

Matthew Branstetter
Dr. Malgorzata Chrzanowska-Jeske
ECE 428 VLSI Design Automation
Fall 2013
Project 1

KL Partitioning Software

This project was a programming assignment. The assignment required creating a partitioning software that used the Kernighan Lin algorithm. The software was generated using C++ in a Linux environment. Two test files were generated from practice exercises in previous homework assignments. The test files can be seen in APPENDIX C, as well as hand generated results from using the Kernighan Algorithm in APPEDIX B. In addition to the two test files generated, a third test file was provided by Dr. Chrzanowska-Jeske. The computer generated results from all three files can be seen in APPEDIX A.

The programming in itself was not very difficult. It required very little research other than a refresher on using C++. Some work was required in researching makefiles. A modest makefile was generated to go along with the software. This would allow the user to compile the program in Linux with a single call to make. In addition, a read me file was included that covered the Copyright Licensing Agreement, Compiling information, running information, and contact information. This was included to allow users in the future to compile, run, modify, the program without any issues. The contact information was provided in the event users needed some assistance. The software was compiled to run on Ubuntu 10.04 or later Linux distributions. I initially included screen generated text throughout the program to help explain any calculations being conducted and debug the program. I used this output to compare to hand generated results in APPENDIX B. I found valgrind to be very useful in addition to gdb to debug any pointer, array, or vector access mistakes in programming. More often the issues were simple grammatical errors. A basic algorithm was generated before programming to help provide an outline. This did require a full understanding of the K-L algorithm. Most of the time on the project was spent ensuring the software married up with actual hand generated results to ensure a complete understanding of the K-L algorithm.

Some modifications for the future would be to include a greedy algorithm for generating the first iteration rather than dividing them up $\{1....n\}$ and $\{n+1.....2n\}$. The code needs to be subdivided up into headers as well to allow for ease in modifications. The use of a class made it simple to access any arrays and also allowed for minimal interaction with them via the user. Two functions were allowed to the public, partition and print the results. This meant a lot of the software was done behind the scenes. Memory usage was not taken into account. Using binary numbers to build the adjacency matrix would cut down on some of the memory usage. Speed or performance was ultimately limited to how the Algorithm works. Utilizing arrays and varying Sorting Algorithms would help in accessing the arrays.

Overall I was very pleased with the outcome. It allowed me to further develop my understanding of the KL algorithm. I would like to have done some work with Simulated Annealing or Fiduccia Matthyses Heuristic. On my own I will refine this program to allow the user to select which algorithm to use.

Appendix A

Computer Generated Results

KL INPUT FILE - "kl.txt"

- a. Iteration number: 1
- b. Partition 1: {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20}
- c. Partition 2: {21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40}
- d. Cost of the partition: 38

- a. Iteration number: 2
- b. Partition 1: {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,17,18,19,20,25}
- c. Partition 2: {16,21,22,23,24,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40}
- d. Cost of the partition: 39

- a. Iteration number: 3
- b. Partition 1: {1,2,3,4,5,6,7,9,10,11,12,13,14,15,17,18,19,20,23,25}
- c. Partition 2: {8,16,21,22,24,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40}
- d. Cost of the partition: 42

- a. Iteration number: 4
- b. Partition 1: {1,2,3,4,6,7,9,10,11,12,13,14,15,17,18,19,20,23,25,26}
- c. Partition 2: {5,8,16,21,22,24,27,28,29,30,31,32,33,34,35,36,37,38,39,40}
- d. Cost of the partition: 44

- a. Iteration number: 5
- b. Partition 1: {1,2,3,4,6,7,8,9,10,11,12,13,14,15,17,18,19,20,23,26}
- c. Partition 2: {5,16,21,22,24,25,27,28,29,30,31,32,33,34,35,36,37,38,39,40}
- d. Cost of the partition: 43

- a. Iteration number: 6
- b. Partition 1: {2,3,4,6,7,8,9,10,11,12,13,14,15,17,18,19,20,23,26,28}
- c. Partition 2: {1,5,16,21,22,24,25,27,29,30,31,32,33,34,35,36,37,38,39,40}
- d. Cost of the partition: 48

- a. Iteration number: 7
- b. Partition 1: {2,3,4,6,7,8,9,10,11,12,13,14,15,18,19,20,23,26,28,36}
- c. Partition 2: {1,5,16,17,21,22,24,25,27,29,30,31,32,33,34,35,37,38,39,40}
- d. Cost of the partition: 49

- a. Iteration number: 8
- b. Partition 1: {2,3,4,6,7,8,9,10,11,12,13,14,15,18,20,22,23,26,28,36}
- c. Partition 2: {1,5,16,17,19,21,24,25,27,29,30,31,32,33,34,35,37,38,39,40}
- d. Cost of the partition: 45

