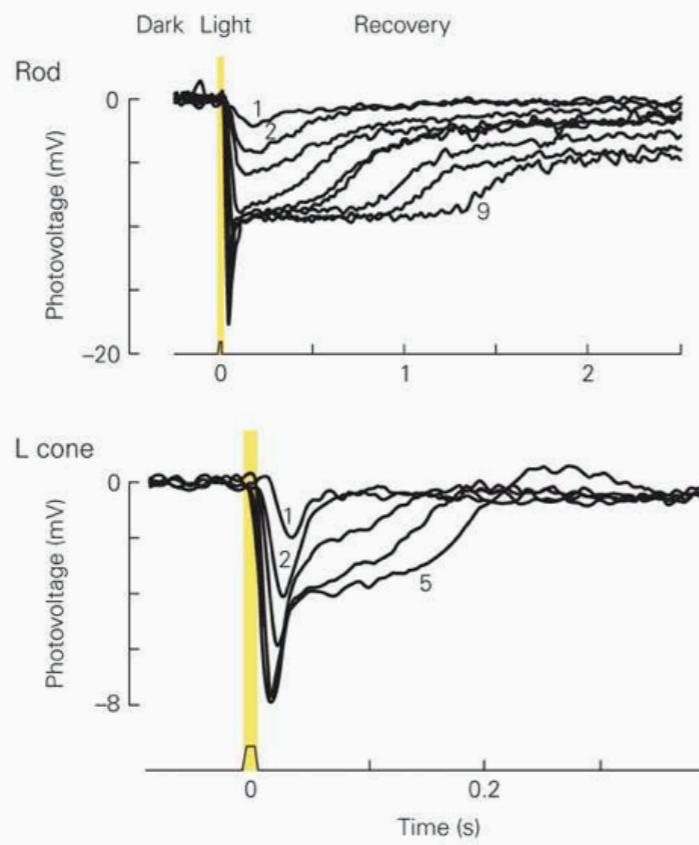
**B Rod signal circuitry**



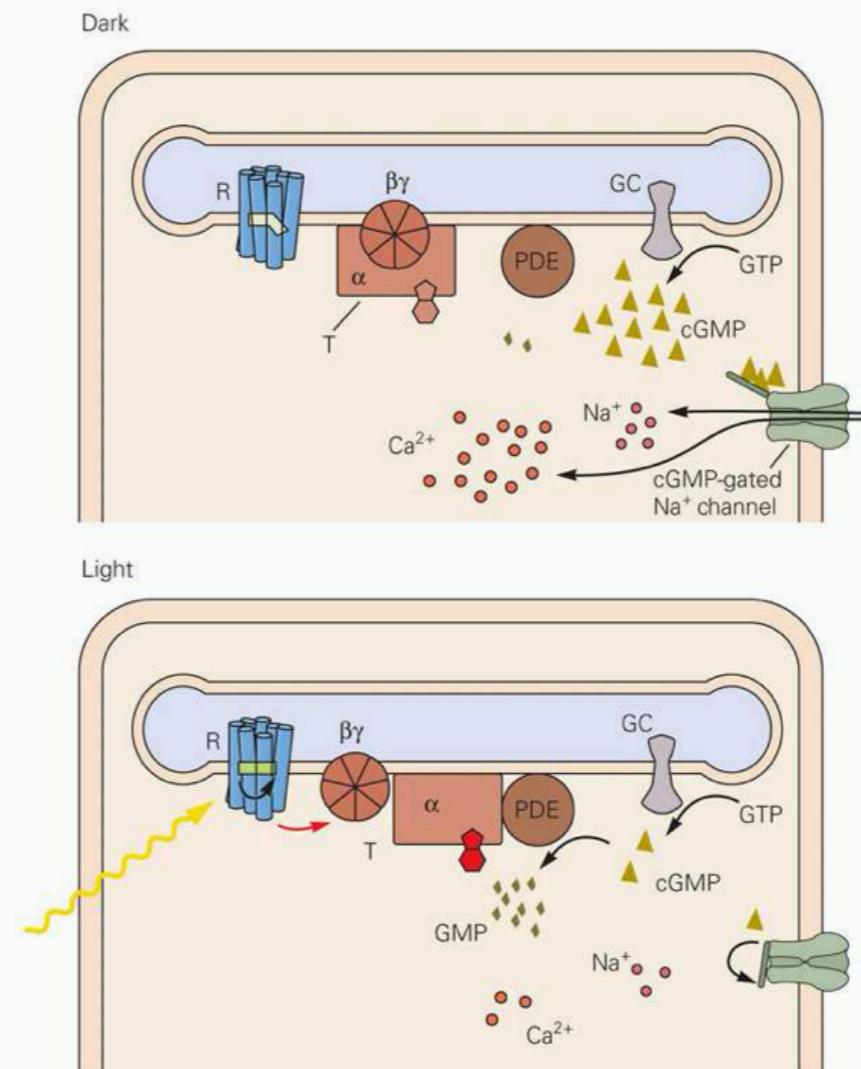
### A Phototransduction and neural signaling



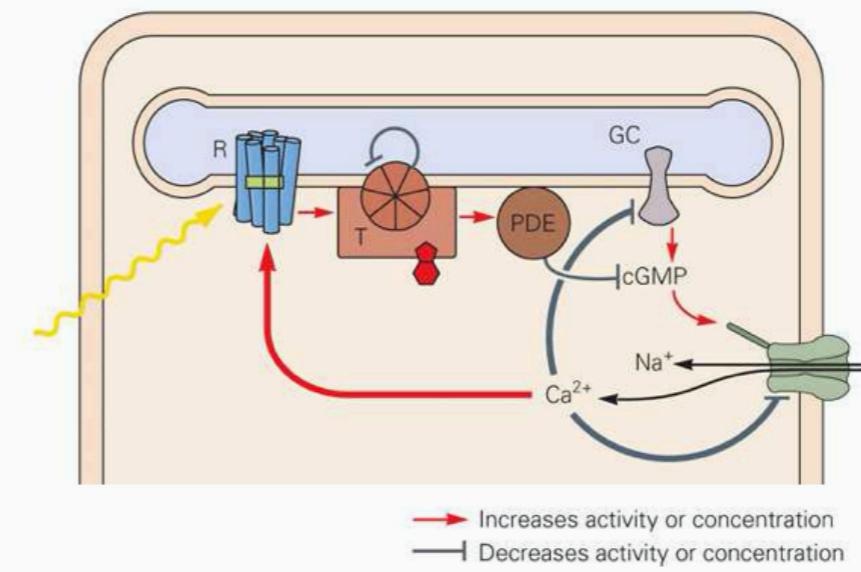
### C Voltage response to light



### B<sub>1</sub> Molecular processes in phototransduction

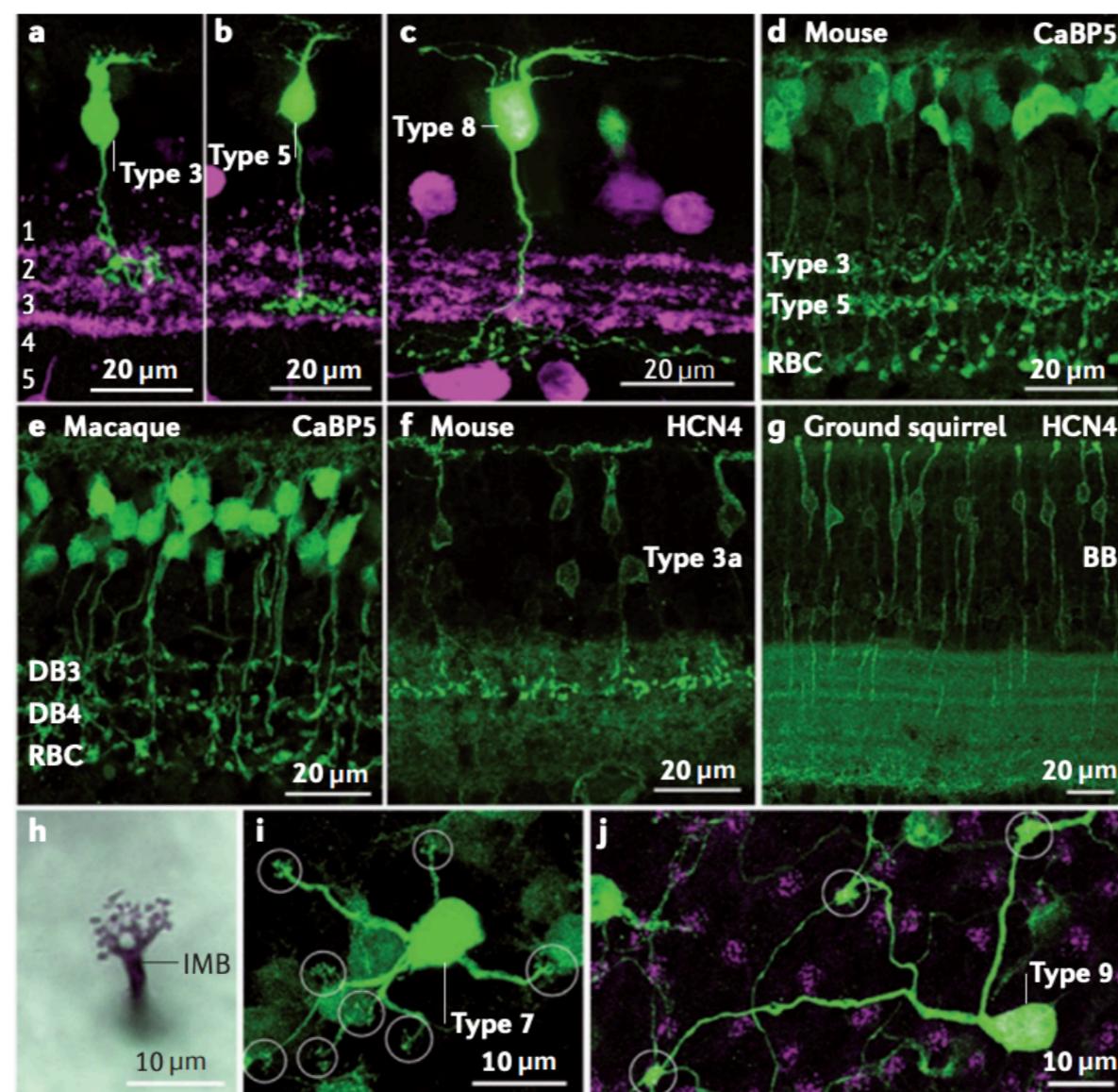
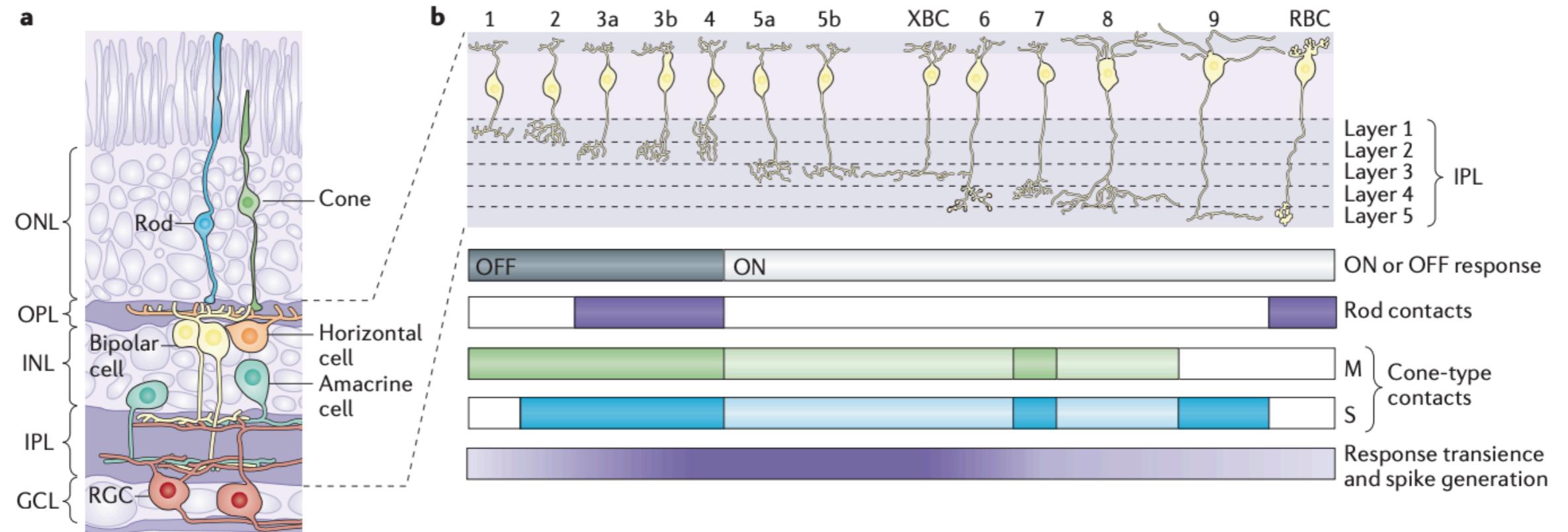


