Calculus - Single Variable Part1 - Functions

1. What is the domain of the function $\ln \sin(x)$?

Sol:

Recall that domain of logarithmic function $\ln x$ is x > 0

=> $\sin(x) > 0$ to meet the domain of $\ln x$

=> $\sin(x) > 0, if\ x \in (k\pi, (k+1)\pi), k \in \mathbb{Z}$

2. Let $f(x) = \frac{1}{x+2}$

   Determine f o f = ?

Sol:

f o f = $f(f(x)) = \frac{1}{\frac{1}{x+2}+2} = \frac{x+2}{1+2(x+2)} = \frac{x+2}{2x+5}$

3. What is the inverse of the function $f(x) = \sin(x^2)$ on some appropriate domain ?

Sol:

$y = \sin(x^2)$

=> $\arcsin(y) = x^2$

=> $\sqrt{\arcsin(y)} = x$

=> $f^{-1}(x) = \sqrt{\arcsin(x)}$

4. What is the inverse of the function $f(x) = \arctan(\ln(3x))$ on some appropriate domain ?

Sol:

$y = \arctan(\ln(3x))$

=> $\tan(y) = \ln(3x)$

=> $e^{\tan(y)} = 3x$

=> $\frac{1}{3}e^{\tan(y)} = x$

=> $f^{-1}(x) = \frac{1}{3}e^{\tan(x)}$