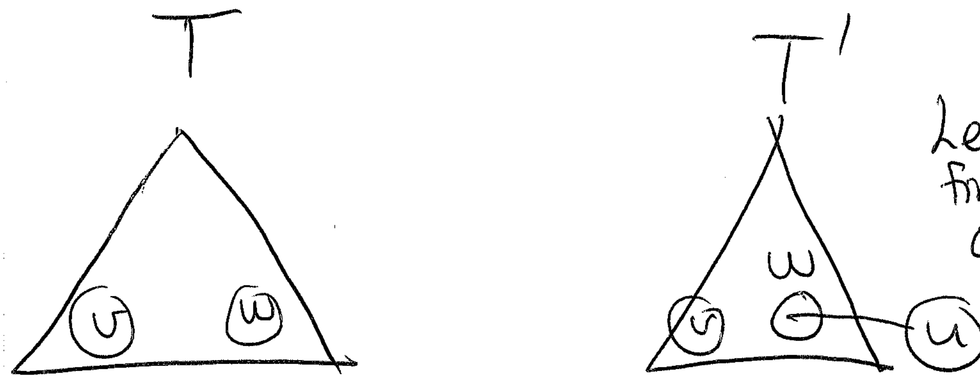

Let w be the first hop of u on T'

We know that the subinstance obtained by choosing only nodes, edges, and paths from $V(T)$ results in a stable SPP instance, call it S, (we assume this).

Once we add another node u, will the new SPP instance S' obtained from the new tree T' be stable? (i.e. if all nodes choose their path along T' then no one can change their chosen paths)

Consider first node u. Will it change its path in S'? (ie its path along T').
Let $T'[u, 0]$ be the path along T' for u.
If u changes to another path P, that implies

$$\lambda(T'[u,0]) < \lambda(P).$$

Note that P is being offered to u by one of its neighbors, say x, on T'. Thus, P is consistent with T'.

From the rules of the tree construction, the parent of u on T' (i.e. w) will be its highest ranked consistent path. Hence,

$$\lambda(T'[u,0]) < \lambda(P)$$

is not possible, and u cannot change its path unless another node changes first.

Consider now a node v which was in ~~on~~ T, and hence, is also in T'.

Will v change its path? v cannot choose a node in T as its new next hop, because we are assuming the SPP instance S is stable

So, if v changes, it has to choose u as its next hop.

Let $Q = (v, u, T'[u, 0])$.

Let T_v be the tree we had when v was <u>added</u> to the tree. Note that T_v is a subgraph of T'.

From how we build trees, the path $T_v[v, 0]$ is higher ranked than any path of v that is consistent with T_v

$\longrightarrow$

Note that Q is consistent with T_v (not direct into T_v, but consistent, since T_v is a subgraph of T)

hence $\lambda(Q) < \lambda(T_v[v,0])$

note that $T_v[v,0] = T'[v,0]$

Hence, v cannot change paths

Hence, S' is stable when all nodes choose their path to be the one along T'.