Undoing quantum measurement must violate either determinism, Lorentz invariance or time-symmetry

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The peaceful coexistence between quantum mechanics and special relativity is maintained due to the unobservability of hidden variables, and hence the intrinsic unobservability of quantum non-locality in real time. I prove that if time-reversing quantum measurement is possible, this coexistence breaks down. Either Lorentz invariance or time-symmetry must be visibly violated, the more radical option being the inexistence of hidden variables.

The EPR argument [1], based on the combined premises of realism, determinism and SR, was in essence an argument for hidden variables (HVs). Then, Bell's theorem [2] proved that if such HVs exist, they must be nonlocal. Yet even after Bell-inequality violations were demonstrated, the violation of SR remained indirect, as no observable superluminal signal can be sent through the EPR measurements. The proverbial "peaceful coexistence" between QM and SR [3] in due to this very non-signaling.

Subsequent theoretical advances related to this issue were relatively minor. Bohm and Hiley [4], seeking to preserve Determinism even under Bell's proof, imposed another restriction on HVs: They must exert their nonlocal effects in some preferred reference frame. This non-relativistic feature eventually became (see below) central to Bohmian mechanics.

On the other hand, Elitzur and Dolev [5] derived from SR that hidden variables must remain hidden *forever*, otherwise their indirect violation of SR would become direct. This restriction, reminiscent of the "cosmic censorship" conjecture [6], equally prohibits the Bohmian preferred reference frame to be ever detected.

Apart from these restrictions on HVs, of which Elitzur and Dolev's [5] renders Bohm's [4] untestable and leaves Bell's [2] as the only one of experimental significance, no other restrictions were made.

In this article, I derive a few new restrictions, leading to a new proof of incompatibility between QM and some basic principles of classical physics.

1. Outline of the Incompatibility Proof

Classical physics postulates

i) **Determinism**

ii) **Lorentz Invariance**

iii) **Time-Symmetry**,

While QM postulates

iv) **The Uncertainty Principle**.

By (*iv*), the EPR argument and Bell's inequality proved

v) **Nonlocality** as an inherent property of QM, although (*i*) was not ruled out and (*ii*) was preserved.

By (*i*) and (*v*), Bohm and Hiley derived a

vi) **Privileged reference frame** as the only frame within which (*v*) can operate.

By (*ii*) and (*iv*), Elitzur and Dolev derived a

vii) **Censorship** on directly observing (*v*) and (*vi*).

I now submit that by (*i*) and (*iii*),

viii) **Unmeasurement** of quantum phenomena is possible, where the quantum state resumes its pre-measurement superposition.

And, by (*i*), (*iii*) and (*v*),

ix) The nonlocal effects of unmeasurement must operate in the same privileged reference frame (*vi*) as measurement.

However,

x) In a moving reference frame, the time-order of two EPR measurements in the privileged frame (*vi*) may differ from that of two consecutive unmeasurements, defying (*vii*) by giving rise to an observable failure of (*viii*) under inertial motion,

Thereby

xi) Violating (*ii*),

or

xii) Violating (*iii*),

or

xiii) Disproving (*i*).

The following sections present the proof in detail. Readers who wish to examine only the main move, namely, the failure of time-reversal under Lorentz-transformation (*x*), may skip to section 6.5.

2. Setting the Stage: "Collapse" *vs*. Hidden Variables

We begin with the Proof's first classical premise (*i*), namely, determinism. Does it still hold beneath quantum randomness? This debate centers on the notion of "collapse." When a particle undergoes, say, a spin measurement, its superposition gives its way to either an "up" or "down" definite state:

(1) $$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle+|\downarrow\rangle\right)\xrightarrow{M}|\uparrow\rangle \text{or } |\downarrow\rangle,$$

rendering the process fundamentally irreversible: While the particle begins as a wave function split into two halves that move along the two arms of the Stern-Gerlach magnet (Fig. 1a), the interaction with the detectors leaves a single particle on one arm, with no trace of the wave function anywhere else (1b). It is this wave function's disappearance, by the collapse interpretations, that makes it impossible to undo the measurement [7].

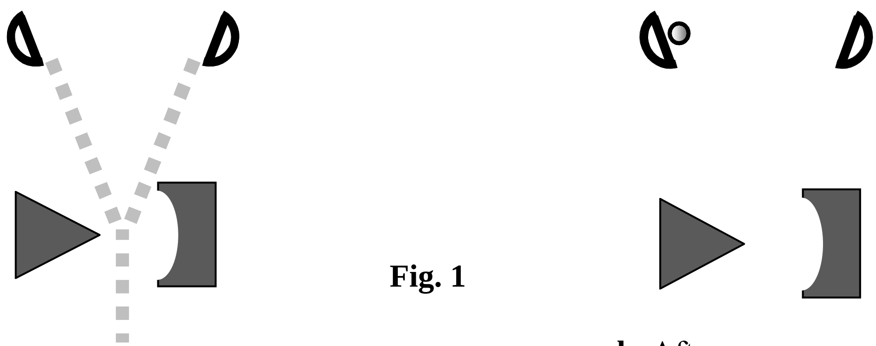

Fig. 1

a. Before measurement **b.** After measurement

Hidden-variables interpretations, in contrast, preserve the undetected parts of the wave function even after measurement. In the many-worlds interpretation [8], the above two possible outcomes are claimed to be two universes splitting from the original one after the measurement. In Bohmian mechanics [4], these are two halves of the guide wave, of which one contains the particle while the other is empty but existing nonetheless. Rather than (1), then, measurement is viewed within hidden-variables theories as

(2) $$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right) \xrightarrow{M} \begin{cases} |\uparrow\rangle \\ + \\ |\downarrow\rangle \end{cases},$$

the + sign between the splitting arrows indicating that both outcomes occur. The single result actually observed, then, is either the newly-branched universe within which the newly-branched observer resides, or the wave-function-half within which the localized particle happened to remain.

