Found Phys (2009) 39: 499–509
DOI 10.1007/s10701-009-9301-1

Can Special Relativity Be Derived from Galilean Mechanics Alone?

Or Sela · Boaz Tamir · Shahar Dolev · Avshalom C. Elitzur

Received: 1 September 2008 / Accepted: 17 March 2009 / Published online: 28 March 2009

Abstract Special relativity is based on the apparent contradiction between two postulates, namely, Galilean vs. c-invariance. We show that anomalies ensue by holding the former postulate alone. In order for Galilean invariance to be consistent, it must hold not only for bodies' motions, but also for the signals and forces they exchange. If the latter ones do not obey the Galilean version of the Velocities Addition Law, invariance is violated. If, however, they do, causal anomalies, information loss and conservation laws' violations are bound to occur. These anomalies are largely remedied by introducing waves and fields that *disobey* Galilean invariance. Therefore, from these inconsistencies within classical mechanics, electromagnetism could be predicted before experiment proved its existence. Special relativity, it might be argued, would then follow naturally, either as a resolution of the resulting conflict or as an extrapolation of the path between the theories. We conclude with a review of earlier attempts to base SR on a single postulate, and point out the originality of the present work.

Keywords Special relativity · Galilean invariance · Electromagnetism

Two postulates underlie SR [1], namely,

(i) The laws of physics are the same in all inertial reference frames,

and

(ii) The speed of light in vacuum has the same value c in all inertial reference frames.

O. Sela
Timorim, Israel

B. Tamir · S. Dolev · A.C. Elitzur (✉)
Iyar, The Israeli Institute for Advanced Research, Rehovot, Israel
e-mail: avshalom.elitzur@weizmann.ac.il

B. Tamir
The Bezalel Academy of Arts and Design, Jerusalem, Israel

It is the contradiction between them, so goes the conventional account, which necessitates relativity theory.

But are both assumptions necessary? We submit that even if (i) alone is held consistently enough, anomalies are bound to occur, from which (ii) would then be predicted by theory before being confirmed by experiment.

The above move is not new. It has been proposed, forgotten and then revisited several times, yielding a variety of insights. We shall review them in Sect. 9 and then point out the novelty of the present one.

1 The GC Assumption: Imposing Galilean Transformations on Signals and Forces

Galilean transformation subjects all bodies' motions to the Velocities Addition Law. But what about the causal effects exerted by these bodies? Unless signals and distant forces act instantaneously (an option to be considered below), they occur after some time interval, hence a consistent transformation must hold for them too.

Suppose, then, that *all* motions, those of bodies, signals and forces, obey Galilean transformations. This is the Galilean Causality assumption, henceforth GC:

Causal effects move according to the same laws governing the motions of the bodies that exchange them.

One advantage of GC immediately appears: It yields greater symmetry than ordinary classical physics. Light's velocity must be added to both the source's and the observer's velocities, as no ether is invoked. Consequently, the Doppler effect for light under GC,

$$f' = f \cdot (1 + v/c), \tag{1}$$

refers to both velocities as v. No relativistic correction is therefore needed.

In other respects, however, GC gives rise to numerous anomalies, of which we point out only a few below. We then show that the introduction of fields, within which signals propagate with finite velocity *disobeying* the Velocities Addition Law, remedies most of these anomalies. Galilean invariance is therefore the sole progenitor of the conflict that would require, in turn, SR for resolution.

2 Time and Position Becoming Velocity Dependent

Under GC, the measurement of a body's position at a certain time strictly depends on the body's velocity, no matter how low.

Consider the simplest method of determining an event's time. Signals from the event, along with signals emitted by a nearby clock, are received by the observer at the same time. Similarly for a position measurement: Signals from the body arrive together with signals emitted from a nearby ruler.

Let the universe, then, be populated with equidistant milestones, each equipped with its own light-source and clock. Similarly let all other bodies have their light-sources and clocks. Let all clocks be synchronized. Finally, let light have a constant velocity c in the absolute rest frame.

