Answers to CE 6305 Homework 4, spring 2002,

1. A residue arithmetic system is to be designed to represent integers in the range -2^{47} to $2^{47} - 1$.

   (a) Develop an efficient set of unrestricted residues for this system. Show your working.

   Answer:

   The product of primes, beginning at 2, that is just larger than 2^{48} is 2*3*5*7*11*13*17*19*23*29*31*37*41 $= 3.0425 \times 10^{14}$, while $2^{48} = 2.815 \times 10^{14}$. The total number of bits needed is 1+2+3+3+4+4+5+5+5+5+5+6+6 = 54 bits. The efficiency is $2^{-6} = 1.56\%$. To increase efficiency, we need to eliminate inefficient columns. For example, 2*3*5=30, which saves 2 columns and requires 5 bits instead of 7, and has efficiency 6.25%. Or we could square the 7 and eliminate the 2 and 3, eliminating 2 columns and saving 2 bits.

   Lots of possible better answers when one column is allowed to go to 7 or more bits width.

   (b) Develop an efficient set of low-cost residues for this system. Show your working.

   Answer: 2^{11}, $2^{11} - 1$, $2^9 - 1$, $2^8 - 1$, $2^7 - 1$, $2^5 - 1$ = 2048, 2047, 511, 255, 127, 31, has range 2.15×10^{15}, requires 11 + 11 + 9 + 8 + 7 + 5= 51 bits and has efficiency 12.5%.

   (c) Estimate the delays in the system of part (b) for addition using adders and using ROMs (use the equations given in the notes for these estimates). Show your working.

Answer:

Adder delay for the 11 bit column $= 5D + (4\lceil log_4\, 11 - 2\rceil)D =$ 9D.

ROM delay $= D\lceil log_4\, 6\rceil + D\lceil log_4\, 5\rceil + D\lceil log_4\, 2\rceil + D = 5D$.

2. Convert the numbers 247 and -118 to the system RNS(16,15,7) assuming the normal way of representing negative values. Show your working.

   Answer: $247 = (7, 7, 2)_{RNS(16,15,7)}$

   $118 = (6, 13, 6)_{RNS(16,15,7)}$, $-118 = (10, 2, 1)_{RNS(16,15,7)}$,

3. Negate the number (8,7,6) in RNS(16,15,7) Show your working.

   Answer:

   $-(8, 7, 6)_{RNS(16,15,7)} = (16-8, 15-7, 7-6)_{RNS(16,15,7)} = (8, 8, 1)_{RNS(16,15,7)}$,

4. Convert (12,7,4) and (1,13,6) in RNS(16,15,7) to decimal by going via the mixed radix system. Show your working.

   Answer:

   $(12, 7, 4)_{RNS(16,15,7)}$. $z_0 = 4$.

   Subtracting 4 from each column gives $(8, 3, 0)$.

   Now divide by the modulus 7 by multiplying by the multiplicative inverse of 7 in each column, i.e. $(7, 13, -)$. (since 7*7 mod 16 = 49 mod 16 = 1, 7*13 mod 15 = 91 mod 15 = 1.)

   $\frac{(8,3,0)}{7} = (8, 3, 0) \times (7, 3, -) = (8, 9, 0)$, $z_1 = 9$.

   Now subtract 9 from each column giving $(15, 0, 0)$.

   Now divide by the modulus 15 by multiplying by the multiplicative inverse of 15 in each cloumn, i.e. $(15, -, -)$). (since 15*15 mod 16 = 225 mod 16 = 1.

   $\frac{(15,0,0)}{15} = (15, 0, 0) \times (15, -.-) = (1, 0, 0)$, $z_2 = 1$.

   The MRS system will have moduli 15*7 = 105, 7, and 1.

   $(1, 9, 4)_{MRS(105,7,1)} = 1 \times 105 + 9 \times 7 + 4 \times 1 = \boxed{172}$.

   $(1, 13, 6)_{RNS(16,15,7)}$. $z_0 = 6$.

   Subtract 6 from each column. Then divide by 7, as above.

   $(11, 7, 0) \rightarrow \frac{(11,7,0)}{7} = (11, 7, 0) \times (7, 13, -) = (13, 1, 0), z_1 = 1$

   Subtract 1 from each column. Then divide by 15, as above.

   $(12, 0, 0) \rightarrow \frac{(12,0,0)}{15} = (12, 0, 0) \times (15, -, -) = (4, 0, 0), z_2 = 4$

   $(4, 1, 6)_{MRS(105,7,1)} = 4 \times 105 + 1 \times 7 + 6 \times 1 = \boxed{433}$