Aharonov-Bohm effect under causality considerations

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We present a setup based on the Aharonov-Bohm effect, which at first sight seem to allow causality violation. We show that it is the dual topological effect, namely the Aharonov-Casher effect, that resolves the paradox in its two dimensional manifestation. Interestingly, the 3-dimensional problem involves many degrees of freedom which makes for a weak back-reaction, and seems to point to a curious quantum-to-classical transition.

Often, upon encountering the EPR paradox [1, 2] for the first time, it appears that a superluminal signal can be sent through this setting in the following way. Let two Spin-half particles, say, $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z^A\uparrow_z^B\rangle + |\downarrow_z^A\downarrow_z^B\rangle)$, be shared by Alice and Bob. Let Bob use an interferometer based on two successive Stern-Gerlach (SG) devices, aligned so as to measure the spin x. Their magnetic fields are pointing in opposite directions such that the second SG erases the measurement of the first, as illustrated in Figure 1 [3]. The particle then enter a third SG, which measures the spin z. If Bob's particle has a determined spin along the z axis, then it will emerge either in detector D_1 or D_2 with 100% probability. Alice chooses how to align her SG so that it can measure either spin x or spin z. If Alice measures the spin x of her particle, then Bob's particle does not travel in both arms of his interferometer. and reach both detectors D_1 and D_2. If, on the other hand, Alice chooses to measure spin z of her particle, then Bob's particle interferes, and thereby reaches only one of the detectors. The resolution is straightforward. In order for Bob to observe the interference of the $|\uparrow_x\rangle$ and $|\downarrow_x\rangle$ paths, he must know whether Alice obtained $|\uparrow_z\rangle$ or $|\downarrow_z\rangle$.

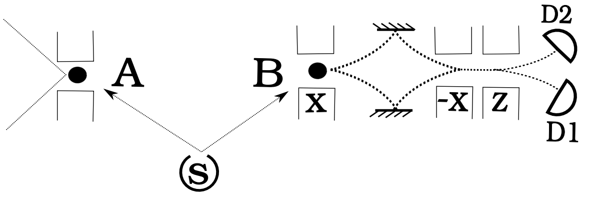

FIG. 1: Alice and Bob share a Bell state of charged particles. Bob inserts his particle into an interferometer that is based on two SG devices, which measure spin x. The magnetic fields of the successive SG point in opposite directions so that the second SG erases the measurement of the first. Then the particle enter a third SG, which measure spin z. Alice uses a single SG device to detect the spin of her particle in either the z or x directions.

In this paper we present a more subtle setting of this kind, using the Aharonov-Bohm (AB) [4] effect. By treating the flux quantum-mechanically, we arrange it to be in a superposed state of being inside the interference loop of a charged particle and outside of it. The charged particle measures the flux only in the case the particle interferences around it. Hence, if the distant party controls whether the charged particle is in a superposed state in both interference arms or localized in one of them, superluminal signaling seems to be enabled. Characteristically to quantum-mechanical paradoxes, we show that it is the nature of the back-reaction of the charged particle on the flux that resolves the paradox. Interestingly, this back-reaction is also a topological one. We shall treat the problem separately in two and three dimensions as they reveal significant differences.

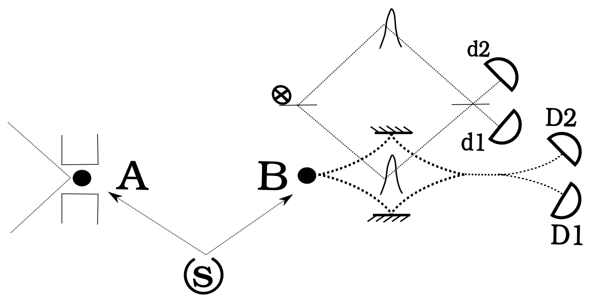

FIG. 2: Two-dimensional setting. A fluxon wave-packet is in superposition of being inside Bob's electron loop and being outside of it. After Bob's particle interferes the fluxon interferes too.

