PR1-P2023

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Práctica – Borja Villena Pardo – Intento 2

- **La resolución de la práctica (memoria técnica detallada). Importante: debéis especificar a que intento de la tabla corresponde.**
- **El código en R.**
- **Las imágenes y/o figuras que se os pidan.**

Datos:

- **id:** identificador de la sección censal.
- **1- rent:** renta bruta por persona.
- **2- inc_sal:** ingresos provenientes del salario.
- **3- inc_ret:** ingresos provenientes de pensiones de jubilación.
- **4- inc_emp:** ingresos provenientes de prestaciones del paro.
- **5- inc_non:** ingresos provenientes de otros tipos de prestaciones.
- **6- inc_oth:** otros ingresos.
- **7- gini:** coeficiente de Gini que mide la desigualdad.
- **8- dist8020:** relación de renta entre el percentil 80 (P80) y el percentil 20 (P20) – P80/P20.
- **9- mean_age:** edad media de la población.
- **10- perc_chil:** porcentaje de población menor de 18 años.
- **11- per_ret:** porcentaje de población mayor de 65 años.
- **12- home_size:** tamaño medio del hogar (m_2).

Pregunta 0. Los resultados del informe corresponden al primer intento:

Respuesta:

- V = gini
- C = 1
- Copiamos código R usado para conseguir la respuesta: NA

Pregunta 1. [10 %] Para empezar, generar la matriz X a partir del *dataframe* **var_df** utilizando, por ejemplo, la instrucción **as.matrix.** Aseguraros de eliminar el valor de la primera columna ('id') ya que no proporciona ninguna información relevante. Comprobar, también, que X es una matriz y no un *dataframe* utilizando la siguiente instrucción:

```
> class (X)
[1] " matrix " " array "
```

Responder: ¿cuántas secciones censales tiene la ciudad?

Respuesta:

- La ciudad tiene **61** secciones censales.
- Copiamos código R usado para conseguir la respuesta:

```
> #Abrimos archivo variables.csv y lo nombramos como var_df
> var_df <- read.csv("C:\\Users\\usuario\\Documents\\4. UOC\\1º Álgebra Lineal\\Reto 4\\variables.csv")
>
> #Imprimimos archivo para ver su contenido
> fix(var_df)
> var_df
>
> #Comprobamos que var_df es de tipo dataframe
> class(var_df)
[1] "data.frame"
>
```

```
> #Eliminamos la primera columna llamada 'id'. Para ello vamos a usar el paquete "dplyr"
> library(dplyr)
> var_df <- select(var_df, -id)
>
> #Comprobamos que realmente se ha eliminado la columna
> fix(var_df)
> var_df
>
> #Convertimos var_df en una matriz
> var_matrix <- as.matrix(var_df)
>
> #Comprobamos que ahora var_df es una matriz
> class(var_matrix)
[1] "matrix" "array"
>
> #Buscamos la dimensión que tiene la matriz, y podemos observar que nos devuelve
> # 61, 12, es decir, contamos con 61 observaciones que coinciden con el número de
> #secciones censales, y 12 columnas que coinciden con el número de variables:
> dim(var_matrix)
[1] 61 12
```

Pregunta 2. [10 %] Como alcaldables, os interesa tener una primera impresión general de las variables medidas y explorar los datos en crudo. Una de las características interesantes a estudiar es la razón (*M/m*) entre el valor máximo (*M)* y el mínimo (*m*) de una variable. Calcular la razón de la variable V.

Respuesta:

