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ECE 428 VLSI Design Automation
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Project 1

KL Partitioning Software

This project was a programming assignment. The assignment required creating partitioning software that used the Kernighan Lin algorithm. The software was generated using C++ in a Linux environment. Four test files were generated from practice exercises in previous homework assignments. The test files can be seen in APPENDIX C, as well as hand generated results from using the Kernighan Algorithm in APPENDIX B. In addition to the test files generated, a fifth test file was provided by Dr. Chrzanowska-Jeske. The computer generated results from all test files can be seen in APPENDIX A.

The programming in itself was not difficult and required very little research other than a refresher on using C++. Some work was required in researching makefiles. A modest makefile was generated to go along with the software. This would allow the user to compile the program in Linux with a single call to make. A read me file was included that covered the Copyright Licensing Agreement, compiling, running, and contact information. The read me file instructs users how to compile, run, and modify the program. The contact information was provided in the event users need some assistance. The software was compiled to run on Ubuntu 10.04 or later Linux distributions. I initially included screen generated text throughout the program to help explain any calculations being conducted and debug the program. I used this output to compare to hand generated results in APPENDIX B. I found valgrind to be very useful in addition to gdb to debug any pointer, array, or vector access mistakes in programming. More often the issues were simple grammatical errors. A basic algorithm was generated before programming to help provide an outline. This did require a full understanding of the K-L algorithm. Most of the time on the project was spent ensuring the software married up with actual hand generated results to ensure a complete understanding of the K-L algorithm.

Some modifications for the future would be to include a greedy algorithm for generating the first iteration rather than dividing V up $\{1....n\}$ and $\{n+1.....2n\}$. The use of a class made it simple to access any arrays and also allowed for minimal interaction with them via the user. Two functions were allowed to the public, `partition()` and `printResults()`. This meant a lot of the work was done behind the scenes. Memory usage was not taken into account. Using binary numbers to build the adjacency matrix would cut down on some of the memory expense. The tradeoff to binary numbers would be handling graphs with weights. Speed or performance was ultimately limited to how the algorithm works. Utilizing arrays with various sorting algorithms would help in accessing the arrays.

Overall I was very pleased with the outcome. It allowed me to further develop my understanding of the KL algorithm. I would like to have done some work with simulated annealing or Fiduccia Matthyyses heuristic. I would have liked to also modify the program to take weighted graphs into account and improve on the error checking. Hypergraphs cannot be handled well with the KL algorithm, so a check to confirm the adjacency matrix is not a hypergraph ahead of time could halt the program sooner. Then displaying an error message in the event a hypergraph was detected would notify the user. Some of these suggestions will be added in future revisions.

Appendix A

Computer Generated Results

KL INPUT FILE - "kl.txt"

a. Iteration number: 1
b. Partition 1: {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20}
c. Partition 2: {21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40}
Cost of the partition: 38

a. Iteration number: 2
b. Partition 1: {2,3,4,5,6,7,8,10,13,14,15,17,20,21,24,25,35,37,38,40}
c. Partition 2: {1,9,11,12,16,18,19,22,23,26,27,28,29,30,31,32,33,34,36,39}
Cost of the partition: 22

a. Iteration number: 3
b. Partition 1: {2,4,6,8,10,12,13,15,18,20,22,23,26,28,30,32,34,36,38,40}
c. Partition 2: {1,3,5,7,9,11,14,16,17,19,21,24,25,27,29,31,33,35,37,39}
Cost of the partition: 9

a. Iteration number: 4
b. Partition 1: {1,3,5,7,9,11,14,16,17,19,21,24,25,27,29,31,33,35,37,39}
c. Partition 2: {2,4,6,8,10,12,13,15,18,20,22,23,26,28,30,32,34,36,38,40}
Cost of the partition: 9

Final Partition 1: {2,4,6,8,10,12,13,15,18,20,22,23,26,28,30,32,34,36,38,40}
Final Partition 2: {1,3,5,7,9,11,14,16,17,19,21,24,25,27,29,31,33,35,37,39}
Cost of the Final partitions: 9

"kl2.txt"

a. Iteration number: 1
b. Partition 1: {1,2,3,4}
c. Partition 2: {5,6,7,8}
Cost of the partition: 4

a. Iteration number: 2
b. Partition 1: {2,3,4,6}
c. Partition 2: {1,5,7,8}
Cost of the partition: 2

a. Iteration number: 3
b. Partition 1: {1,3,4,6}
c. Partition 2: {2,5,7,8}
Cost of the partition: 2

Final Partition 1: {2,3,4,6}
Final Partition 2: {1,5,7,8}
Cost of the Final partitions: 2

"kl3.txt"

a. Iteration number: 1
b. Partition 1: {1,2,3,4}
c. Partition 2: {5,6,7,8,9}
Cost of the partition: 5

a. Iteration number: 2
b. Partition 1: {1,2,3,5,6}
c. Partition 2: {4,7,8,9}
Cost of the partition: 2

a. Iteration number: 3
b. Partition 1: {1,2,5,6}
c. Partition 2: {3,4,7,8,9}
Cost of the partition: 4

Final Partition 1: {1,2,3,5,6}
Final Partition 2: {4,7,8,9}
Cost of the Final partitions: 3

“kl4.txt”

a. Iteration number: 1
b. Partition 1: {1,2,3,4}
c. Partition 2: {5,6,7,8}
Cost of the partition: 5

a. Iteration number: 2
b. Partition 1: {1,2,6,7}
c. Partition 2: {3,4,5,8}
Cost of the partition: 1

a. Iteration number: 3
b. Partition 1: {3,4,5,8}
c. Partition 2: {1,2,6,7}
Cost of the partition: 1

Final Partition 1: {1,2,6,7}
Final Partition 2: {3,4,5,8}
Cost of the Final partitions: 1

“kl5.txt”

a. Iteration number: 1
b. Partition 1: {1,2,3,4}
c. Partition 2: {5,6,7,8}
Cost of the partition: 9

a. Iteration number: 2
b. Partition 1: {1,2,5,6}
c. Partition 2: {3,4,7,8}
Cost of the partition: 1

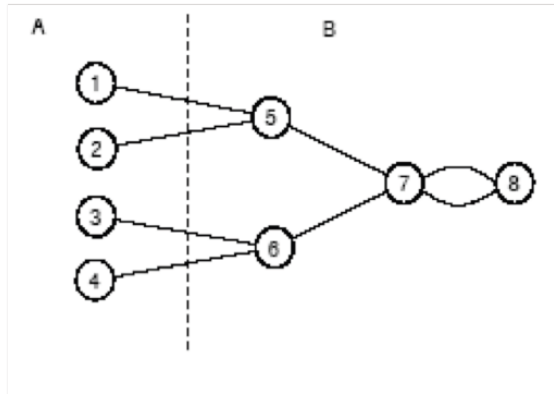
a. Iteration number: 3
b. Partition 1: {3,4,7,8}
c. Partition 2: {1,2,5,6}
Cost of the partition: 1

Final Partition 1: {1,2,5,6}
Final Partition 2: {3,4,7,8}
Cost of the Final partitions: 1

Appendix B – Hand Generated Results

“kl2.txt”

Problem 4.



STEP 1.