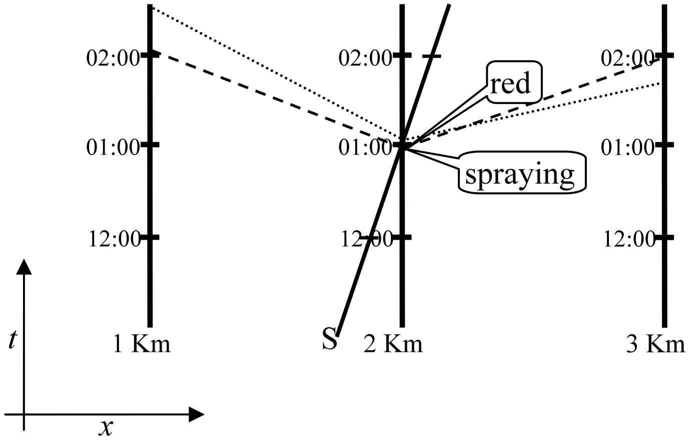

Fig. 1 Measuring position and time. A network of three equidistant milestones + clocks enables determining the position of spacecraft S at a certain instant. The interaction between moving and the stationary bodies is represented by the milestone spraying the spacecraft in red. Signals from cause and effect are represented by dashed and dotted lines, respectively

Now consider a spacecraft in motion, no matter how slow. Its position at a certain time is determined by the juxtaposition of its light signals with those of the nearby milestone, as well as the readings of their two clocks. Assume also that there is some physical interaction between the two bodies. E.g., the spacecraft is sprayed by the milestone in red when passing by it (Fig. 1).

By GC, the spacecraft emits light signals to all directions with velocity $\vec{c} + \vec{v}$, where $\vec{c}$ is a radial vector with speed c, and $\vec{v}$ the spacecraft's velocity, required by GC to be add to $\vec{c}$, the resultant vector of the signal towards the observer being $\vec{u}$.

The time interval Δt between the two causally-related events as seen by the observer, namely, the milestone's spraying and the spacecraft's reddening, varies according to the spacecraft's velocity:

$$\Delta t = |\vec{r}| \cdot \left(\frac{1}{c} - \frac{1}{|\vec{u}|} \right), \tag{2}$$

where $\vec{r}$ is the spacecraft's location vector relative to the observer (the observer being at the origin). The resultant $|\vec{u}|$ is

$$|\vec{u}| = c\sqrt{1 - \frac{|\vec{r} \times \vec{v}|^2}{c^2 \cdot |\vec{r}|^2}} - \frac{\vec{v} \cdot \vec{r}}{|\vec{r}|}. \tag{3}$$

Thus, under GC, even events occurring at the same location cannot be perceived as simultaneous if they have different velocities. This becomes especially paradoxical when the spacecraft moves towards the observer: The spacecraft's reddening appears before the milestone's spraying, causality thereby appearing reversed.

Consider next a body's physical effects, such as its electric or gravitational field. It does not matter whether the field is conveyed by particles or waves (see [2]). Our only concern is, Do changes in the field's position spread from the mass/charge at a velocity that is added to that of the mass/charge's velocity? GC entails a positive answer. Consequently, the timing of all physical effects varies according to the masses' and charges' velocities.

To summarize, a body's real position at a certain time is a function of its position as it appears to an observer and its velocity,

$$\vec{r}_{real} = \vec{r}_{ob} + \vec{v} \cdot \frac{|\vec{r}_{ob}|^2}{\sqrt{|\vec{r}_{ob}|^2 c^2 - |\vec{r}_{ob} \times \vec{v}|^2} - \vec{r}_{ob} \cdot \vec{v}}, \tag{4}$$

position and time measurements thus becoming velocity dependent.

3 Violation of Rotational Invariance and Apparent Causality Reversal as a Result of Acceleration

When acceleration is considered within GC, one would expect only Galilean invariance to be violated. In reality, however, severe causal anomalies ensue.

Let a body commence motion towards an observer O_1. By GC, the first signals emitted during the body's motion reach observer before the last signals emitted during its rest, giving rise to a double image (Fig. 2). As the same holds for any force exerted by B (see above), the observer also experiences B's forces twice as much. And as the signals indicating motion come before the signal indicating the motion's commencement at the end of the rest period, causality reversal appears here as well.

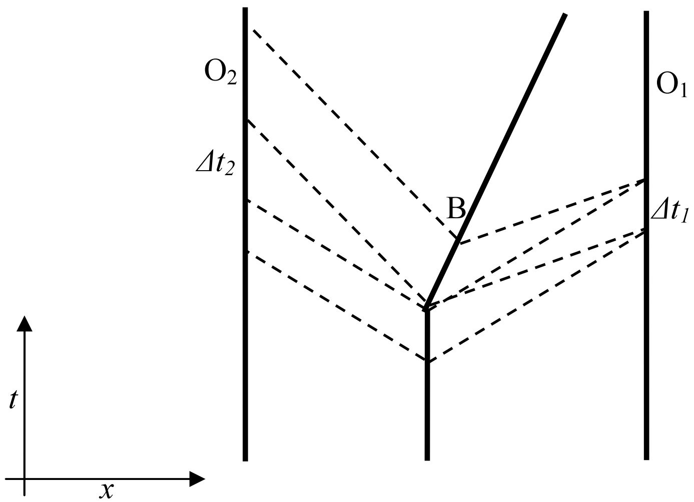

Fig. 2 Two observers see body B. O_1 observes during Δt_1 a double image and causality violations while O_2 observes during Δt_2 a momentary disappearance of B