- a. Iteration number: 9
- b. Partition 1: {2,3,4,6,7,8,9,10,12,13,14,15,18,20,22,23,26,28,36,38}
- c. Partition 2: {1,5,11,16,17,19,21,24,25,27,29,30,31,32,33,34,35,37,39,40}
- d. Cost of the partition: 40

- a. Iteration number: 10
- b. Partition 1: {2,3,4,6,7,8,9,10,12,13,15,18,20,22,23,26,28,36,38,40}
- c. Partition 2: {1,5,11,14,16,17,19,21,24,25,27,29,30,31,32,33,34,35,37,39}
- d. Cost of the partition: 34

- a. Iteration number: 11
- b. Partition 1: {2,3,4,6,8,9,10,12,13,15,18,20,22,23,26,28,32,36,38,40}
- c. Partition 2: {1,5,7,11,14,16,17,19,21,24,25,27,29,30,31,33,34,35,37,39}
- d. Cost of the partition: 29

- a. Iteration number: 12
- b. Partition 1: {2,3,4,6,8,10,12,13,15,18,20,22,23,26,28,30,32,36,38,40}
- c. Partition 2: {1,5,7,9,11,14,16,17,19,21,24,25,27,29,31,33,34,35,37,39}
- d. Cost of the partition: 20

- a. Iteration number: 13
- b. Partition 1: {2,4,6,8,10,12,13,15,18,20,22,23,26,28,30,32,34,36,38,40}
- c. Partition 2: {1,3,5,7,9,11,14,16,17,19,21,24,25,27,29,31,33,35,37,39}
- d. Cost of the partition: 9

- a. Iteration number: 14
- b. Partition 1: {2,4,5,6,8,10,12,13,15,18,20,22,23,26,30,32,34,36,38,40}
- c. Partition 2: {1,3,7,9,11,14,16,17,19,21,24,25,27,28,29,31,33,35,37,39}
- d. Cost of the partition: 11

Final Partition 1: {2,4,6,8,10,12,13,15,18,20,22,23,26,28,30,32,34,36,38,40}

Final Partition 2: {1,3,5,7,9,11,14,16,17,19,21,24,25,27,29,31,33,35,37,39}

“kl2.txt”

- a. Iteration number: 1
- b. Partition 1: {1,2,3,4}
- c. Partition 2: {5,6,7,8}
- d. Cost of the partition: 4

- a. Iteration number: 2
- b. Partition 1: {2,3,4,6}
- c. Partition 2: {1,5,7,8}
- d. Cost of the partition: 2

- a. Iteration number: 3
- b. Partition 1: {1,3,4,6}
- c. Partition 2: {2,5,7,8}
- d. Cost of the partition: 2

Final Partition 1: {2,3,4,6}
Final Partition 2: {1,5,7,8}

“kl3.txt”

- a. Iteration number: 1
- b. Partition 1: {1,2,3,4}
- c. Partition 2: {5,6,7,8,9}
- d. Cost of the partition: 5

- a. Iteration number: 2
- b. Partition 1: {1,2,3,6}
- c. Partition 2: {4,5,7,8,9}
- d. Cost of the partition: 4

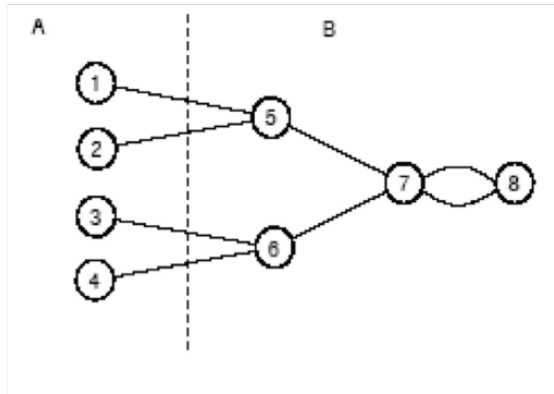
- a. Iteration number: 3
- b. Partition 1: {1,2,5,6}
- c. Partition 2: {3,4,7,8,9}
- d. Cost of the partition: 4

Final Partition 1: {1,2,3,6}
Final Partition 2: {4,5,7,8,9}

Appendix B – Hand Generated Results

“kl2.txt”

Problem 4.



This was problem 4 from Homework 5.

Step 1. Initialization

Iteration number 1 performed was a selection of chosen vertices in increasing order $\{1, \dots, n\}$ and $\{n+1, \dots, 2n\}$, where $2n$ was the total number of vertices.

