CE 6305, spring 2002, Homework 1 Answers

1. In a k bit 2's complement binary adder, there are k identical stages. Stage i has inputs x_i, y_i, and carry in c_i. It produces outputs sum_i and c_{i+1} according to the following truth table:

c_{i+1}	sum_i	x_i	y_i	c_i
0	0	0	0	0
0	1	0	0	1
0	1	0	1	0
1	0	0	1	1
0	1	1	0	0
1	0	1	0	1
1	0	1	1	0
1	1	1	1	1

Consider a radix -2 adder, where the position weights are

$$\{(-2)^{k-1}, (-2)^{k-2}, \cdots (-2)^1, (-2)^0\}$$

Decide on the inter-stage signal(s) for this adder and give a corresponding truth table. The inter-stage signal(s) must not skip over stages.
Answer:

b_{i+1}	c_{i+1}	s_i	x_i	y_i	b_i	c_i
0	0	0	0	0	0	0
0	0	1	0	1	0	0
0	0	1	1	0	0	0
1	0	0	1	1	0	0
0	0	1	0	0	0	1
1	0	0	0	1	0	1
1	0	0	1	0	0	1
1	0	1	1	1	0	1
0	1	1	0	0	1	0
0	0	0	0	1	1	0
0	0	0	1	0	1	0
0	0	1	1	1	1	0

2. Prove that a value V represented by the 2's complement bit vector $\langle a_{k-1}, a_{k-2}, \cdots, a_1, a_0 \rangle$ has value

$$V = -a_{k-1} \times 2^{k-1} + \sum_{i=0}^{k-2} a_i 2^i$$

.

Answer:

Say v is a positive value, $0 \leq v < 2^{k-1}$

It's representation in a 2's complement system is $\langle 0 v_{k-2} v_{k-3} \cdots v_1 v_0 \rangle$.

Going to the value from this representation, we get

$$\text{value} = \sum_{i=0}^{k-2} v_i 2^i$$

The representation of $-v$ is, by definition, $2^k - v$. It looks like: $\langle 1 v_{k-2} v_{k-3} \cdots v_1 v_0 \rangle$.

Going to the value from this representation, we get

$$\begin{aligned} \text{value} &= -(2^k - \sum_{i=0}^{k-1} v_i 2^i) \\ &= -(2^k - 2^{k-1} - \sum_{i=0}^{k-1} v_i 2^i) \\ &= -2^{k-1} - \sum_{i=0}^{k-1} v_i 2^i \end{aligned}$$

3. Say that a value V is represented in a k bit 2's complement system. Show the steps necessary to transform this representation of V into a 2's complement representation in $2k$ bits.

Answer:

All we need to do is to sign-extend the representation. That means copy the sign bit into the bits of the upper-half of the new representation.

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0abcdefg -> 000000000abcdefg
1abcdefg -> 111111111abcdefg
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4. Instead of using the 2's complement system for representing negative values, someone suggests a *bias* scheme, where a bias B is added to any positive or negative value V before conversion to binary. Discuss the choice of B necessary to simplify the conversion process and to balance the negative and positive ranges.

   How are values added and negated within the bias representation?

   Answer:
   The range of the unsigned system is 0 to $2^k - 1$. We need to share this range as evenly as possible between positive and negative values. A value x is represented in the biased system by $x+b$. If $b = 2^{k-1}$ then the range of x is -2^{k-1} to $+2^{k-1}-1$, the same as a 2's complement system in k bits. Indeed, the representation only differs from the 2's complement representation in the sign bit. For the biased representation, the sign bit is 1 for the non-negative range 0 to $2^{k-1} - 1$ and 0 for all negative values.

   To negate a biased value, simply complement all the bits and add 1 to the result (as with 2's complement).

   When we add two values x and y represented in biased notation, the result is $x + y + 2b$. We must subtract b from the result. But note that the bias value is 100...0, a 1 in the sign bit position. This should enable very fast subtraction of the bias by a simple logic circuit.