

NON-SEQUENTIAL BEHAVIOR OF THE WAVE FUNCTION

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An experiment is presented in which the alleged progression of a photon's wave function is "measured" by a row of superposed atoms. The photon's wave function affects only one out of the atoms, regardless of its position within the row. It also turns out that, out of n atoms, each one has a probability which is higher than the classical probability $1/n$ to be the single affected one. These results indicate that the wave function manifests not only non-local but also non-sequential characteristics.

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I. INTRODUCTION

When a single photon goes through a Mach-Zehnder Interferometer, its behavior indicates that it has somehow traversed both arms. However, when its position is measured during this passage, it turns out to have traversed only one arm. This is one of the notable manifestations of the measurement problem, for which several competing interpretations have been proposed. These can be crudely divided into two groups: "collapse" (*e.g.* Copenhagen, GRW) and "non-collapse" (*e.g.* Guide Wave, Many Worlds) interpretations.

Both groups, however, seem to share one assumption. The photon – whether in the form of wave-plus-particle or of a wave function evenly spread over all available positions – is believed to proceed from the source to the detector sequentially through space-time. Hence, if a few objects are placed along its path, the photon is expected to interact with them one after another, according to the order of their positions.

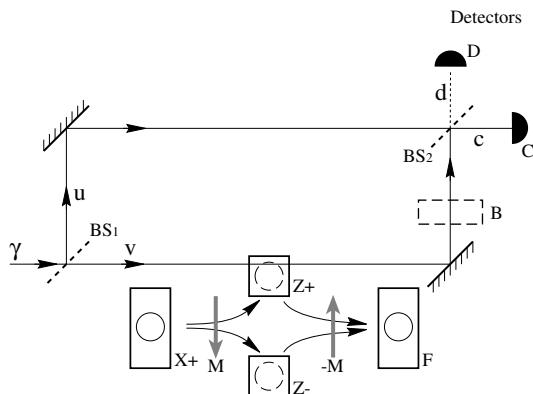


FIG. 1: A photon in a Mach-Zehnder Interferometer interacting with a superposed atom.

In this Letter we present an experiment in which space-time sequentiality does not seem to hold.

II. MUTUAL IFM

Interaction-Free Measurement (IFM) [?] highlights the way two interferometer arms, or even a myriad of them [?], are "felt" by a single particle. Its essence lies in an exchange of roles: the quantum object, rather than being the subject of measurement, becomes the measuring apparatus itself, whereas the macroscopic detector (or super-sensitive bomb in the original version) is the object to be measured.

In their paper [?], Elitzur and Vaidman (EV) mentioned the possibility of an IFM in which both objects, the measuring and the measured, are single particles, in which case even more intriguing effects can appear. This proposition was taken up in a seminal paper by Hardy [?]. He considered an EV device (Fig. 1) where a single photon traverses a Mach-Zehnder Interferometer (MZI) and interacts with an atom in the following way: A spin $1/2$ atom is prepared in a spin state $|X+\rangle$ (that is, $\sigma_x = +1$), and split by a non-uniform magnetic field M into its two Z components. The box is then carefully split into two halves, each containing either the $|Z+\rangle$ or the $|Z-\rangle$ part, while preserving their superposition state:

$$\Psi = |\gamma\rangle \cdot \frac{1}{\sqrt{2}}(|Z+\rangle + |Z-\rangle). \quad (1)$$

Now let the photon be split by BS_1 :

$$\Psi = \frac{1}{2}(i|u\rangle + |v\rangle) \cdot (|Z+\rangle + |Z-\rangle). \quad (2)$$

The boxes are transparent for the photon but opaque for the atom. The atom's $Z+$ box is positioned across the photon's v path in such a way that the photon can pass through the box and interact with the atom inside in 100% efficiency. Discarding all cases of the photon's scattering by the atom (25%) removes the term $|v\rangle|Z+\rangle$, leaving:

$$\Psi = \frac{1}{2}(i|u\rangle|Z+\rangle + i|u\rangle|Z-\rangle + |v\rangle|Z-\rangle) + |\text{scattering}\rangle. \quad (3)$$