$A\{1,2,3,4\}$

$B\{5,6,7,8\}$

Step 2. Calculated D Values

$$D1 = E + I = 1$$

$$D2 = 1$$

$$D3 = 1$$

$$D4 = 1$$

$$D5 = 1$$

$$D6 = 1$$

$$D7 = -4$$

$$D8 = -2$$

Step 3. Compute gains (g).

$$g_{15} = D1 + D5 + c_{15} = 1 + 1 - 2 * 1 = 0$$

$$g_{16} = 1 + 1 - 2 * 0 = 2$$

$$g_{17} = 1 + -4 - 2 * 0 = -3$$

$$g_{18} = 1 + -2 - 2 * 0 = -1$$

$$g_{25} = 1 + 1 - 2 * 1 = 0$$

$$g_{26} = 1 + 1 - 2 * 0 = 2$$

$$g_{27} = 1 + -4 - 2 * 0 = -3$$

$g_{28} = 1 + -2 - 2 * 0 = -1$
 $g_{35} = 1 + 1 - 2 * 0 = 2$
 $g_{36} = 1 + 1 - 2 * 1 = 0$
 $g_{37} = 1 + -4 - 2 * 0 = -3$
 $g_{38} = 1 + -2 - 2 * 0 = -1$
 $g_{45} = 1 + 1 - 2 * 0 = 2$
 $g_{46} = 1 + 1 - 2 * 1 = 0$
 $g_{47} = 1 + -4 - 2 * 0 = -3$
 $g_{48} = 1 + -2 - 2 * 0 = -1$

g16 had largest gain = 2

New partitions:

$A'\{2,3,4, \text{locked } 6\}$
 $B'\{5,7,8, \text{locked } 1\}$

Step 4. Updated D values:

$D1' = D1 + 2c_i - 2c_i = 1$
 $D2' = D2 + 2c_i - 2c_i = 1$
 $D3' = D3 + 2c_i - 2c_i = -1$
 $D4' = D4 + 2c_i - 2c_i = -1$
 $D5' = D5 + 2c_i - 2c_i = -1$
 $D6' = D6 + 2c_i - 2c_i = 1$
 $D7' = D7 + 2c_i - 2c_i = -2$
 $D8' = D8 + 2c_i - 2c_i = -2$

Step 3. Update g values:

$g_{25} = 1 + -1 - 2 * 1 = -2$
 $g_{27} = 1 + -2 - 2 * 0 = -1$
 $g_{28} = 1 + -2 - 2 * 0 = -1$
 $g_{35} = -1 + -1 - 2 * 0 = -2$
 $g_{37} = -1 + -2 - 2 * 0 = -3$
 $g_{38} = -1 + -2 - 2 * 0 = -3$
 $g_{45} = -1 + -1 - 2 * 0 = -2$
 $g_{47} = -1 + -2 - 2 * 0 = -3$
 $g_{48} = -1 + -2 - 2 * 0 = -3$

g27 had largest gain: -1

$A'\{3,4, 6, \text{locked } 7\}$
 $B'\{5,8, 1, \text{locked } 2\}$

Step 4. Update D values

$$\begin{aligned}
D1' &= D1 + 2c_i - 2c_i = 1 \\
D2' &= D2 + 2c_i - 2c_i = 1 \\
D3' &= D3 + 2c_i - 2c_i = -1 \\
D4' &= D4 + 2c_i - 2c_i = -1 \\
D5' &= D5 + 2c_i - 2c_i = -1 \\
D6' &= D6 + 2c_i - 2c_i = 1 \\
D7' &= D7 + 2c_i - 2c_i = -2 \\
D8' &= D8 + 2c_i - 2c_i = 2
\end{aligned}$$

Step 3. Compute gains.

$$\begin{aligned}
g_{35} &= -1 + -1 - 2 * 0 = -2 \\
g_{38} &= -1 + 2 - 2 * 0 = 1 \\
g_{45} &= -1 + -1 - 2 * 0 = -2 \\
g_{48} &= -1 + 2 - 2 * 0 = 1
\end{aligned}$$

g_{38} had the largest gain = 1

$A'\{4, 6, 7, \text{locked } 8\}$
 $B'\{5, 1, 2, \text{locked } 3\}$

Step 4. Update D Values

$$\begin{aligned}
D1' &= D1 + 2c_i - 2c_i = 1 \\
D2' &= D2 + 2c_i - 2c_i = 1 \\
D3' &= D3 + 2c_i - 2c_i = -1 \\
D4' &= D4 + 2c_i - 2c_i = -1 \\
D5' &= D5 + 2c_i - 2c_i = -1 \\
D6' &= D6 + 2c_i - 2c_i = 1 \\
D7' &= D7 + 2c_i - 2c_i = -2 \\
D8' &= D8 + 2c_i - 2c_i = 2
\end{aligned}$$

Step 5. Determine k.