- La razón de la variable V es igual a **1.65**
- Copiamos código R usado para conseguir la respuesta:

```
> #En este intento, la variable V es igual a la variable 'gini'. Para calcular
> #la razón de 'gini' vamos a detectar los valores máximo y mínimo de la variable.

> #Asignamos a V la columna 'gini' de la matriz var_df
> V <- var_matrix[,7]

1    2    3    4    5    6    7    8    9    10   11   12   13   14   15   16   17   18
39.7 35.6 31.8 32.5 34.2 28.5 33.3 35.1 33.4 33.7 30.0 34.0 35.5 32.9 29.3 27.2 30.4 32.1

19   20   21   22   23   24   25   26   27   28   29   30   31   32   33   34   35   36
37.0 31.4 30.3 30.5 32.1 26.4 27.3 38.8 30.9 29.6 36.9 33.9 33.4 27.5 27.7 32.2 33.1 30.4

37   38   39   40   41   42   43   44   45   46   47   48   49   50   51   52   53   54
30.5 31.1 31.8 31.5 28.9 32.4 32.2 31.4 29.0 29.8 29.2 30.6 37.0 39.3 31.9 31.6 28.0 28.3

55   56   57   58   59   60   61
33.7 25.4 24.9 25.4 24.1 28.1 26.5

> #Definimos máximo como el valor 'máximo' de V, y definimos 'mínimo'
> #como el valor mínimo de V
> maximo <- max(V)
> minimo <- min(V)
> maximo
[1] 39.7
> minimo
[1] 24.1

> #Calculamos la razón entre el valor Máximo y el valor Mínimo de V (M/m).
>#Guardamos valor en 'razon'
> razon <- maximo/minimo
> razon
[1] 1.647303

> #Redondeamos resultado a dos decimales
> round(razon, digits=2)
[1] 1.65
```

Pregunta 3. [15 %] Para poder realizar el análisis de componentes principales debéis, inicialmente, normalizar los datos y guardarlos a la variable ***Xs***, como se muestra a la Sección 2.1 de los apuntes del módulo. Una vez normalizados, calcular la matriz de covarianzas de los datos; guardarla en la variable ***CXs*** y adjuntarla 1 como una imagen. Finalmente, indicar cuales son el par de variables (distintas) con mayor covarianza (en valor absoluto) y el par con menor covarianza (en valor absoluto).

Respuesta:

- Los pares con MAYOR varianza en valores absolutos son: {1, 2}.
- Los pares con MENOR varianza en valores absolutos son: {5, 11}
- Copiamos código R usado para conseguir la respuesta:

```
> #Calculamos la matriz de datos normalizada y la guardamos en la variable Xs
> Xs <- as.matrix(scale(var_matrix, center = TRUE, scale = TRUE))
> Xs

rent      inc_sal      inc_ret       inc_emp       inc_non      inc_oth        gini
1    0.11215097 -0.00395886 -0.931139719  2.418143014 -0.214973914  0.79806710  2.39053026
2   -0.08473303 -0.55542215  0.645412259 -1.550917436 -0.530200087  0.35484048  1.21938116
3    0.21537779  0.44131517 -0.007907388 -0.167760007 -0.341064383 -0.06529216  0.13392589
4   -0.64918611 -0.56595790 -0.712153958  0.644093267 -0.175570643 -0.51384816  0.33387817
5   -2.27947972 -2.66395824 -1.775799086 -3.144555344  5.057183822 -1.27284043  0.81947658
6   -1.53712508 -1.81416690 -0.840198788 -0.438377765  2.094057800 -1.03079784 -0.80870632
7    1.72393277  1.23981404  0.982984995 -0.829270082  0.147536184  2.32315748  0.56239507
8    0.98942212  1.12225726  0.258367657 -0.007393928  1.408440874  0.76653494  1.07655810
9    0.97593047  0.26664354  1.488616571 -0.949544641  0.320910579  1.45091592  0.59095969
10   0.77794832  0.27579300  1.026636642 -0.438377765  0.005684406  1.15380309  0.67665352
11   1.07350963  0.91514662  1.108119716 -0.137691367  1.219305171  0.83404040 -0.38023714
12   0.42010577 -0.03584336  1.236892074 -0.929498881 -0.916352148  0.58533609  0.76234736
13   1.02989708  0.93538635  0.627951601 -0.668904003  0.336671888  1.04543907  1.19081654
14   0.96008562  0.54279107  0.863670494 -0.157737127 -0.081002791  1.33322549  0.44813662
15   0.07449984  0.22505505  0.297654139  0.503772948 -0.017957557 -0.35219035 -0.58018942
16   0.08077503  0.66034784 -0.311286334  0.203086551 -0.742977753 -0.60577993 -1.18004628
17  -0.34154506  0.25971212 -0.987159333  1.065054224 -0.798142334 -0.78164941 -0.26597869
18   0.75849524  0.22893665  2.109197484 -1.079842080  0.234223381  0.52848939  0.21961972
19   0.47234671 -0.26374825  1.600655798  0.082811992 -0.735097099  0.82071696  1.61928573
20  -0.66801167 -0.64636230 -0.689600607  0.814482226  0.975004887 -0.52495103  0.01966744
21   1.48108303  1.83868820  0.624313963 -0.508537924 -0.845426260  0.93707505 -0.29454330
22   2.39835851  1.78351415  3.336536288 -2.823823186 -1.389191407  2.09976771 -0.23741407
23   0.80587290  1.06625144  0.140508211  0.323361110 -0.979397383  0.52893350  0.21961972
24   0.51250790  1.15469628 -0.245081336  0.453658549 -1.302504210 -0.19541781 -1.40856318
25   0.48630900  1.29748341 -0.899856039 -0.087576967 -0.806022988 -0.10259781 -1.15148167
26   1.91579662  1.43666620  0.721075114 -1.089864960 -0.199212606  2.74151367  2.13344875
27   0.17192212  0.37837793  0.312932216 -0.418332005  0.060848987 -0.29445542 -0.12315562
28  -0.55223447  0.21562833 -1.842004084  0.283269590  0.202700764 -0.80829630 -0.49449558
29   0.54482512  0.06507802  0.556653911  0.764367826 -0.356825692  1.08452117  1.59072112
30   1.13830094  0.53807770  1.193967955  0.744322066 -0.151928680  1.60768846  0.73378275
31   0.43234239  0.54694991 -0.301828477  0.032697592  0.573091517  0.49828958  0.59095969
32   0.97091032  1.18269919  0.276555843 -1.180070879 -0.262257840  0.75321150 -1.09435244
33   1.17077503  1.35487551  0.842572198 -1.370505597 -0.301661112  0.70746767 -1.03722322
34  -0.13666020 -0.21716915  0.804013243 -1.069819200  0.975004887 -0.53738624  0.24818434
35  -1.11794258 -1.11575764 -0.549915337 -0.318103205 -0.703574482 -0.98771870  0.50526585
36  -0.92388242 -0.61253700 -1.046089056  0.714253427 -0.790261679 -0.98283344 -0.26597869
37  -1.56191207 -1.66112129 -0.952238015 -0.298057446 -0.388348309 -1.14360301 -0.23741407
38  -0.88152491 -0.67491972 -0.873665051  0.443635669  0.281507307 -0.91666032 -0.06602640
39  -1.24940775 -1.21667903 -0.938414994  0.583955988 -0.632648593 -1.00503918  0.13392589
40  -1.22603268 -1.24135486 -0.757260659  0.654116147 -0.735097099 -1.00725975  0.04823205
41  -1.39640401 -1.60650174 -0.493895723 -0.237920166  0.510046282 -1.09563861 -0.69444787
42  -1.43264322 -1.35807987 -1.216330479 -0.318103205 -0.813903642 -1.07742990  0.30531356
43  -0.88199555 -1.10854897  0.248909800  0.062766232 -0.530200087 -0.84515783  0.24818434
44  -0.63553758 -0.77750465  0.445342211  0.343406870 -0.538080741 -0.81007276  0.01966744
45  -0.85548288 -0.53213260 -1.071552517  0.363452629 -0.396228964 -0.90822214 -0.66588326
46  -1.06868237 -0.70735874 -1.433133658  1.024962704 -0.813903642 -1.01658616 -0.43736636
47   0.54968839  1.04407092 -0.269817270  0.082811992  0.100252258  0.03951894 -0.60875403
48   0.51078223  0.73160278  0.758906541  0.183040791  0.431239739 -0.22162059 -0.20884946
49   0.06508706  0.12718349 -1.156673228  2.809035331 -1.397072061  0.64129456  1.61928573
50  -0.20427534 -0.17114456 -0.733979781  1.085099984  0.210581419  0.08393043  2.27627181