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Next, let us reunite the photon by BS_2 :

$$|v\rangle \xrightarrow{BS_2} \frac{1}{\sqrt{2}}(|d\rangle + i|c\rangle) \quad (4)$$

$$|u\rangle \xrightarrow{BS_2} \frac{1}{\sqrt{2}}(|c\rangle + i|d\rangle), \quad (5)$$

so that

$$\Psi = \frac{1}{\sqrt{2}^3} \cdot [i|c\rangle(|Z+\rangle + 2|Z-\rangle) - |d\rangle|Z+\rangle]. \quad (6)$$

Once the photon reaches one of the detectors, the atom's Z boxes are joined and a reverse magnetic field $-M$ is applied to bring the atom to its final state $|F\rangle$. Measuring F 's σ_x gives:

$$\begin{aligned} \Psi &= \frac{1}{4}|c\rangle \cdot (3|X+\rangle - |X-\rangle) \\ &\quad - \frac{1}{4}|d\rangle \cdot (|X+\rangle + |X-\rangle). \end{aligned} \quad (7)$$

Here, it can happen that the photon hits detector D , while the atom is found in a final spin state of $|X-\rangle$ rather than its initial state $|X+\rangle$. In such a case, both particles performed IFM on one another, destroying each other's interference. Nevertheless, the photon has not been scattered, so no interaction between the photon and the atom seems to have taken place.

Hardy's analysis revealed the striking consequence of this result: The atom can be regarded as EV's "bomb" as long as it is in a superposition, whereas a measurement that forces it to assume a definite Z spin (to "collapse") amounts to "detonating" it. However, the photon's hitting detector D indicates that it has been disturbed too. And yet, in the absence of scattering, no interaction seems to have occurred between the photon and the atom. That seems to indicate that the photon has traversed the u arm of the MZI while "detonating" the atom on the other arm, forcing it to assume (as measurement indeed confirms) a definite $Z+$ spin!

Hardy argued that this case supports the guide-wave interpretation of QM. His reasoning was that the photon's corpuscle plus half-wave took the u arm of the MZI while its other, empty half-wave took the v arm and broke the atom's superposition. However, Clifton [?] and Pagonis [?] argued that the result is no less consistent with the "collapse" interpretation. Griffiths [?], employing the "consistent histories" interpretation, argued that the result indicates that the particle might have taken the v arm as well, and Dewdney *et. al.* [?] reached the same conclusion using Bohmian mechanics.

All the above analyses, however, seem to assume space-time sequentiality. To show how this assumption can become strained, let us reconsider Hardy's experiment with a slight yet crucial addition. Let a macroscopic object be placed after the atom on the v arm of the photon MZI ("B" on Fig. 1). Here Eq. (2) becomes:

$$\Psi = \frac{i}{2}|u\rangle \cdot (|Z+\rangle + |Z-\rangle), \quad (8)$$

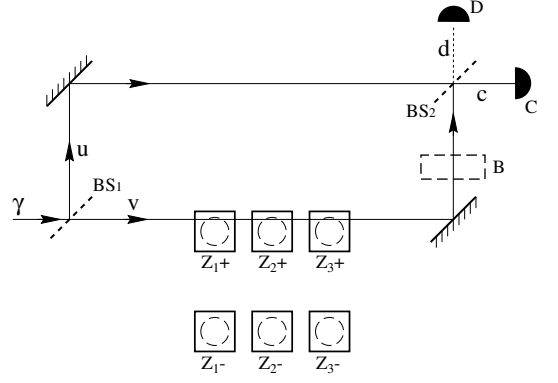


FIG. 2: One photon MZI with several interacting atoms.

and consequently Eq. (7) changes into

$$\Psi = \frac{1}{2}(i|c\rangle - |d\rangle) \cdot |X+\rangle. \quad (9)$$

The atom has retained its $|X+\rangle$ state, indicating that the peculiar effect pointed out by Hardy can appear only if the two halves of the photon's wave function are allowed to reunite. In other words, the alleged "empty guide wave" or "collapsing wave function" will not exert their effect unless path v is allowed, *later*, to reach BS_2 . Here, ordinary temporal notions are defied, and this defiance will become more prominent in what follows.