By thus preserving the wave-function's otherwise mute part, the hidden-variables models enable in principle a closer look into the question of measurement's undoing.

3. Introducing Unmeasurement

Is it possible, then, to be "unmeasure" a particle, *i.e.*, to time-reverse its measurement such that the particle resumes its initial superposition? While a definite answer is beyond this paper's scope, the consequences of both "yes" and "no" are obvious:

- If quantum measurement cannot be undone *in principle*, this would be a fundamental irreversibility, negating assumptions (*i*) and/or (*iii*) of the Incompatibility Proof and indicating a microscopic origin of other time-asymmetries [9].
- If, on the other hand, unmeasurement is possible, time-symmetry violation return below with vengeance, the other option being Lorentz-invariance violation.

The issue, also, is not too remote from present-day technology. A procedure known as "quantum erasure" [10] purports to undo quantum measurement: A particle undergoing interference is entangled with another superposed particle while passing through the two routes. Measurement of the latter particle can reveal which path was taken by the former. Consequently, the original particle's interference is destroyed. However, by subjecting the other particle to an appropriate measurement analogous to interference, each particle gives a "key" that enables observing the other's interference. Upon a closer inspection, however, no real undoing of measurement is attained with this method, because no real measurement was made in the first place. Rather, entanglement occurred, followed (but not undone) by a more subtle measurement.

Elitzur and Dolev [11] proposed a different method of unmeasurement based on interaction-free measurement. They showed, for the first time, that just like measurement, *unmeasurement too exerts a non-local effect in the EPR experiment*: Unmeasuring one measured particle forces the distant particle to re-assume its pre-measurement state. Another advance of their technique is that it is not subject to the no-recording restriction (see section 4 below). Katz *et al*. [12] presented a realization of this method, although not employing it on the EPR setting.

However, from the HVs viewpoint, one might argue that both the partial measurement and its undoing involve irreversible absorption of two opposite portions of the "empty" wave function, hence there was no undoing in the full, time-reversal sense.

Other methods, still far from technical feasibility, have been studied [13] [14]. Like earlier *gedankenexperiments*, their ability to yield novel insights gives an incentive to experimenters to make them realizable.

4. Only Unrecorded Measurement can be Undone

Annoyingly, even at the gedanken level, perfect unmeasurement carries yet another price of a "Faustian" nature: *Unmeasurement requires total elimination of any record of the outcome, including the observer's memory.* The smallest trace of a measurement left unreversed would ruin the entire procedure. This, indeed, is the case with [10], where the measurement's outcome remains unknown.

This restriction threatens to make the whole issue non-scientific. If "unperformed measurements have no results" as Peres [15] dryly pointed out, then so much the more for measurements that were performed and then *un*performed! Is there any point in considering unmeasurement if the outcome to be undone must never be known?

5. Unmeasurement Applied to the EPR

Fortunately, the EPR setting enables replying to the above question with a resounding "Yes." Simply, let a *pair* of measurements is carried out on two entangled particles. Here, there is no need to know the outcomes that we seek to undo [16]. *Suffice it that we can see, following the unmeasurement of two previously measured particles, whether they are entangled again, regaining their initial Bell-state.*

Fig. 3 depicts such an EPR experiment where the two particles are measured along the same spin axis, and then undergo *unmeasurement*. Our question now takes a simple form: Would the particles resume their entangled state?

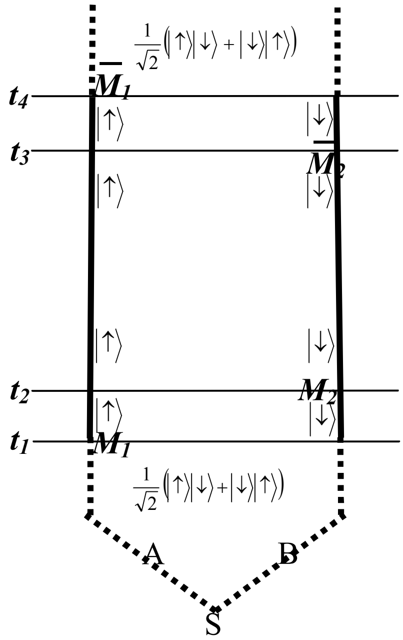

Fig. 3: EPR with time-reversal. On each particle's world-line, the dotted segment represents the time interval during which the particle is superposed, whereas the solid segment depicts its having a definite spin.