A{1,2,3,4}

B{5,6,7,8}

STEP 2.

$$D1 = E - I = 1$$

$$D2 = E - I = 1$$

$$D3 = E - I = 1$$

$$D4 = E - I = 1$$

$$D5 = E - I = 1$$

$$D6 = E - I = 1$$

$$D7 = E - I = -4$$

$$D8 = E - I = -2$$

STEP 3.

$$g_{15} = 1 + 1 - 2 * 1 = 0$$

$$g_{16} = 1 + 1 - 2 * 0 = 2$$

$$g_{17} = 1 + -4 - 2 * 0 = -3$$

$$g_{18} = 1 + -2 - 2 * 0 = -1$$

$$g_{25} = 1 + 1 - 2 * 1 = 0$$

$$g_{26} = 1 + 1 - 2 * 0 = 2$$

$$g_{27} = 1 + -4 - 2 * 0 = -3$$

$$g_{28} = 1 + -2 - 2 * 0 = -1$$

$$g_{35} = 1 + 1 - 2 * 0 = 2$$

$$g_{36} = 1 + 1 - 2 * 1 = 0$$

$$g_{37} = 1 + -4 - 2 * 0 = -3$$

$$g_{38} = 1 + -2 - 2 * 0 = -1$$

$$g_{45} = 1 + 1 - 2 * 0 = 2$$

$$g_{46} = 1 + 1 - 2 * 1 = 0$$

$$g_{47} = 1 + -4 - 2 * 0 = -3$$

$$g_{48} = 1 + -2 - 2 * 0 = -1$$

The max gain is 2 at vertices 1 and 6

Locking Vertices: 1 6

STEP 4.

$$1 + 2 * 0 - 2 * 0 = 1$$

$$1 + 2 * 0 - 2 * 1 = -1$$

$$1 + 2 * 0 - 2 * 1 = -1$$

$$1 + 2 * 0 - 2 * 1 = -1$$

$$-4 + 2 * 1 - 2 * 0 = -2$$

$$-2 + 2 * 0 - 2 * 0 = -2$$

$$D1' = D1 + 2 * ci - 2 * ci = 1$$

$$D2' = D2 + 2 * ci - 2 * ci = 1$$

$$D3' = D3 + 2 * ci - 2 * ci = -1$$

$$D4' = D4 + 2 * ci - 2 * ci = -1$$

$$D5' = D5 + 2 * ci - 2 * ci = -1$$

$$D6' = D6 + 2 * ci - 2 * ci = 1$$

$$D7' = D7 + 2 * ci - 2 * ci = -2$$

$$D8' = D8 + 2 * ci - 2 * ci = -2$$

STEP 3.

$$g_{25} = 1 + -1 - 2 * 1 = -2$$

$$g_{27} = 1 + -2 - 2 * 0 = -1$$

$$g_{28} = 1 + -2 - 2 * 0 = -1$$

$$g_{35} = -1 + -1 - 2 * 0 = -2$$

$$g_{37} = -1 + -2 - 2 * 0 = -3$$

$$g_{38} = -1 + -2 - 2 * 0 = -3$$

$$g_{45} = -1 + -1 - 2 * 0 = -2$$

$$g_{47} = -1 + -2 - 2 * 0 = -3$$

$$g_{48} = -1 + -2 - 2 * 0 = -3$$

The max gain is -1 at vertices 2 and 7

Locking Vertices: 2 7

STEP 4.

$$-1 + 2 * 0 - 2 * 0 = -1$$

$$-1 + 2 * 0 - 2 * 0 = -1$$

$$-1 + 2 * 1 - 2 * 1 = -1$$

$$-2 + 2 * 2 - 2 * 0 = 2$$

$$D1' = D1 + 2 * ci - 2 * ci = 1$$

$$D2' = D2 + 2 * ci - 2 * ci = 1$$

$$D3' = D3 + 2 * ci - 2 * ci = -1$$

$$D4' = D4 + 2 * ci - 2 * ci = -1$$

$$D5' = D5 + 2 * ci - 2 * ci = -1$$

$$D6' = D6 + 2 * ci - 2 * ci = 1$$

$$D7' = D7 + 2 * ci - 2 * ci = -2$$

$$D8' = D8 + 2 * ci - 2 * ci = 2$$

STEP 3.

$$g_{35} = -1 + -1 - 2 * 0 = -2$$

$$g_{38} = -1 + 2 - 2 * 0 = 1$$

$$g_{45} = -1 + -1 - 2 * 0 = -2$$

$$g_{48} = -1 + 2 - 2 * 0 = 1$$

The max gain is 1 at vertices 3 and 8

Locking Vertices: 3 8

STEP 4.

$$-1 + 2 * 0 - 2 * 0 = -1$$

$$-1 + 2 * 0 - 2 * 0 = -1$$

$$D1' = D1 + 2 * c_i - 2 * c_i = 1$$

$$D2' = D2 + 2 * c_i - 2 * c_i = 1$$

$$D3' = D3 + 2 * c_i - 2 * c_i = -1$$

$$D4' = D4 + 2 * c_i - 2 * c_i = -1$$

$$D5' = D5 + 2 * c_i - 2 * c_i = -1$$

$$D6' = D6 + 2 * c_i - 2 * c_i = 1$$

$$D7' = D7 + 2 * c_i - 2 * c_i = -2$$

$$D8' = D8 + 2 * c_i - 2 * c_i = 2$$

Last vertices to swap are : 4 and 5

$$g_{45} = -2$$

STEP 5.

$$k_0 = 2$$

$$k_1 = 1$$

$$k_2 = 2$$

$$k_3 = 0$$

The maximum G is when $k = 0$ and the gain is 2

$$A\{2,3,4,6\}$$

$$B\{1,5,7,8\}$$

STEP 2.

$$D1 = E - I = -1$$

$$D2 = E - I = 1$$

$$D3 = E - I = -1$$

$$D4 = E - I = -1$$

$$D5 = E - I = -1$$

$$D6 = E - I = -1$$

$$D7 = E - I = -2$$

$$D8 = E - I = -2$$

STEP 3.

$$g_{21} = 1 + -1 - 2 * 0 = 0$$

$$\begin{aligned}
g_{25} &= 1 + -1 - 2 * 1 = -2 \\
g_{27} &= 1 + -2 - 2 * 0 = -1 \\
g_{28} &= 1 + -2 - 2 * 0 = -1 \\
g_{31} &= -1 + -1 - 2 * 0 = -2 \\
g_{35} &= -1 + -1 - 2 * 0 = -2 \\
g_{37} &= -1 + -2 - 2 * 0 = -3 \\
g_{38} &= -1 + -2 - 2 * 0 = -3 \\
g_{41} &= -1 + -1 - 2 * 0 = -2 \\
g_{45} &= -1 + -1 - 2 * 0 = -2 \\
g_{47} &= -1 + -2 - 2 * 0 = -3 \\
g_{48} &= -1 + -2 - 2 * 0 = -3 \\
g_{61} &= -1 + -1 - 2 * 0 = -2 \\
g_{65} &= -1 + -1 - 2 * 0 = -2 \\
g_{67} &= -1 + -2 - 2 * 1 = -5 \\
g_{68} &= -1 + -2 - 2 * 0 = -3
\end{aligned}$$

The max gain is 0 at vertices 2 and 1
Locking Vertices: 2 1

STEP 4.

$$\begin{aligned}
-1 + 2 * 0 - 2 * 0 &= -1 \\
-1 + 2 * 0 - 2 * 0 &= -1 \\
-1 + 2 * 0 - 2 * 0 &= -1 \\
-1 + 2 * 1 - 2 * 1 &= -1 \\
-2 + 2 * 0 - 2 * 0 &= -2 \\
-2 + 2 * 0 - 2 * 0 &= -2 \\
D1' &= D1 + 2 * c_i - 2 * c_i = -1 \\
D2' &= D2 + 2 * c_i - 2 * c_i = 1 \\
D3' &= D3 + 2 * c_i - 2 * c_i = -1 \\
D4' &= D4 + 2 * c_i - 2 * c_i = -1 \\
D5' &= D5 + 2 * c_i - 2 * c_i = -1 \\
D6' &= D6 + 2 * c_i - 2 * c_i = -1 \\
D7' &= D7 + 2 * c_i - 2 * c_i = -2 \\
D8' &= D8 + 2 * c_i - 2 * c_i = -2
\end{aligned}$$

STEP 3.

$$\begin{aligned}
g_{35} &= -1 + -1 - 2 * 0 = -2 \\
g_{37} &= -1 + -2 - 2 * 0 = -3 \\
g_{38} &= -1 + -2 - 2 * 0 = -3 \\
g_{45} &= -1 + -1 - 2 * 0 = -2 \\
g_{47} &= -1 + -2 - 2 * 0 = -3 \\
g_{48} &= -1 + -2 - 2 * 0 = -3 \\
g_{65} &= -1 + -1 - 2 * 0 = -2 \\
g_{67} &= -1 + -2 - 2 * 1 = -5 \\
g_{68} &= -1 + -2 - 2 * 0 = -3
\end{aligned}$$

The max gain is -2 at vertices 3 and 5
Locking Vertices: 3 5

STEP 4.