### B<sub>2</sub> Reaction network in phototransduction

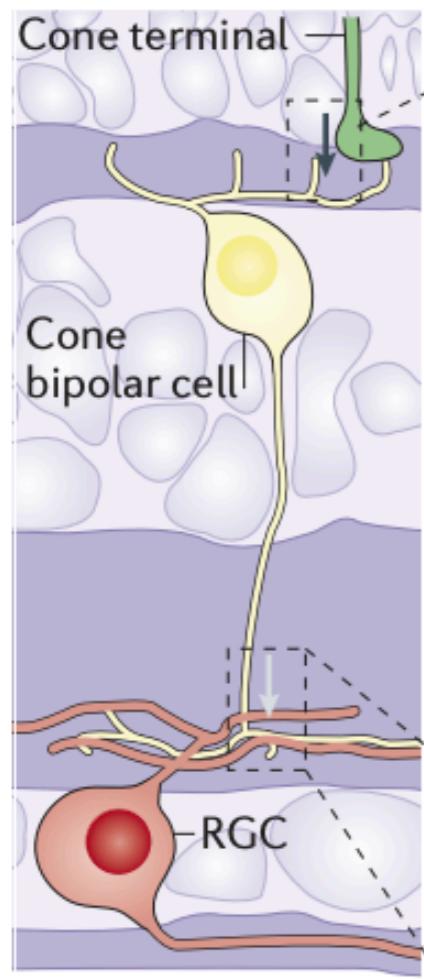


# Three types of synapse

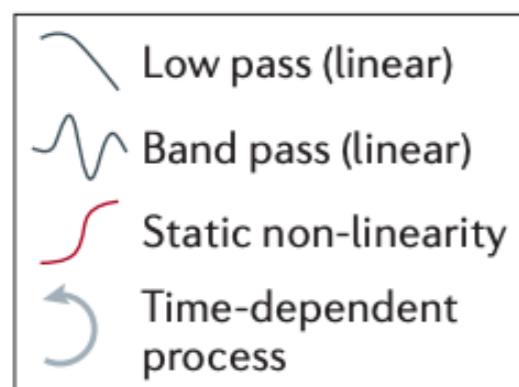
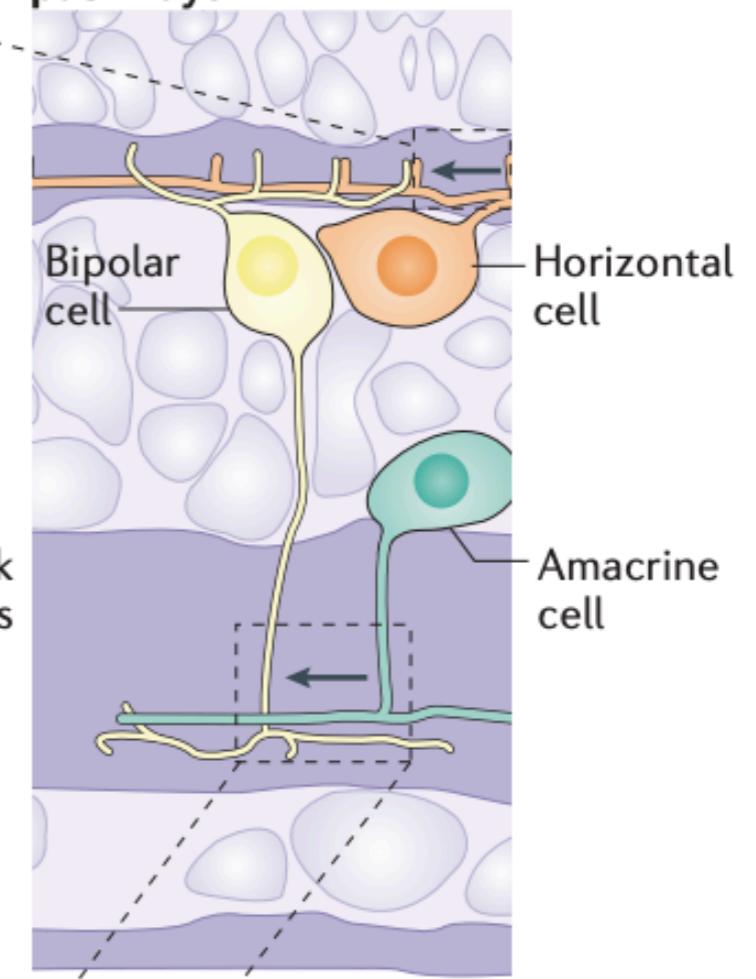
- Chemical synapse, digital output
- Ribbon synapse, analog output
- Gap junction, bidirectional analog output



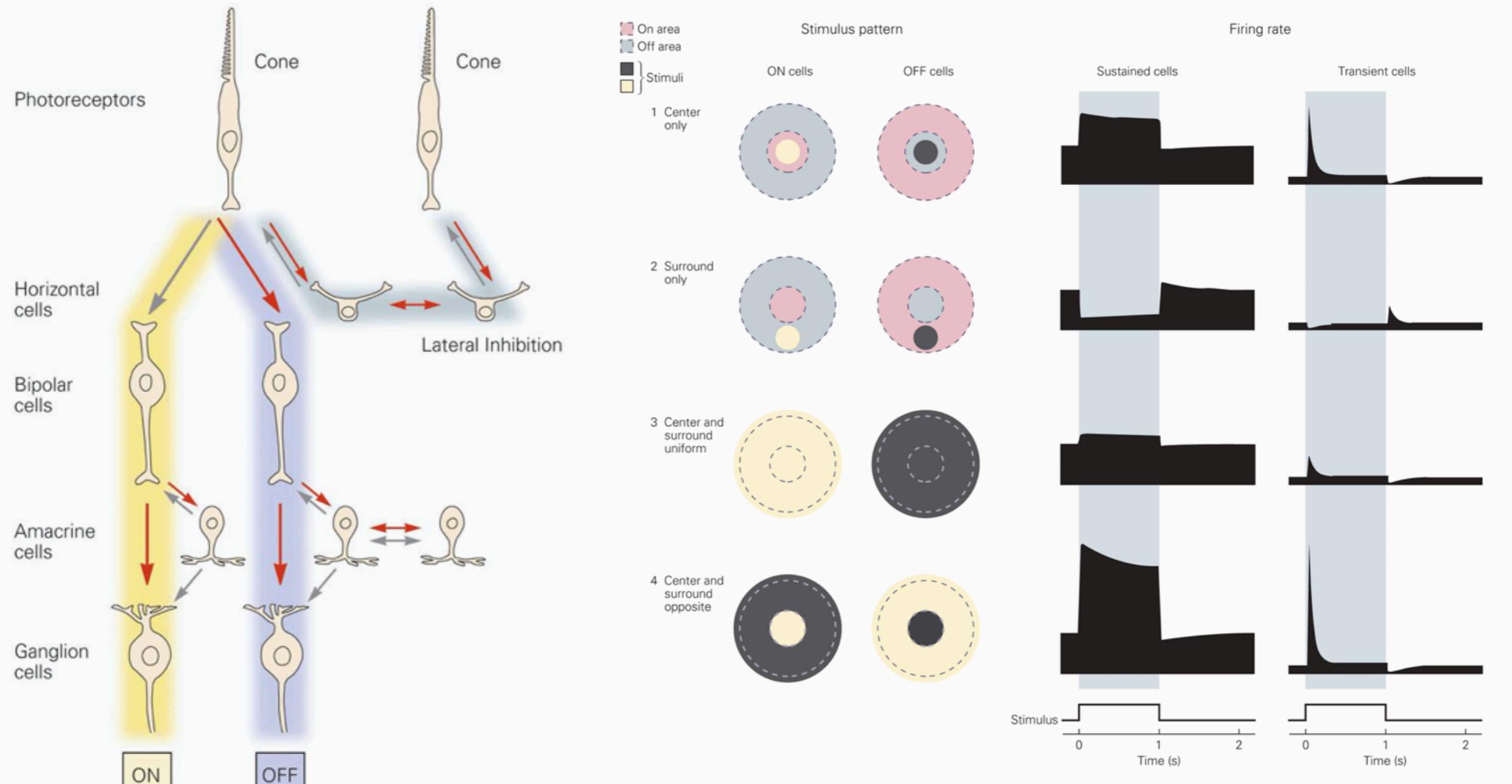
## Vertical pathway

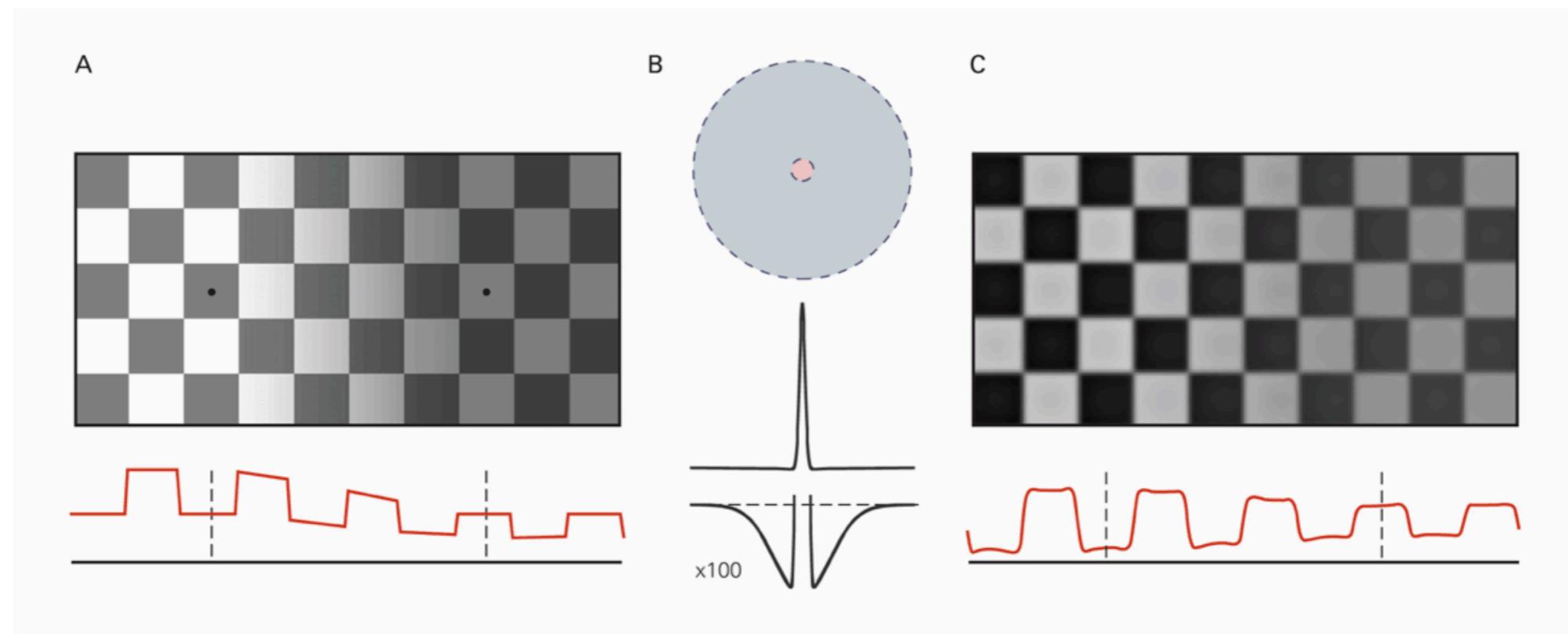
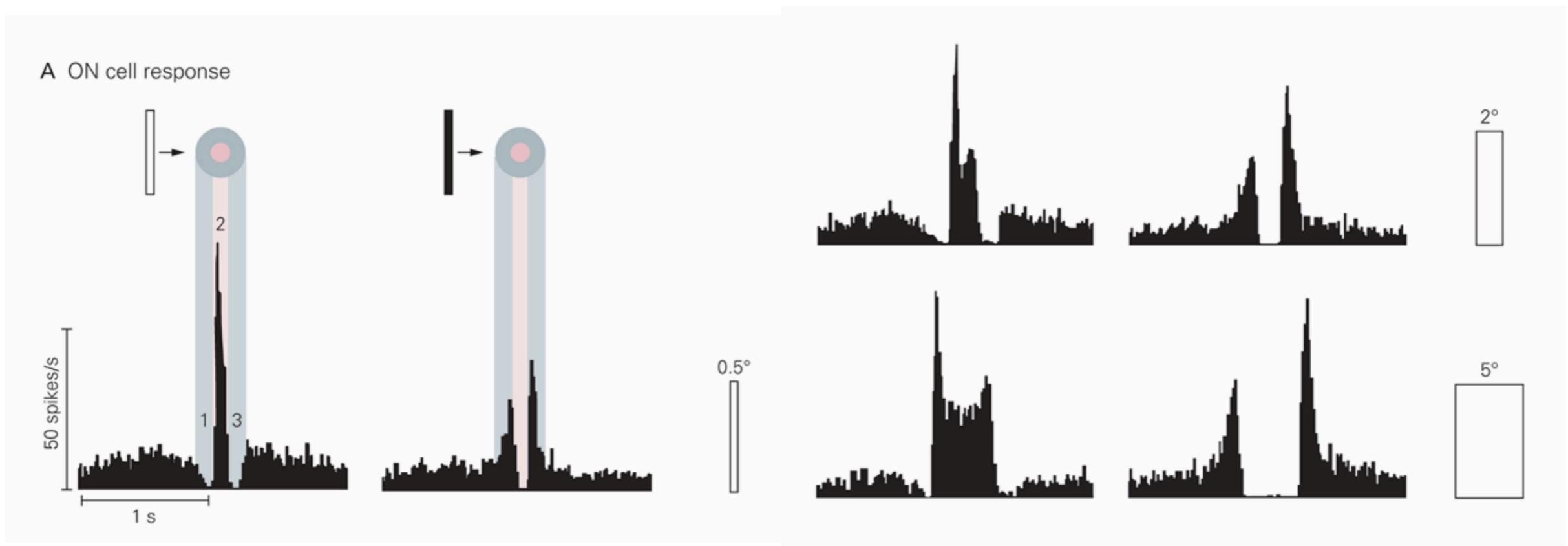


