

Calculus - Single Variable Part1 -Exponential

1. Find all possible solutions to the equation $e^{ix} = i$

Sol:

$$e^{ix} = \cos x + i \sin x = i$$

$\Rightarrow \cos x = 0$ for real part, and $\sin x = 1$ for imaginary part

$$\Rightarrow x = \frac{\pi}{2} + 2\pi k, k \in \mathbb{Z}$$

$$\Rightarrow x = \frac{\pi + 4\pi k}{2}, k \in \mathbb{Z}$$

$$\Rightarrow x = \frac{\pi(4k+1)}{2}, k \in \mathbb{Z}$$

2. Calculate $\sum_{k=0}^{\infty} (-1)^k \frac{(\ln 4)^k}{k!}$

Sol:

$$\sum_{k=0}^{\infty} (-1)^k \frac{(\ln 4)^k}{k!} = \sum_{k=0}^{\infty} \frac{(-\ln 4)^k}{k!} = e^{-\ln 4} = e^{\ln 4^{-1}} = e^{\ln(\frac{1}{4})}$$

$$= \frac{1}{4}$$

3. Calculate $\sum_{k=0}^{\infty} (-1)^k \frac{\pi^{2k}}{(2k)!}$

Sol:

$$\sum_{k=0}^{\infty} (-1)^k \frac{\pi^{2k}}{(2k)!} = \cos(\pi) = -1$$

4. Write out the first four terms of the sum $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 2^k}{2k-1}$

Sol:

$$\frac{1 \cdot 2}{1} + \frac{-1 \cdot 4}{3} + \frac{1 \cdot 8}{5} + \frac{-1 \cdot 16}{7}$$

5. Write out the first four terms of the sum $\sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k}}{k!(2k+1)}$

Sol:

$$\frac{1 \cdot 1}{1 \cdot 1} + \frac{-1 \cdot \pi^2}{1 \cdot 3} + \frac{1 \cdot \pi^4}{2 \cdot 5} + \frac{-1 \cdot \pi^6}{6 \cdot 7}$$

6. What is the expression describes the sum $\frac{e}{2} - \frac{e^2}{4} + \frac{e^3}{6} - \frac{e^4}{8} + \dots$?

Sol:

$$\sum_{k=0}^{\infty} \frac{e^{k+1}}{2(k+1)} (-1)^k$$

,or

$$\sum_{k=1}^{\infty} \frac{e^k}{2k} (-1)^{k+1}$$

7. What is the expression describes the sum $-1 + \frac{x}{2!} - \frac{x^2}{3!} + \frac{x^3}{4!} + \dots$?

Sol:

$$\sum_{k=0}^{\infty} \frac{x^k}{(k+1)!} (-1)^{k+1}$$

, or

$$\sum_{k=1}^{\infty} \frac{x^{(k-1)}}{(k)!} (-1)^k$$

8. Given $y = \ln x$, and $z = \log x$, eliminate x and generate a equation using by y and z

Sol:

$$z = \log x$$

$$\Rightarrow z = \frac{\ln x}{\ln 10} = \frac{y}{\ln 10}$$

$$\Rightarrow y = z \cdot \ln 10$$

$$\Rightarrow y = z \cdot \frac{\log 10}{\log e} = \frac{z}{\log e}$$

$$\Rightarrow y = z \cdot \ln 10, \text{ or } y = \frac{z}{\log e}$$