CE 6305, Homework 2 Answers

1. Perform redundant BSD addition to do the following sums, and give the intermediate values and answer, just as they are displayed by the applet:

   (a) $1\bar{1}\bar{1}110 + 011010$

   (b) $1\bar{1}1\bar{1}\bar{1}0 + 001\bar{1}01$

   (c) $100110 + 011\bar{1}10$

   Comment on the results obtained in each case.

   Answer:

   (a)
   X: $1\ \bar{1}\ \bar{1}\ 1\ 1\ 0$: 14
   Y: $0\ 1\ 1\ 0\ 1\ 0$: 26
   e: $1\ 1\ 0\ 0\ 0\ 0$
   s: $1\ 0\ 0\ \bar{1}\ 0\ 0$
   c: $0\ 0\ 1\ 1\ 0\ 0$
   Z: $1\ 0\ 1\ 0\ 0\ 0$: 40

   Here, the e_i signal is coded so that it is a 1 when $c_i \in \{0, \bar{1}\}$. Note that X=1,Y=$\bar{1}$, or vice versa, causes $e = 1$.

   (b)
   X: $1\ \bar{1}\ 1\ \bar{1}\ \bar{1}\ 0$: 18
   Y: $0\ 0\ 1\ \bar{1}\ 0\ 1$: 5
   e: $1\ 0\ 1\ 1\ 0\ 0$
   s: $1\ \bar{1}\ 0\ 0\ \bar{1}\ \bar{1}$
   c: $0\ 1\ \bar{1}\ 0\ 1\ 0$
   Z: $1\ 0\ \bar{1}\ 0\ 0\ \bar{1}$: 23

   Nothing very interesting happens here.

(c)
X: 1 0 0 1 1 0 : 38
Y: 0 1 1 $\bar{1}$ 1 0 : 22
e: 0 0 1 0 0 0
s: $\bar{1}$ $\bar{1}$ 1 0 0 0
c: 1 0 0 1 0 0
Z: 0 $\bar{1}$ 1 1 0 0 : -4

The answer is incorrect because there was a carry-out from the most significant column. The answer does, however, fit in the range, which is $[-63, 63]$. Any carry-out from the left hand end of the adder can be used to signal overflow. It might, however, be possible to apply a correction step to the result when an overflow occurs. For example, in the result obtained above, the carry-out together with the two MS digits of the answer are $10\bar{1}$ and this can be changed to 011. Here are some transformations that can be applied:

$$
\begin{array}{rcl}
\bar{1}1 & \rightarrow & 0\bar{1} \\
1\bar{1} & \rightarrow & 01 \\
\bar{1}01 & \rightarrow & 0\bar{1}\bar{1} \\
10\bar{1} & \rightarrow & 011 \\
\bar{1}11 & \rightarrow & 001 \\
1\bar{1}\bar{1} & \rightarrow & 00\bar{1} \\
\bar{1}001 & \rightarrow & 0\bar{1}\bar{1}\bar{1} \\
100\bar{1} & \rightarrow & 0111 \\
\bar{1}011 & \rightarrow & 0\bar{1}0\bar{1} \\
10\bar{1}\bar{1} & \rightarrow & 0101 \\
\bar{1}111 & \rightarrow & 0001 \\
1\bar{1}\bar{1}\bar{1} & \rightarrow & 000\bar{1}
\end{array}
$$

2. Design an unlimited carry-free addition system for radix = 3, and digit set $d_i \in [-4, 5]$.

   (a) Give suitable values for λ and μ.

   Answer:

$$
\begin{aligned}
\lambda &\geq \frac{\alpha}{r-1} \\
&\geq \frac{4}{2} \\
\lambda &= 2 \\
\mu &\geq \frac{\beta}{r-1} \\
&\geq \frac{5}{2} \\
\mu &= 3
\end{aligned}
$$

   (b) Determine the range of values for the transfer digits for each intermediate sum value, $p_i \in [-8, 10]$.

   Answer:

t[i+1]	-2	-1	0	1	2	3
p[i]	[-8,-4]	[-5,-1]	[-2,2]	[1,5]	[4,8]	[7,10]

   (c) If there are 8 digits in a number in this system, what is the corresponding range?

   Answer:

   [-13120,16400]

(d) Show the system ay work by giving the working to the addition: [-3,4,4,-1] + [5,4,-2,-4].

Answer:

X: -3 4 4 -1 = -34
Y: 5 4 -2 -4 = 161

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p: 2 8 2 -5
w: 2 -1 2 1
t: 3 0 -2 -

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s: 5 -1 0 1 = 127

3. In which of the following is an unlimited carr-free system possible?

   (a) $r = 3, \quad \alpha = 1, \quad \beta = 2$

   (b) $r = 2, \quad \alpha = 1, \quad \beta = 1$

   (c) $r = 10, \quad \alpha = 4, \quad \beta = 5$

   (d) $r = 4, \quad \alpha = 2, \quad \beta = 2$

   Answer:

   The rules are:
   (i) $(r > 2) \wedge (\rho \geq 3)$
   (ii) $(r > 2) \wedge (\rho = 2) \wedge (\alpha \neq 1) \wedge (\beta \neq 1)$
   where $\rho = \alpha + \beta + 1 - r$

   (a) $\rho = 1$, both rules fail
   (b) $r = 2$, both rules fail
   (c) $\rho = 0$, both rules fail
   (d) $\rho = 1$, both rules fail

   None of these systems permits unlimited carry-free addition.