These anomalies occur during the time interval

$$\Delta t_1 = \frac{Dv}{c(c+v)}, \tag{5}$$

where D is the distance between the body and the observer prior to the motion, and v the final velocity once acceleration is over.

Now consider the same process from the viewpoint of another observer O_2 located at the body's opposite side, such that the body moves away from her. She sees the body virtually disappearing for a while and then re-appearing. This disappearance occurs during the time interval

$$\Delta t_2 = \frac{Dv}{c(c-v)}. \tag{6}$$

Thus, "dark bodies," the progenitors of the modern black holes, which have already been shown by Mitchell to plague the Newtonian theory of light [4], would be much more commonplace in classical physics: not only with massive bodies but with every receding body.

The case portrayed in Fig. 2 exhibits an even more strident result: Two observers, observing the same process *within the same reference frame*, get different causal accounts. Unlike ordinary deceptive results, such as aberration, that can be corrected, a body's virtual disappearance leaves no signals for correction. While Galilean invariance is preserved in this case, a more fundamental invariance, namely, invariance under rotation, is violated.

4 Disappearance and Phantom Images

Next consider superluminal motions. Let an observer at rest observe a body moving towards her faster than light. The light signals emitted from the body move at $c + v$. Once the body has passed the observer, it virtually vanishes for her, its light and forces no longer reaching her, as $c - v < 0$. On the other hand, the body's effects on other bodies on its way continue to be observed, as those bodies keep emitting normal signals giving rise to ghost-like effects.

The superluminal body reappears, however, in a most peculiar way. Earlier signals, transmitted from its *rear* before it has passed the observer, keep moving towards her at $v - c > 0$, in a reverse order to that of their emission (see Fig. 3). The body therefore appears to have "turned back." This phantom image, however, does not interact with the other bodies on its "way," e.g., going through walls, etc.

One might object that superluminal velocities give rise to paradoxes even in Maxwellian physics. But there is an essential difference: In Galilean physics, all these anomalies arise every time the *relative* velocity between two bodies surpasses c (Lévy-Leblond [3] makes a similar observation), whereas in Maxwell's theory, only *absolute* superluminal motions allow only *some* of these anomalies.

Here, the introduction of Maxwellian waves remedies only some of the anomalies entailed by GC. This partial success is significant in itself, as shown below.

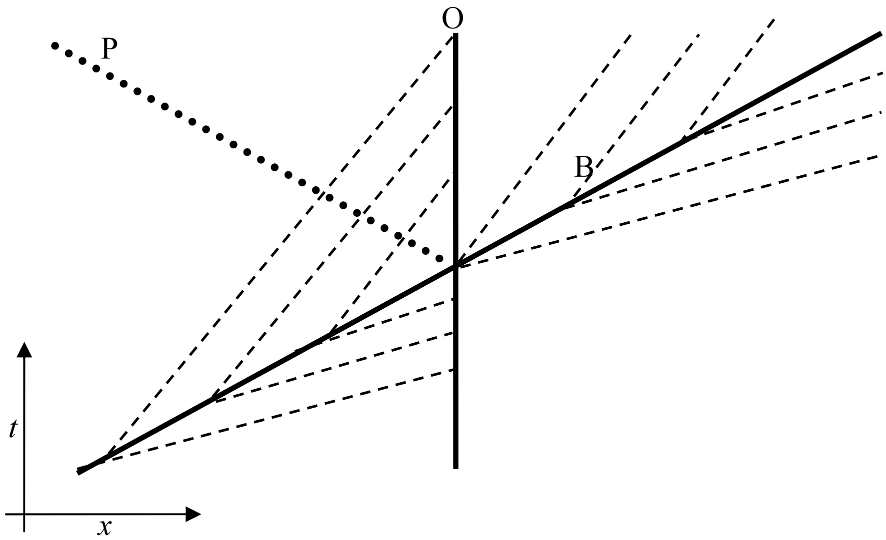

Fig. 3 A body B moving faster than the speed of its signals at rest, hence emitting them at $c+v$ towards O. After the body passes O, it disappears for her, but the phantom image P of its rear appears to be turning back