Consider, then, the following standard EPR experiment. An atom with spin 0 ejects two spin-½ particles, A and B, that fly far apart towards two measuring devices. These are Stern-Gerlach magnets, randomly aligned along one out of three coplanar spatial directions α, β and γ, perpendicular to the particle's flight. The angles between these directions are chosen so as to maximize violation of Bell's inequality: α=0º, β=60º, γ=120º. Each SG magnet measures whether the particle's spin is ½ ("up," ↑) or –½ ("down," ↓) along the direction chosen. The probability for correlation between the spin results of each pair (A↑,B↓ or A↓,B↑) depends on the relative angle θ between the measured spins: $p = \cos^2 \frac{\theta}{2}$. This corresponds to a correlation of $\langle \sigma_{"x"} \sigma_{"y"} \rangle = 1 - 2p = -\cos\theta$.

We envisage, furthermore, an idealized, small and perfectly isolated lab, within which the above EPR pair undergoes two measurements, without anyone from outside inspecting the outcomes. Next, the entire process is "undone" within the lab. Finally, the two particles come out of the lab. If the unmeasurements were indeed successful, the two particles should be as entangled as they were upon entering the lab.

6. Restrictions on Hidden Variables

With the above setting we shall first study the restrictions that HVs must obey under an ordinary pair of measurements. Next we shall consider the case where these measurements are followed by a pair of two unmeasurements.

6.1. Hidden Variables must be Deterministic

The first restriction may look trivial, but will prove essential for the next steps. Consider a simple EPR experiment in which both particles undergo spin measurements in the same direction α. Suppose further that particle A is measured long before B, its spin turning out to be $\alpha_{\uparrow}$. B's spin, by conservation, must be $\alpha_{\downarrow}$.

Whence this correlation? The spin value obeys the uncertainty principle, which makes it apparently random. It is against this randomness that hidden variables were invoked. Let, then, $HV\uparrow$ denote the hidden variable whose presence in the EPR measurement causes the particle to assume the spin $\alpha_{\uparrow}$, and $HV\downarrow$ the variable which brings about the opposite spin.

In order to comply with observational data, $HV\uparrow$ must be present in 50% of all measurements and $HV\downarrow$ in the other 50%. This randomness of the $HV\uparrow/\downarrow$ distribution, so go HVs theories, is not fundamental but rather apparent, like the statistical distribution of classical dice throws. Each such a variable is causally determined by the previous event, as well as causally determining the consequent spin. Ergo,

<u>Restriction 1:</u> *Hidden variables must, by assumption, be deterministic.*

6.2. Hidden Variables must be Nonlocal

It is to this implicit determinism that Bell amended the next, celebrated restriction: The EPR correlations cannot be pre-established within the particles themselves, hence they must be determined by the measurement events. In terms of Restriction 6.1, the presence of our presumed factor $HV\uparrow/\downarrow$ in one measurement affects not only the nearby particle but both of them, however mutually remote. Hence,

<u>Restriction 2:</u> *Hidden variables must be nonlocal.*

6.3. Hidden Variables must Exert their Nonlocal Effects in some Privileged Reference Frame

Once nonlocal effects were made inescapable by Bell, all realistic accounts had to assign them a paradox-free evolution. This is the motivation for Bohm and Hiley's [4] restriction on the reference frame within which HVs exert their effects.

Notice, first, that the uncertainty relations underlying the eigenstates

(3)

$$
\begin{aligned}
&\left|x\uparrow\right\rangle=\frac{1}{\sqrt{2}}\left(\left|z\uparrow\right\rangle+\left|z\downarrow\right\rangle\right)\ ,\ \left|x\downarrow\right\rangle=\frac{1}{\sqrt{2}}\left(\left|z\uparrow\right\rangle-\left|z\downarrow\right\rangle\right)\\
&\left|z\uparrow\right\rangle=\frac{1}{\sqrt{2}}\left(\left|x\uparrow\right\rangle+\left|x\downarrow\right\rangle\right)\ ,\ \left|z\downarrow\right\rangle=\frac{1}{\sqrt{2}}\left(\left|x\uparrow\right\rangle-\left|x\downarrow\right\rangle\right)\\
&\left|y\uparrow\downarrow\right\rangle=\frac{1}{\sqrt{2}}\left(\left|x\uparrow\right\rangle\pm i\left|x\downarrow\right\rangle\right)
\end{aligned}
$$

inject a subtle time-asymmetry: Measuring the spin along one direction makes uncertain the outcome of the *subsequent* measurement along other directions. This feature gains further emphasis in the HVs models. In Bohmian mechanics, the particle "chooses" one Stern-Gerlach arm at the moment the guide wave is split. Subjecting the outgoing particle to another measurement amounts to loosing the guide wave's empty half. Similarly in Everett's model, consecutive splits of the universe occur *within one another*, (Fig. 2) such that within any branch, part of the wave function is lost. Both models, therefore, introduce a preferred time-direction to quantum mechanics.

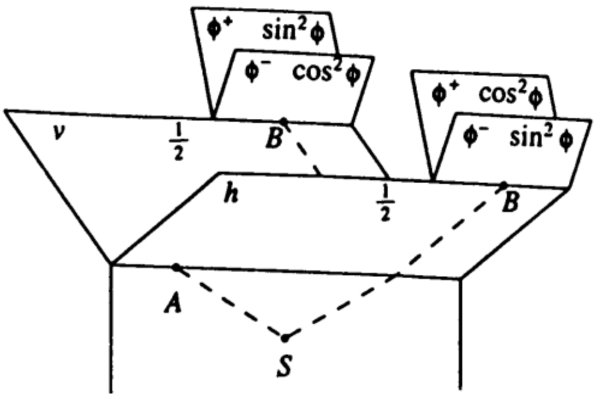

Fig. 2. Branching of the universe during an EPR experiment according to the Many-Worlds interpretation (from [17]).

