Calculus - Single Variable Part1 -Exponential

1. Find all possible solutions to the equation $\mathrm{e}^{ix} = \mathrm{i}$

Sol:

$\mathrm{e}^{ix} = \cos x + \mathrm{i}\sin x = \mathrm{i}$

=> $\cos x = 0$ for real part, and $\sin x = 1$ for imaginary part

=> $x = \frac{\pi}{2} + 2\pi k, k \in \mathbb{Z}$

=> $x = \frac{\pi + 4\pi k}{2}, k \in \mathbb{Z}$

=> $x = \frac{\pi(4k+1)}{2}, k \in \mathbb{Z}$

2. Calculate $\sum_{k=0}^{\infty}(-1)^k \frac{(\ln 4)^k}{k!}$

Sol:

$\sum_{k=0}^{\infty}(-1)^k \frac{(\ln 4)^k}{k!} = \sum_{k=0}^{\infty} \frac{(-\ln\ \ )^k}{k!} = \mathrm{e}^{-\ln 4} = \mathrm{e}^{\ln 4^{-1}} = \mathrm{e}^{\ln(\frac{1}{4})}$

$= \frac{1}{4}$

3. Calculate $\sum_{1\Leftarrow 0}^{\infty}(-1)^k \frac{\pi^{2\mathrm{k}}}{(2k)!}$

Sol:

$\sum_{1\Leftarrow 0}^{\infty}(-1)^k \frac{\pi^{2\mathrm{k}}}{(2k)!} = \cos(\pi)$ = -1

4. Write out the first four terms of the sum $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}2^k}{2k-1}$

Sol:

$\frac{1\cdot 2}{1} + \frac{-1\cdot 4}{3} + \frac{1\cdot 8}{5} + \frac{-1\cdot 16}{7}$

5. Write out the first four terms of the sum $\sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k}}{k!(2k+1)}$

Sol:

$\frac{1\cdot 1}{1\cdot 1} + \frac{-1\cdot\pi^2}{1\cdot 3} + \frac{1\cdot\pi^4}{2\cdot 5} + \frac{-1\cdot\pi^6}{6\cdot 7}$

6. What is the expression describes the sum $\frac{e}{2} - \frac{e^2}{4} + \frac{e^3}{6} - \frac{e^4}{8} + \cdots$?

Sol:

$\sum_{k=0}^{\infty} \frac{e^{k+1}}{2(k+1)} (-1)^k$

,or

$\sum_{k=1}^{\infty} \frac{e^k}{2k} (-1)^{k+1}$

7. What is the expression describes the sum $-1+\frac{x}{2!}-\frac{x^2}{3!}+\frac{x^3}{4!}+\cdots$?

Sol:

$$\sum_{k=0}^{\infty}\frac{x^k}{(k+1)!}(-1)^{k+1}$$

, or

$$\sum_{k=1}^{\infty}\frac{x^{(k-1)}}{(k)!}(-1)^{k}$$

8. Given $y=\ln x$, $and\ z=\log x$, eliminate x and generate a equation using by y and z

Sol:

$z=\log x$

$=> z=\frac{\ln}{\ln 10}=\frac{y}{\ln 10}$

$=> y=z\cdot\ln 10$

$=> y=z\cdot\frac{\log 10}{\log \mathrm{e}}=\frac{z}{\log \mathrm{e}}$

$=>\ \ y=z\cdot\ln 10$, or $y=\frac{z}{\log \mathrm{e}}$