5 Information Loss, Irreversibility and Violations of Conservation Laws

Finally, let a body commence motion at $v>c$ at some distance from an observer. For the observer to determine its position, (4) should be used. If $\vec{r}_{ob} \times \vec{v} = 0$, the quotient in (4) becomes negative, but then this quotient equals the time it takes the signal to reach the observer, hence it cannot be negative. Similarly, if $\vec{r}_{ob} \cdot \vec{v} = 0$, the expression under the root sign becomes negative. Physically, the body's real position in both cases becomes undefined. i.e., it disappears.

Should the body stop at some later time, it would reappear to the observer at $t = D/c$. However, the signals which did not reach her during the superluminal travel would never return. The body's effects on other bodies during its superluminal travel give only partial information. A fundamental loss of information thus occurs.

The body's disappearance creates a virtual violation of mass conservation, and, if the body is charged, violation of charge conservation as well. These violations are local and temporal, as observers at other sites and times will see double or phantom images as described above. But this global conservation is attained at the cost of violating rotational invariance, shown in Sect. 3 above.

Moreover, although the observer may retrieve the lost signals simply by running after them, *other* bodies will consequently be lost for her. Violations of conservation laws, over large areas and extended periods, are therefore unavoidable under GC every time a body moves at $v>c$.

6 Action at a Distance?

Where, then, did GC go wrong? First, the idea that causal influences are mediated by particle-like agents seems to violate conservation laws. It would be better, then, to conceive of a field, in the form of a sphere surrounding the body, moving with it as it moves, thus carrying its effect. Let us explore this option.

The crucial question is, At what velocity should this field transmit the body's force? Here a simple option comes to mind that might restore GC with no anomalies: The field should enable instantaneous actions. As $\infty + v = \infty$, infinite velocity obeys Galilean transformations!

Consider, e.g., a charge surrounded by a large field. Far away there is much smaller charge of the same sign, lying just outside the former's noticeable influence. Now let the big charge slightly move towards the small. Once the big charge's field engulfs the small charge, kinetic energy is transmitted and pushes the latter.

Our question then takes the following form: Could the big charge's field move with it instantaneously? A positive answer would end our quest and leave Galilean invariance paradox-free, while a negative answer would render electromagnetism the better option. Note that relativistic considerations should not be employed for this purpose: Infinite velocity should be outlawed on purely classical grounds.

Consider first a mass moving at infinite speed. In fact this is not real motion but an instantaneous appearance and disappearance of the mass at numerous locations, violating mass conservation. Also, it requires infinite energy to be initiated. Finally, a ball bouncing back and forth between two walls at infinite velocity gives rise to infinite mass at every location, as well as a variety of causal loops. This rules out infinite kinetic energy. Other instantaneous energy transfers can be dismissed similarly.

Infinite velocity, therefore, can be ruled out within GC due to the causal anomalies it entails even on the classical, nonrelativistic level.

7 The Remedy

We are therefore left with the elastic, non-rigid field known from electromagnetism as the only anomalies-free means of causal effects. Consider a body surrounded by a field, commencing motion. Its signals are sent through the field. *The field's distant parts are still at rest, hence the effect propagates in them as in the rest frame* (see Fig. 4). In other words, the body's effect on distant bodies does *not* obey Galileo's law of velocity addition. Similarly for light signals: If they are waves, their medium is not affected by the source's motion. Thanks to this exemption, no anomalies occur.

Note that the introduction of Maxwellian waves does not eliminate all the anomalies associated with superluminal motions. For example, the Galilean Doppler effect (1) allows disappearance of a body when either the body or the observer outraces c. This is because both source's and the observer's velocities must be added to that of light. In contrast, the Maxwellian Doppler is