This time-asymmetry equally holds whether the two consecutive measurements are carried out on the same particle or on two distant EPR particles. The correlation between two spin measurements depends only on the relative angle between them. Therefore, by Bell's inequality, the spin value yielded by one particle's measurement depends on the choice of spin direction used in the other particle's measurement. The bearing of this effect on hidden-variables theories, imposed by Bohm and Hiley [4], is straightforward: *For any EPR pair of spin measurements, the individual "up" and "down" results depend on the time-order of the measurement.*

True, Lorentz invariance and quantum indeterminism conspire to camouflage this temporal feature in the EPR case: It must be impossible to tell which measurement occurred first. Still, the temporal feature is inescapable for *any* realistic account, even in all the realistic "collapse" models (see below). To quote Bohm and Hiley [4]: "Briefly, what this means is that there is always a unique frame in which the nonlocal connections operate instantaneously (p. 285)." Albert [18, pp. 155-160] is more straightforward: If we measure, say, the α-spin of both EPR particles, then *the individual outcomes depend on which particle is measured first*. The same conclusion is rigorously derived by Barrett [19, p. 141]: "*So the temporal order of the measurements determines the outcome*" (italics original). See also Rae [20, p. 304], Callender [21 and references therein] and Craig [22].

But, if the two EPR measurements are spacelike separated, what do "first" and "second" mean? Here hidden variables proponents, perhaps reluctantly, had to go further: Bohm and Hiley [4] invoked, contra SR, a *privileged* reference frame. Popper [23] was quick to point out this move's implications:

> It is only now, in the light of the new experiments stemming from Bell's work, that the suggestion of replacing Einstein's interpretation by Lorentz's can be made. If there is action at a distance, then there is something like absolute space. If we now have theoretical reasons from quantum theory for introducing absolute simultaneity, then we would have to go back to Lorentz's interpretation (p. 30).

Bell himself was even more explicit:

> The reason I want to go back to the idea of an aether here is because in these EPR experiments there is the suggestion that behind the scenes something is going faster than light. Now if all Lorentz frames are equivalent, that also means that things can go backwards in time… [this] introduces great problems, paradoxes of causality, and so on. And so it is precisely to avoid these that I want to say there is a real causal sequence which is defined in the aether (Quoted in [22], p. 31)

Maudlin [24] concludes:

> Since the exact outcome of the experiment depends on which […] measurement is made first, the notion of "first" and "second" has an ineliminable physical role in Bohm's theory. In the non-relativistic theory, which measurement comes first and which second is determined by absolute simultaneity. And if one is to transfer the Bohmian dynamic to a spacetime with a Lorentzian structure, one needs there to be something fit to play the same dynamical role. Since no such structure is determined by the Lorentzian metric, the simplest thing to do is to add the required structure: to add a foliation relative to which the relevant sense of "first" and "second" is (or "before" and "after") is defined. The foliation would then be invoked in the statement of the fundamental dynamical law governing the particle (p. 162).

Let us formulate this reasoning in terms of our hypothetical $HV\uparrow$. By probability, this variable is assumed to be present in 50% of the measurements. Hence it must be present in both EPR measurements in 25% of the cases. However, by angular momentum conservation, the two spins must be opposite. Therefore, to be paradox-free, the causal priority must go to one measurement, the second being able only to amplify the previously-determined spin.

Which measurement, then, is "first" and which "second" when they are timelike-separated? Fig. 4 shows how all alternatives to non-relativistic simultaneity either clash with experiment or lead to paradox (see also [25]). The nonlocal influence cannot go with any finite superluminal speed, lest spin-conservation violations ensue. Neither can the nonlocal influence remain "instantaneous" under Lorentz-transformation. Note that setting 4b has already been tested by Gisin *et al*. [26] and no Lorentz transformations were observed on the nonlocal correlations (see also [27]). The debate on "Sutherland's paradox" [28, 29], as well as the Maudlin-Kastner controversy [30]-[31], show that for any realistic temporal account that allows EPR particles to affect each other's past, back-and-forth effects will give rise to causal loops. Bohm's [4] privileged reference frame, then, remains the lesser evil.