Compute final gain.

$$g_{45} = 0$$

$$\text{step k1 } G = g_{16} = 2$$

$$\text{step k2 } G = g_{16} + g_{27} = 2 - 1 = 1$$

$$\text{step k3 } G = g_{16} + g_{27} + g_{38} = 2 - 1 + 1 = 2$$

$$\text{step k4 } G = g_{16} + g_{27} + g_{38} + g_{45} = 2 - 1 + 1 + 0 = 2$$

The max G was step k1 = 2, swap vertices 2 and 1 and begin new iteration.

Step 1.

$A\{2,3,4,6\}$

$B\{1,5,7,8\}$

Step 2.

$$D1 = E + I = -1$$

$$D2 = 1$$

$$D3 = -1$$

$$D4 = -1$$

$$D5 = -1$$

$$D6 = 1$$

$$D7 = -2$$

$$D8 = -2$$

Step 3.

$$g21 = 1 + -1 - 2 * 0 = 0$$

$$g25 = 1 + -1 - 2 * 1 = -2$$

$$g27 = 1 + -2 - 2 * 0 = -1$$

$$g28 = 1 + -2 - 2 * 0 = -1$$

$$g31 = -1 + -1 - 2 * 0 = -2$$

$$g35 = -1 + -1 - 2 * 0 = -2$$

$$g37 = -1 + -2 - 2 * 0 = -3$$

$$g38 = -1 + -2 - 2 * 0 = -3$$

$$g41 = -1 + -1 - 2 * 0 = -2$$

$$g45 = -1 + -1 - 2 * 0 = -2$$

$$g47 = -1 + -2 - 2 * 0 = -3$$

$$g48 = -1 + -2 - 2 * 0 = -3$$

$$g61 = -1 + -1 - 2 * 0 = -2$$

$$g65 = -1 + -1 - 2 * 0 = -2$$

$$g67 = -1 + -2 - 2 * 1 = -5$$

$$g68 = -1 + -2 - 2 * 0 = -3$$

$$g21 = 0$$

$A'\{3,4,6, \text{locked } 1\}$

$B'\{5,7,8, \text{locked } 2\}$

Step 4.

$$D1' = D1 + 2ci - 2ci = -1$$

$$D2' = D2 + 2ci - 2ci = 1$$

$$D3' = D3 + 2ci - 2ci = -1$$

$$D4' = D4 + 2ci - 2ci = -1$$

$$D5' = D5 + 2ci - 2ci = -1$$

$$D6' = D6 + 2ci - 2ci = -1$$

$$D7' = D7 + 2ci - 2ci = -2$$

$$D8' = D8 + 2ci - 2ci = -2$$

Step 3.

$$g_{35} = -1 + -1 - 2 * 0 = -2$$

$$g_{37} = -1 + -2 - 2 * 0 = -3$$

$$g_{38} = -1 + -2 - 2 * 0 = -3$$

$$g_{45} = -1 + -1 - 2 * 0 = -2$$

$$g_{47} = -1 + -2 - 2 * 0 = -3$$

$$g_{48} = -1 + -2 - 2 * 0 = -3$$

$$g_{65} = -1 + -1 - 2 * 0 = -2$$

$$g_{67} = -1 + -2 - 2 * 1 = -5$$

$$g_{68} = -1 + -2 - 2 * 0 = -3$$

$$g_{35} = -2$$

A'{4,6,1, locked 5}

B'{7,8,2, locked 3}

Step 4.

$$D1' = D1 + 2ci - 2ci = -1$$

$$D2' = D2 + 2ci - 2ci = 1$$

$$D3' = D3 + 2ci - 2ci = -1$$

$$D4' = D4 + 2ci - 2ci = -1$$

$$D5' = D5 + 2ci - 2ci = -1$$

$$D6' = D6 + 2ci - 2ci = 1$$

$$D7' = D7 + 2ci - 2ci = 0$$

$$D8' = D8 + 2ci - 2ci = -2$$

Step 3.

$$g_{47} = -1 + 0 - 2 * 0 = -1$$

$$g_{48} = -1 + -2 - 2 * 0 = -3$$

$$g_{67} = 1 + 0 - 2 * 1 = -1$$

$$g_{68} = 1 + -2 - 2 * 0 = -1$$

$$g_{47} = -1$$

A'{6,1,5, locked 7}

B'{8,2,3, locked 4}

Step 4.