$$f' = f\frac{c + v_o}{c - v_s}, \tag{7}$$

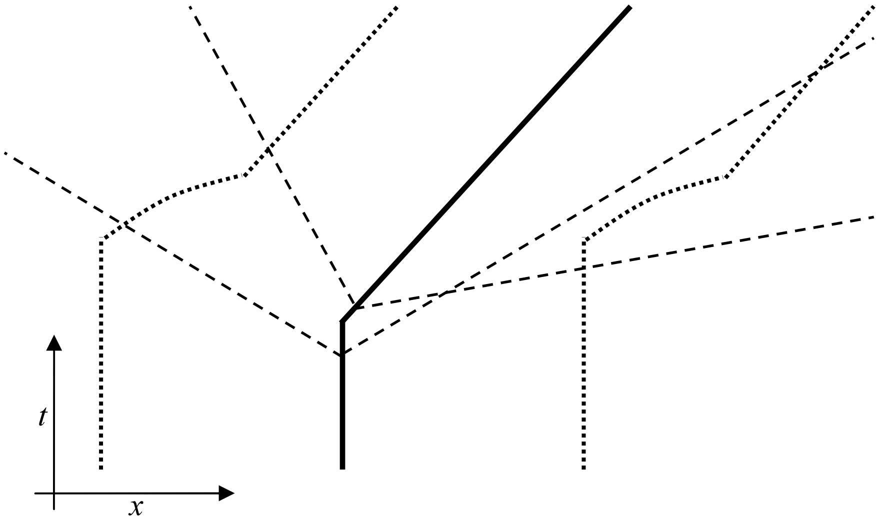

Fig. 4 Anomalies like those portrayed in Figs. 2 and 3 vanish once an elastic field is invoked. The body's motion's effect on the field (*dotted line*) differs from that of particle signals (*dashed lines*), hence the field eventually resumes its normal form around the moving body

where v_o and v_s are the positive motions of the observer and source, respectively, towards one another. Only the observer's velocity is added to that of light. Consequently, disappearance occurs only when the observer outraces c. This "half-way" resolution by electromagnetism is significant, as shown in the conclusions below.

8 A Natural Evolution of the Invariance Principle: Galilean, Electromagnetic and Relativistic

We are now ready to put in the new perspective the relativistic Doppler

$$f' = f\frac{1-\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}, \tag{8}$$

where v is the relative velocity between source and observer. The three Doppler version, namely, The Galilean (1), The Maxwellian (7) and the relativistic Doppler (8), are based on three accounts of light velocity. Let c_{rel} denote light's relative velocity, c light's velocity in the absolute rest frame, and v_s and v_o, respectively, the source's and the observer's velocities. The Galilean version assumes

$$c_{rel} = c + v_s + v_o, \tag{9}$$

adding the velocities of both source and observer. The Maxwellian Doppler (7) is based on a different assumption,

$$c_{rel} = c + v_o, \tag{10}$$

making the source's velocity ineffective. Finally the relativistic Doppler (8) assumes

$$c_{rel} = c. \tag{11}$$

As reasonable (9) seems to be, it gives rise to the plethora of anomalies dealt with in this article. Equation (10) removes most of these, but at the cost of asymmetry, while (11) comes full circle, restoring symmetry and eliminating all anomalies.

We have shown that the Maxwellian (10) can be arrived at as a means of eliminating the Galilean (9)'s anomalous consequences, SR's (11) then arising from the conflict between them. A more direct reasoning is also possible: If (10) still leaves some of (9)'s anomalies unresolved, then, by extrapolation, (11) suggests itself as completion.

9 History of the Idea

Lévy-Leblond [3] ended his illuminating article with the following "*Note added in proof:*"

> This work was already submitted for publication when a closely related investigation by R. Lee and T.M. Kalotas appeared in this Journal... These authors also point out the very early roots of such considerations, going back to several papers more than 60 years old, and long forgotten or neglected.

Some thirty years later, we found ourselves in Lévy-Leblond's position. A reference suggested by one of our referees, as well as a comment by the Editor, made us realize that what we regarded as an original idea has been raised several times in the past, yielding many insights into relativity. In what follows we review these works and point out the present one's novelty.

9.1 Subjecting Light to the Velocites Addition Law

Galileo's Velocity Addition Law was applied to signals at the heyday of Newtonian physics [4], but this move was soon abandoned with the advent of electromagnetism. More strikingly, GC has been considered some time before 1905 by no other than Einstein. He has toyed with an "emission theory" in which light's velocity was constant with respect to the source. He eventually abandoned it in favor of SR, due to the former's incompatibility with known phenomena like polarization and aberration [5]. Ritz, however, kept endorsing an emission theory version of his own [6].

It is striking that Einstein, famous for his high regard for elegance and simplicity, discarded the emission theory on purely empirical grounds, based on what he knew on electromagnetic effects. The move in this article, therefore, is more strictly Einsteinian: GC should be rejected because it eventually ruins the invariance which is supposed to be Galilean physics' greatest virtue.