## Modulation by lateral pathways



### A Cone signal circuitry





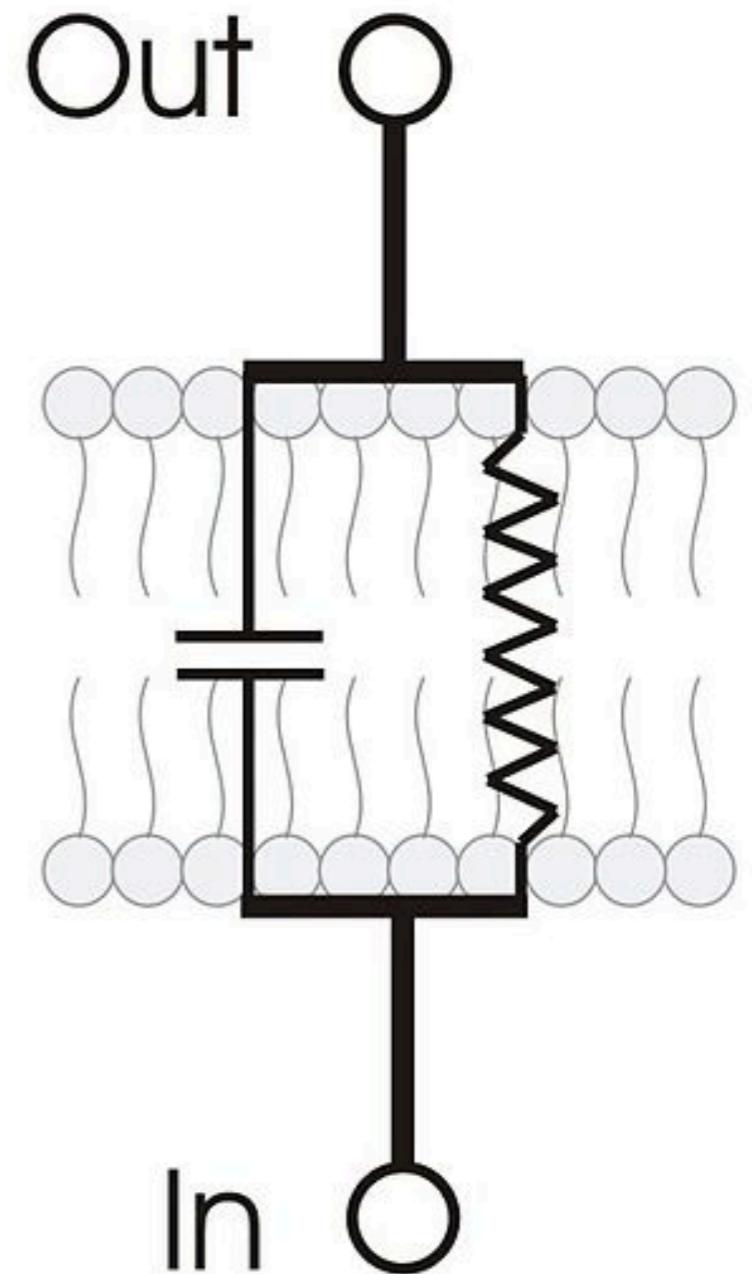
# RC Circuit

- We can treat membrane as a RC circuit
- Ex: Hodgkin-Huxley model with voltage-dependent resistance

- $CV = Q \Rightarrow C \frac{dV}{dt} = I_C$

- $I_C + I_R - I = 0$

- $C \frac{dV}{dt} + \frac{V - V_m}{R} - I = 0$



# Solution

- $\tau \frac{dV}{dt} = -V + V_m + RI(t), \tau = RC$
- $V(t) = e^{-\frac{t}{\tau}} \left( V(0) + \int_{s=0}^t e^{\frac{s}{\tau}} \frac{V_m + RI(s)}{\tau} ds \right)$
- When the input current is a constant we have
- $V(t) = e^{-\frac{t}{\tau}} V(0) + (1 - e^{-\frac{t}{\tau}})(V_m + RI)$

# Convolution View

- In the above equation, set the membrane potential and initial voltage as 0. We get
- $V(t) = \int_0^t e^{\frac{s-t}{\tau}} \frac{RI}{\tau} ds$ , which we can rewrite as
- $V(t) = \frac{R}{\tau} e^t * I(t)$

# Adaptation of the retina

- Recall that by efficient coding theory, we derive that the connection of the retina should be PCA, which induces center surround receptive field.
- This is working under the assumption that the correlation of natural image statistics are high for nearby pixels and inverse proportion to the square of the distance.
- By subtracting the nearby values, we only transmit what is different from expectation and hence can more efficiently communicate. Ex: edge detection

**What happens if the  
environment is different?**

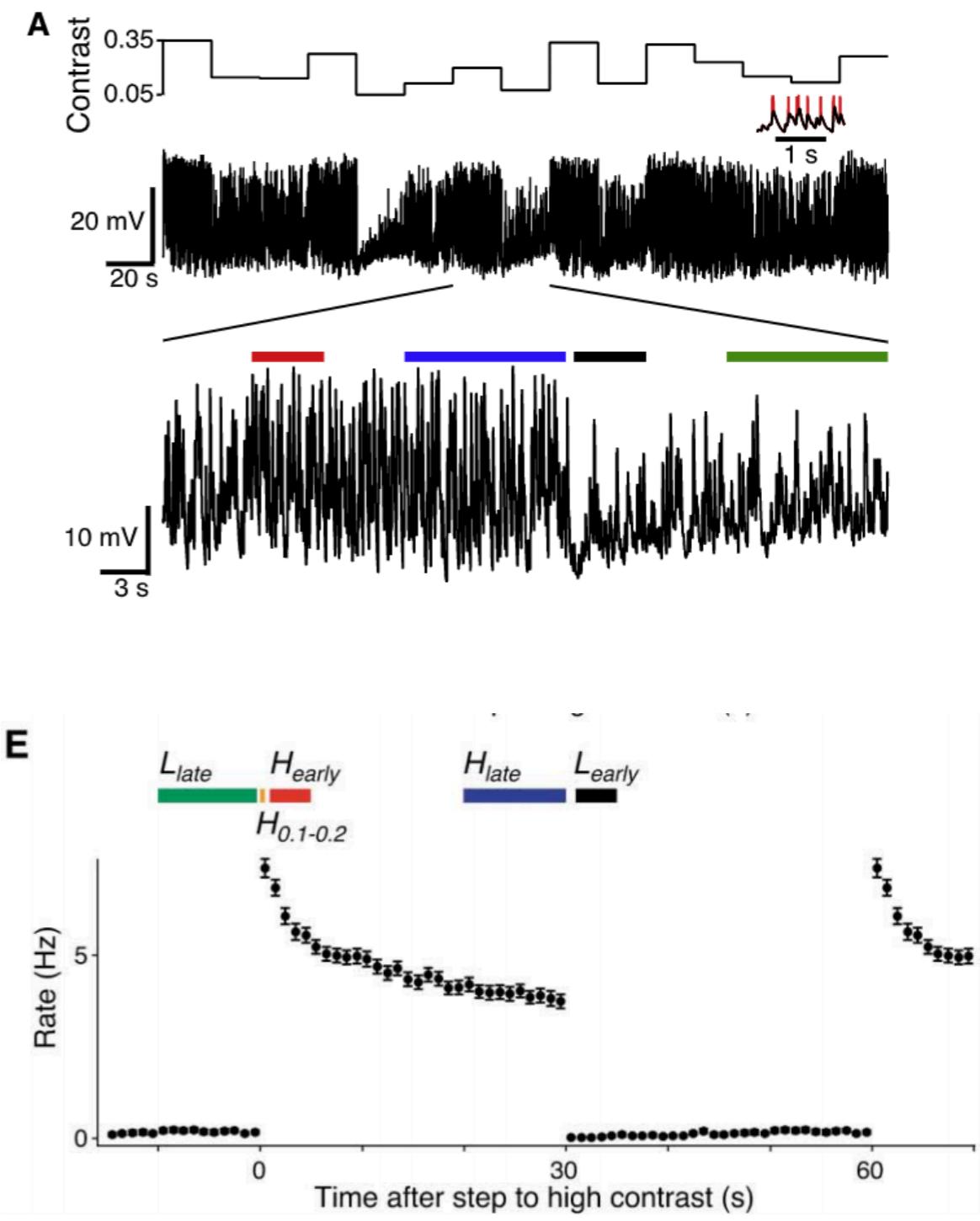
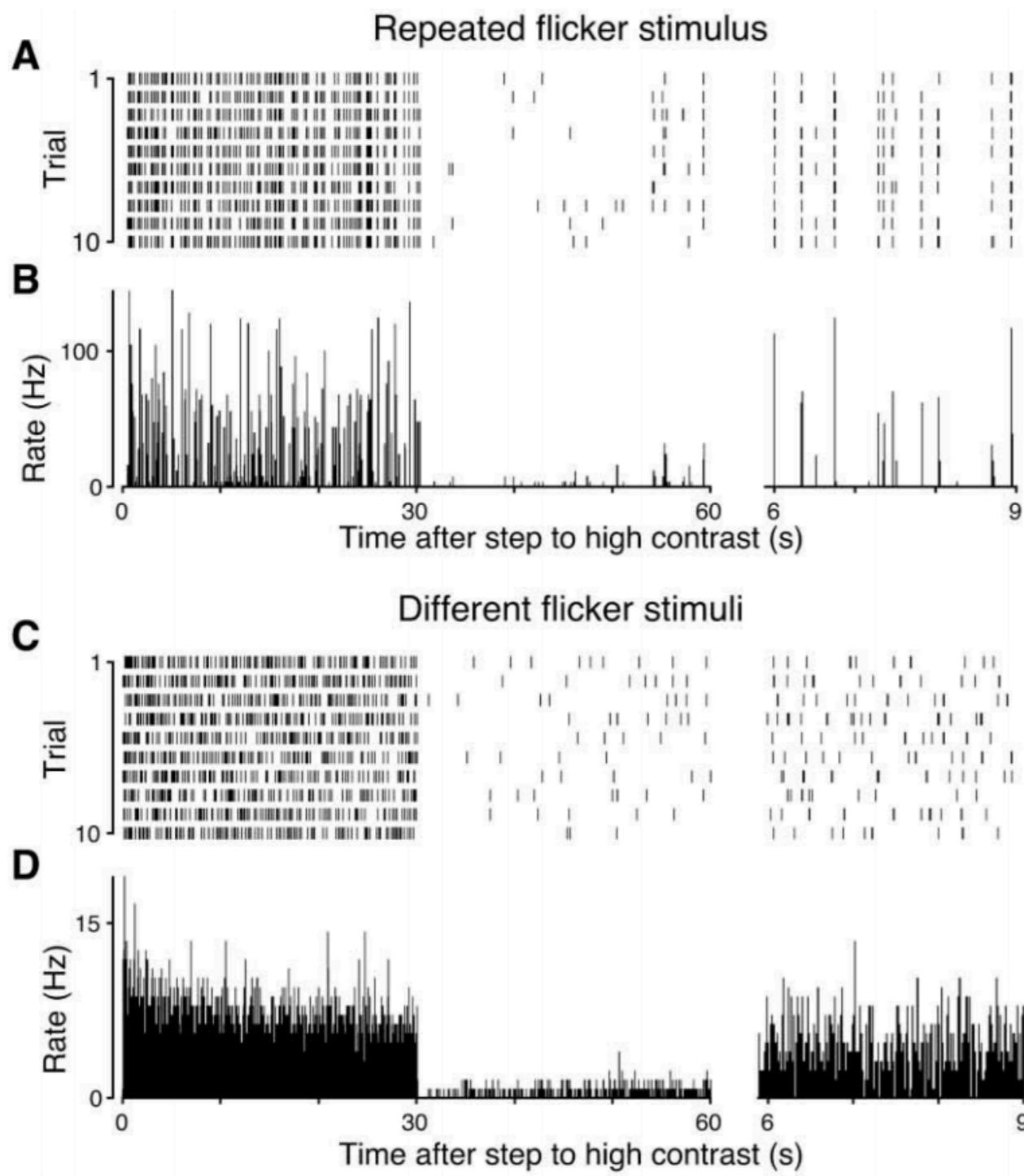
# Different adaptation

