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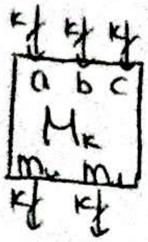
HW 6

CE6305.501

1. Let us consider a building block M_k, that produces

$m = a \times b + c$, where a, b, c are k-bit & m $2k$-bit.



where m_u are k-MSB of m &

m_l are k-LSB of m.

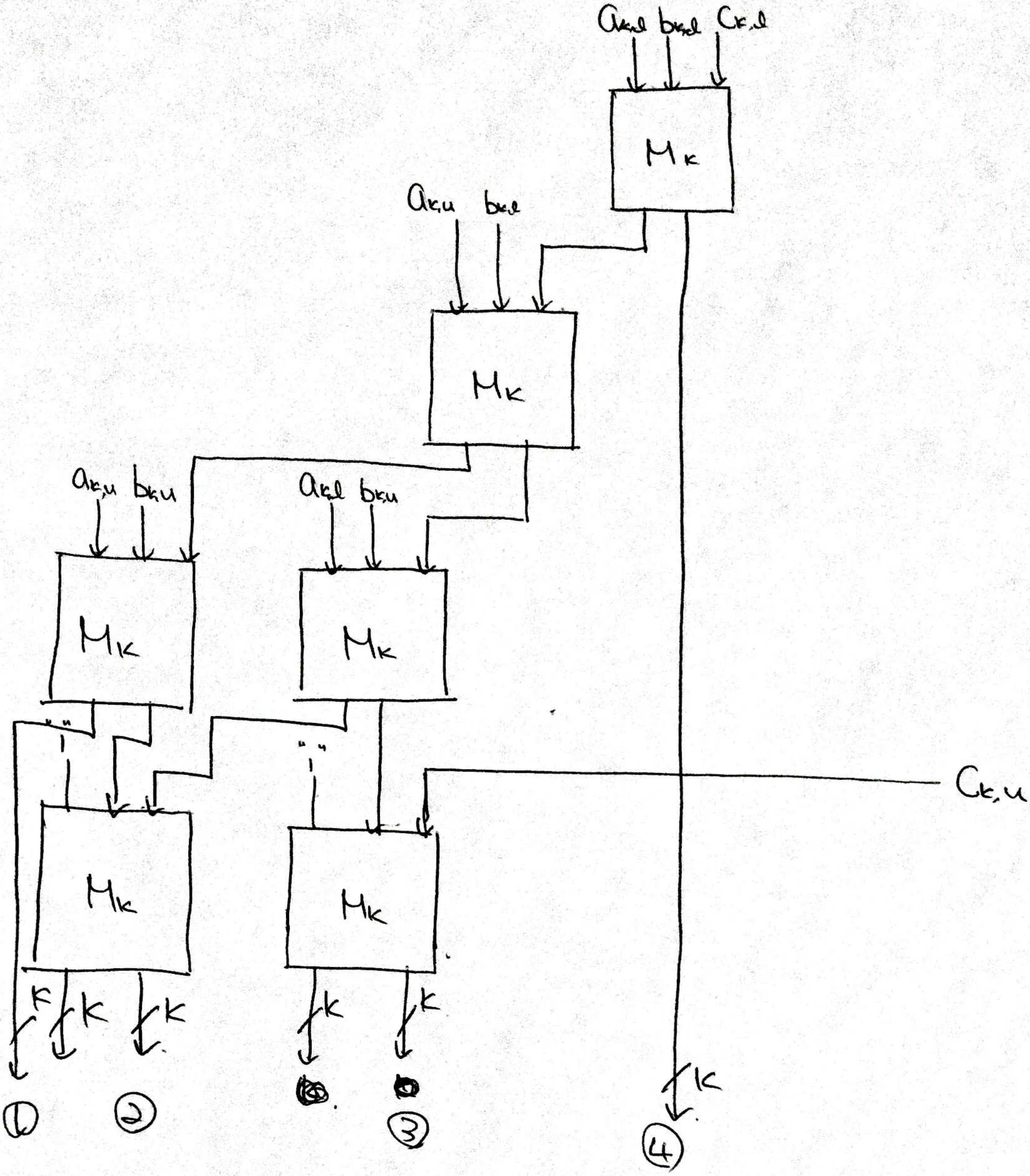
Now, consider $a_{2k} \times b_{2k} + c_{2k}$, where subscript denotes the number of bits. Then, we have

$$
\begin{array}{cccc}
 & & a_{k,u} & a_{k,l} \\
\times & & b_{k,u} & b_{k,l} \\
\hline
 & & a_{k,u}\, b_{k,l} & a_{k,l} \cdot b_{k,l} \\
 & a_{k,u}\, b_{k,u} & a_{k,l} \cdot b_{k,u} & \\
+ & & c_{k,u} & c_{k,l} \\
\hline
\end{array}
$$

($a_{k,l}$ means "lower" k bits)
(& $a_{k,u}$ " "upper" ")

∴ Yet, each product $a_i \cdot b_i$ is $2k$-bit result.