$$D1' = D1 + 2ci - 2ci = -1$$

$$D2' = D2 + 2ci - 2ci = 1$$

$$D3' = D3 + 2ci - 2ci = -1$$

$$D4' = D4 + 2ci - 2ci = -1$$

$$D5' = D5 + 2ci - 2ci = -1$$

$$D6' = D6 + 2ci - 2ci = 1$$

$$D7' = D7 + 2c_i - 2c_i = 0$$

$$D8' = D8 + 2c_i - 2c_i = 2$$

Step 5.

$$g_{68} = 0$$

$$\text{step k1 } G = g_{21} = 0$$

$$\text{step k2 } G = g_{21} + g_{35} = 0 - 2 = -2$$

$$\text{step k3 } G = g_{21} + g_{35} + g_{47} = 0 - 2 - 1 = -3$$

$$\text{step k4 } G = g_{21} + g_{35} + g_{47} + g_{68} = 0 - 2 - 1 + 0 = -3$$

Since the maximum G value for any k step was 0, then the previous iteration is the best choice and can stop calculations.

Iterations listed with partitions and cost of partitions.

a. Iteration number: 1

b. Partition 1: {1,2,3,4}

c. Partition 2: {5,6,7,8}

d. Cost of the partition: 4

a. Iteration number: 2

b. Partition 1: {2,3,4,6}

c. Partition 2: {1,5,7,8}

d. Cost of the partition: 2

a. Iteration number: 3

b. Partition 1: {1,3,4,6}

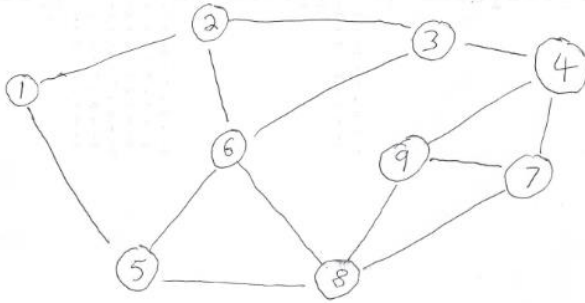
c. Partition 2: {2,5,7,8}

d. Cost of the partition: 2

Final Partition 1: {2,3,4,6}

Final Partition 2: {1,5,7,8}

"kl3.txt"



Step 1. Initialization

Iteration number 1 performed was a selection of chosen vertices in increasing order $\{1, \dots, n\}$ and $\{n+1, \dots, 2n\}$, where $2n$ was the total number of vertices.

Step 1.

Iteration number: 1.

$\{1, 2, 3, 4\}$

$\{5, 6, 7, 8, 9\}$

Step 2.

$$D1' = E + I = 0$$

$$D2' = -1$$

$$D3' = -1$$

$$D4' = 1$$

$$D5' = -1$$

$$D6' = 0$$

$$D7' = -2$$

$$D8' = -3$$

$$D9' = -1$$

Step 3.

$$g_{15} = 0 + -1 - 2 * 1 = -3$$

$$g_{16} = 0 + 0 - 2 * 0 = 0$$

$$g_{17} = 0 + -2 - 2 * 0 = -2$$

$$g_{18} = 0 + -3 - 2 * 0 = -3$$

$$g_{19} = 0 + -1 - 2 * 0 = -1$$

$$g_{25} = -1 + -1 - 2 * 0 = -2$$

$$g_{26} = -1 + 0 - 2 * 1 = -3$$

$$g_{27} = -1 + -2 - 2 * 0 = -3$$

$$g_{28} = -1 + -3 - 2 * 0 = -4$$

$$g_{29} = -1 + -1 - 2 * 0 = -2$$

$g_{35} = -1 + -1 - 2 * 0 = -2$
 $g_{36} = -1 + 0 - 2 * 1 = -3$
 $g_{37} = -1 + -2 - 2 * 0 = -3$
 $g_{38} = -1 + -3 - 2 * 0 = -4$
 $g_{39} = -1 + -1 - 2 * 0 = -2$
 $g_{45} = 1 + -1 - 2 * 0 = 0$
 $g_{46} = 1 + 0 - 2 * 0 = 1$
 $g_{47} = 1 + -2 - 2 * 1 = -3$
 $g_{48} = 1 + -3 - 2 * 0 = -2$
 $g_{49} = 1 + -1 - 2 * 1 = -2$

$g_{46} = 1$

$A'\{1,2,3, \text{locked } 6\}$
 $B'\{5,7,8,9, \text{locked } 4\}$

Step 4.

$D1' = D1 + 2ci - 2ci = 0$
 $D2' = D2 + 2ci - 2ci = -3$
 $D3' = D3 + 2ci - 2ci = -1$
 $D4' = D4 + 2ci - 2ci = 1$
 $D5' = D5 + 2ci - 2ci = 1$
 $D6' = D6 + 2ci - 2ci = 0$
 $D7' = D7 + 2ci - 2ci = -2$
 $D8' = D8 + 2ci - 2ci = -3$
 $D9' = D9 + 2ci - 2ci = -3$

Step 3.