51   1.84535764  1.39369143  1.801453373 -1.671191995 -0.167689989  1.97585967  0.16249050
52  -0.80355571 -0.68157388 -0.591384401  0.754344946  0.368194505 -0.87624587  0.07679666
53  -0.90348806 -0.89312063 -0.660499509  0.944779665 -0.033718865 -0.76388482 -0.95152938
54  -0.01554909 -0.11513874  0.218353648  0.233155190 -0.427751581  0.02086612 -0.86583555
55  -1.15276987 -1.41713552 -0.552097919  0.624047507  1.471486109 -0.76610539  0.67665352
```

```
56  0.69448833  1.53925112 -0.539002425  0.994894065 -0.932113457 -0.16122097 -1.69420930
57 -0.23769071  0.07200944  0.013190907 -0.237920166  0.313029925 -0.80296692 -1.83703237
58 -0.59945525 -0.29591001 -0.798729724  0.934756785  0.935601615 -0.82872558 -1.69420930
59 -0.66440344 -0.48832606 -0.406592430  0.373475509 -0.214973914 -0.85492836 -2.06554926
60 -0.40288501 -0.29008762 -0.212342601  0.062766232  0.714943295 -0.58890357 -0.92296477
61 -0.89438904 -1.14847392  0.256185075  0.523818708  1.715786392 -0.96817765 -1.37999857
```

```
     dist8020   mean_age    perc_chil      per_ret   home_size
1   1.7676480 -0.5671366 -1.512406683 -1.169959532 -1.54065428
2   2.5907512  0.7141721  0.414628464  1.302649642  0.66528253
3  -0.4272940  0.1650398 -0.458559337  0.119178584 -0.87537175
4   0.6701770 -0.2010484 -0.368229565 -0.303489651 -0.91038662
5   0.1214415 -3.7887128  4.599907925 -2.268896942  2.97626394
6   0.1214415 -0.3840925  1.378146038  0.288245878  0.63026766
7  -0.1529263  1.1534779 -0.037020399  1.281516230 -0.70029740
8   1.2189125  0.2748662 -0.428449413 -0.028755298 -1.08546097
9  -0.1529263  1.2999132 -0.428449413  1.535117171 -0.73531227
10  0.9445447  1.3365220 -0.880098276  1.492850347 -0.91038662
11 -0.1529263  0.8606074 -0.488669261  1.239249406 -1.22552045
12  0.9445447  1.0436515 -0.548889110  1.218115995 -0.80534201
13  0.6701770  0.2748662 -0.247789868  0.182578819 -0.17507435
14 -0.4272940  0.7873897 -0.669328806  0.584113643 -0.49020818
15 -0.4272940  0.6409544 -1.602736456 -0.176689180 -0.38516357
16 -1.2503973 -0.2010484 -0.037020399 -0.599357415 -0.73531227
17 -0.1529263 -1.2993131  0.565178085 -1.317893414 -0.35014870
18  0.3958092  2.1419161 -1.120977669  2.591787758 -0.80534201
19  1.7676480  1.0436515 -0.428449413  1.598517406 -0.98041636
20  0.3958092 -0.2376573  0.083419298 -0.303489651  0.21008922
21 -0.9760295  0.0552133  0.053309374 -0.134422357  0.59525279
22 -0.4272940  1.4463485 -1.030647896  1.196982583  1.40059480
23 -0.4272940  0.2016486  0.324298692  0.267112466  1.50563941
24 -1.2503973 -0.2376573  0.836167403 -0.409156709 -0.17507435
25 -1.2503973 -1.2993131  1.588915507 -1.423560473  0.17507435
26  0.9445447  0.1650398 -0.247789868  0.161445408  0.80534201
27 -0.1529263  0.2382574 -0.006910474  0.267112466 -0.42017844
28 -0.7016618 -2.8368835  2.311553687 -2.332297178  0.07002974
29  2.0420157  0.2382574 -0.910208200  0.309379290 -1.36557993
30  0.1214415  0.9704338 -1.482296759  0.816581172 -1.61068401
31  1.2189125 -0.2742661 -0.609108958 -0.514823768 -0.87537175
32 -0.9760295  0.3846927 -0.338119640 -0.007621886 -0.35014870
33 -0.9760295  0.0552133  0.294188767 -0.218956004 -0.28011896
34  0.6701770  0.6409544  0.053309374  0.816581172  0.03501487
35  0.3958092 -0.1644396 -0.247789868 -0.049888710  0.28011896
36 -0.1529263 -0.7501807 -0.187570019 -0.937492003  0.42017844
37 -0.1529263 -0.8233984  0.986717024 -0.240089415  1.82077323
38 -0.1529263 -0.9332248  0.474848312 -0.789558121  1.19050558
39 -0.4272940 -1.0796601  0.866277327 -0.641624238  1.22552045
40  0.6701770 -0.7867896  0.595288009 -0.578224003  1.75074349
41 -0.1529263 -0.3108749  0.595288009  0.309379290  1.50563941
42  0.6701770 -1.4457483  0.806057478 -1.085425885  1.96083271
43  0.6701770  0.3480839 -0.488669261  0.816581172 -0.14005948
44 -0.1529263  0.6409544 -0.639218882  0.710914113  0.35014870
45 -0.4272940 -0.4939190  0.233968919 -1.022025650  0.28011896
46  0.3958092 -1.4457483  0.986717024 -1.508094120  0.77032714
47 -0.9760295 -0.5671366  0.113529222 -0.916358591 -0.42017844
48 -0.1529263  0.4579103 -0.849988351  0.288245878 -0.17507435
49  1.7676480 -0.6403543 -1.452186835 -1.423560473 -1.78575836
50  1.7676480 -0.1278308 -0.458559337 -0.620490827 -1.22552045
51 -0.7016618  1.1534779 -0.368229565  1.154715760  0.84035688
52  0.1214415  0.3114750 -0.669328806  0.098045173 -1.01543123
53 -0.1529263 -0.4207014  0.384518540 -0.493690356  0.07002974
54 -1.2503973  0.6043456  0.565178085  0.626380466 -0.14005948
55  0.9445447  0.1284309  0.655507858  0.499579996  0.84035688
56 -1.7991328 -0.4573102  0.504958237 -1.212226355  0.42017844
57 -1.7991328  0.3480839 -0.067130323 -0.282356239  0.28011896
58 -1.5247650 -0.4573102  0.956607099 -0.345756474  0.31513383
59 -1.7991328  0.1284309 -0.127350171 -0.240089415  0.49020818
60 -0.7016618  0.2016486  0.053309374 -0.197822592  0.49020818
61 -1.2503973  1.7758279 -1.843615849  2.021185641 -1.68071375
```