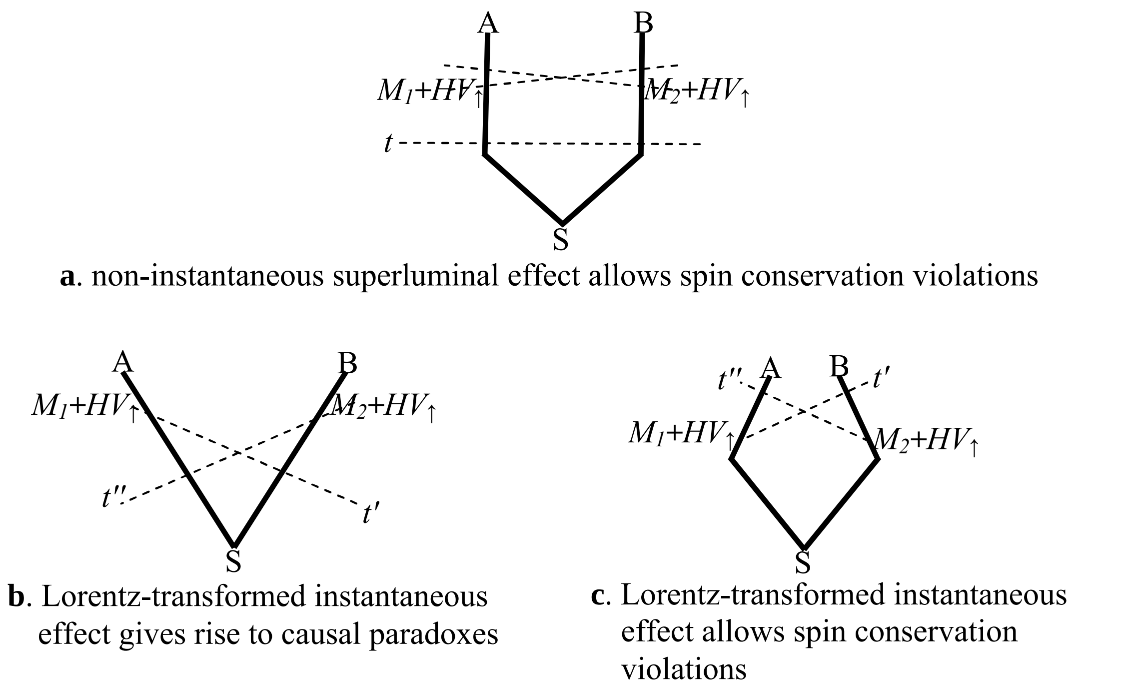

Fig. 4. EPR experiments where the hypothetical hidden variable *HV*↑ is present in both measurements (Following [25])

Admittedly, the question "what would the results be if the measurements' time-order was reversed" is a counterfactual, a method often considered dubious in QM. Recall Peres' [15] "unperformed measurements have no results" remark. While Peres considered the imperative "Thou shalt not think" as a possible remedy, he acknowledged that the EPR argument is based upon this very kind of counterfactuals. Obviously, it was worth thinking!

So, by Restriction 2 + spin conservation + causality,

Restriction 3: *Hidden variables must exert instantaneous effects within a preferred reference frame*.

Our challenge, now, is to extricate the measurement's temporal order from the realm of counterfactuals into that of experiments.

6.4. Hidden Variables must be Hidden Forever

While the experimental validation of Bell's proof entailed only an indirect violation of SR, the Bohmian preferred frame brings it closer to a manifest violation. This seems to add rigor to Elitzur and Dolev's [5] next restriction, namely that HVs are hidden not merely for some technical reason. Rather, relativity dictates that they forever *remain* what their name implies.

Consider again the hypothetical $HV\uparrow$, whose presence in the measurement causes the particle to assume the spin $\alpha_\uparrow$. By Restriction 2 (non-locality), $HV\uparrow$ may reside either in the measuring apparatus (say, its being positioned on an even or odd number of nanometers from the source) or in the interaction between the apparatus and the particle (say, its being performed at an even or odd number of nanoseconds since the particle's emission from the source).

Now, as, by Restriction 1, $HV\uparrow$ is deterministic, the implicit violation of relativity becomes straightforward: By merely *detecting* $HV\uparrow$'s presence or absence, the decision whether or not to perform a measurement enables enforcing the particle to assume a certain spin, thereby sending a superluminal signal to the other measurement. Valentini's [32] and Elitzur [33] reached a similar conclusion.[1]

Hence, by SR + Restriction 2,

<u>Restriction 4</u>: *Hidden variables must be hidden forever.*

Next, however, I show that, even without being detectable, hidden variables may lead to violations of SR.

6.5. Hidden Variables, if Time-Symmetric, must Violate Lorentz Invariance

It is now time to employ the tool of unmeasurement introduced in Section 3. If we cannot detect HVs, let us only time-reverse the measurements in which they are involved.

Let, then, EPR particles A and B undergo spacelike separated measurements, M_1 and M_2, of their α spins, with a small time interval (in the absolute frame of Restriction 3) between them. Recall that in 25% of the cases, the presumed spin-determining variable $HV_\uparrow$ is present in both M_1 and M_2. As M_1 is slightly "earlier" than M_2, then,

[1] Valentini [32] suggested that quantum nonlocality is hidden only because, in the remote past, the universe relaxed into statistical equilibrium at the HVs level. To this he added the even bolder prediction that once a way is found to isolate some exotic particles that escaped the primordial heat death, superluminal signaling would become possible, as well as entropy decrease. Elitzur [33] reached the diametrically-opposed conclusion: The three prohibitions – *i*) the relativistic prohibition on superluminal velocities, *ii*) the quantum mechanical prohibition on certainty and *iii*) the thermodynamic prohibition on entropy decrease – preserve one another in a still-unfathomed way. Both works, however, agree on one point: If HVs cease to be hidden, SR will be disproved.

by Restriction 4, only M_1 can affect A's spin to be $\alpha_\uparrow$, thereby forcing B to be $\alpha_\downarrow$. Consequently, M_2 can only amplify the state $\alpha_\downarrow$ imposed by M_1, despite the presence of $HV_\uparrow$ in M_2 too (Fig. 5a).