$g_{15} = 0 + 1 - 2 * 1 = -1$
 $g_{17} = 0 + -2 - 2 * 0 = -2$
 $g_{18} = 0 + -3 - 2 * 0 = -3$
 $g_{19} = 0 + -3 - 2 * 0 = -3$
 $g_{25} = -3 + 1 - 2 * 0 = -2$
 $g_{27} = -3 + -2 - 2 * 0 = -5$
 $g_{28} = -3 + -3 - 2 * 0 = -6$
 $g_{29} = -3 + -3 - 2 * 0 = -6$
 $g_{35} = -1 + 1 - 2 * 0 = 0$
 $g_{37} = -1 + -2 - 2 * 0 = -3$
 $g_{38} = -1 + -3 - 2 * 0 = -4$
 $g_{39} = -1 + -3 - 2 * 0 = -4$

$g_{35} = 0$

$A'\{1,2,6, \text{locked } 5\}$
 $B'\{7,8,9,4 \text{ locked } 3\}$

Step 4.

$$\begin{aligned}D1' &= D1 + 2ci - 2ci = -2 \\D2' &= D2 + 2ci - 2ci = -1 \\D3' &= D3 + 2ci - 2ci = -1 \\D4' &= D4 + 2ci - 2ci = 1 \\D5' &= D5 + 2ci - 2ci = 1 \\D6' &= D6 + 2ci - 2ci = 0 \\D7' &= D7 + 2ci - 2ci = -2 \\D8' &= D8 + 2ci - 2ci = -1 \\D9' &= D9 + 2ci - 2ci = -3\end{aligned}$$

Step 3.

$$\begin{aligned}g17 &= -2 + -2 - 2 * 0 = -4 \\g18 &= -2 + -1 - 2 * 0 = -3 \\g19 &= -2 + -3 - 2 * 0 = -5 \\g27 &= -1 + -2 - 2 * 0 = -3 \\g28 &= -1 + -1 - 2 * 0 = -2 \\g29 &= -1 + -3 - 2 * 0 = -4\end{aligned}$$

$$g28 = -1$$

A'{1,6,5, locked 8}
B'{7,9,4,3, locked 2}

Step 4.

$$\begin{aligned}D1' &= D1 + 2ci - 2ci = 0 \\D2' &= D2 + 2ci - 2ci = -1 \\D3' &= D3 + 2ci - 2ci = -1 \\D4' &= D4 + 2ci - 2ci = 1 \\D5' &= D5 + 2ci - 2ci = 1 \\D6' &= D6 + 2ci - 2ci = 0 \\D7' &= D7 + 2ci - 2ci = 0 \\D8' &= D8 + 2ci - 2ci = -1 \\D9' &= D9 + 2ci - 2ci = -1\end{aligned}$$

Step 3.

$$\begin{aligned}g17 &= 0 + 0 - 2 * 0 = 0 \\g19 &= 0 + -1 - 2 * 0 = -1\end{aligned}$$

$$g17 = 0$$

A'{6,5,8, locked 7}
B'{9,4,3,2, locked 1}

Step 4.

$$D1' = D1 + 2c_i - 2c_i = 0$$

$$D2' = D2 + 2c_i - 2c_i = -1$$

$$D3' = D3 + 2c_i - 2c_i = -1$$

$$D4' = D4 + 2c_i - 2c_i = 1$$

$$D5' = D5 + 2c_i - 2c_i = 1$$

$$D6' = D6 + 2c_i - 2c_i = 0$$

$$D7' = D7 + 2c_i - 2c_i = 0$$

$$D8' = D8 + 2c_i - 2c_i = -1$$

$$D9' = D9 + 2c_i - 2c_i = 1$$

Step 5.

$$g_9 = 1$$

$$\text{step k1 } G = g_{46} = 1$$

$$\text{step k2 } G = g_{46} + g_{35} = 1 + 0 = 1$$

$$\text{step k3 } G = g_{46} + g_{35} + g_{28} = 1 + 0 - 1 = 0$$

$$\text{step k4 } G = g_{46} + g_{35} + g_{28} + g_{17} = 1 + 0 - 1 + 0 = 0$$

$$\text{step k5 } G = g_{46} + g_{35} + g_{28} + g_{17} + g_9 = 1 + 0 - 1 + 0 + 1 = 1$$

The max G was step k1 = 1, swap vertices 4 and 6 and begin new iteration.

Iteration number: 2.