To start with let us demonstrate the paradox in the two-dimensional setting. In the three-dimensional magnetic AB effect a charged particle's wave packet interferes around an infinite solenoid, feeling no electromagnetic force during its travel. In the case where the solenoid carries a half integer magnetic flux $\phi = (N + 1/2)\frac{2\pi\hbar}{e}$, the particle acquires a relative phase of π, which changes its interference pattern. Here instead of the solenoid Bob has a "fluxon", a particle which can be thought of as an infinitesimal current ring. The fluxon carries half a magnetic flux quanta. Now assume that such a fluxon enters a superposed state of being inside Bob's interferometer and outside of it, as illustrated schematically in figure 2.

After Bob's particle wave packet interferes, the fluxon's wave packet interferes by itself too, where the phases are arranged such that without taking into account Bob's particle the fluxon must enter detector d_1. Superluminal signaling, alas, is "achieved" due to the following reasoning. Suppose Alice measures spin z of her particle. Then Bob's particle is in one of the superpositions of spin x states: $\frac{1}{\sqrt{2}}(|\uparrow_x\rangle \pm |\downarrow_x\rangle)$. Therefore it interferes and acquires an AB phase, if the fluxon is in $|\text{in}\rangle$ state. That is, the fluxon's path is measured by Bob's particle so it collapses and may enter detector d_2 with non-vanishing probability. If, however, Alice measures spin x of her particle, Bob's particle is in $|\uparrow_x\rangle$ or $|\downarrow_x\rangle$ states. It does not interfere, and therefore can say nil about the path of the fluxon. Therefore, the fluxon should enter only detector d_1.

It is common to quantum-mechanical "paradoxes" to be resolved due to the back-reaction, as for example demonstrated in [5]. Our paradox is resolved due to the existence of a topological effect dual to the AB effect, the Aharonov-Casher (AC) effect [6] – a neutral particle that carries a magnetic dipole acquires a relative phase as it interferes around an infinite charged wire in three-dimensional space. In two-dimensional space the AC effect is straightforwardly obtained from the linearity of quantum-mechanics: the initial state of both particles and the fluxon after it enters its interferometer (and stay there) is

$$|\psi_{in}\rangle = \frac{1}{2}(|\text{in}\rangle + |\text{out}\rangle) \otimes (|\uparrow_z^A\rangle|\uparrow_z^B\rangle + |\downarrow_z^A\rangle|\downarrow_z^B\rangle). \quad (1)$$

This state then evolves into

$$\begin{aligned}|\psi_{fi}\rangle = 2^{-\frac{3}{2}}\Big[&|\text{in}\rangle\otimes|\uparrow_z^A\rangle(|\uparrow_x^B\rangle-|\downarrow_x^B\rangle)+|\downarrow_z^A\rangle(|\uparrow_x^B\rangle+|\downarrow_x^B\rangle)+\\ &|\text{out}\rangle\otimes|\uparrow_z^A\rangle(|\uparrow_x^B\rangle+|\downarrow_x^B\rangle)+|\downarrow_z^A\rangle(|\uparrow_x^B\rangle-|\downarrow_x^B\rangle)\Big]\\ = \frac{1}{2}\Big[&|\text{in}\rangle\otimes(|\uparrow_z^A\rangle|\downarrow_z^B\rangle+|\downarrow_z^A\rangle|\uparrow_z^B\rangle)+\\ &|\text{out}\rangle\otimes(|\uparrow_z^A\rangle|\uparrow_z^B\rangle+|\downarrow_z^A\rangle|\downarrow_z^B\rangle)\Big],\end{aligned} \quad (2)$$

where a relative phase of π is added to Bob's particle in the case the flux is in the $|\text{in}\rangle$ state. Indeed the identity of the Bell state Alice and Bob posses depends on the fluxon's path. However, Eq. 2 also equals

$$|\psi_{fi}\rangle = \frac{1}{2}(|\text{in}\rangle+|\text{out}\rangle)\otimes|\uparrow_x^A\rangle|\uparrow_x^B\rangle-(|\text{in}\rangle-|\text{out}\rangle)\otimes|\downarrow_x^A\rangle|\downarrow_x^B\rangle. \quad (3)$$

That is, if Alice measures spin x of her particle, then the fluxon acquires a phase of π given that Bob's particle is inside the fluxon's interferometer – this is the two-dimensional manifestation of the AC effect. Therefore, Bob may find the fluxon in detector d_2 no matter what Alice measures.