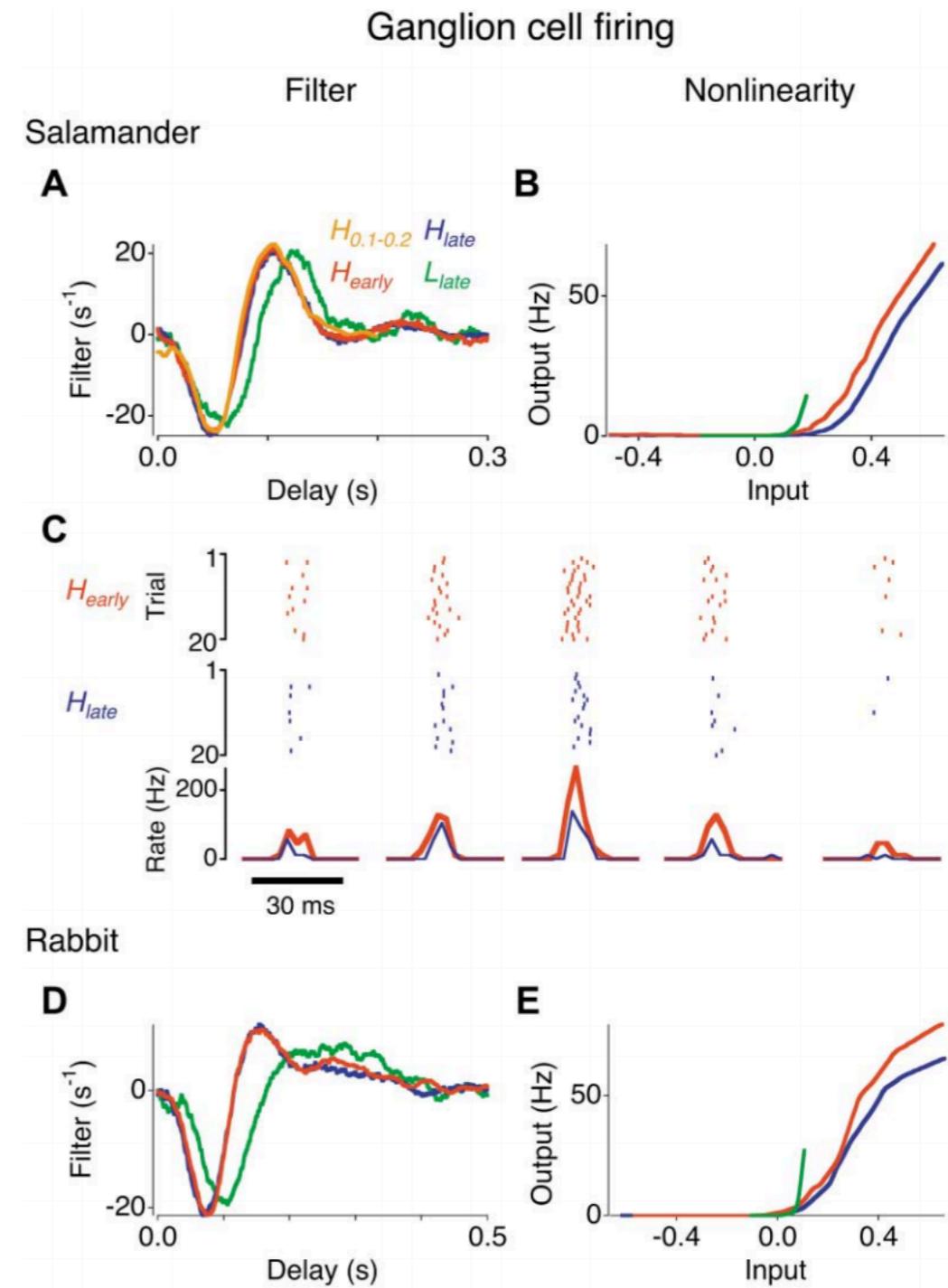
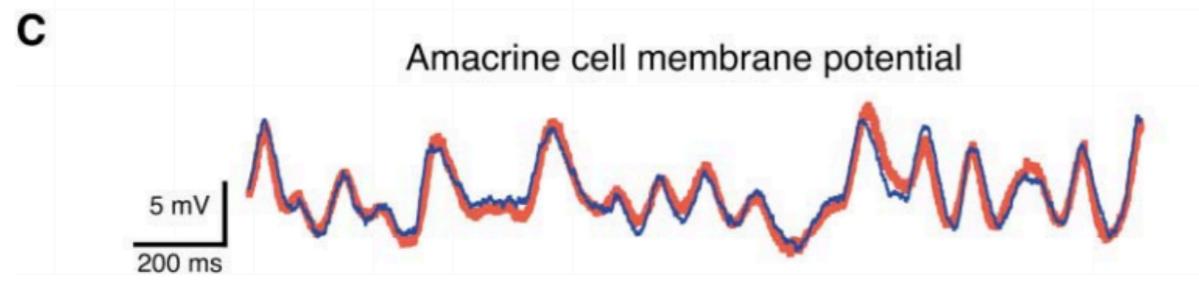
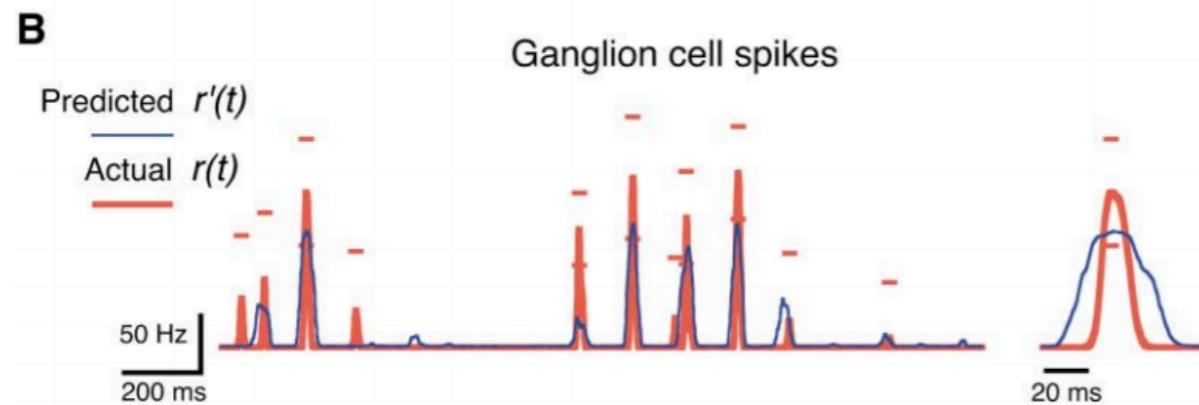
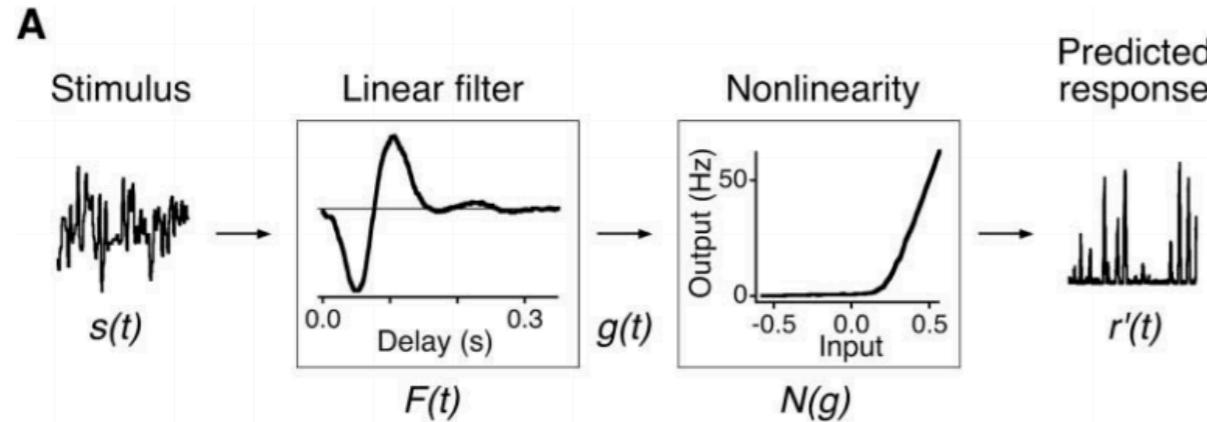
- Contrast adaptation
- Spatial frequency adaptation
- Orientation adaptation
- Temporal sequence adaptation
- Proposal of the underlying mechanism: Short term learning with vesicles and desensitization with AMPA

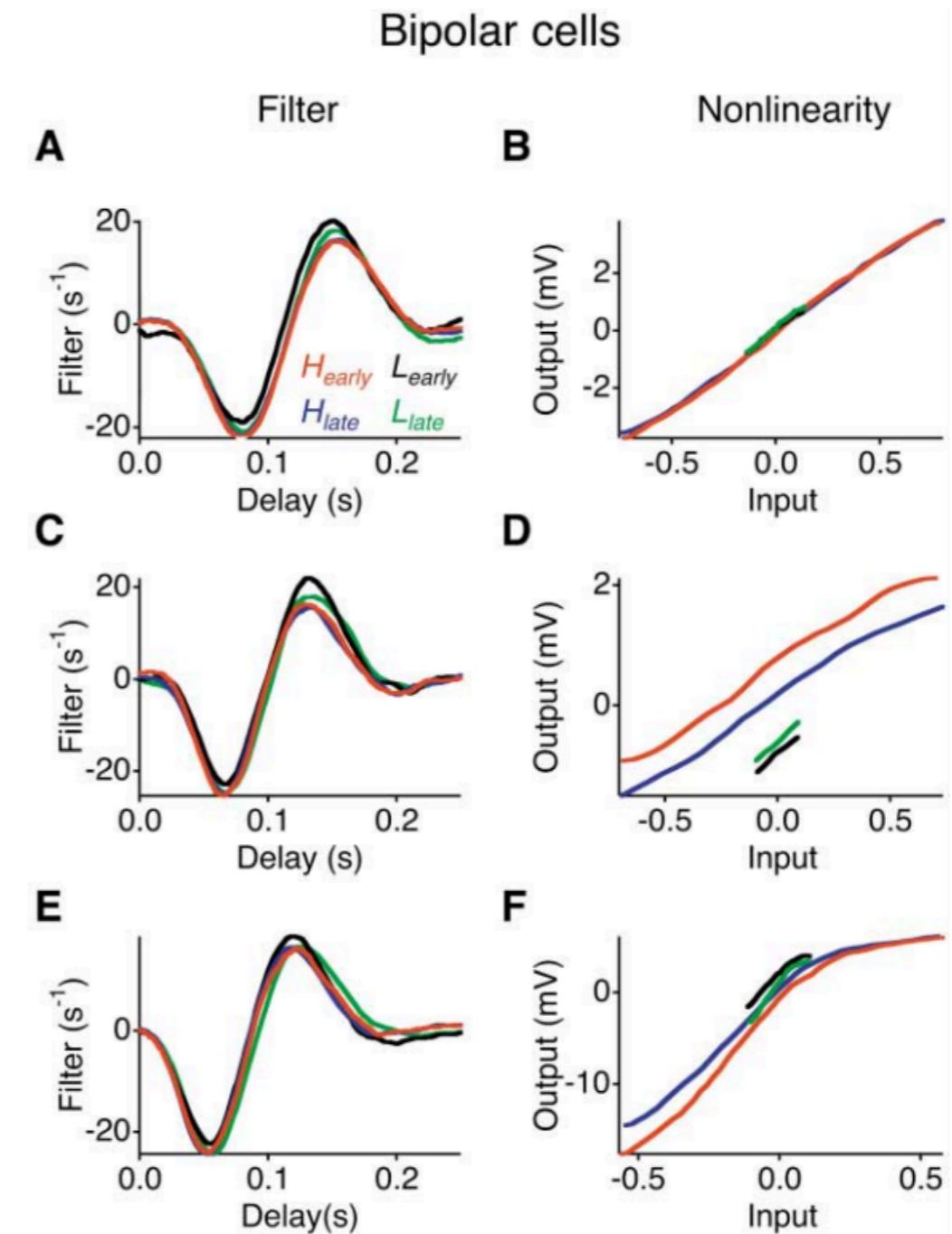
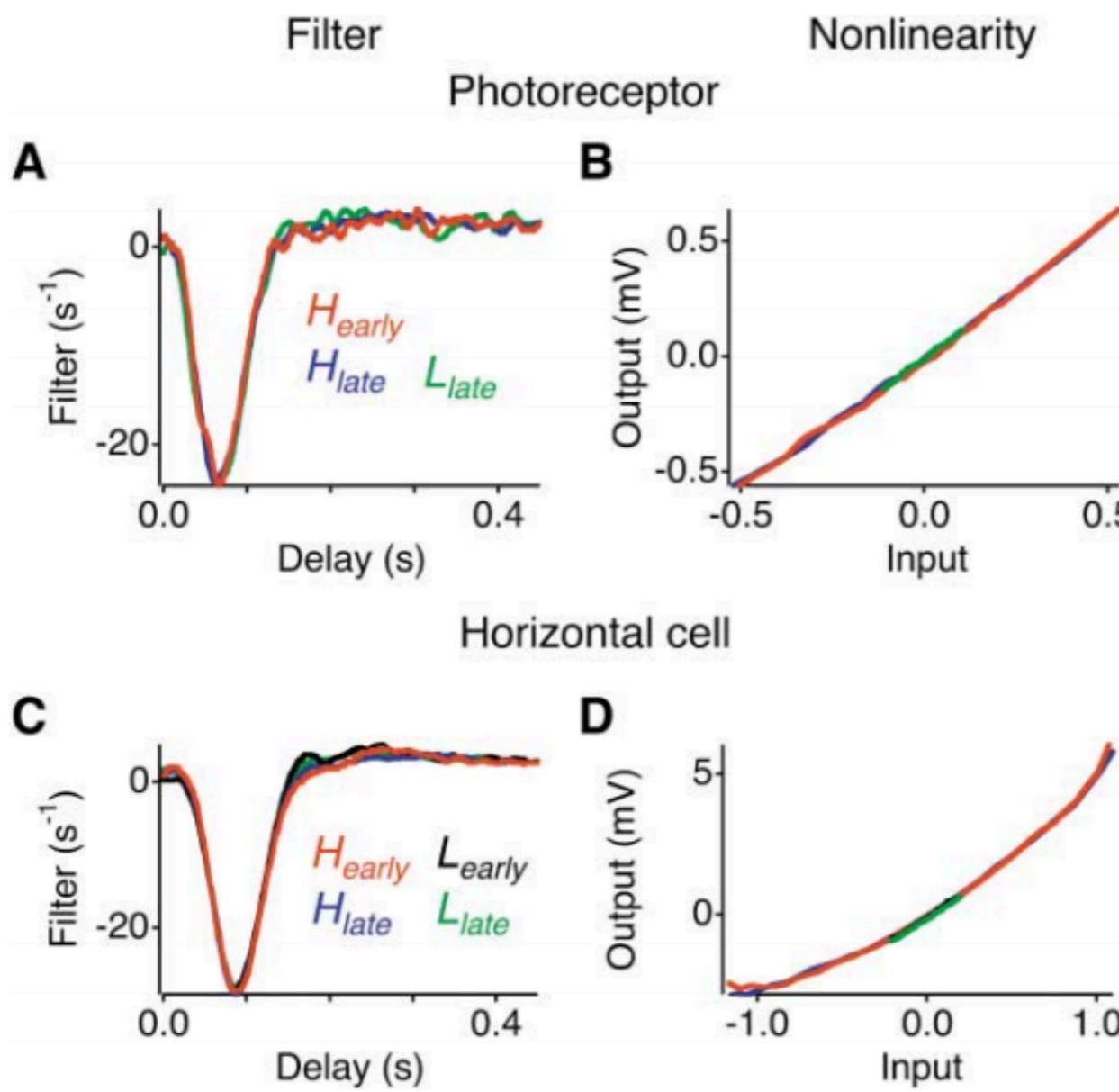