> #Comprobamos que realmente el valor de la media de los datos de Xs es
> #prácticamente cero y el valor de la desviación típica prácticamente 1.

```
> mean(Xs)
[1] 4.598972e-19
> sd(Xs)
[1] 0.9924475
> round(mean(Xs), digits = 0)
[1] 0
> round(sd(Xs), digits = 0)
[1] 1
>
```

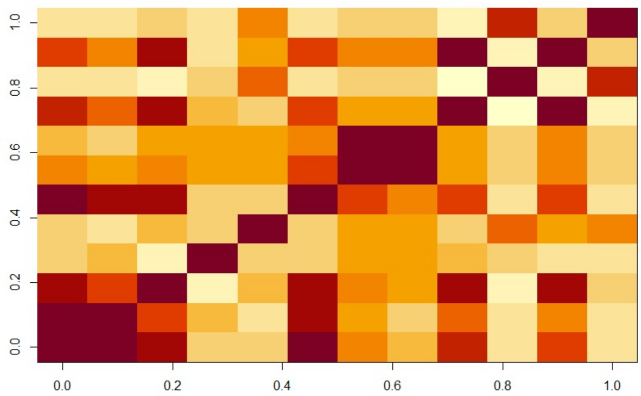
> #Ahora vamos a calcular la matriz de covarianza de los datos Xs y la vamos a
> #guardar en la variable CXs.

```
> CXs <- cov(Xs)
> fix(CXs)
> CXs
```