So, constructing a hardware to manage this,



* we only use 1, 2, 3, 4 (assume that no "overflow" occurs on last blocks)

Since we can build M_{2k} from M_k,

We can also build M_{4k} w/ M_{2k} w/ same recursive

Schematic

2. a) for $gb \times gb$, not considering carry, the height of the matrix will be g. Yet the column left to it has $(g-1)$ elements, which mean that there can be $(g-1)$ carry.

$\therefore$ The height: $\boxed{2g-1}$.

b) if $gb * hb$, there are three cases to consider.

$\begin{cases} ① \ h>g & : \text{there can be no more than } g \text{, height of column} : 2g \\ ② \ h=g & : \text{above case} : 2g-1 \\ ③ \ h<g & : \text{now you can't have column height more than } h : 2h \end{cases}$

ex) $6b \times 4b$

$\therefore 6>4 : 4\times 2 = \boxed{8}$.

3. In order for division to hold w/o overflow,

$z < 2^k d$ must be true.

4. a) $z = 00.0110\ 1101\ 00$ $\quad d = 0.10011$

$-d = 1.01101$

$s^{(0)} = 00.0110\ 1101\ 00$
$2s^{(0)} = 00.1101\ 1010\ 00$ $\quad > 0 \quad : q_{-1} = 1$
$-d = 1.01101$

$s^{(1)} = 10.0100\ 0010\ 0$
$2s^{(1)} = 00.1000\ 0100\ 0$ $\quad > 0 \quad : q_{-2} = 1$
$-d = 1.01101$

$s^{(2)} = 01.1110\ 1100$
$2s^{(2)} = 11.1101\ 1000.$ $\quad < 0 \quad : q_{-3} = \bar{1}$
$d = 0.10011$

$s^{(3)} = 00.0111\ 000$
$2s^{(3)} = 00.1110\ 000.$ $\quad > 0 \quad : q_{-4} = 1$
$-d = 1.01101$

$s^{(4)} = \cancel{01}\,10.010010.$
$2s^{(4)} = 00.1001\ 00.$ $\quad > 0 \quad : q_{-5} = 1$
$-d = 1.01101$

$s^{(5)} = 01.11111$ $\quad < 0.$
$+d = 00.10011$ $\quad$ correction, since $\text{sign}(s) \neq \text{sign}(z)$.

$10.|10010$

$s = \boxed{10010}$

$q = 11\bar{1}10 = \boxed{10110}$

b) $Z = 00.0110\ 1101\ 00$ $\quad d = 0.10011$

$-d = 1.01101$

$S^{(0)} = 00.0110\ 1101.\ 00$

$2S^{(0)} = 00.1101\ 1010\ 00.$ $\quad > d \ : q_{-1} = 1$

$-d = 1.0110.\ 1$

$S^{(1)} = 10.0100\ 0010\ 0.$

$2S^{(1)} = 00.1000.\ 0100\ 0.$ $\quad [-d, d] \ : q_{-2} = 0$

$S^{(2)} = 00.1000.\ 0100$

$2S^{(2)} = 01.0000\ 1000$ $\quad > d \ : q_{-3} = 1$

$-d = 01.0110\ 1$

$S^{(3)} = 10.0111\ 000$

$2S^{(3)} = 00.1110\ 000$ $\quad > d \ : q_{-4} = 1$

$-d = 01.0110\ 1$

$S^{(4)} = 10.0100\ 10$

$2S^{(4)} = 00.1001\ 00$ $\quad [-d, d) \ : q_{-5} = 0$

$S^{(5)} = 00.1001\ 0.$

$\therefore$ $\boxed{\begin{array}{l} q = 10110. \\ S = 10010. \end{array}}$

c) $z = 00.0110\ 1101\ 00$ $\quad d = 0.10011$

$-d = 1.01101$

$$
\begin{array}{lll}
S^{(0)} = 00.0110\ 1101\ 00 & & \\
2S^{(0)} = 00.1101\ 1010\ 00 & \geq 1/2 & : q_1 = 1 \\
-d = 1.0110\ 1 & & \\
\hline
S^{(1)} = 10.0100\ 0010\ 0 & & \\
2S^{(1)} = 00.1000\ 0100\ 0 & \geq 1/2 & : q_2 = 1 \\
-d = 1.0110\ 1 & & \\
\hline
S^{(2)} = 01.1110\ 1100 & & \\
2S^{(2)} = 11.1101\ 1000 & [-1/2, 1/2) & : q_3 = 0 \\
\hline
S^{(3)} = 11.1101\ 100 & & \\
2S^{(3)} = 11.1011\ 000 & [-1/2, 1/2) & : q_4 = 0 \\
\hline
2S^{(4)} = 11.0110\ 00 & < -1/2 & : q_5 = \bar{1} \\
d = 0.1001\ 1 & & \\
\hline
11.1111\ 1 & \leftarrow \text{correction} & \\
+d = .1001\ 1 & & \\
\hline
00.1001\ 0 & &
\end{array}
$$

$S = 10010$

$q = 1100\bar{1} = 10110$