Now let us consider the alleged paradox in the three-dimensional case. Bob uses two infinite identical

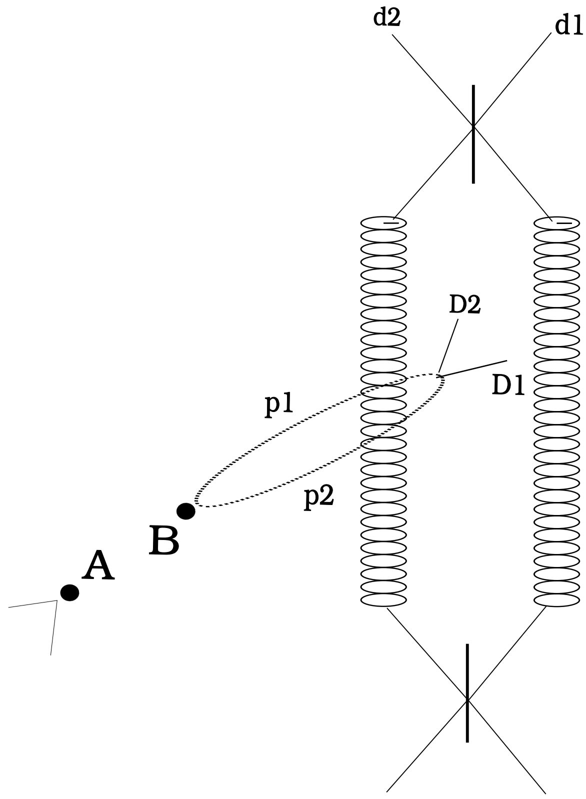

FIG. 3: Three-dimensional setting. Current carriers enter a superposed state of traveling along the right and left solenoids. Without taking into account Bob's particle they all come out at d_1.

solenoids, carrying interfering currents. No external magnetic fields are applied – the current carriers induce the magnetic flux inside the solenoids. Let us assume that if all current carriers travel in one of the solenoids they induce a full flux quanta in it $\frac{2\pi\hbar}{e}$ [7]. In this case Bob's interfering particle acquires a meaningless relative phase of 2π and there is no interaction. This macroscopic interference seems very unrealistic [8], as the interactions between the current carriers should destroy the interference. But, assume for the moment that we can perform many single electron interferences that are separated enough so that their interaction is negligible, yet they appear frequently enough so that they constitute a current. Then, with no other particles around we arrange that all current carriers should enter d_1, as illustrated in figure 3. Ignoring back-reaction, if Bob's particle is localized so that it travel either in path $p1$ or $p2$ the interference of the current carriers should not be effected. However, if Bob's particle interferes around one of the solenoids, it measures the AB flux in it and therefore reduces the wave-packets of the relevant current carriers to be either in the left or right solenoids. Then, the interference of the current carriers should be broken.

The paradox is here resolved through a careful consideration of the nature of the supposed collapse. Let us take a simplified view in which we have a very large number N

of non-interacting interfering current carries. If all pass through the left solenoid then each of them contribute $\delta\Phi = \frac{2\pi\hbar}{Ne}$ to the overall flux. Bob's interferometer, in this case, measures only the number of current carriers passing through the left solenoid n, and not the position of any of them individually, and thus acquires a phase of $2\pi n/N$. In the case $N \to \infty$ this measurement is a weak measurement [9], which changes the wave function of the current carriers only infinitesimally in the following manner. The state of the current carriers without taking into account Bob's particle is:

$$|\psi_0^f\rangle = \left(\frac{|\text{in}\rangle + |\text{out}\rangle}{\sqrt{2}}\right)^{\otimes N}. \quad (4)$$