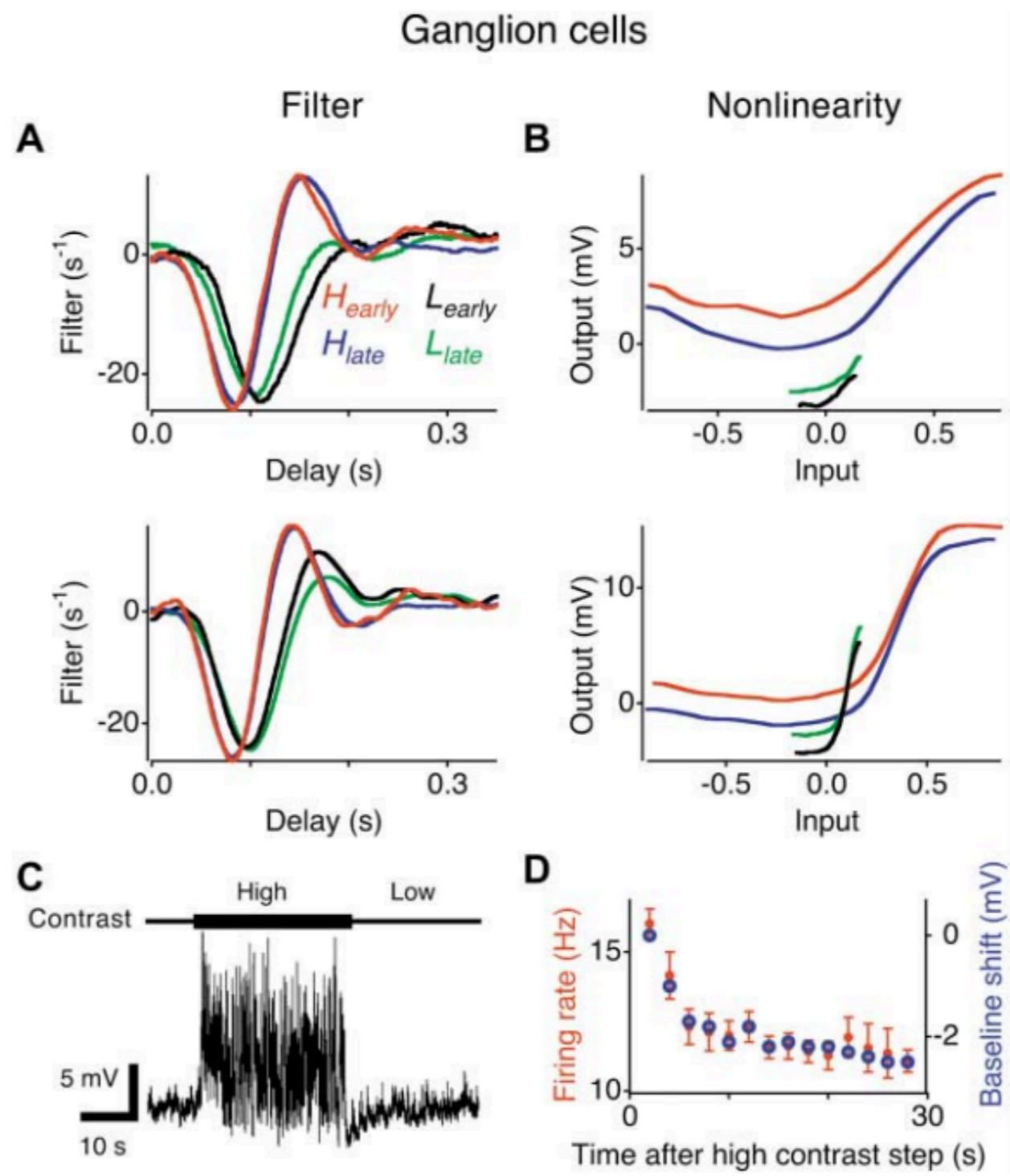
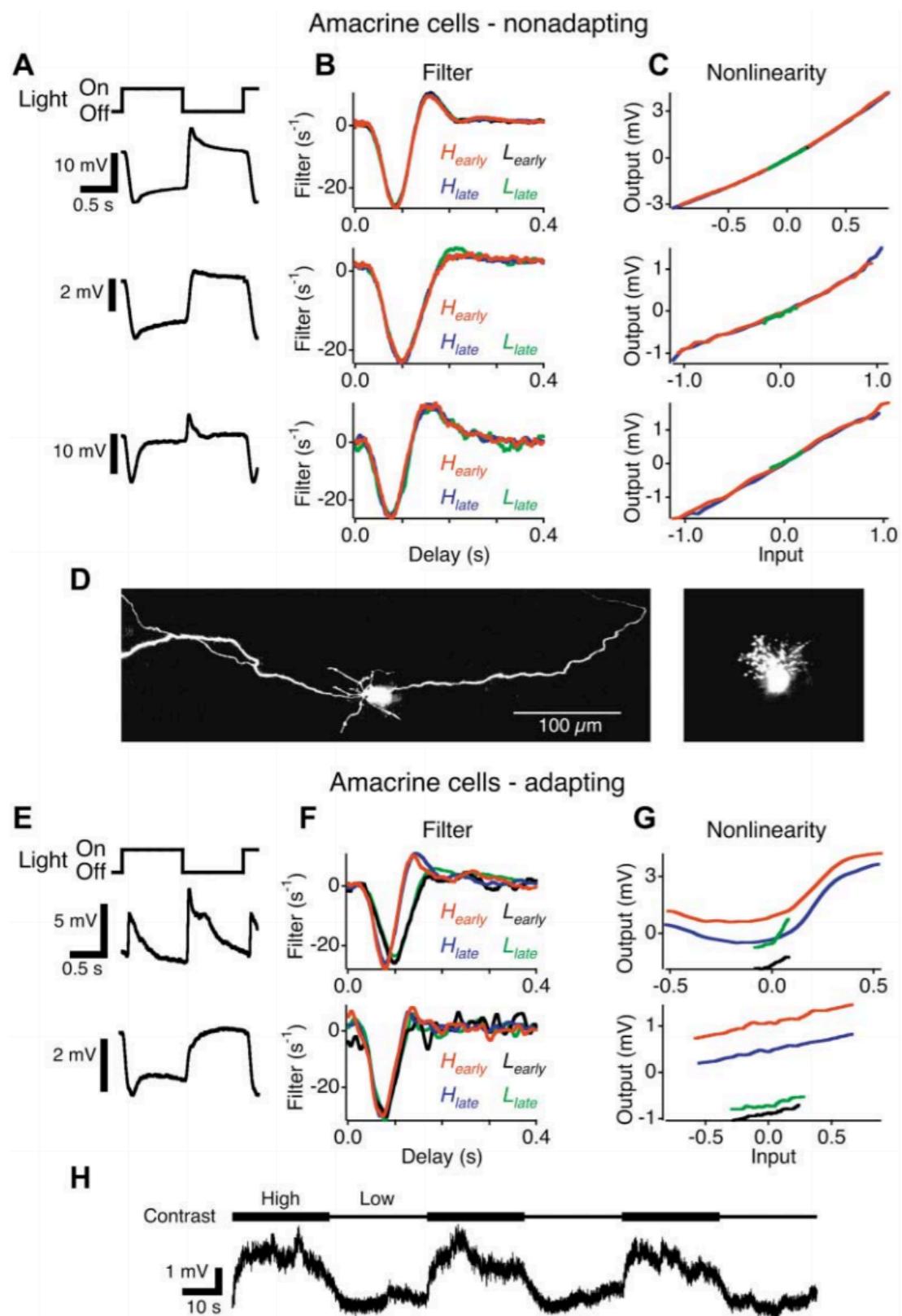
# Contrast adaptation

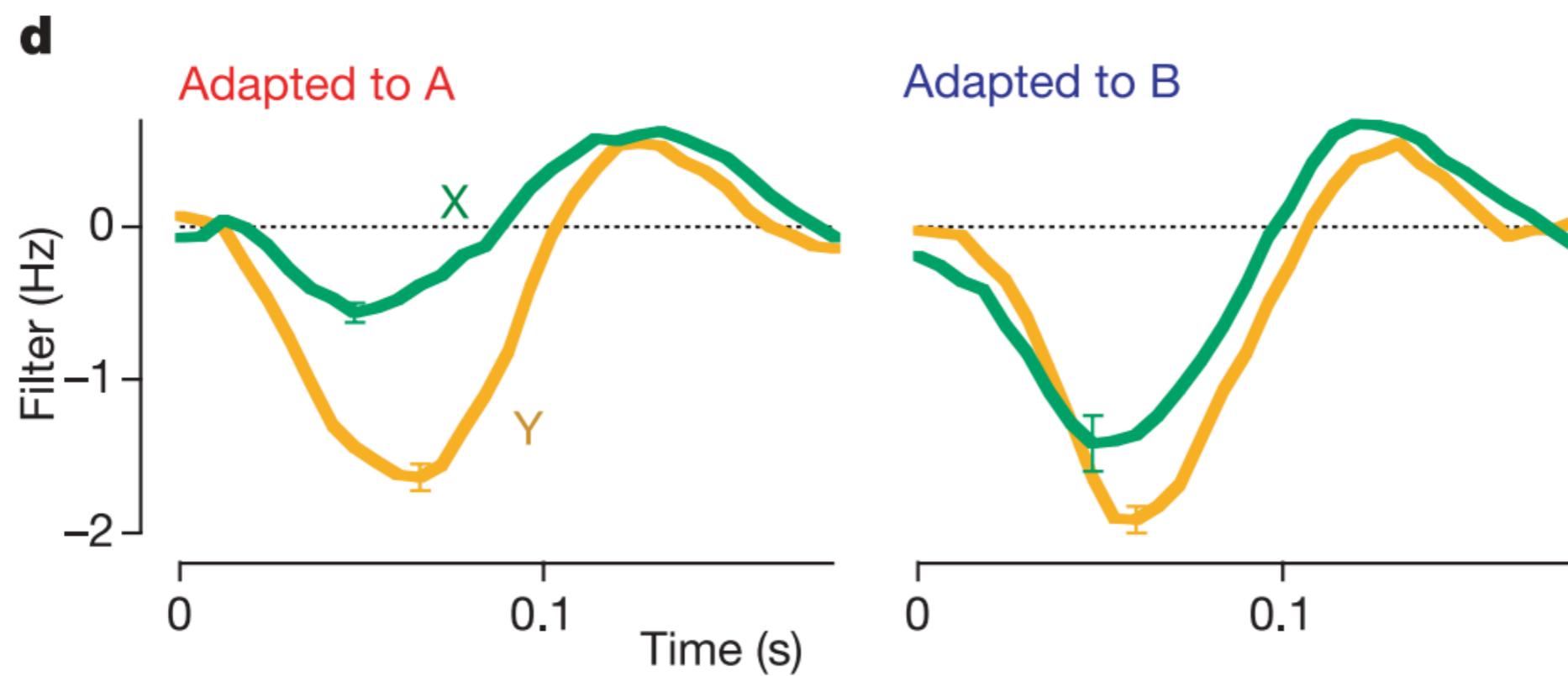
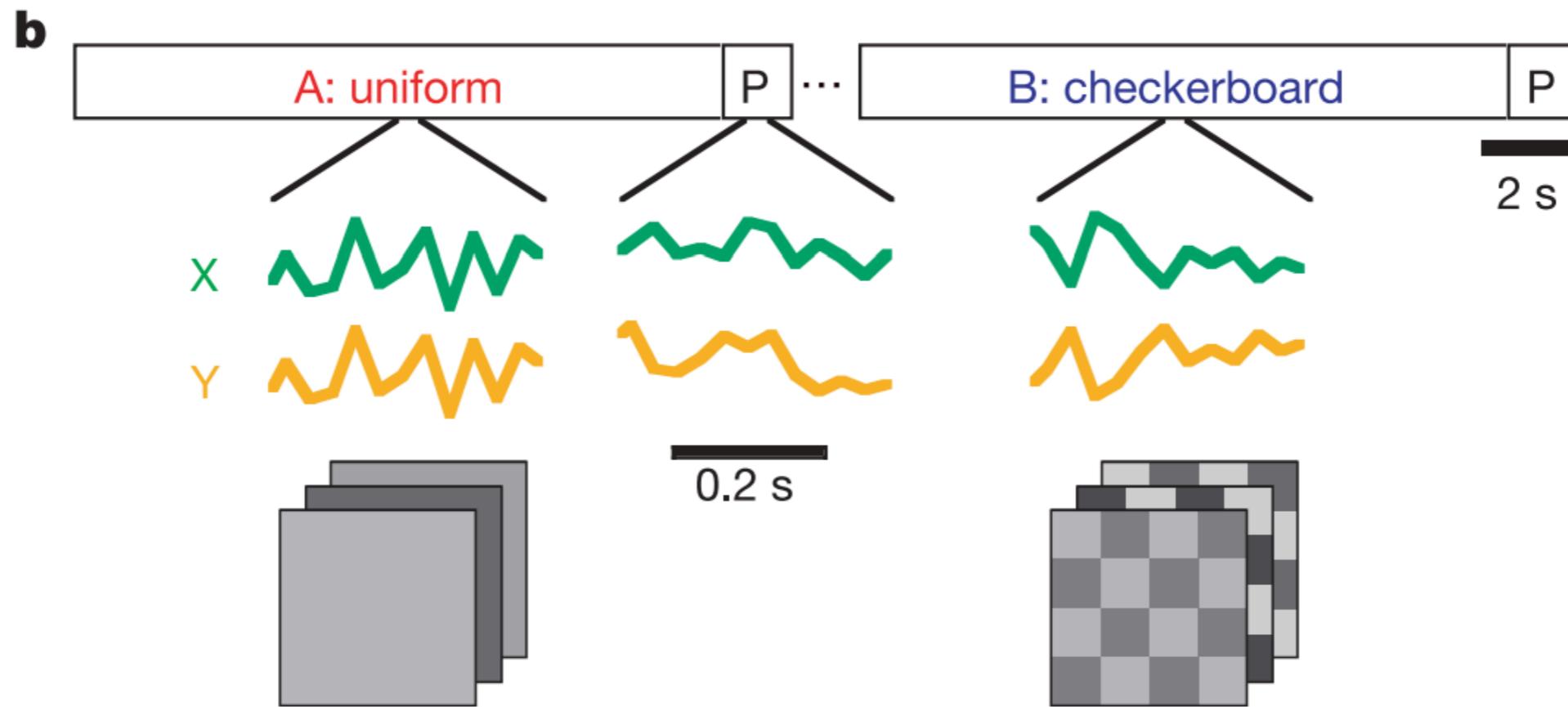
- When you are walking in MIT, because the scene contains wide range of illumination, the retina decreases the sensitivity to cover the whole range. Therefore it is much harder to notice a random person.
- When you are skiing in Killington, because the scene has very low contrast, the retina increases its sensitivity to distinguish different shades of the white. Therefore it is much easier to notice other skiers.