$$-1 + 2 * 0 - 2 * 0 = -1$$

$$-1 + 2 * 1 - 2 * 0 = 1$$

$$-2 + 2 * 1 - 2 * 0 = 0$$

$$-2 + 2 * 0 - 2 * 0 = -2$$

$$D1' = D1 + 2 * ci - 2 * ci = -1$$

$$D2' = D2 + 2 * ci - 2 * ci = 1$$

$$D3' = D3 + 2 * ci - 2 * ci = -1$$

$$D4' = D4 + 2 * ci - 2 * ci = -1$$

$$D5' = D5 + 2 * ci - 2 * ci = -1$$

$$D6' = D6 + 2 * ci - 2 * ci = 1$$

$$D7' = D7 + 2 * ci - 2 * ci = 0$$

$$D8' = D8 + 2 * ci - 2 * ci = -2$$

STEP 3.

$$g_{47} = -1 + 0 - 2 * 0 = -1$$

$$g_{48} = -1 + -2 - 2 * 0 = -3$$

$$g_{67} = 1 + 0 - 2 * 1 = -1$$

$$g_{68} = 1 + -2 - 2 * 0 = -1$$

The max gain is -1 at vertices 4 and 7

Locking Vertices: 4 7

STEP 4.

$$1 + 2 * 1 - 2 * 1 = 1$$

$$-2 + 2 * 2 - 2 * 0 = 2$$

$$D1' = D1 + 2 * ci - 2 * ci = -1$$

$$D2' = D2 + 2 * ci - 2 * ci = 1$$

$$D3' = D3 + 2 * ci - 2 * ci = -1$$

$$D4' = D4 + 2 * ci - 2 * ci = -1$$

$$D5' = D5 + 2 * ci - 2 * ci = -1$$

$$D6' = D6 + 2 * ci - 2 * ci = 1$$

$$D7' = D7 + 2 * ci - 2 * ci = 0$$

$$D8' = D8 + 2 * ci - 2 * ci = 2$$

Last vertices to swap are : 6 and 8

$$g_{68} = 3$$

STEP 5.

$$k_0 = 0$$

$$k_1 = -2$$

$$k_2 = -3$$

$$k_3 = 0$$

The maximum G is when $k = 0$ and the gain is 0

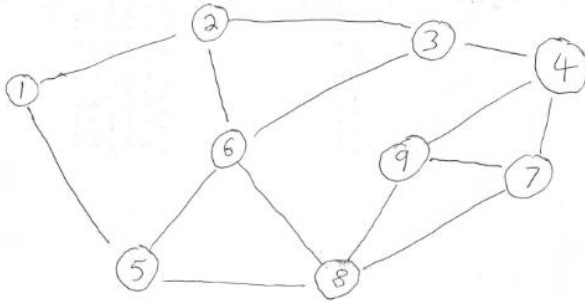
a. Iteration number: 1
b. Partition 1: {1,2,3,4}
c. Partition 2: {5,6,7,8}
Cost of the partition: 4

a. Iteration number: 2
b. Partition 1: {2,3,4,6}
c. Partition 2: {1,5,7,8}
Cost of the partition: 2

a. Iteration number: 3
b. Partition 1: {1,3,4,6}
c. Partition 2: {2,5,7,8}
Cost of the partition: 2

Final Partition 1: {2,3,4,6}
Final Partition 2: {1,5,7,8}
Cost of the Final partitions: 2

"kl3.txt"



STEP 1.

$A\{1,2,3,4\}$

$B\{5,6,7,8,9\}$

STEP 2.

$$D1 = E - I = 0$$

$$D2 = E - I = -1$$

$$D3 = E - I = -1$$

$$D4 = E - I = 1$$

$$D5 = E - I = -1$$

$$D6 = E - I = 0$$

$$D7 = E - I = -1$$

$$D8 = E - I = -3$$

$$D9 = E - I = -1$$

STEP 3.

$$g_{15} = 0 + -1 - 2 * 1 = -3$$

$$g_{16} = 0 + 0 - 2 * 0 = 0$$

$$g_{17} = 0 + -1 - 2 * 0 = -1$$

$$g_{18} = 0 + -3 - 2 * 0 = -3$$

$$g_{19} = 0 + -1 - 2 * 0 = -1$$

$$g_{25} = -1 + -1 - 2 * 0 = -2$$

$$g_{26} = -1 + 0 - 2 * 1 = -3$$

$$g_{27} = -1 + -1 - 2 * 0 = -2$$

$$g_{28} = -1 + -3 - 2 * 0 = -4$$

$$g_{29} = -1 + -1 - 2 * 0 = -2$$

$$g_{35} = -1 + -1 - 2 * 0 = -2$$

$$g_{36} = -1 + 0 - 2 * 1 = -3$$

$$g_{37} = -1 + -1 - 2 * 0 = -2$$

$$g_{38} = -1 + -3 - 2 * 0 = -4$$

$$g_{39} = -1 + -1 - 2 * 0 = -2$$

$$g_{45} = 1 + -1 - 2 * 0 = 0$$

$$g_{46} = 1 + 0 - 2 * 0 = 1$$

$$g_{47} = 1 + -1 - 2 * 1 = -2$$

$$\begin{aligned}
g_{48} &= 1 + -3 - 2 * 0 = -2 \\
g_{49} &= 1 + -1 - 2 * 1 = -2 \\
g_{-15} &= 0 + -1 - 2 * 0 = -1 \\
g_{-16} &= 0 + 0 - 2 * 0 = 0 \\
g_{-17} &= 0 + -1 - 2 * 0 = -1 \\
g_{-18} &= 0 + -3 - 2 * 0 = -3 \\
g_{-19} &= 0 + -1 - 2 * 0 = -1
\end{aligned}$$

The max gain is 1 at vertices 4 and 6
Locking Vertices: 4 6

STEP 4.

$$\begin{aligned}
0 + 2 * 0 - 2 * 0 &= 0 \\
-1 + 2 * 0 - 2 * 1 &= -3 \\
-1 + 2 * 1 - 2 * 1 &= -1 \\
-1 + 2 * 1 - 2 * 0 &= 1 \\
-1 + 2 * 0 - 2 * 1 &= -3 \\
-3 + 2 * 1 - 2 * 0 &= -1 \\
-1 + 2 * 0 - 2 * 1 &= -3 \\
D1' &= D1 + 2 * ci - 2 * ci = 0 \\
D2' &= D2 + 2 * ci - 2 * ci = -3 \\
D3' &= D3 + 2 * ci - 2 * ci = -1 \\
D4' &= D4 + 2 * ci - 2 * ci = 1 \\
D5' &= D5 + 2 * ci - 2 * ci = 1 \\
D6' &= D6 + 2 * ci - 2 * ci = 0 \\
D7' &= D7 + 2 * ci - 2 * ci = -3 \\
D8' &= D8 + 2 * ci - 2 * ci = -1 \\
D9' &= D9 + 2 * ci - 2 * ci = -3
\end{aligned}$$

STEP 3.

$$\begin{aligned}
g_{15} &= 0 + 1 - 2 * 1 = -1 \\
g_{17} &= 0 + -3 - 2 * 0 = -3 \\
g_{18} &= 0 + -1 - 2 * 0 = -1 \\
g_{19} &= 0 + -3 - 2 * 0 = -3 \\
g_{25} &= -3 + 1 - 2 * 0 = -2 \\
g_{27} &= -3 + -3 - 2 * 0 = -6 \\
g_{28} &= -3 + -1 - 2 * 0 = -4 \\
g_{29} &= -3 + -3 - 2 * 0 = -6 \\
g_{35} &= -1 + 1 - 2 * 0 = 0 \\
g_{37} &= -1 + -3 - 2 * 0 = -4 \\
g_{38} &= -1 + -1 - 2 * 0 = -2 \\
g_{39} &= -1 + -3 - 2 * 0 = -4 \\
g_{-15} &= 0 + 1 - 2 * 0 = 1 \\
g_{-17} &= 0 + -3 - 2 * 0 = -3 \\
g_{-18} &= 0 + -1 - 2 * 0 = -1 \\
g_{-19} &= 0 + -3 - 2 * 0 = -3
\end{aligned}$$

The max gain is 1 at vertices -1 and 5

Locking Vertices: -1 5

STEP 4.