We shall next point out a more peculiar effect of the wave function for which all the above interpretations, due to their sequentiality assumption, seem to be insufficient.

III. IFM WITH ONE PHOTON AND SEVERAL ATOMS

Consider the setup given in Fig. 2. Here too, one photon traverses the MZI, but now it interacts with three superposed atoms rather than one. Formally:

$$\Psi = |\gamma\rangle|X_1+\rangle|X_2+\rangle|X_3+\rangle. \quad (10)$$

After the photon's passage through BS_1 and the atoms' splitting according to σ_z :

$$\begin{aligned} \Psi &= \frac{1}{4}(i|u\rangle + |v\rangle) \cdot (|Z_1+\rangle + |Z_1-\rangle) \\ &\quad \cdot (|Z_2+\rangle + |Z_2-\rangle) \cdot (|Z_3+\rangle + |Z_3-\rangle). \end{aligned} \quad (11)$$

Let us denote:

$$\psi^\equiv = |Z_1-\rangle|Z_2-\rangle|Z_3-\rangle, \quad (12)$$

$$\begin{aligned} \phi &= (|Z_1+\rangle + |Z_1-\rangle) \cdot (|Z_2+\rangle + |Z_2-\rangle) \\ &\quad \cdot (|Z_3+\rangle + |Z_3-\rangle). \end{aligned} \quad (13)$$

As in the previous experiment, we discard all the cases (44%) in which scattering occurs:

$$\Psi = \frac{1}{4}(i|u\rangle \cdot \phi + |v\rangle \cdot \psi^\equiv) + |\text{scattering}\rangle. \quad (14)$$

Note that the ϕ part – where each atom has a 50/50 probability to be found in the $Z+$ or the $Z-$ box – is coupled to the photon traversing the upper route u , while ψ^\equiv – where all three atoms are found in the $Z-$ state – is attached to the photon traversing the lower route v .

Now let us pass the photon through BS_2 and select only these cases in which it has lost its interference, hitting detector D :

$$\Psi = \frac{1}{4\sqrt{2}} \cdot |d\rangle \cdot (\psi^\equiv - \phi). \quad (15)$$

Measuring the three atoms' spins now will yield, with a uniform probability, all possible results, except for the case where all the atoms are found in their $|Z-\rangle$ boxes, which will never occur due to a destructive interference between ψ^\equiv and ϕ .

Reuniting the atoms' Z boxes and measuring their σ_x will yield all possible combinations of $|X+\rangle$ and $|X-\rangle$ in uniform probability, except the case of all three atoms measuring $X+$ which has a higher probability. This is not surprising, as these atoms are supposed to have interacted either with the guide wave, or with the real particle itself (see [?]), or with the uncollapsed wave function [?]].

Let us, however, return to the stage before uniting the Z boxes (as per Eq. (15)). We know that at least one atom must be in the $|Z+\rangle$ box to account for the loss of the photon's interference. Let us, then, measure atom 2's spin, and proceed only if it is found to be $|Z_2+\rangle$ (57% of the cases):

$$\begin{aligned} \Psi = \frac{1}{4\sqrt{2}} \cdot |d\rangle \cdot (|Z_1+\rangle + |Z_1-\rangle) \\ \cdot |Z_2+\rangle \cdot (|Z_3+\rangle + |Z_3-\rangle). \end{aligned} \quad (16)$$

Now unite the Z boxes of atoms 1 and 3 and apply the reverse magnetic field $-M$:

$$\Psi = \frac{1}{2\sqrt{2}} \cdot |d\rangle \cdot |X_1+\rangle \cdot |Z_2+\rangle \cdot |X_3+\rangle. \quad (17)$$

Contrary to classical intuition, these atoms will *always* exhibit their original spin undisturbed, just as if no photon has ever interacted with them.