Step 1.

$$A\{1,2,3,6\}$$

$$B\{4,5,7,8,9\}$$

Step 2.

$$D1' = D1 + 2c_i - 2c_i = 0$$

$$D2' = D2 + 2c_i - 2c_i = -3$$

$$D3' = D3 + 2c_i - 2c_i = -1$$

$$D4' = D4 + 2c_i - 2c_i = -1$$

$$D5' = D5 + 2c_i - 2c_i = 1$$

$$D6' = D6 + 2c_i - 2c_i = 0$$

$$D7' = D7 + 2c_i - 2c_i = -2$$

$$D8' = D8 + 2c_i - 2c_i = -3$$

$$D9' = D9 + 2c_i - 2c_i = -3$$

Step 3.

$$g_{14} = 0 + -1 - 2 * 0 = -1$$

$$g_{15} = 0 + 1 - 2 * 1 = -1$$

$g_{17} = 0 + -2 - 2 * 0 = -2$
 $g_{18} = 0 + -3 - 2 * 0 = -3$
 $g_{19} = 0 + -3 - 2 * 0 = -3$
 $g_{24} = -3 + -1 - 2 * 0 = -4$
 $g_{25} = -3 + 1 - 2 * 0 = -2$
 $g_{27} = -3 + -2 - 2 * 0 = -5$
 $g_{28} = -3 + -3 - 2 * 0 = -6$
 $g_{29} = -3 + -3 - 2 * 0 = -6$
 $g_{34} = -1 + -1 - 2 * 1 = -4$
 $g_{35} = -1 + 1 - 2 * 0 = 0$
 $g_{37} = -1 + -2 - 2 * 0 = -3$
 $g_{38} = -1 + -3 - 2 * 0 = -4$
 $g_{39} = -1 + -3 - 2 * 0 = -4$
 $g_{64} = 0 + -1 - 2 * 0 = -1$
 $g_{65} = 0 + 1 - 2 * 1 = -1$
 $g_{67} = 0 + -2 - 2 * 1 = -4$
 $g_{68} = 0 + -3 - 2 * 0 = -3$
 $g_{69} = 0 + -3 - 2 * 0 = -3$

$g_{35} = 0$

$A'\{1,2,6 \text{ locked } 5\}$
 $B'\{4,7,8,9, \text{ locked } 3\}$

Step 4.

$D1' = D1 + 2ci - 2ci = -2$
 $D2' = D2 + 2ci - 2ci = -1$
 $D3' = D3 + 2ci - 2ci = -1$
 $D4' = D4 + 2ci - 2ci = -3$
 $D5' = D5 + 2ci - 2ci = 1$
 $D6' = D6 + 2ci - 2ci = 0$
 $D7' = D7 + 2ci - 2ci = -2$
 $D8' = D8 + 2ci - 2ci = -1$
 $D9' = D9 + 2ci - 2ci = -3$

Step 3.

$g_{14} = -2 + -3 - 2 * 0 = -5$
 $g_{17} = -2 + -2 - 2 * 0 = -4$
 $g_{18} = -2 + -1 - 2 * 0 = -3$
 $g_{19} = -2 + -3 - 2 * 0 = -5$
 $g_{24} = -1 + -3 - 2 * 0 = -4$
 $g_{27} = -1 + -2 - 2 * 0 = -3$
 $g_{28} = -1 + -1 - 2 * 0 = -2$
 $g_{29} = -1 + -3 - 2 * 0 = -4$
 $g_{64} = 0 + -3 - 2 * 0 = -3$
 $g_{67} = 0 + -2 - 2 * 1 = -4$

$$g_{68} = 0 + -1 - 2 * 0 = -1$$

$$g_{69} = 0 + -3 - 2 * 0 = -3$$

$$g_{68} = -1$$

A'{1,2,5, locked 8}

B'{4,7,9,3, locked 6}

Step 4.

$$D1' = D1 + 2ci - 2ci = -2$$

$$D2' = D2 + 2ci - 2ci = 1$$

$$D3' = D3 + 2ci - 2ci = -1$$

$$D4' = D4 + 2ci - 2ci = -3$$

$$D5' = D5 + 2ci - 2ci = 1$$

$$D6' = D6 + 2ci - 2ci = 0$$

$$D7' = D7 + 2ci - 2ci = -2$$

$$D8' = D8 + 2ci - 2ci = -1$$

$$D9' = D9 + 2ci - 2ci = -1$$

Step 3.