$$0 + 2 * 0 - 2 * 1 = -2$$

$$-3 + 2 * 0 - 2 * 0 = -3$$

$$-1 + 2 * 0 - 2 * 0 = -1$$

$$-3 + 2 * 0 - 2 * 0 = -3$$

$$-1 + 2 * 0 - 2 * 0 = -1$$

$$-3 + 2 * 0 - 2 * 0 = -3$$

$$D1' = D1 + 2 * ci - 2 * ci = -2$$

$$D2' = D2 + 2 * ci - 2 * ci = -3$$

$$D3' = D3 + 2 * ci - 2 * ci = -1$$

$$D4' = D4 + 2 * ci - 2 * ci = 1$$

$$D5' = D5 + 2 * ci - 2 * ci = 1$$

$$D6' = D6 + 2 * ci - 2 * ci = 0$$

$$D7' = D7 + 2 * ci - 2 * ci = -3$$

$$D8' = D8 + 2 * ci - 2 * ci = -1$$

$$D9' = D9 + 2 * ci - 2 * ci = -3$$

STEP 3.

$$g_{17} = -2 + -3 - 2 * 0 = -5$$

$$g_{18} = -2 + -1 - 2 * 0 = -3$$

$$g_{19} = -2 + -3 - 2 * 0 = -5$$

$$g_{27} = -3 + -3 - 2 * 0 = -6$$

$$g_{28} = -3 + -1 - 2 * 0 = -4$$

$$g_{29} = -3 + -3 - 2 * 0 = -6$$

$$g_{37} = -1 + -3 - 2 * 0 = -4$$

$$g_{38} = -1 + -1 - 2 * 0 = -2$$

$$g_{39} = -1 + -3 - 2 * 0 = -4$$

The max gain is -2 at vertices 3 and 8

Locking Vertices: 3 8

STEP 4.

$$-2 + 2 * 0 - 2 * 0 = -2$$

$$-3 + 2 * 1 - 2 * 0 = -1$$

$$-3 + 2 * 1 - 2 * 0 = -1$$

$$-3 + 2 * 1 - 2 * 0 = -1$$

$$D1' = D1 + 2 * ci - 2 * ci = -2$$

$$D2' = D2 + 2 * ci - 2 * ci = -1$$

$$D3' = D3 + 2 * ci - 2 * ci = -1$$

$$D4' = D4 + 2 * ci - 2 * ci = 1$$

$$D5' = D5 + 2 * ci - 2 * ci = 1$$

$$D6' = D6 + 2 * ci - 2 * ci = 0$$

$$D7' = D7 + 2 * ci - 2 * ci = -1$$

$$D8' = D8 + 2 * ci - 2 * ci = -1$$

$$D9' = D9 + 2 * ci - 2 * ci = -1$$

STEP 3.

$$g_{17} = -2 + -1 - 2 * 0 = -3$$

$$g_{19} = -2 + -1 - 2 * 0 = -3$$

$$g_{27} = -1 + -1 - 2 * 0 = -2$$

$$g_{29} = -1 + -1 - 2 * 0 = -2$$

The max gain is -2 at vertices 2 and 7

Locking Vertices: 2 7

STEP 4.

$$-2 + 2 * 1 - 2 * 0 = 0$$

$$-1 + 2 * 1 - 2 * 0 = 1$$

$$D1' = D1 + 2 * ci - 2 * ci = 0$$

$$D2' = D2 + 2 * ci - 2 * ci = -1$$

$$D3' = D3 + 2 * ci - 2 * ci = -1$$

$$D4' = D4 + 2 * ci - 2 * ci = 1$$

$$D5' = D5 + 2 * ci - 2 * ci = 1$$

$$D6' = D6 + 2 * ci - 2 * ci = 0$$

$$D7' = D7 + 2 * ci - 2 * ci = -1$$

$$D8' = D8 + 2 * ci - 2 * ci = -1$$

$$D9' = D9 + 2 * ci - 2 * ci = 1$$

Last vertices to swap are : 1 and 9

$$g_{19} = 1$$

STEP 5.

$$k_0 = 1$$

$$k_1 = 2$$

$$k_2 = 0$$

$$k_3 = -2$$

$$k_4 = -1$$

The maximum G is when $k = 1$ and the gain is 2

$A\{1,2,3,5,6\}$

$B\{4,7,8,9\}$

STEP 2.

$$D1 = E - I = -2$$

$$D2 = E - I = -3$$

$$D3 = E - I = -1$$

$$D4 = E - I = -1$$

$$D5 = E - I = -1$$

$$D6 = E - I = -2$$

$$D7 = E - I = -3$$

$$D8 = E - I = -1$$

$$D9 = E - I = -3$$

STEP 3.

$$\begin{aligned}g_{14} &= -2 + -1 - 2 * 0 = -3 \\g_{17} &= -2 + -3 - 2 * 0 = -5 \\g_{18} &= -2 + -1 - 2 * 0 = -3 \\g_{19} &= -2 + -3 - 2 * 0 = -5 \\g_{1-1} &= -2 + 0 - 2 * 0 = -2 \\g_{24} &= -3 + -1 - 2 * 0 = -4 \\g_{27} &= -3 + -3 - 2 * 0 = -6 \\g_{28} &= -3 + -1 - 2 * 0 = -4 \\g_{29} &= -3 + -3 - 2 * 0 = -6 \\g_{2-1} &= -3 + 0 - 2 * 0 = -3 \\g_{34} &= -1 + -1 - 2 * 1 = -4 \\g_{37} &= -1 + -3 - 2 * 0 = -4 \\g_{38} &= -1 + -1 - 2 * 0 = -2 \\g_{39} &= -1 + -3 - 2 * 0 = -4 \\g_{3-1} &= -1 + 0 - 2 * 0 = -1 \\g_{54} &= -1 + -1 - 2 * 0 = -2 \\g_{57} &= -1 + -3 - 2 * 0 = -4 \\g_{58} &= -1 + -1 - 2 * 1 = -4 \\g_{59} &= -1 + -3 - 2 * 0 = -4 \\g_{5-1} &= -1 + 0 - 2 * 0 = -1 \\g_{64} &= -2 + -1 - 2 * 0 = -3 \\g_{67} &= -2 + -3 - 2 * 0 = -5 \\g_{68} &= -2 + -1 - 2 * 1 = -5 \\g_{69} &= -2 + -3 - 2 * 0 = -5 \\g_{6-1} &= -2 + 0 - 2 * 0 = -2\end{aligned}$$

The max gain is -1 at vertices 3 and -1

Locking Vertices: 3 -1

STEP 4.

$$\begin{aligned}-2 + 2 * 0 - 2 * 0 &= -2 \\-3 + 2 * 1 - 2 * 0 &= -1 \\-1 + 2 * 0 - 2 * 0 &= -1 \\-2 + 2 * 1 - 2 * 0 &= 0 \\-1 + 2 * 0 - 2 * 1 &= -3 \\-3 + 2 * 0 - 2 * 0 &= -3 \\-1 + 2 * 0 - 2 * 0 &= -1 \\-3 + 2 * 0 - 2 * 0 &= -3 \\D1' &= D1 + 2 * c_i - 2 * c_i = -2 \\D2' &= D2 + 2 * c_i - 2 * c_i = -1 \\D3' &= D3 + 2 * c_i - 2 * c_i = -1 \\D4' &= D4 + 2 * c_i - 2 * c_i = -3 \\D5' &= D5 + 2 * c_i - 2 * c_i = -1 \\D6' &= D6 + 2 * c_i - 2 * c_i = 0 \\D7' &= D7 + 2 * c_i - 2 * c_i = -3 \\D8' &= D8 + 2 * c_i - 2 * c_i = -1 \\D9' &= D9 + 2 * c_i - 2 * c_i = -3\end{aligned}$$

STEP 3.