In other words, only one atom is affected by the photon in the way pointed out by Hardy, but that atom does not have to be the first one, nor the last; it can be any one out of the atoms. The other atoms, whose half wave functions intersected the MZI arm before or after that particular atom, remain unaffected.

We can prove, however, that although atoms 1 and 3 seem to be totally unaffected by the photon, *something* must have passed through them. As in the previous section, let a macroscopic object be placed further along the v route, after the three atoms (object “B” on Fig. 2). The above results will never show up. Here, all the atoms will give either $Z-$ (when the photon hits the obstacle), or $X+$ (when it does not). Hence, something must have

passed through all three atoms, yet it has left the first and last unaffected.

Moreover, that “something” that seems to have passed through all the atoms must have done that at the precise moment. Let us place the atoms within sealed boxes, with apertures which open to the v path only for the minute interval during which the photon's wave function is supposed to pass through them. The slightest failure in the timing of any aperture's opening will ruin the predicted result.

The next result will deal the final blow on any realistic account in which a particular atom is affected by the photon at the moment of their interaction. We noted above that if we pick one atom, measure its position and find it in the $Z+$ box, then that measurement will disentangle the two other atoms, hence their spins will reveal no trace of interaction with the photon. One might think that there is, prior to measurements of the atoms, one particular atom that “has been” affected, and that the experimenter only has to be lucky to pick up that “right” atom that yields $|Z+\rangle$. Not so: rather than the normal 33% probability to find the “right” atom, expected when there are 3 atoms, the probability is 56%!

In other words, every atom chosen by the experimenter, regardless of its position within the row, has the 57% probability to be “the only atom that has been affected by the photon.” And once this atom gives this result, the other atoms will become nonlocally disentangled [1].

Note that the above analysis does not depend on the number of atoms or the index of the tested atom. For n atoms, the probability for any atom to be “the right atom” is $P = \frac{2^{(n-1)}}{2^n - 1}$ instead of the expected $P = 1/n$, approaching $1/2$ as n increases.

IV. CONCLUSIONS

From a formalist's point of view, it seems to be no surprise that the photon's v branch disappeared, leaving only part of the ϕ state, where the unmeasured atoms remain undisturbed. This is because the photon's v branch was coupled to the term ψ^\equiv (all three atoms are in the $Z-$ state) which contains Z_2- . By post-selecting Z_2+ we eliminated this term altogether, leaving the others atoms superposed.

However, it is the attempt to reconstruct a comprehensible scenario from these correlations that gives a highly counterintuitive picture. For, if it is the measurement of the second atom that have cancelled the photon's v term, then, for the photon to reach that atom, it must have first pass through the first atom, and, later, through the third as well. If one tries to visualize this result obliged by the formalism, then, a single photon's wave function seems to “skip” a few atoms that it encounters, then disturb the m^{th} atom, and then again leave all next atoms undisturbed. Ordinary concepts of motion, which sometimes remain implicit within prevailing interpretations, are in-

adequate to explain this behavior. The most prudent description of this result is that a wave function, when interacting with a row of other wave functions one after another, does not seem to comply with ordinary notion of causality, space and time [2].

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- [1] No superluminal communication is entailed. In all cases in which the particular atom is not detected in the intersecting path, the probability for one of the other atoms to reside in that path increases to 1. The overall result is Lorentz-invariant. Still, the correlation is Bell-like.
- [2] This is, in fact, one out of a family of peculiar effects

yielded by experiments of this kind, when a quantum mechanical object interacts with the measuring apparatus not directly but through another, intermediate quantum mechanical object. For an “inverse EPR” effect obtained by this method see [?]