$$g_{14} = -2 + -3 - 2 * 0 = -5$$

$$g_{17} = -2 + -2 - 2 * 0 = -4$$

$$g_{19} = -2 + -1 - 2 * 0 = -3$$

$$g_{24} = 1 + -3 - 2 * 0 = -2$$

$$g_{27} = 1 + -2 - 2 * 0 = -1$$

$$g_{29} = 1 + -1 - 2 * 0 = 0$$

$$g_{29} = 0$$

A'{1,5,8, locked 9}

B'{4,7,3,6, locked 2}

Step 4.

$$D1' = D1 + 2ci - 2ci = 0$$

$$D2' = D2 + 2ci - 2ci = 1$$

$$D3' = D3 + 2ci - 2ci = -1$$

$$D4' = D4 + 2ci - 2ci = -1$$

$$D5' = D5 + 2ci - 2ci = 1$$

$$D6' = D6 + 2ci - 2ci = 0$$

$$D7' = D7 + 2ci - 2ci = 0$$

$$D8' = D8 + 2ci - 2ci = -1$$

$$D9' = D9 + 2ci - 2ci = -1$$

Step 3.

$$g_{14} = 0 + -1 - 2 * 0 = -1$$

$$g_{17} = 0 + 0 - 2 * 0 = 0$$

$$g_{17} = 0$$

A' {5,8,9, locked 7}

B' {4,3,6,2, locked 1}

$$D1' = D1 + 2c_i - 2c_i = 0$$

$$D2' = D2 + 2c_i - 2c_i = 1$$

$$D3' = D3 + 2c_i - 2c_i = -1$$

$$D4' = D4 + 2c_i - 2c_i = 1$$

$$D5' = D5 + 2c_i - 2c_i = 1$$

$$D6' = D6 + 2c_i - 2c_i = 0$$

$$D7' = D7 + 2c_i - 2c_i = 0$$

$$D8' = D8 + 2c_i - 2c_i = -1$$

$$D9' = D9 + 2c_i - 2c_i = -1$$

$$g_4 = 1$$

$$\text{step } k1 \quad G = g_{35} = 0$$

$$\text{step } k2 \quad G = g_{35} + g_{68} = 0 - 1 = -1$$

$$\text{step } k3 \quad G = g_{35} + g_{68} + g_{29} = 0 - 1 + 0 = -1$$

$$\text{step } k4 \quad G = g_{35} + g_{68} + g_{29} + g_{17} = 0 - 1 + 0 + 0 = -1$$

$$\text{step } k5 \quad G = g_{35} + g_{68} + g_{29} + g_{17} + g_4 = 0 - 1 + 0 + 0 + 1 = 0$$

Since the maximum G value for any k step was 0, then the previous iteration is the best choice and can stop calculations.

Iterations listed with partitions and cost of partitions.

a. Iteration number: 1

b. Partition 1: {1,2,3,4}

c. Partition 2: {5,6,7,8,9}

Cost of the partition: 5

a. Iteration number: 2

b. Partition 1: {1,2,3,6}

c. Partition 2: {4,5,7,8,9}

Cost of the partition: 4

a. Iteration number: 3

b. Partition 1: {1,2,5,6}

c. Partition 2: {3,4,7,8,9}

Cost of the partition: 4

Final Partition 1: {1,2,3,6}

Final Partition 2: {4,5,7,8,9}

Appendix C - Input Files

“kl.txt”

40
1,11,19,23,27
2,5,13,18,20,36,40
3,7,14,17,25,33
4,8,13,15,20,37,38,40
5,2,7,8,11,24
6,10,13,20,34,40
7,3,5,16,21,29
8,4,5,17,30,32
9,11,17,31,33
10,6,15,17,20,34
11,1,5,9,37
12,18,22,34
13,2,4,6,36,40
14,3,19,35,37
15,4,10,30,38
16,7,19,24,29,39
17,3,8,9,10,21,24,37
18,2,12,22,28
19,1,14,16,39
20,2,4,6,10,34
21,7,17,25,39
22,12,18,25,26
23,1,26,30,32
24,5,16,17,35
25,3,21,22,35
26,22,23,38
27,1,31,39
28,18,29,30,34,37
29,7,16,28,31,33
30,8,15,23,28,36
31,9,27,29,37
32,8,23,34,36
33,3,9,29,39
34,6,10,12,20,28,32
35,14,24,25
36,2,13,30,32
37,4,11,14,17,28,31
38,4,15,26
39,16,19,21,27,33

40,2,4,6,13

“kl2.txt”

8
1,5
2,5
3,6
4,6
5,1,2,7
6,3,4,7
7,5,6,8,8
8,7,7

“kl3.txt”

9
1,2,5
2,1,3,6
3,2,4,6
4,3,7,9
5,1,6,8
6,2,3,5,7
7,4,6,8,9
8,5,7,9
9,4,7,8