$$\begin{aligned}g_{14} &= -2 + -3 - 2 * 0 = -5 \\g_{17} &= -2 + -3 - 2 * 0 = -5 \\g_{18} &= -2 + -1 - 2 * 0 = -3 \\g_{19} &= -2 + -3 - 2 * 0 = -5 \\g_{24} &= -1 + -3 - 2 * 0 = -4 \\g_{27} &= -1 + -3 - 2 * 0 = -4 \\g_{28} &= -1 + -1 - 2 * 0 = -2 \\g_{29} &= -1 + -3 - 2 * 0 = -4 \\g_{54} &= -1 + -3 - 2 * 0 = -4 \\g_{57} &= -1 + -3 - 2 * 0 = -4 \\g_{58} &= -1 + -1 - 2 * 1 = -4 \\g_{59} &= -1 + -3 - 2 * 0 = -4 \\g_{64} &= 0 + -3 - 2 * 0 = -3 \\g_{67} &= 0 + -3 - 2 * 0 = -3 \\g_{68} &= 0 + -1 - 2 * 1 = -3 \\g_{69} &= 0 + -3 - 2 * 0 = -3\end{aligned}$$

The max gain is -2 at vertices 2 and 8
Locking Vertices: 2 8

STEP 4.

$$\begin{aligned}-2 + 2 * 1 - 2 * 0 &= 0 \\-1 + 2 * 0 - 2 * 1 &= -3 \\0 + 2 * 1 - 2 * 1 &= 0 \\-3 + 2 * 0 - 2 * 0 &= -3 \\-3 + 2 * 1 - 2 * 0 &= -1 \\-3 + 2 * 1 - 2 * 0 &= -1 \\D1' &= D1 + 2 * c_i - 2 * c_i = 0 \\D2' &= D2 + 2 * c_i - 2 * c_i = -1 \\D3' &= D3 + 2 * c_i - 2 * c_i = -1 \\D4' &= D4 + 2 * c_i - 2 * c_i = -3 \\D5' &= D5 + 2 * c_i - 2 * c_i = -3 \\D6' &= D6 + 2 * c_i - 2 * c_i = 0 \\D7' &= D7 + 2 * c_i - 2 * c_i = -1 \\D8' &= D8 + 2 * c_i - 2 * c_i = -1 \\D9' &= D9 + 2 * c_i - 2 * c_i = -1\end{aligned}$$

STEP 3.

$$\begin{aligned}g_{14} &= 0 + -3 - 2 * 0 = -3 \\g_{17} &= 0 + -1 - 2 * 0 = -1 \\g_{19} &= 0 + -1 - 2 * 0 = -1 \\g_{54} &= -3 + -3 - 2 * 0 = -6 \\g_{57} &= -3 + -1 - 2 * 0 = -4 \\g_{59} &= -3 + -1 - 2 * 0 = -4 \\g_{64} &= 0 + -3 - 2 * 0 = -3 \\g_{67} &= 0 + -1 - 2 * 0 = -1\end{aligned}$$

$$g_{69} = 0 + -1 - 2 * 0 = -1$$

The max gain is -1 at vertices 1 and 7

Locking Vertices: 1 7

STEP 4.

$$-3 + 2 * 1 - 2 * 0 = -1$$

$$0 + 2 * 0 - 2 * 0 = 0$$

$$-3 + 2 * 1 - 2 * 0 = -1$$

$$-1 + 2 * 1 - 2 * 0 = 1$$

$$D1' = D1 + 2 * ci - 2 * ci = 0$$

$$D2' = D2 + 2 * ci - 2 * ci = -1$$

$$D3' = D3 + 2 * ci - 2 * ci = -1$$

$$D4' = D4 + 2 * ci - 2 * ci = -1$$

$$D5' = D5 + 2 * ci - 2 * ci = -1$$

$$D6' = D6 + 2 * ci - 2 * ci = 0$$

$$D7' = D7 + 2 * ci - 2 * ci = -1$$

$$D8' = D8 + 2 * ci - 2 * ci = -1$$

$$D9' = D9 + 2 * ci - 2 * ci = 1$$

STEP 3.

$$g_{54} = -1 + -1 - 2 * 0 = -2$$

$$g_{59} = -1 + 1 - 2 * 0 = 0$$

$$g_{64} = 0 + -1 - 2 * 0 = -1$$

$$g_{69} = 0 + 1 - 2 * 0 = 1$$

The max gain is 1 at vertices 6 and 9

Locking Vertices: 6 9

STEP 4.

$$-1 + 2 * 1 - 2 * 0 = 1$$

$$-1 + 2 * 1 - 2 * 0 = 1$$

$$D1' = D1 + 2 * ci - 2 * ci = 0$$

$$D2' = D2 + 2 * ci - 2 * ci = -1$$

$$D3' = D3 + 2 * ci - 2 * ci = -1$$

$$D4' = D4 + 2 * ci - 2 * ci = 1$$

$$D5' = D5 + 2 * ci - 2 * ci = 1$$

$$D6' = D6 + 2 * ci - 2 * ci = 0$$

$$D7' = D7 + 2 * ci - 2 * ci = -1$$

$$D8' = D8 + 2 * ci - 2 * ci = -1$$

$$D9' = D9 + 2 * ci - 2 * ci = 1$$

Last vertices to swap are : 5 and 4

$$g_{54} = 2$$

STEP 5.

$$k_0 = -1$$

$$k_1 = -3$$

$k_2 = -4$

$k_3 = -3$

$k_4 = -1$

The maximum G is when $k = 0$ and the gain is -1

a. Iteration number: 1

b. Partition 1: $\{1,2,3,4\}$

c. Partition 2: $\{5,6,7,8,9\}$

Cost of the partition: 5

a. Iteration number: 2

b. Partition 1: $\{1,2,3,5,6\}$

c. Partition 2: $\{4,7,8,9\}$

Cost of the partition: 3

a. Iteration number: 3

b. Partition 1: $\{1,2,5,6\}$

c. Partition 2: $\{3,4,7,8,9\}$

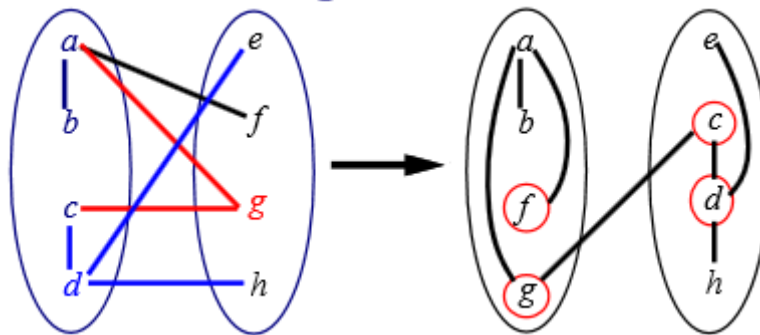
Cost of the partition: 4

Final Partition 1: $\{1,2,3,5,6\}$

Final Partition 2: $\{4,7,8,9\}$

Cost of the Final partitions: 3

“kl4.txt”



STEP 1.

A{1,2,3,4}

B{5,6,7,8}

STEP 2.

D1 = E - I = 1

D2 = E - I = -1

D3 = E - I = 0

D4 = E - I = 1

D5 = E - I = 1

D6 = E - I = 1

D7 = E - I = 2

D8 = E - I = 1

STEP 3.

g 15 = 1 + 1 - 2 * 0 = 2

g 16 = 1 + 1 - 2 * 1 = 0

g 17 = 1 + 2 - 2 * 1 = 1

g 18 = 1 + 1 - 2 * 0 = 2

g 25 = -1 + 1 - 2 * 0 = 0

g 26 = -1 + 1 - 2 * 0 = 0

g 27 = -1 + 2 - 2 * 0 = 1

g 28 = -1 + 1 - 2 * 0 = 0

g 35 = 0 + 1 - 2 * 0 = 1

g 36 = 0 + 1 - 2 * 0 = 1

g 37 = 0 + 2 - 2 * 1 = 0

g 38 = 0 + 1 - 2 * 0 = 1

g 45 = 1 + 1 - 2 * 1 = 0

g 46 = 1 + 1 - 2 * 0 = 2

g 47 = 1 + 2 - 2 * 0 = 3

g 48 = 1 + 1 - 2 * 1 = 0

The max gain is 3 at vertices 4 and 7

Locking Vertices: 4 7

STEP 4.