9.2 Deriving C from Galilean Invariance

Similarly, the derivation of SR from Galileo's invariance alone has precedents too [7–13 and references therein, 14 and references therein]. References [11, 12] deviate from other concepts of relativity and are therefore controversial. The other authors, who sought only to narrow the axiomatic basis of SR, employed homogeneity and isotropy of space and time as auxiliary assumptions to Galilean invariance [3, 8, 9]. Mermin [10] voiced a common opinion that relativity "is not a branch of electromagnetism and the subject should be developed without any reference whatever to light" (p. 119).

None of these authors, however, pursued GC to the end in order to examine whether it eventually yields a paradox-free physics.

9.3 The Novelty of the Present Proposal

The novelty of the present work lies, therefore, in the following:

Simplicity Most of the previous works [e.g., 3, 7–9] competed on simplicity. The present one boils down to the following proof: "*If* Galileo's Velocities Addition Law holds for signals and forces, *then*, when pursued consistently, effects appear before their causes, and bodies temporarily cease to exert their forces." This proof is logical, needing no more than basic algebra to be phrased quantitatively.

Elegance As Galileo's invariance makes physical law simple and elegant, it is striking that, when pursued consistently, it gives rise to such awkward results as causal anomalies and rotation invariance violation. The aesthetic criterion, so central to Einstein's thinking, serves in our proposal as the main guide.

Ether naturalized Due to the above anomalies, the ether, for the first time, is no more a surplus construction but rather an essential remedy. Physics, as it were, yearns for some medium in order to restore causality and invariance. Only later will this remedy be discarded in favor of greater invariance.

10 Conclusions

Historically, fields and waves were introduced into physics by experiment and observation rather than by theoretical considerations. We have shown, however, that they could have been theoretically anticipated by the anomalies stemming from an all-inclusive Galilean transformation, such that later experiments would confirm them, as in other cases in physics' history. To be free of causal paradoxes, Galilean transformations must hold for masses and charges, and at the same time *not* hold for the causal effect that they exchange.

Two possible moves, as shown at the end of Sect. 8, lead from this step to SR. Either way, and only with the cheap advantage of hindsight, relativity could be attained from Galilean invariance's bearing the seeds of its own destruction.

Acknowledgements We are deeply indebted to Galina Weinstein for her invaluable help and criticism. It is a pleasure to thank Joe Rosen for enlightening discussions. A.C.E. wishes to thank Oded Maler and Université Joseph Fourier, Grenoble, where this work has been carried out. Thanks are due also to two referees and the Editor for helpful comments.

References

1. Einstein, A.: The Meaning of Relativity. London, Methuen (1951)
2. Petkov, V.: Relativity and the Nature of Spacetime. Springer, New York (2005)
3. Lévy-Leblond, J.-M.: One more derivation of the Lorentz transformation. Am. J. Phys. **43**, 434 (1975). doi:10.1119/1.9807
4. Eisenstaedt, J.: From Newton to Einstein: A forgotten relativistic optics of moving bodies. Am. J. Phys. **75**, 741 (2007). doi:10.1119/1.2742398
5. Norton, J.: Einstein's investigations of Galilean covariant electrodynamics prior to 1905. Arch. Hist. Exact Sci. **59**, 45 (2004). doi:10.1007/s00407-004-0085-6
6. Martínez, A.A.: Ritz, Einstein, and the Emission hypothesis. Phys. Perspect. **6**, 4 (2004). doi:10.1007/s00016-003-0195-6
7. Mitvalský, V.: Special relativity without the postulate of constancy of light. Am. J. Phys. **34**, 825 (1966). doi:10.1119/1.1973524
8. Lee, A.R., Kalotas, T.M.: Lorentz transformations from the first postulate. Am. J. Phys. **44**, 271 (1976). doi:10.1119/1.10231
9. Behera, H.: A comment on the invariance of the speed of light. http://arxiv.org/abs/physics/0304087v1 (2003)
10. Mermin, N.D.: Relativity without light. Am. J. Phys. **52**, 120 (1984)
11. Hsu, J.-P., Hsu, L.: A Broader View of Relativity: General Implications of Lorentz and Poincare Invariance. World Scientific, Singapore (2006)
12. Sfarti, A.: Single-postulate theory of relativity and generalization of the clock synchronization principle. Phys. Essays **19**, 355 (2006)
13. Feigenbaum, M.J.: The theory of relativity—Galileo's child. http://arxiv.org/abs/0806.1234v1 (2008)
14. Gannett, J.W.: Nothing but relativity, redux. Eur. J. Phys. **28**, 1145 (2007). doi:10.1088/0143-0807/28/6/011