Next, perform two unmeasurements $\overline{M}_1$ and $\overline{M}_2$ on the two particles. By Restriction 1, we assume that $HV_\uparrow$, present in both measuring devices, takes part also in the two unmeasurements. As M_1 has occurred before M_2, now $\overline{M}_2$ must occur before $\overline{M}_1$. This, by definition, is true for every deterministic time-reversal. This is also what is required when the two measurements are performed on the same particle.

We have, then, two α-spin measurements followed by two unmeasurements, restoring the original Bell-state:

(4)

$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle+|\downarrow\rangle|\uparrow\rangle\right)\xrightarrow{M_1}\begin{array}{c} |\alpha_{A}\downarrow\rangle|\alpha_{B}\uparrow\rangle \xrightarrow{M_2} |\alpha_{A}\downarrow\rangle|\alpha_{B}\uparrow\rangle \xrightarrow{\overline{M}_2} |\alpha_{A}\downarrow\rangle|\alpha_{B}\uparrow\rangle \\ + \\ |\alpha_{A}\uparrow\rangle|\alpha_{B}\downarrow\rangle \xrightarrow{M_2} |\alpha_{A}\uparrow\rangle|\alpha_{B}\downarrow\rangle \xrightarrow{\overline{M}_2} |\alpha_{A}\uparrow\rangle|\alpha_{B}\downarrow\rangle \end{array}\xrightarrow{\overline{M}_1}\frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle+|\downarrow\rangle|\uparrow\rangle\right)$$

So far so good (Fig. 5a). Things do not go well, however, when the experiment's frame is *moving* (5b). The experimental setup (e.g., rulers and clocks) undergoes Lorentz transformations, but, by Restriction 3, the nonlocal effects must *not*.

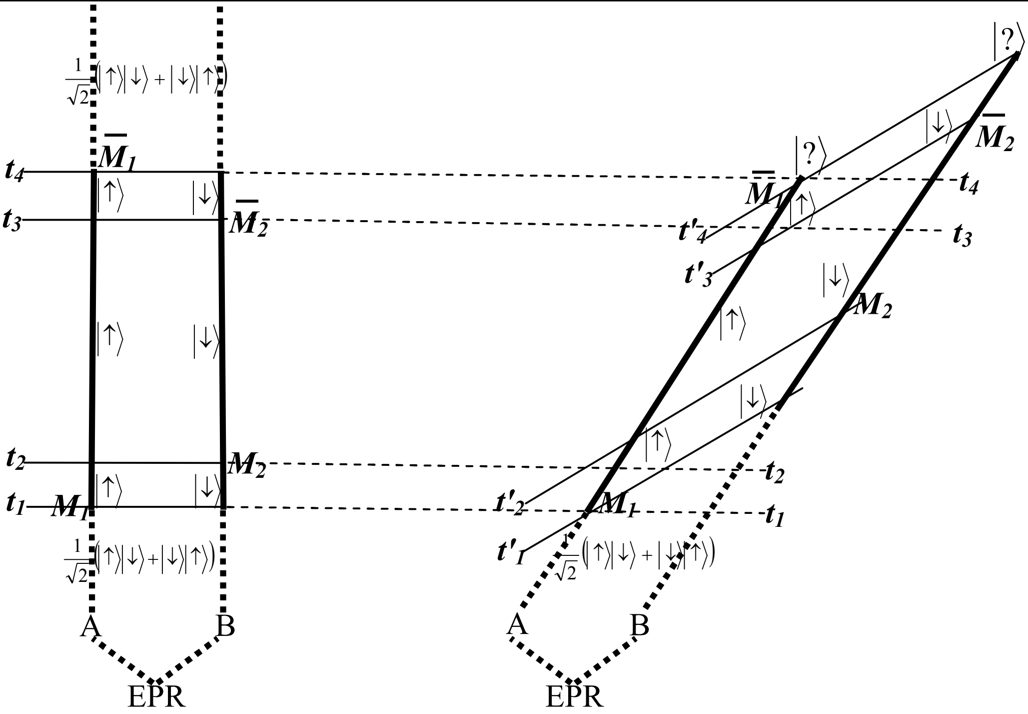

Fig. 5

a. An EPR experiment undergoing time-reversal. Notice that the order of the unmeasurements is opposite to that of the measurements.

b. The same experiment in a moving frame. The time interval between measurements only dilates, while that between unmeasurements is reversed.

The clash is imminent. By Restriction 3 on HVs, the nonlocal effects of the measurements and unmeasurements are exerted along the privileged time-planes t_1, t_2, t_3, t_4. When the frame moves, the simultaneity planes t'_1, t'_2, t'_3, t'_4 of the moving frame become Lorentz-inclined, deviating from the privileged planes. Consequently, the time-interval between the spacelike-separated M_1 and M_2 merely dilates, but the $\overline{M}_2$-$\overline{M}_1$ interval is *reversed*. The symmetric sequence M_1-M_2-$\overline{M}_2$-$\overline{M}_1$ now changes to the asymmetric M_1-M_2-$\overline{M}_1$-$\overline{M}_2$, which will ruin the overall time-reversal of the EPR experiment.