$$1 + 2 * 0 - 2 * 1 = -1$$

$$-1 + 2 * 0 - 2 * 0 = -1$$

$$0 + 2 * 1 - 2 * 1 = 0$$

$$1 + 2 * 0 - 2 * 1 = -1$$

$$1 + 2 * 0 - 2 * 0 = 1$$

$$1 + 2 * 0 - 2 * 1 = -1$$

$$D1' = D1 + 2 * ci - 2 * ci = -1$$

$$D2' = D2 + 2 * ci - 2 * ci = -1$$

$$D3' = D3 + 2 * ci - 2 * ci = 0$$

$$D4' = D4 + 2 * ci - 2 * ci = 1$$

$$D5' = D5 + 2 * ci - 2 * ci = -1$$

$$D6' = D6 + 2 * ci - 2 * ci = 1$$

$$D7' = D7 + 2 * ci - 2 * ci = 2$$

$$D8' = D8 + 2 * ci - 2 * ci = -1$$

STEP 3.

$$g_{15} = -1 + -1 - 2 * 0 = -2$$

$$g_{16} = -1 + 1 - 2 * 1 = -2$$

$$g_{18} = -1 + -1 - 2 * 0 = -2$$

$$g_{25} = -1 + -1 - 2 * 0 = -2$$

$$g_{26} = -1 + 1 - 2 * 0 = 0$$

$$g_{28} = -1 + -1 - 2 * 0 = -2$$

$$g_{35} = 0 + -1 - 2 * 0 = -1$$

$$g_{36} = 0 + 1 - 2 * 0 = 1$$

$$g_{38} = 0 + -1 - 2 * 0 = -1$$

The max gain is 1 at vertices 3 and 6

Locking Vertices: 3 6

STEP 4.

$$-1 + 2 * 0 - 2 * 1 = -3$$

$$-1 + 2 * 0 - 2 * 0 = -1$$

$$-1 + 2 * 0 - 2 * 0 = -1$$

$$-1 + 2 * 0 - 2 * 0 = -1$$

$$D1' = D1 + 2 * ci - 2 * ci = -3$$

$$D2' = D2 + 2 * ci - 2 * ci = -1$$

$$D3' = D3 + 2 * ci - 2 * ci = 0$$

$$D4' = D4 + 2 * ci - 2 * ci = 1$$

$$D5' = D5 + 2 * ci - 2 * ci = -1$$

$$D6' = D6 + 2 * ci - 2 * ci = 1$$

$$D7' = D7 + 2 * ci - 2 * ci = 2$$

$$D8' = D8 + 2 * ci - 2 * ci = -1$$

STEP 3.

$$g_{15} = -3 + -1 - 2 * 0 = -4$$

$$g_{18} = -3 + -1 - 2 * 0 = -4$$

$$g_{25} = -1 + -1 - 2 * 0 = -2$$

$$g_{28} = -1 + -1 - 2 * 0 = -2$$

The max gain is -2 at vertices 2 and 5

Locking Vertices: 2 5

STEP 4.

$$-3 + 2 * 1 - 2 * 0 = -1$$

$$-1 + 2 * 0 - 2 * 0 = -1$$

$$D1' = D1 + 2 * c_i - 2 * c_i = -1$$

$$D2' = D2 + 2 * c_i - 2 * c_i = -1$$

$$D3' = D3 + 2 * c_i - 2 * c_i = 0$$

$$D4' = D4 + 2 * c_i - 2 * c_i = 1$$

$$D5' = D5 + 2 * c_i - 2 * c_i = -1$$

$$D6' = D6 + 2 * c_i - 2 * c_i = 1$$

$$D7' = D7 + 2 * c_i - 2 * c_i = 2$$

$$D8' = D8 + 2 * c_i - 2 * c_i = -1$$

Last vertices to swap are : 1 and 8

$$g_{18} = -2$$

STEP 5.

$$k_0 = 3$$

$$k_1 = 4$$

$$k_2 = 2$$

$$k_3 = 0$$

The maximum G is when $k = 1$ and the gain is 4

$A\{1,2,6,7\}$

$B\{3,4,5,8\}$

STEP 2.

$$D1 = E - I = -3$$

$$D2 = E - I = -1$$

$$D3 = E - I = 0$$

$$D4 = E - I = -3$$

$$D5 = E - I = -1$$

$$D6 = E - I = -1$$

$$D7 = E - I = 0$$

$$D8 = E - I = -1$$

STEP 3.

$$g_{13} = -3 + 0 - 2 * 0 = -3$$

$$g_{14} = -3 + -3 - 2 * 0 = -6$$

$$g_{15} = -3 + -1 - 2 * 0 = -4$$

$$g_{18} = -3 + -1 - 2 * 0 = -4$$

$$\begin{aligned}
g_{23} &= -1 + 0 - 2 * 0 = -1 \\
g_{24} &= -1 + -3 - 2 * 0 = -4 \\
g_{25} &= -1 + -1 - 2 * 0 = -2 \\
g_{28} &= -1 + -1 - 2 * 0 = -2 \\
g_{63} &= -1 + 0 - 2 * 0 = -1 \\
g_{64} &= -1 + -3 - 2 * 0 = -4 \\
g_{65} &= -1 + -1 - 2 * 0 = -2 \\
g_{68} &= -1 + -1 - 2 * 0 = -2 \\
g_{73} &= 0 + 0 - 2 * 1 = -2 \\
g_{74} &= 0 + -3 - 2 * 0 = -3 \\
g_{75} &= 0 + -1 - 2 * 0 = -1 \\
g_{78} &= 0 + -1 - 2 * 0 = -1
\end{aligned}$$

The max gain is -1 at vertices 2 and 3
Locking Vertices: 2 3

STEP 4.

$$\begin{aligned}
-3 + 2 * 1 - 2 * 0 &= -1 \\
-1 + 2 * 0 - 2 * 0 &= -1 \\
0 + 2 * 0 - 2 * 1 &= -2 \\
-3 + 2 * 1 - 2 * 0 &= -1 \\
-1 + 2 * 0 - 2 * 0 &= -1 \\
-1 + 2 * 0 - 2 * 0 &= -1 \\
D1' &= D1 + 2 * c_i - 2 * c_i = -1 \\
D2' &= D2 + 2 * c_i - 2 * c_i = -1 \\
D3' &= D3 + 2 * c_i - 2 * c_i = 0 \\
D4' &= D4 + 2 * c_i - 2 * c_i = -1 \\
D5' &= D5 + 2 * c_i - 2 * c_i = -1 \\
D6' &= D6 + 2 * c_i - 2 * c_i = -1 \\
D7' &= D7 + 2 * c_i - 2 * c_i = -2 \\
D8' &= D8 + 2 * c_i - 2 * c_i = -1
\end{aligned}$$

STEP 3.

$$\begin{aligned}
g_{14} &= -1 + -1 - 2 * 0 = -2 \\
g_{15} &= -1 + -1 - 2 * 0 = -2 \\
g_{18} &= -1 + -1 - 2 * 0 = -2 \\
g_{64} &= -1 + -1 - 2 * 0 = -2 \\
g_{65} &= -1 + -1 - 2 * 0 = -2 \\
g_{68} &= -1 + -1 - 2 * 0 = -2 \\
g_{74} &= -2 + -1 - 2 * 0 = -3 \\
g_{75} &= -2 + -1 - 2 * 0 = -3 \\
g_{78} &= -2 + -1 - 2 * 0 = -3
\end{aligned}$$

The max gain is -2 at vertices 1 and 4
Locking Vertices: 1 4

STEP 4.

$$-1 + 2 * 1 - 2 * 0 = 1$$

$$-2 + 2 * 1 - 2 * 0 = 0$$

$$-1 + 2 * 1 - 2 * 0 = 1$$

$$-1 + 2 * 1 - 2 * 0 = 1$$

$$D1' = D1 + 2 * ci - 2 * ci = -1$$

$$D2' = D2 + 2 * ci - 2 * ci = -1$$

$$D3' = D3 + 2 * ci - 2 * ci = 0$$

$$D4' = D4 + 2 * ci - 2 * ci = -1$$

$$D5' = D5 + 2 * ci - 2 * ci = 1$$

$$D6' = D6 + 2 * ci - 2 * ci = 1$$

$$D7' = D7 + 2 * ci - 2 * ci = 0$$

$$D8' = D8 + 2 * ci - 2 * ci = 1$$

STEP 3.