	row.names	rent	inc_sal	inc_ret	inc_emp	inc_non	inc_oth	gini	dist8020	mean_age	perc_chil	per_ret	home_size
1	rent	1	0.9267303	0.7612249	-0.2836566	-0.3159901	0.9130952	0.1801722	-0.04145366	0.575407	-0.471699	0.4162707	-0.3950107
2	inc_sal	0.9267303	1	0.5298288	-0.1325474	-0.3995027	0.7277385	-0.03060094	-0.2353366	0.3706067	-0.3548565	0.1432096	-0.3646303
3	inc_ret	0.7612249	0.5298288	1	-0.5115812	-0.1609269	0.7283159	0.1627369	0.05805812	0.8461552	-0.5501134	0.833983	-0.2723374
4	inc_emp	-0.2836566	-0.1325474	-0.5115812	1	-0.2921928	-0.30488	0.01873346	0.09633633	-0.1687072	-0.2774858	-0.3558086	-0.3969711
5	inc_non	-0.3159901	-0.3995027	-0.1609269	-0.2921928	1	-0.2019849	0.02066379	0.02290216	-0.1966636	0.3776646	-0.00395581	0.1294757
6	inc_oth	0.9130952	0.7277385	0.7283159	-0.30488	-0.2019849	1	0.4573695	0.2185647	0.538953	-0.4420784	0.4569156	-0.3588892
7	gini	0.1801722	-0.03060094	0.1627369	0.01873346	0.02066379	0.4573695	1	0.8793883	0.07983447	-0.2432927	0.1451232	-0.2259754
8	dist8020	-0.04145366	-0.2353366	0.05805812	0.09633633	0.02290216	0.2185647	0.8793883	1	0.05947531	-0.2204091	0.1483152	-0.2113529
9	mean_age	0.575407	0.3706067	0.8461552	-0.1687072	-0.1966636	0.538953	0.07983447	0.05947531	1	-0.7753111	0.9151241	-0.482575
10	perc_chil	-0.471699	-0.3548565	-0.5501134	-0.2774858	0.3776646	-0.4420784	-0.2432927	-0.2204091	-0.7753111	1	-0.5509099	0.687172
11	per_ret	0.4162707	0.1432096	0.833983	-0.3558086	-0.00395581	0.4569156	0.1451232	0.1483152	0.9151241	-0.5509099	1	-0.3038743
12	home_size	-0.3950107	-0.3646303	-0.2723374	-0.3969711	0.1294757	-0.3588892	-0.2259754	-0.2113529	-0.482575	0.687172	-0.3038743	1

> #Visualizamos la matriz como una imagen y la guardamos en formato .jpeg

```
> image(CXs)
> jpeg('CXs.jpeg')
> image(CXs)
> dev.off()
```



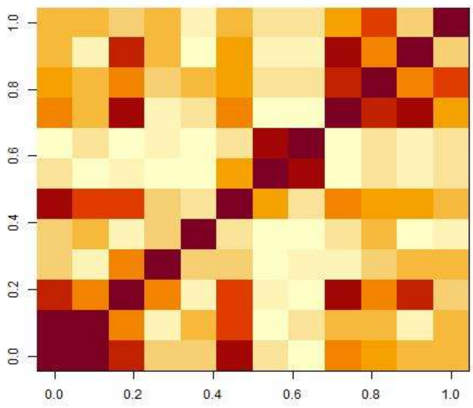
> #Buscamos el nombre (índice) de las variables que contienen el mayor y menor
> #valor absoluto de covarianza.

> #Primero mostramos la matriz CXs en valores absolutos

```
> CXs_ab <- abs(CXs)
> CXs_ab
```

> #Visualizamos la matriz de covarianzas definida positiva como una imagen y
> #la guardamos en formato .jpeg

```
> image(CXs_ab)
> jpeg('CXs_ab.jpeg')
> image(CXs_ab)
> dev.off()
```



> #Mostramos el par de valores máximo y mínimo de los valores absolutos de la
> #matriz CXs_ab.

```
> par_max = tail(sort(CXs_ab), 2)
> par_min = head(sort(CXs_ab), 2)
> par_max
[1] 1 1
> par_min
[1] 0.00395581 0.00395581
```