Let say Bob's particle is initially in the unentangled state $|\psi^B\rangle = \frac{1}{\sqrt{2}}(|\uparrow_x^B\rangle + |\downarrow_x^B\rangle)$, such that without any fluxes it should be detected at D_1. The whole state then involves into a sum of states spanning the subspaces with different number of current carriers in $|\text{in}\rangle$:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \sum_{j=0}^{N} d_j |\psi_j^f\rangle \otimes \left(|\uparrow_x^B\rangle + e^{i\frac{2\pi j}{N}} |\downarrow_x^B\rangle\right), \quad (5)$$

where $|\psi_j^f\rangle$ are equally superposed normalized states spanning $|\text{in}\rangle^{\otimes j}|\text{out}\rangle^{\otimes N-j}$ with all the permutations taken in and

$$d_j = \sqrt{\frac{\binom{N}{j}}{2^N}}. \quad (6)$$

As $N \to \infty$, Bob's particle will measure $n = N/2$ with probability infinitesimally close to 1, so it will be detected at D_2. The current carriers wave-function reduces to the state of the reasonable subspace $|\psi_{N/2}^f\rangle$, where $|\langle\psi_0^f|\psi_{N/2}^f\rangle|^2 \to 1$. Therefore, up to logarithmic correction all N current carriers enter d_1 and there is no destruction of the current-carriers interference. This setup demonstrates a quantum to classical transition, where in the two-dimensional case, the charged particle and the fluxon act one on each other on equal footing, where in the 3d many body setting, Bob's charged particle evolves into an orthogonal state, while the current carriers remain the same.

Now let us examine the setting in the case Bob's particle is localized (Alice measured x). First of all it is clear that if we treat all current carriers as a wave-function of an infinitely long single particle carrying half a flux quanta, then we recover the AC effect in a somewhat different version, in which the quantum interfering flux is infinitely long and the classical (localized) charge is point-like. The net effect of such an effect is identical to the original AC effect, and therefore coincides with the 2d case. Now, the fact that the flux is physically described by a many-body wave function means that if Bob's particle is in the $|\downarrow_x^B\rangle$ state (see Eq. 3), then each of the current-carriers acquires a small phase of $2\pi/N$:

$$|\psi_{AC}^f\rangle = \left(\frac{|\text{in}\rangle + e^{i\frac{2\pi}{N}}|\text{out}\rangle}{\sqrt{2}}\right)^{\otimes N}. \quad (7)$$

However, the current carriers measure Bob's particle only weakly: $\langle\psi_0^f|\psi_{AC}^f\rangle \to -1$. That is, the wave functions are almost the same (up to a global phase) and again all current carrier enter d_1.

Note that in the case where Bob's particle is interfering $|\psi^B\rangle$, the weak topological AC back-reaction is described by:

$$\begin{aligned} |\psi\rangle &= \frac{1}{\sqrt{2}}\left(|\psi_0^f\rangle \otimes |\uparrow_x^B\rangle + |\psi_{AC}^f\rangle \otimes |\downarrow_x^B\rangle\right) \\ &\approx \frac{1}{\sqrt{2}}|\psi_0^f\rangle \otimes \left(|\uparrow_x^B\rangle - |\downarrow_x^B\rangle\right), \end{aligned} \quad (8)$$

in consistence with the AB picture.

Finally, we would like to note that a $2d$ manifestation of the AC effect has been realized [10], where the electric charge inside the loop was taken as classical. Mach-Zehnder interferometer for single electrons encircling magnetic flux in two dimensions has been realized too [11] (but not with SG devices). Though it may be extremely difficult to combine these two experimental settings so that both the charge and flux are treated quantum-mechanically, it seems in-principle feasible. The three-dimensional setting is kept only as a thought experiment.

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[7] Note the change in the settings from the 2-dimensional case.
[8] One can argue that the time it takes the relevant particles in the solenoid to get into the detectors should be very long so that we can regard the setup as a sufficient

accurate simulation of the AB effect. The time interval may be so long that any discussion regarding superluminality is irrelevant. This argument, however, does not treat causality formally and we will not discuss it.

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