$$g_{65} = 1 + 1 - 2 * 0 = 2$$

$$g_{68} = 1 + 1 - 2 * 0 = 2$$

$$g_{75} = 0 + 1 - 2 * 0 = 1$$

$$g_{78} = 0 + 1 - 2 * 0 = 1$$

The max gain is 2 at vertices 6 and 5

Locking Vertices: 6 5

STEP 4.

$$0 + 2 * 0 - 2 * 0 = 0$$

$$1 + 2 * 0 - 2 * 0 = 1$$

$$D1' = D1 + 2 * ci - 2 * ci = -1$$

$$D2' = D2 + 2 * ci - 2 * ci = -1$$

$$D3' = D3 + 2 * ci - 2 * ci = 0$$

$$D4' = D4 + 2 * ci - 2 * ci = -1$$

$$D5' = D5 + 2 * ci - 2 * ci = 1$$

$$D6' = D6 + 2 * ci - 2 * ci = 1$$

$$D7' = D7 + 2 * ci - 2 * ci = 0$$

$$D8' = D8 + 2 * ci - 2 * ci = 1$$

Last vertices to swap are : 7 and 8

$$g_{78} = 1$$

STEP 5.

$$k_0 = -1$$

$$k_1 = -3$$

$$k_2 = -1$$

$$k_3 = 0$$

The maximum G is when $k = 3$ and the gain is 0

a. Iteration number: 1

b. Partition 1: {1,2,3,4}

c. Partition 2: {5,6,7,8}

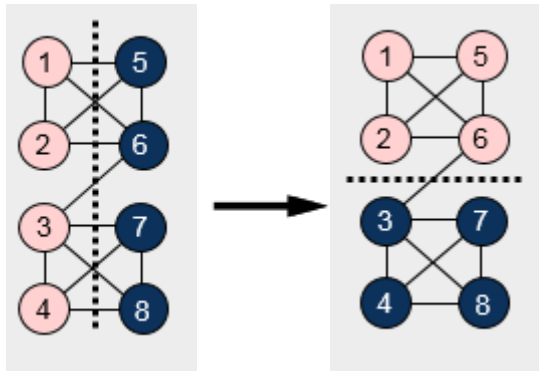
Cost of the partition: 5

a. Iteration number: 2
b. Partition 1: {1,2,6,7}
c. Partition 2: {3,4,5,8}
Cost of the partition: 1

a. Iteration number: 3
b. Partition 1: {3,4,5,8}
c. Partition 2: {1,2,6,7}
Cost of the partition: 1

Final Partition 1: {1,2,6,7}
Final Partition 2: {3,4,5,8}
Cost of the Final partitions: 1

“kl5.txt”



STEP 1.

$A\{1,2,3,4\}$

$B\{5,6,7,8\}$

STEP 2.

$D1 = E - I = 1$

$D2 = E - I = 1$

$D3 = E - I = 2$

$D4 = E - I = 1$

$D5 = E - I = 1$

$D6 = E - I = 2$

$D7 = E - I = 1$

$D8 = E - I = 1$

STEP 3.

$g_{15} = 1 + 1 - 2 * 1 = 0$

$g_{16} = 1 + 2 - 2 * 1 = 1$

$g_{17} = 1 + 1 - 2 * 0 = 2$

$g_{18} = 1 + 1 - 2 * 0 = 2$

$g_{25} = 1 + 1 - 2 * 1 = 0$

$g_{26} = 1 + 2 - 2 * 1 = 1$

$g_{27} = 1 + 1 - 2 * 0 = 2$

$g_{28} = 1 + 1 - 2 * 0 = 2$

$g_{35} = 2 + 1 - 2 * 0 = 3$

$g_{36} = 2 + 2 - 2 * 1 = 2$

$g_{37} = 2 + 1 - 2 * 1 = 1$

$g_{38} = 2 + 1 - 2 * 1 = 1$

$g_{45} = 1 + 1 - 2 * 0 = 2$

$g_{46} = 1 + 2 - 2 * 0 = 3$

$g_{47} = 1 + 1 - 2 * 1 = 0$

$g_{48} = 1 + 1 - 2 * 1 = 0$

The max gain is 3 at vertices 3 and 5

Locking Vertices: 3 5

STEP 4.

$$\begin{aligned}
1 + 2 * 0 - 2 * 1 &= -1 \\
1 + 2 * 0 - 2 * 1 &= -1 \\
1 + 2 * 1 - 2 * 0 &= 3 \\
2 + 2 * 1 - 2 * 1 &= 2 \\
1 + 2 * 0 - 2 * 1 &= -1 \\
1 + 2 * 0 - 2 * 1 &= -1 \\
D1' &= D1 + 2 * ci - 2 * ci = -1 \\
D2' &= D2 + 2 * ci - 2 * ci = -1 \\
D3' &= D3 + 2 * ci - 2 * ci = 2 \\
D4' &= D4 + 2 * ci - 2 * ci = 3 \\
D5' &= D5 + 2 * ci - 2 * ci = 1 \\
D6' &= D6 + 2 * ci - 2 * ci = 2 \\
D7' &= D7 + 2 * ci - 2 * ci = -1 \\
D8' &= D8 + 2 * ci - 2 * ci = -1
\end{aligned}$$

STEP 3.

$$\begin{aligned}
g_{16} &= -1 + 2 - 2 * 1 = -1 \\
g_{17} &= -1 + -1 - 2 * 0 = -2 \\
g_{18} &= -1 + -1 - 2 * 0 = -2 \\
g_{26} &= -1 + 2 - 2 * 1 = -1 \\
g_{27} &= -1 + -1 - 2 * 0 = -2 \\
g_{28} &= -1 + -1 - 2 * 0 = -2 \\
g_{46} &= 3 + 2 - 2 * 0 = 5 \\
g_{47} &= 3 + -1 - 2 * 1 = 0 \\
g_{48} &= 3 + -1 - 2 * 1 = 0
\end{aligned}$$

The max gain is 5 at vertices 4 and 6
Locking Vertices: 4 6

STEP 4.

$$\begin{aligned}
-1 + 2 * 0 - 2 * 1 &= -3 \\
-1 + 2 * 0 - 2 * 1 &= -3 \\
-1 + 2 * 0 - 2 * 1 &= -3 \\
-1 + 2 * 0 - 2 * 1 &= -3 \\
D1' &= D1 + 2 * ci - 2 * ci = -3 \\
D2' &= D2 + 2 * ci - 2 * ci = -3 \\
D3' &= D3 + 2 * ci - 2 * ci = 2 \\
D4' &= D4 + 2 * ci - 2 * ci = 3 \\
D5' &= D5 + 2 * ci - 2 * ci = 1 \\
D6' &= D6 + 2 * ci - 2 * ci = 2 \\
D7' &= D7 + 2 * ci - 2 * ci = -3 \\
D8' &= D8 + 2 * ci - 2 * ci = -3
\end{aligned}$$

STEP 3.

$$\begin{aligned}
g_{17} &= -3 + -3 - 2 * 0 = -6 \\
g_{18} &= -3 + -3 - 2 * 0 = -6 \\
g_{27} &= -3 + -3 - 2 * 0 = -6 \\
g_{28} &= -3 + -3 - 2 * 0 = -6
\end{aligned}$$

The max gain is -6 at vertices 1 and 7

Locking Vertices: 1 7

STEP 4.

$$-3 + 2 * 1 - 2 * 0 = -1$$

$$-3 + 2 * 1 - 2 * 0 = -1$$

$$D1' = D1 + 2 * ci - 2 * ci = -3$$

$$D2' = D2 + 2 * ci - 2 * ci = -1$$

$$D3' = D3 + 2 * ci - 2 * ci = 2$$

$$D4' = D4 + 2 * ci - 2 * ci = 3$$

$$D5' = D5 + 2 * ci - 2 * ci = 1$$

$$D6' = D6 + 2 * ci - 2 * ci = 2$$

$$D7' = D7 + 2 * ci - 2 * ci = -3$$

$$D8' = D8 + 2 * ci - 2 * ci = -1$$

Last vertices to swap are : 2 and 8

$$g_{28} = -2$$

STEP 5.

$$k_0 = 3$$

$$k_1 = 8$$

$$k_2 = 2$$

$$k_3 = 0$$

The maximum G is when $k = 1$ and the gain is 8

$$A\{1,2,5,6\}$$

$$B\{3,4,7,8\}$$

STEP 2.