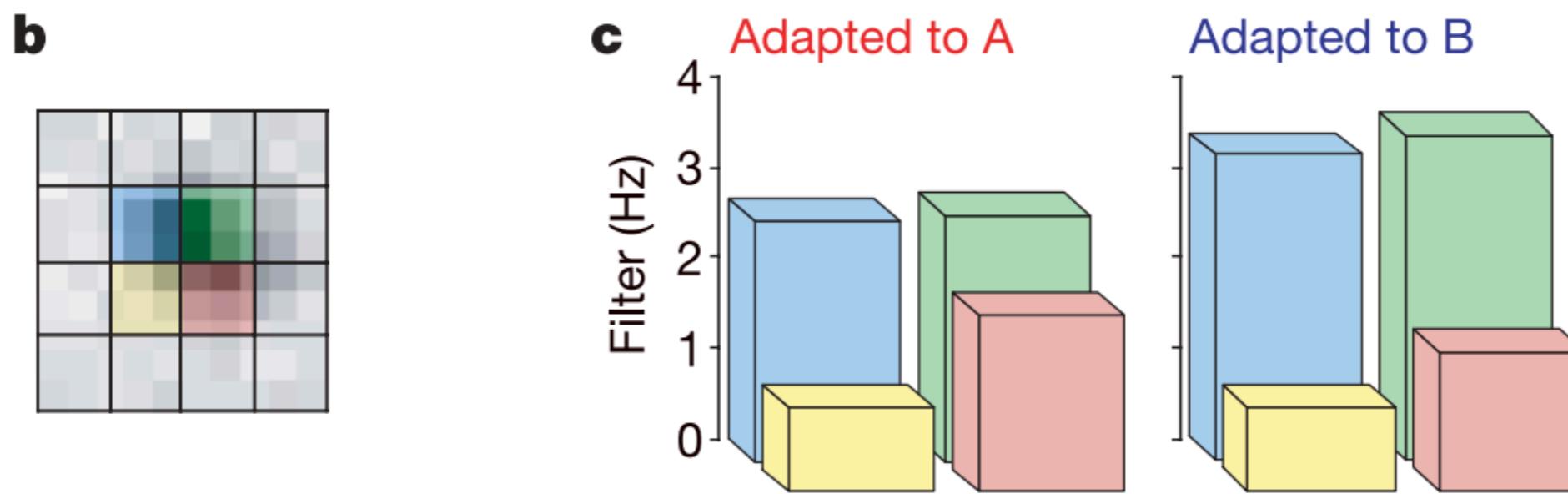
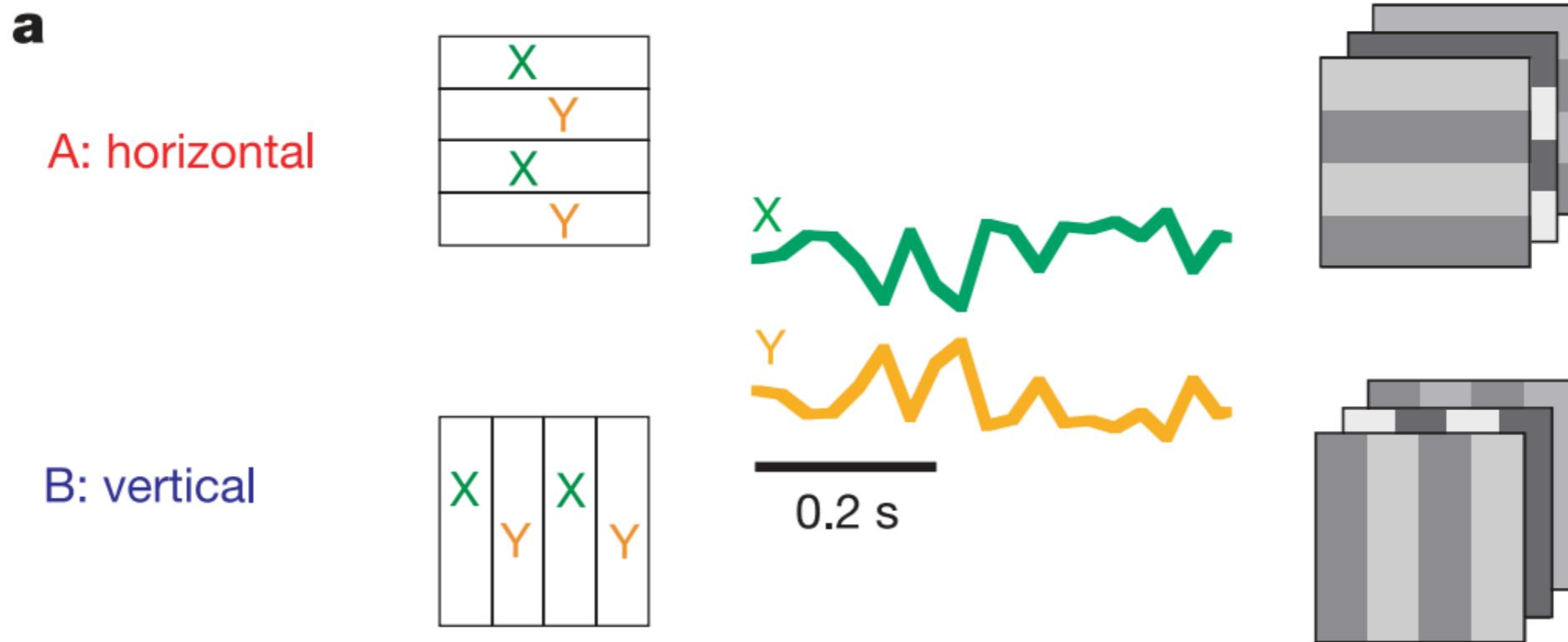


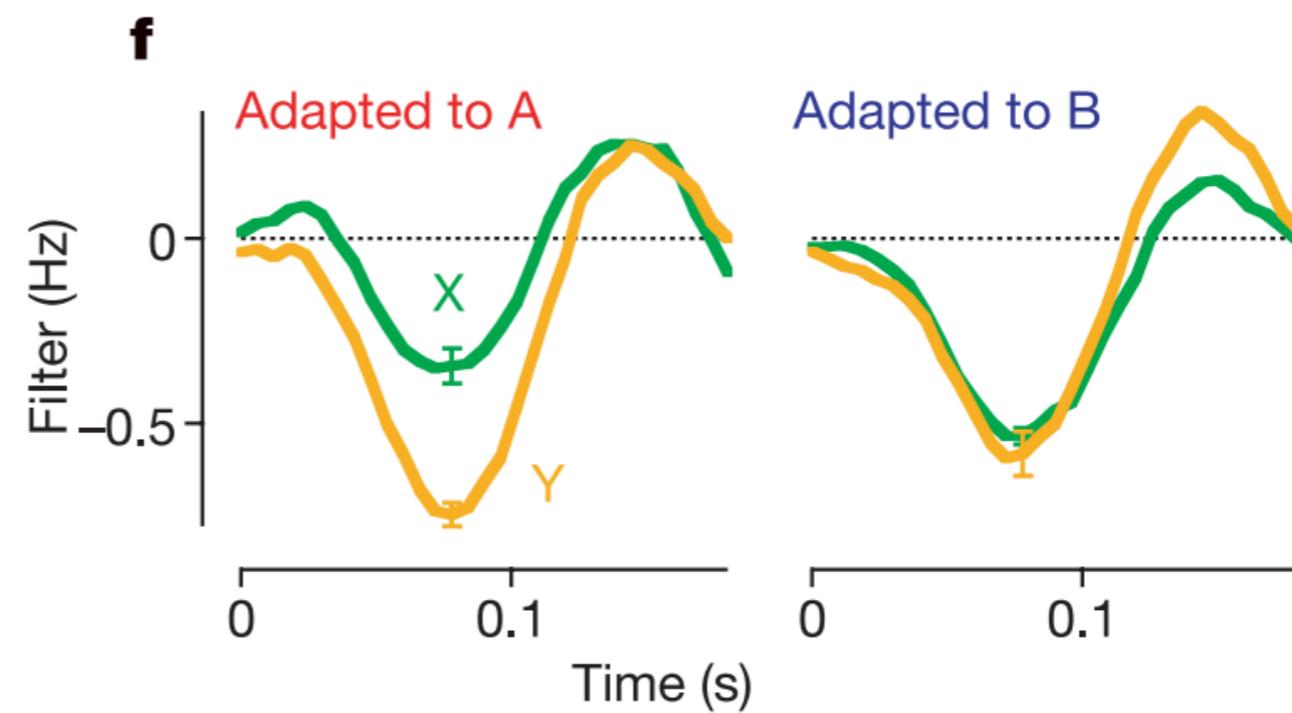
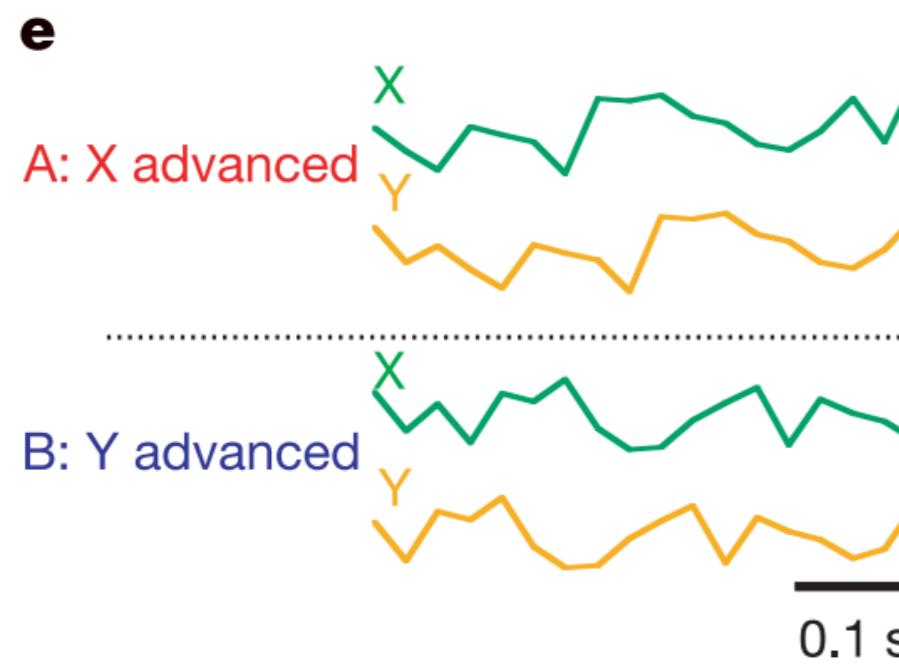
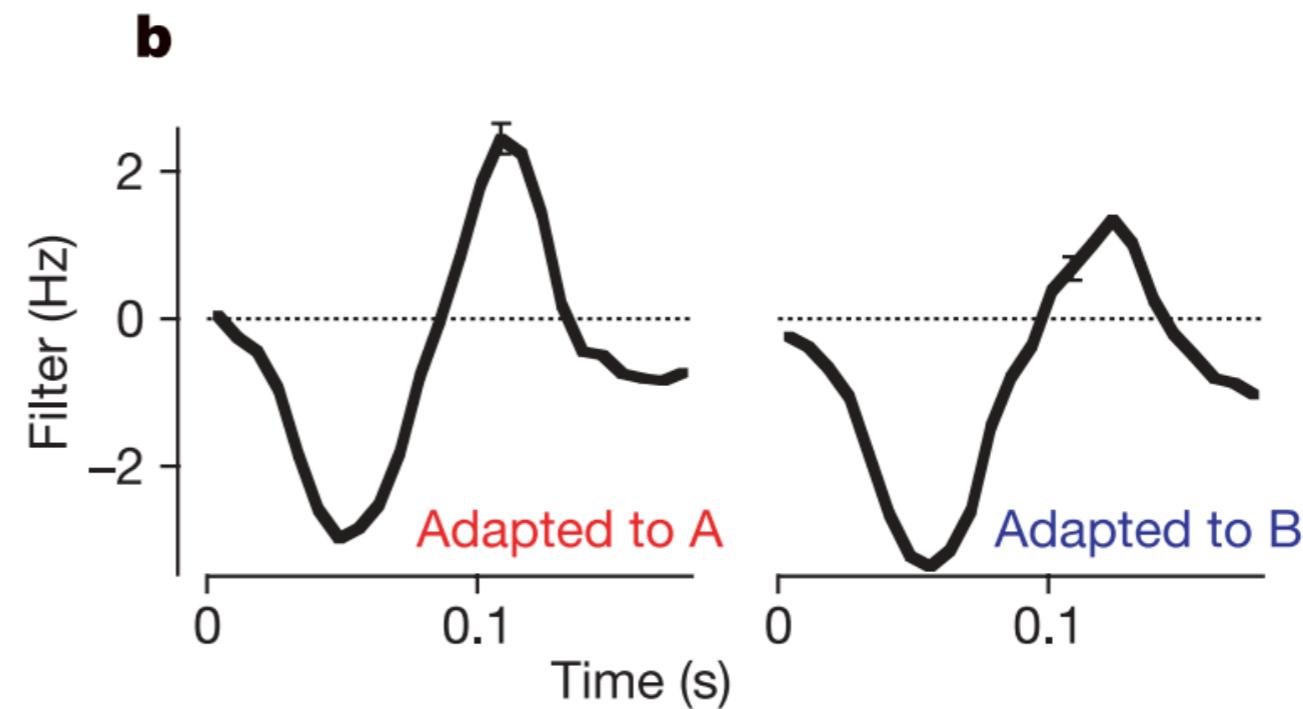
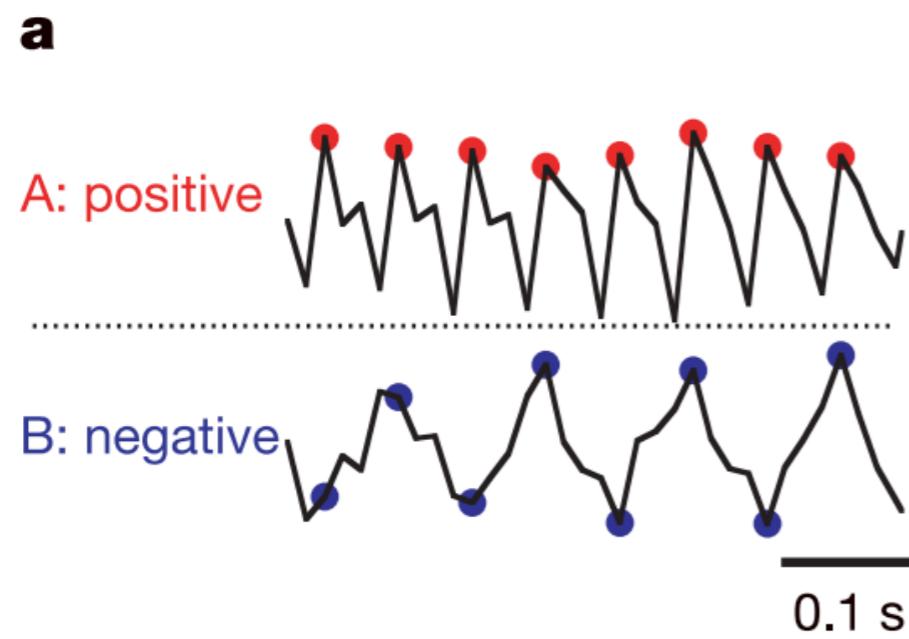