> #Viendo un resumen de los datos podemos observar que el valor máximo puede
> #corresponder a cualquier variable, ya que todas comparten como valor máximo 1.
> # Como valor mínimo podemos observar que las variables son inc_non(5) y per_ret(11)

```
> summary(CXs_ab)

         rent              inc_sal            inc_ret             inc_emp               inc_non             inc_oth             gini              dist8020
 Min.   :0.04145   Min.   :0.0306   Min.   :0.05806   Min.   :0.01873   Min.   :0.003956   Min.   :0.2020   Min.   :0.01873   Min.   :0.02290
 1st Qu.:0.30791   1st Qu.:0.2123   1st Qu.:0.24494   1st Qu.:0.15967   1st Qu.:0.102832   1st Qu.:0.3454   1st Qu.:0.06753   1st Qu.:0.05912
 Median :0.44398   Median :0.3676   Median :0.53997   Median :0.28792   Median :0.199324   Median :0.4571   Median :0.17145   Median :0.17983
 Mean   :0.52339   Mean   :0.4346   Mean   :0.53461   Mean   :0.31991   Mean   :0.260160   Mean   :0.5291   Mean   :0.28699   Mean   :0.26597
 3rd Qu.:0.79919   3rd Qu.:0.5793   3rd Qu.:0.77941   3rd Qu.:0.36610   3rd Qu.:0.331409   3rd Qu.:0.7279   3rd Qu.:0.29681   3rd Qu.:0.22414
 Max.   :1.00000   Max.   :1.0000   Max.   :1.00000   Max.   :1.00000   Max.   :1.000000   Max.   :1.0000   Max.   :1.00000   Max.   :1.00000
    mean_age          perc_chil           per_ret            home_size
 Min.   :0.05948   Min.   :0.2204   Min.   :0.003956   Min.   :0.1295
 1st Qu.:0.18967   1st Qu.:0.3355   1st Qu.:0.147517   1st Qu.:0.2607
 Median :0.51076   Median :0.4569   Median :0.386040   Median :0.3618
 Mean   :0.50073   Mean   :0.4959   Mean   :0.439458   Mean   :0.4024
 3rd Qu.:0.79302   3rd Qu.:0.5850   3rd Qu.:0.621678   3rd Qu.:0.4184
 Max.   :1.00000   Max.   :1.0000   Max.   :1.000000   Max.   :1.0000
```

> #Comprobamos que efectivamente los datos de maximo concuerdan con las variables
> # 1 y 2, y los datos de mínimo con 5 y 11

```
> CXs_ab[1,1]
[1] 1
> CXs_ab[2,2]
[1] 1
> CXs_ab[11,5]
[1] 0.00395581
> CXs_ab[5,11]
[1] 0.00395581
```

Pregunta 4. [5 %] Finalmente, calcular la descomposición en componentes principales de la matriz de covarianzas ***CXs***. Dibujar la distribución de la varianza explicada en porcentaje (eje de ordenadas) para cada componente principal (eje de abscisas) respecto la variancia total de los datos.

