## Calculus - Single Variable Part1 - Functions

1. What is the domain of the function  $\sqrt{2x-x^3}$ ?

Sol:

The value inside square root must be positive or 0

$$=> 2x - x^3 \ge 0$$

$$=> -x(x^2-2) \ge 0$$

$$=>-x(x-\sqrt{2})(x+\sqrt{2})\geq 0$$

$$=>x(x-\sqrt{2})(x+\sqrt{2})\leq 0$$

$$\Rightarrow x \in (-\infty, -\sqrt{2}] \text{ or } x \in (0, \sqrt{2}]$$

2. What is the domain of the function  $\frac{x-3}{x^2-4} \ln x$ ?

Sol:

The denominator part must be non-zero by the rule of division. (2-a) In addition, x must be > 0 for logarithmic function  $\ln x$ . (2-b)

=> From (2-a), we have 
$$x^2 - 4 \neq 0$$

$$=> (x-2)(x+2) \neq 0$$

$$=> x \neq 2$$
 and  $x \neq -2$ 

$$=>$$
 From (2-b), we have  $x>0$ 

=> From the result of (2-a) and (2-b), we have  $x \in (0,2) \cup (2,\infty)$ 

3. What is the domain of the function  $\arcsin \frac{x-2}{3}$ ?

Sol:

Recall that the range of sin(x) is [-1, 1].

Therefore, the domain function inverse  $\arcsin(x)$  is [-1, 1]

$$=>-1\leq \frac{x-2}{3}\leq 1$$

- => we have  $-1 \le \frac{x-2}{3}$  on the left-hand side, and  $\frac{x-2}{3} \le 1$  on the right-hand side.
- $=>-1 \le x$  on the left-hand side, and  $x \le 5$  on the right-hand side.
- => Put them together, we get  $x \in [-1, 5]$

4. What is the range of the function  $-x^2 + 1$ ?

Sol:

$$f(x) = -x^2 + 1 \le 1$$
, and  $f(x)$  goes lower when  $x \ne 0$ 

$$\Rightarrow f(x) \in (-\infty, 1)$$

5. What is range of the function  $ln(1 + x^2)$ ?

Sol:

Recall the logarithmic function  $\ln x$  is increasing function from negative infinity to positive infinity.

From  $1 + x^2$ , we have minimize value 1, and  $1 + x^2$  goes upper when  $x \neq 0$ 

- => minimal value of  $ln(1 + x^2) = ln(1) = 0$
- => Thus, we have  $f(x) \in [0,\infty)$

6. What is the range of the function arctan( cos(x) ) ? (i.e., the inverse of the tangent function with the parameter cos x)

Recall the domain of tan(x) is  $x \in \{x | x \neq k\pi + \frac{\pi}{2}, k \in Z\}$ .

Also, the range of cos(x) is [-1, 1]

- => With the rule of thumb,  $\tan\left(\frac{\pi}{4}\right) = 1$  and  $\tan\left(\frac{-\pi}{4}\right) = -1$
- => Range of arctan(cos(x))  $\in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

7. If  $f(x) = 4x^3 + 1$  and  $g(x) = \sqrt{x+3}$ , compute (fog)(x) and (gof)(x),

Sol:

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x+3}) = 4*(\sqrt{x+3})^3 + 1 = 4*(x+3)^{\frac{3}{2}} + 1$$

$$(g \circ f)(x) = g(f(x)) = g(4x^3 + 1) = \sqrt{(4x^3 + 1) + 3} = \sqrt{4x^3 + 4} = \sqrt{4(x^3 + 1)}$$

$$=2\sqrt{x^3+1}=2\cdot(x^3+1)^{\frac{1}{2}}$$

8. What is the inverse function  $f(x) = e^{2x}$ ?

Sol:

$$v = e^{2x}$$

$$=> \ln y = \ln e^{2x} = 2x$$

$$=> x = \frac{\ln y}{2} = \frac{1}{2} \ln y = \ln y^{\frac{1}{2}} = \ln \sqrt{y}$$

$$=> f^{-1}(x) = \frac{1}{2} \ln x = \ln \sqrt{x}$$