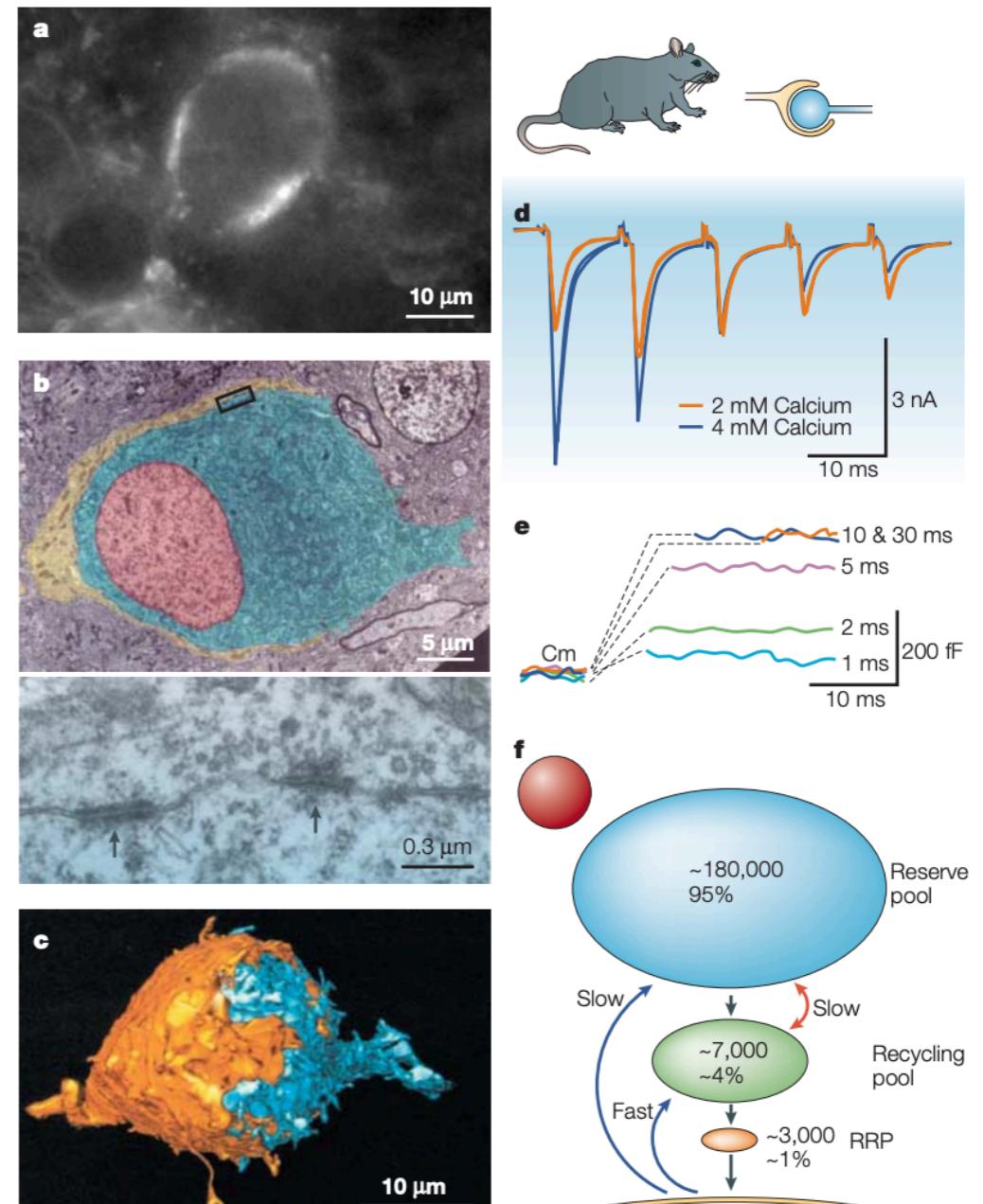






# Vesicle release

- In addition to the long term Hebbian plasticity, there is also short term plasticity which does not change the synaptic weight
- There are three pools: reserve, recycling and ready release pools
- When RRP and recycling pools deplete, the synapse becomes desensitized

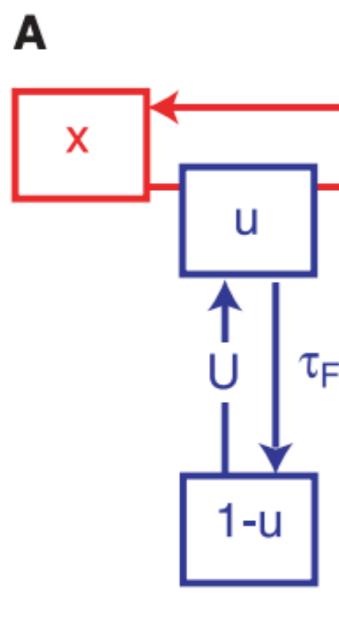


# STP vs STD

- There are actually two competing factors here
- The increase release probability from accumulation of calcium
- The depletion of synaptic resources from release of RRP and recycling pools
- So for a synapse with low release probability, we can actually observe short term potentiation

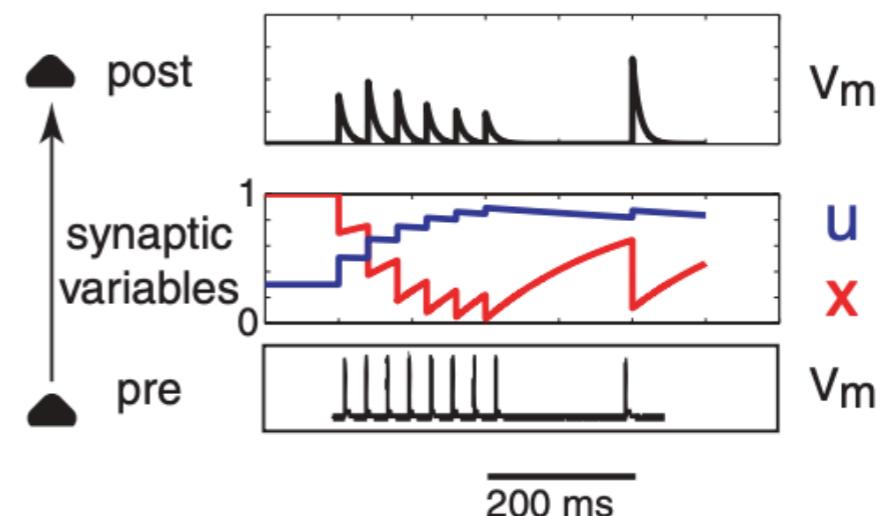
# Simplified STL

- So instead of modeling three vesicle pools, we can just model synapse resources and synapse utilization
- Usually utilization is a slower constant while resources is a faster constant



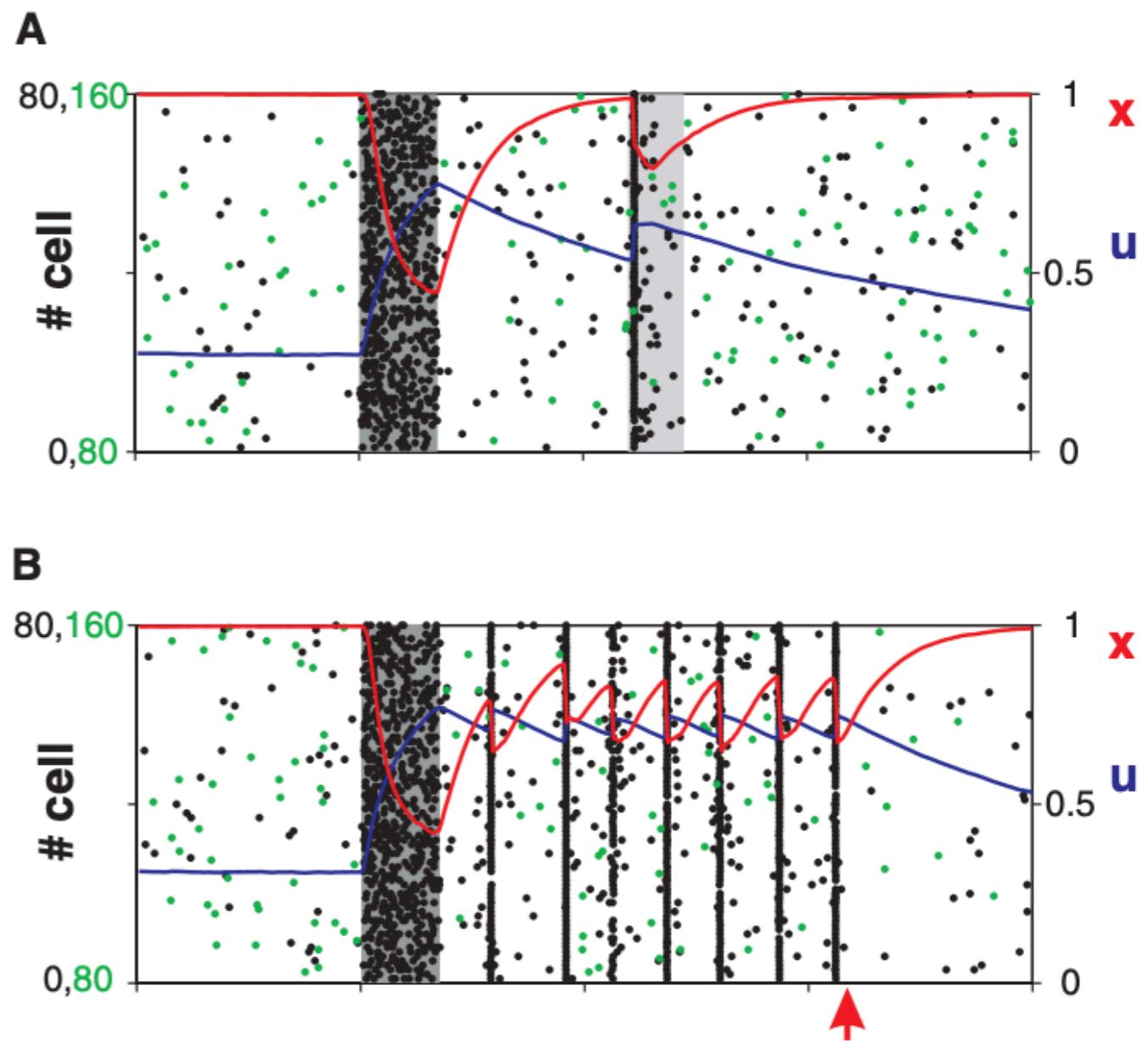
$$\frac{dx}{dt} = \frac{1-x}{\tau_D} - u \ x \ \delta(t-t_{sp})$$