It should again be pointed out that the meticulous experiment performed by Gisin and co-workers [26] has tested quantum nonlocality under relativistic motions of the two particles involved. The results accord with quantum theory. The present *gedankenexperiment* adds the yet-unfeasible challenge of unmeasurement, but now the clash with relativity becomes inescapable:

Restriction 5: *Hidden variables, if time-symmetric ,must violate Lorentz invariance.*

6.6. Hidden Variables, if Lorentz Invariant, must Violate Time-Symmetry

The above anomaly arose from the reasonable assumption that the time-order of the two unmeasurements must be opposite to that of the earlier measurements. What if we drop this assumption? A complementary anomaly is imminent.

Consider, then, the alternative possibility: The individual results of each pair of measurements depend, by Restriction 3, on the measurements' time-order, yet the time order of *un*measurements makes no difference. Whatever spins are measured and whatever their results, unmeasurement always leads back to $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle+|\downarrow\rangle|\uparrow\rangle\right)$.

This assumption is clearly odd, because it is certainly incorrect with respect to a single particle: If one particle undergoes, say, an α and then a β-spin measurements

(5) $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle+|\downarrow\rangle\right)\xrightarrow{M(\alpha)}|\alpha\uparrow\rangle\xrightarrow{M(\beta)}|\beta\uparrow\rangle$,

it is impossible to unmeasure α before β, as α↑ is no longer valid! It is therefore hard to believe that the situation will be different with a pair of EPR measurements.

Even worse: If, in order to preserve Lorentz invariance, we do not assign importance to the time-order of the unmeasurements, a time-asymmetry ensues:

The time-order of the measurements matters in determining the individual spin values (restriction 3), but the time-order of the unmeasurements does not matter in bringing them back to their initial state.

Hence:

W*hile the measurements of some spin directions are non-commuting, their respective unmeasurements always commute.*

Let us inspect this asymmetry in detail. Consider first the time-symmetric evolution in section 6.5:

(6)

$$
\begin{tikzcd}[column sep=small]
 & |\alpha_A\uparrow\rangle|\alpha_B\downarrow\rangle \arrow[r, "M_2(\beta)"] & {\begin{matrix}|\alpha_A\uparrow\rangle|\beta_B\uparrow\rangle \\ + \\ |\alpha_A\uparrow\rangle|\beta_B\downarrow\rangle\end{matrix}} \arrow[r, "\overline{M}_2(\beta)"] & |\alpha_A\uparrow\rangle|\alpha_B\downarrow\rangle \arrow[dr] & \\
\frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle+|\downarrow\rangle|\uparrow\rangle\right) \arrow[ur, "M_1(\alpha)"] \arrow[dr, "+"'] & & & & + \xrightarrow{\overline{M}_1(\alpha)} \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle+|\downarrow\rangle|\uparrow\rangle\right) \\
 & |\alpha_A\downarrow\rangle|\alpha_B\uparrow\rangle \arrow[r, "M_2(\beta)"] & {\begin{matrix}|\alpha_A\downarrow\rangle|\beta_B\uparrow\rangle \\ + \\ |\alpha_A\downarrow\rangle|\beta_B\downarrow\rangle\end{matrix}} \arrow[r, "\overline{M}_2(\beta)"] & |\alpha_A\downarrow\rangle|\alpha_B\uparrow\rangle \arrow[ur] &
\end{tikzcd}
$$

This sequence remains invariant under *T*-reversal: Just read the temporal sequence backwards. Under this operation, every measurement turns into an unmeasurement and *vice versa*, the above sequence remaining unaffected.

Now assume that the time-order of the unmeasurements is *not* important, such that, even under the asymmetric sequence M_1-M_2-$\overline{M}_1$-$\overline{M}_2$, the same final superposition is obtained:

(7)

$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle+|\downarrow\rangle|\uparrow\rangle\right)\xrightarrow{M_1(\alpha)}\begin{array}{c}|\alpha_A\uparrow\rangle|\alpha_B\downarrow\rangle\xrightarrow{M_2(y)}\begin{array}{c}|\alpha_A\uparrow\rangle|\beta_B\uparrow\rangle\\+\\|\alpha_A\uparrow\rangle|\beta_B\downarrow\rangle\end{array}\\+\\|\alpha_A\downarrow\rangle|\alpha_B\uparrow\rangle\xrightarrow{M_2(y)}\begin{array}{c}|\alpha_A\downarrow\rangle|\beta_B\uparrow\rangle\\+\\|\alpha_A\downarrow\rangle|\beta_B\downarrow\rangle\end{array}\end{array}\begin{array}{c}\xrightarrow{\overline{M}_1(\alpha)}|y_A\downarrow\rangle|\beta_B\uparrow\rangle\\+\\\xrightarrow{\overline{M}_1(\alpha)}|y_A\uparrow\rangle|\beta_B\downarrow\rangle\end{array}\xrightarrow{\overline{M}_2(\beta)}\frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle+|\downarrow\rangle|\uparrow\rangle\right)$$

Under *T*-reversal, it is the time-order of the *measurements* which is reversed:

(8)