$$D1 = E - I = -3$$

$$D2 = E - I = -3$$

$$D3 = E - I = -2$$

$$D4 = E - I = -3$$

$$D5 = E - I = -3$$

$$D6 = E - I = -2$$

$$D7 = E - I = -3$$

$$D8 = E - I = -3$$

STEP 3.

$$g_{13} = -3 + -2 - 2 * 0 = -5$$

$$g_{14} = -3 + -3 - 2 * 0 = -6$$

$$g_{17} = -3 + -3 - 2 * 0 = -6$$

$$g_{18} = -3 + -3 - 2 * 0 = -6$$

$$g_{23} = -3 + -2 - 2 * 0 = -5$$

$$g_{24} = -3 + -3 - 2 * 0 = -6$$

$$g_{27} = -3 + -3 - 2 * 0 = -6$$

$$\begin{aligned}
g_{28} &= -3 + -3 - 2 * 0 = -6 \\
g_{53} &= -3 + -2 - 2 * 0 = -5 \\
g_{54} &= -3 + -3 - 2 * 0 = -6 \\
g_{57} &= -3 + -3 - 2 * 0 = -6 \\
g_{58} &= -3 + -3 - 2 * 0 = -6 \\
g_{63} &= -2 + -2 - 2 * 1 = -6 \\
g_{64} &= -2 + -3 - 2 * 0 = -5 \\
g_{67} &= -2 + -3 - 2 * 0 = -5 \\
g_{68} &= -2 + -3 - 2 * 0 = -5
\end{aligned}$$

The max gain is -5 at vertices 1 and 3
Locking Vertices: 1 3

STEP 4.

$$\begin{aligned}
-3 + 2 * 1 - 2 * 0 &= -1 \\
-3 + 2 * 1 - 2 * 0 &= -1 \\
-2 + 2 * 1 - 2 * 1 &= -2 \\
-3 + 2 * 1 - 2 * 0 &= -1 \\
-3 + 2 * 1 - 2 * 0 &= -1 \\
-3 + 2 * 1 - 2 * 0 &= -1 \\
D1' &= D1 + 2 * c_i - 2 * c_i = -3 \\
D2' &= D2 + 2 * c_i - 2 * c_i = -1 \\
D3' &= D3 + 2 * c_i - 2 * c_i = -2 \\
D4' &= D4 + 2 * c_i - 2 * c_i = -1 \\
D5' &= D5 + 2 * c_i - 2 * c_i = -1 \\
D6' &= D6 + 2 * c_i - 2 * c_i = -2 \\
D7' &= D7 + 2 * c_i - 2 * c_i = -1 \\
D8' &= D8 + 2 * c_i - 2 * c_i = -1
\end{aligned}$$

STEP 3.

$$\begin{aligned}
g_{24} &= -1 + -1 - 2 * 0 = -2 \\
g_{27} &= -1 + -1 - 2 * 0 = -2 \\
g_{28} &= -1 + -1 - 2 * 0 = -2 \\
g_{54} &= -1 + -1 - 2 * 0 = -2 \\
g_{57} &= -1 + -1 - 2 * 0 = -2 \\
g_{58} &= -1 + -1 - 2 * 0 = -2 \\
g_{64} &= -2 + -1 - 2 * 0 = -3 \\
g_{67} &= -2 + -1 - 2 * 0 = -3 \\
g_{68} &= -2 + -1 - 2 * 0 = -3
\end{aligned}$$

The max gain is -2 at vertices 2 and 4
Locking Vertices: 2 4

STEP 4.

$$\begin{aligned}
-1 + 2 * 1 - 2 * 0 &= 1 \\
-2 + 2 * 1 - 2 * 0 &= 0 \\
-1 + 2 * 1 - 2 * 0 &= 1 \\
-1 + 2 * 1 - 2 * 0 &= 1
\end{aligned}$$

$$\begin{aligned}
D1' &= D1 + 2*ci - 2*ci = -3 \\
D2' &= D2 + 2*ci - 2*ci = -1 \\
D3' &= D3 + 2*ci - 2*ci = -2 \\
D4' &= D4 + 2*ci - 2*ci = -1 \\
D5' &= D5 + 2*ci - 2*ci = 1 \\
D6' &= D6 + 2*ci - 2*ci = 0 \\
D7' &= D7 + 2*ci - 2*ci = 1 \\
D8' &= D8 + 2*ci - 2*ci = 1
\end{aligned}$$

STEP 3.

$$\begin{aligned}
g_{57} &= 1 + 1 - 2 * 0 = 2 \\
g_{58} &= 1 + 1 - 2 * 0 = 2 \\
g_{67} &= 0 + 1 - 2 * 0 = 1 \\
g_{68} &= 0 + 1 - 2 * 0 = 1
\end{aligned}$$

The max gain is 2 at vertices 5 and 7
Locking Vertices: 5 7

STEP 4.

$$\begin{aligned}
0 + 2 * 1 - 2 * 0 &= 2 \\
1 + 2 * 1 - 2 * 0 &= 3 \\
D1' &= D1 + 2*ci - 2*ci = -3 \\
D2' &= D2 + 2*ci - 2*ci = -1 \\
D3' &= D3 + 2*ci - 2*ci = -2 \\
D4' &= D4 + 2*ci - 2*ci = -1 \\
D5' &= D5 + 2*ci - 2*ci = 1 \\
D6' &= D6 + 2*ci - 2*ci = 2 \\
D7' &= D7 + 2*ci - 2*ci = 1 \\
D8' &= D8 + 2*ci - 2*ci = 3
\end{aligned}$$

Last vertices to swap are : 6 and 8
 $g_{68} = 5$

STEP 5.

$$\begin{aligned}
k_0 &= -5 \\
k_1 &= -7 \\
k_2 &= -5 \\
k_3 &= 0
\end{aligned}$$

The maximum G is when $k = 3$ and the gain is 0

a. Iteration number: 1
b. Partition 1: {1,2,3,4}
c. Partition 2: {5,6,7,8}
Cost of the partition: 9

a. Iteration number: 2
b. Partition 1: {1,2,5,6}

c. Partition 2: {3,4,7,8}
Cost of the partition: 1

a. Iteration number: 3
b. Partition 1: {3,4,7,8}
c. Partition 2: {1,2,5,6}
Cost of the partition: 1

Final Partition 1: {1,2,5,6}
Final Partition 2: {3,4,7,8}
Cost of the Final partitions: 1

Appendix C - Input Files

“kl.txt”

40
1,11,19,23,27
2,5,13,18,20,36,40
3,7,14,17,25,33
4,8,13,15,20,37,38,40
5,2,7,8,11,24
6,10,13,20,34,40
7,3,5,16,21,29
8,4,5,17,30,32
9,11,17,31,33
10,6,15,17,20,34
11,1,5,9,37
12,18,22,34
13,2,4,6,36,40
14,3,19,35,37
15,4,10,30,38
16,7,19,24,29,39
17,3,8,9,10,21,24,37
18,2,12,22,28
19,1,14,16,39
20,2,4,6,10,34
21,7,17,25,39
22,12,18,25,26
23,1,26,30,32
24,5,16,17,35
25,3,21,22,35
26,22,23,38
27,1,31,39
28,18,29,30,34,37
29,7,16,28,31,33
30,8,15,23,28,36
31,9,27,29,37
32,8,23,34,36
33,3,9,29,39
34,6,10,12,20,28,32
35,14,24,25
36,2,13,30,32
37,4,11,14,17,28,31
38,4,15,26
39,16,19,21,27,33
40,2,4,6,13

“kl2.txt”

8
1,5
2,5
3,6
4,6
5,1,2,7
6,3,4,7
7,5,6,8,8
8,7,7

“kl3.txt”

9
1,2,5
2,1,3,6
3,2,4,6
4,3,7,9
5,1,6,8
6,2,3,5,7
7,4,6,8,9
8,5,7,9
9,4,7,8

“kl4.txt”

8
1,2,6,7
2,1
3,4,7
4,3,5,8
5,4
6,1
7,1,3
8,4

“kl5.txt”

8
1,2,5,6
2,1,5,6
3,4,6,7,8
4,3,7,8
5,1,2,6
6,1,2,3,5
7,3,4,8
8,3,4,7