$$\frac{du}{dt} = \frac{U - u}{\tau_F} + U (1-u) \ \delta(t-t_{sp})$$



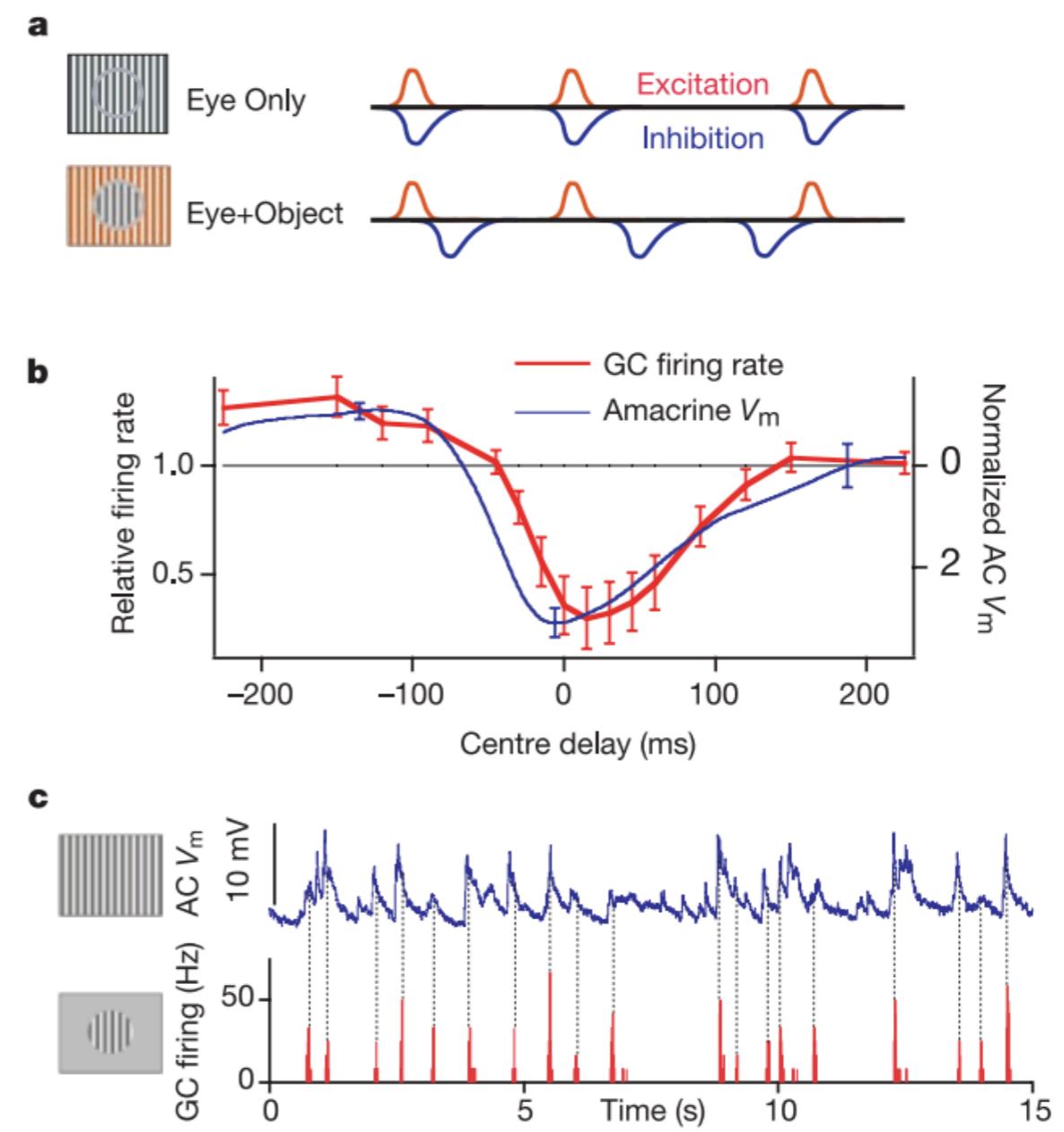
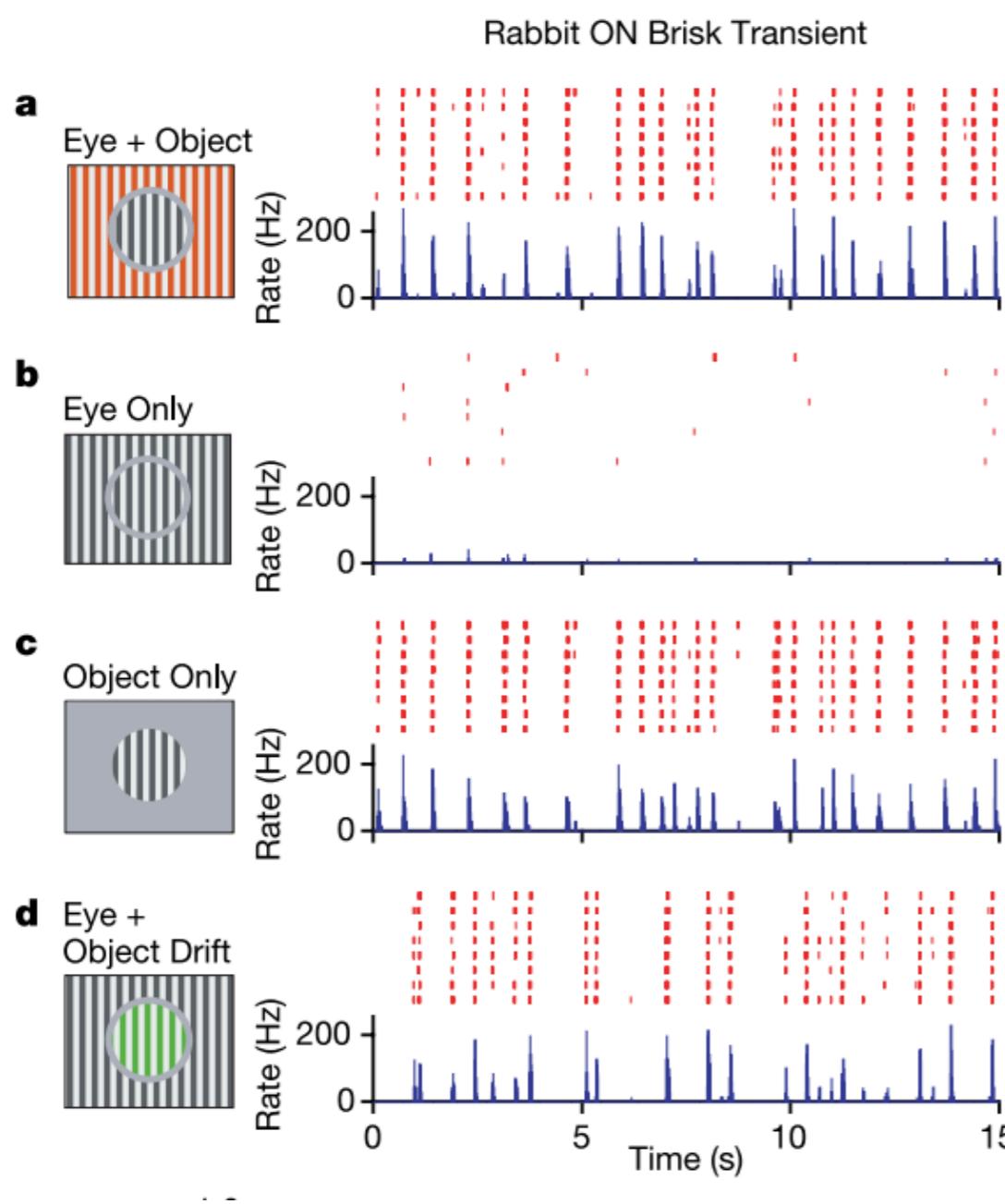
# Assembly without LTP

- Even without LTP, we are able to demonstrate cell assembly
- Generally, with STDP alone, it is very unstable to form assembly. Only with the aids of STL, STDP can stabilize more robustly
- So maybe STL is the main driving force while LTP is a mechanism to consolidate?



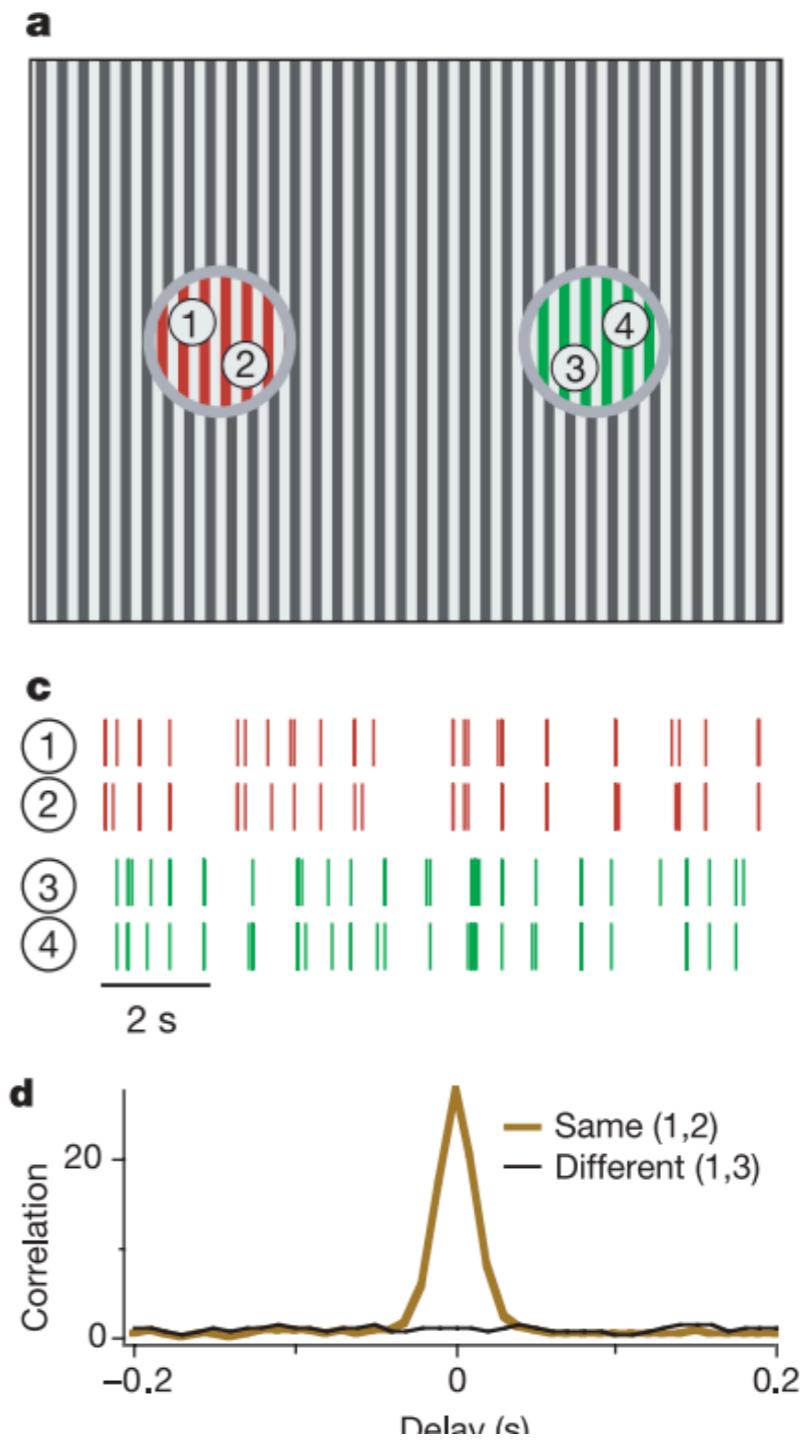
# Pixel vs Object

- Object is a higher level visual feature. Does the retina understand object?
- We can define a simple rough definition of the object—matter that moves together.
- We already mentioned that the receptive fields of the retina will emphasize the boundary of the moving objects.
- But what if there are many objects are moving?



# Vision is dynamic

- Different objects move at different directions.
- So different directions generate different spike sequences.
- RGC already understands object.
- Normal ML trains on static images. However, since brain receives dynamic input, it has extra information to work with.



# Summary

- Retina develops a center surround receptive field for natural image statistics.
- Retina adapts to new statistics to code more efficiently.
- Short term learning has two competing factors, synaptic resource and synaptic utilization.
- The motion of the scene helps the retina understand what an object is.