$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle+|\downarrow\rangle|\uparrow\rangle\right)\xrightarrow{M_1(\beta)}\begin{array}{c}\beta_A\downarrow\rangle|\beta_B\uparrow\rangle\xrightarrow{M_2(\alpha)}\begin{array}{c}|\beta_B\uparrow\rangle|\alpha_A\uparrow\rangle\\+\\|\beta_B\downarrow\rangle|\alpha_A\downarrow\rangle\end{array}\\+\\|\beta_A\uparrow\rangle|\beta_B\downarrow\rangle\xrightarrow{M_2(\alpha)}\begin{array}{c}|\beta_B\uparrow\rangle|\alpha_A\downarrow\rangle\\+\\|\beta_B\uparrow\rangle|\alpha_A\downarrow\rangle\end{array}\end{array}\begin{array}{c}\xrightarrow{\overline{M}_1(\beta)}|\alpha_A\uparrow\rangle|\alpha_B\downarrow\rangle\\+\\\xrightarrow{\overline{M}_1(\beta)}|\alpha_A\downarrow\rangle|\alpha_B\uparrow\rangle\end{array}\xrightarrow{\overline{M}_2(\alpha)}\frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle+|\downarrow\rangle|\uparrow\rangle\right),$$

and yet the two measurements' results $|x_A\uparrow\rangle$ and $|y_B\uparrow\rangle$ are identical to those of (7).

Restriction 3, therefore, holds only in the forward time-direction. Worse, noncommuting measurements turn out to be commuting!

Admittedly, the Elitzur-Dolev method [11] as well as the experimental realization by Katz *et al.* [12], are immune to the order of unmeasurements (the order of measurements, of course, being equally devoid of experimental significance). However, as pointed out above, these methods involve an irreversible element and may therefore not count as true unmeasurements by hidden variables theories.

Still, compared to the manifest Lorentz invariance violation posed by Restriction 5, a measurement-unmeasurement asymmetry might be the lesser evil:

<u>Restriction 6</u>: *Hidden variables, if Lorentz invariant, must violate time-symmetry.*

7. Aiming Higher: Perhaps Question Determinism?

Is the choice between the non-Lorentz-invariant Scylla and the time-asymmetric Charybdis forced on us? Not necessarily, for we may go further back along the Incompatibility Proof and dismiss one of its even more fundamental assumptions.

First, one may question the most fundamental assumption of classical physics, namely, realism. In other words, I could add (*iv*) "realism" as a fourth postulate in the above Incompatibility Proof, and then mention its dismissal it as a fourth option at the Proof's end. I confess that I didn't find this option worth considering. Even within the Copenhagen school, where quantum phenomena are denied objective existence, the *relations between them* must comply with all physical principles. Bohr's [34] hasty reply to the EPR argument makes it clear that even if quantum phenomena are observer dependent, they must never violate special relativity.

What, then, about the next dreaded price, namely, relinquishing *determinis*m? Perhaps, so goes this option, the above derivations of Lorentz-non-invariance or time-asymmetry violation merely indicate that *hidden variables do not exist*?

Personally, I always found this option the most tenable one, as well as its broader implications. Recall Restriction 6.4, namely, "Hidden variables must be hidden forever." This restriction renders HVs a more ghostly entity than the pre-relativistic ether. The latter was considered merely hard to observe but there was no theoretical *prohibition* on its detection.[2] HVs, in contrast, are entities which SR *forbids* to be detected or observed. This places them in a position closer to religion than to science (see *Exodus* 33, 20).

If, then, HVs follow the ether to oblivion – if every measurement's outcome is *genuinely* random – then unmeasurement is impossible in the first place: One cannot undo a measurement if there is no spin-determining variable, hidden or manifest, which can take part in the undoing. If this is the case, then *it is this basic irreversibility involved in any quantum interaction that constitutes the long sought-for macroscopic origin of time's arrow*.

Indeed, even authors working within the collapse framework, such as McCall [17], Leggett [35] and Gisin *et al.* [26] conclude that collapse entails a non-Lorentz-

[2] Significantly, Bell himself, who endorsed HVs, explicitly likened them to the ether [22]

invariant physics, hidden forever beneath Restriction 4, in addition to the penalty of abandoning time-symmetry.

Finally, let us consider Cramer's transactional interpretation [36] and Aharonov's two-vector formalism [25], which describe the EPR correlations as a spacetime zigzag between the two measurements via the source in the past. How would these models describe *unmeasurement*? The literature related to this model has not dealt with this question yet, but one thing seems to be clear: *Like measurement, the effect of unmeasurement should also go backwards in time, all the way down till the last measurement*. The conceptual price of this account, however, is high: *Different histories override one another along the same time-sequence every time an unmeasurement is carried out*. Therefore, while these models may preserve the entire sacred trio of determinism, Lorentz invariance and time-symmetry, this will demand a radical revision of the very notion of time.

8. Summary

HVs and time-symmetry are two of QM's most profound unresolved issues. This paper discusses gedankenexperiments which combine measurements with unmeasurements. The results pose two new restrictions on HVs, namely, that they must manifestly violate either Lorentz invariance or time-symmetry. This unappetizing choice may be escaped by abandoning HVs (hence determinism) altogether, embracing instead genuine randomness as a fundamental aspect of physical reality. This would indicate that the future, just as intuition keeps telling us, is indeed open, fundamentally nonexistent, subject to some kind of Becoming [37] alien to Relativity. Either way, something very essential seems to be missing in modern physics' picture of time.

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