<u>Respuesta:</u>

\>

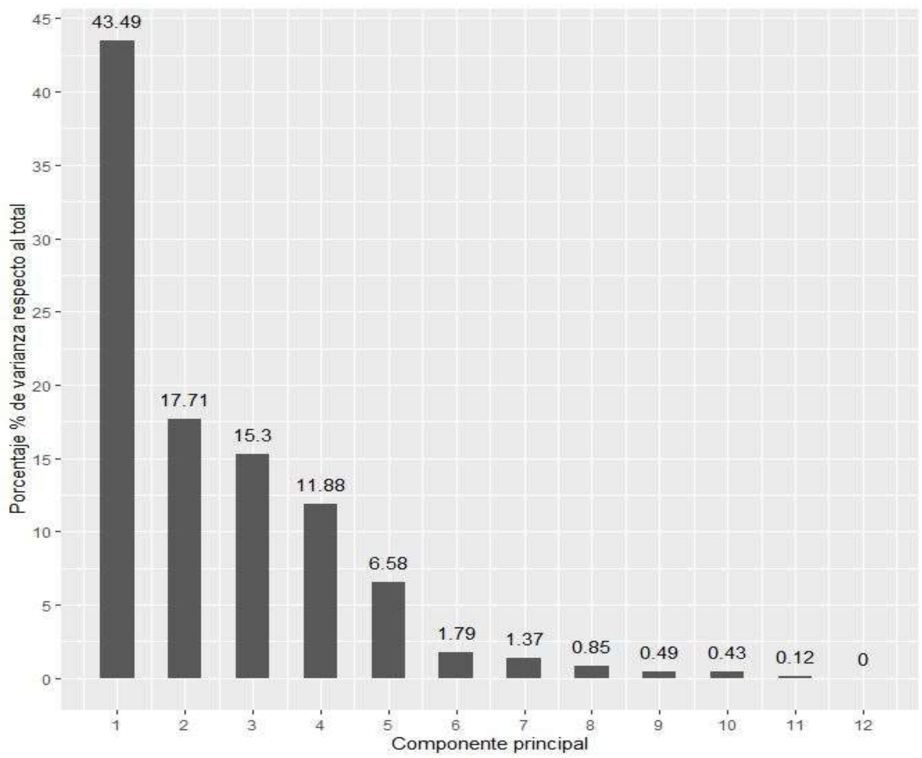
> #Calculamos lo componentes principales de la matriz de datos normalizada Xs

```
> comp_prin <- prcomp(Xs, scale = TRUE, center = TRUE)
```

> #Comprobamos que la media es prácticamente cero y la varianza uno

```
> comp_prin$center
```

```
rent           inc_sal        inc_ret        inc_emp        inc_non
-2.096740e-16 1.458732e-16 -1.292227e-16 -3.150940e-16 -8.502989e-18

inc_oth        gini           dist8020       mean_age       perc_chil
9.577949e-17 -3.434184e-16 2.684556e-16 -4.436342e-16 4.555498e-16

per_ret        home_size
1.550302e-16 3.347732e-16

> comp_prin$scale
rent    inc_sal   inc_ret    inc_emp   inc_non  inc_oth   gini   dist8020
 1        1         1          1         1        1         1       1
mean_age perc_chil per_ret home_size
    1        1         1         1

> comp_prin$sdev^2
[1] 5.218483e+00 2.125341e+00 1.836521e+00 1.425191e+00 7.895196e-01
[6] 2.145532e-01 1.647613e-01 1.015204e-01 5.855235e-02 5.127084e-02
[11] 1.428599e-02 2.020722e-07

> #Ahora dibujamos la distribución de la varianza explicada en porcentaje (eje
> #de ordenadas) para cada componente principal (eje de abscisas) respecto a la
> #varianza total de los datos.
> #Calculamos el % de cada valor respecto al total
> porc_varianza <- abs(comp_prin$sdev^2)*100/sum(abs(comp_prin$sdev^2))
> porc_varianza

[1] 4.348736e+01 1.771117e+01 1.530434e+01 1.187660e+01 6.579330e+00
[6] 1.787943e+00 1.373010e+00 8.460034e-01 4.879362e-01 4.272570e-01
[11] 1.190499e-01 1.683935e-06
>
> #Importamos paquete ggplot2 con el que vamos a generar la gráfica.
> library(ggplot2)
> library(crayon)
Attaching package: 'crayon'
The following object is masked from 'package:ggplot2':
%+%
> ggplot(data = data.frame(porc_varianza, pc = 1:12),
+ aes(x = pc, y = porc_varianza)) + geom_col(width = 0.5) +
+ geom_text(aes(label = round(porc_varianza, digits = 2)), vjust = -1,
+ colour = "black") + ylim(c(0, 100)) +
+ scale_x_continuous(breaks = seq(1, 12, by = 1)) +
+ scale_y_continuous(breaks = seq(0, 60, by = 5)) +
+ labs(x = "Componente principal",
+ y = "Porcentaje % de varianza respecto al total")
```



Pregunta 5. [20 %] Como habéis visto, la mayor parte de la varianza queda concentrada en unas pocas componentes principales. Por esto, podemos reducir la dimensión del subespacio, proyectar nuestros datos allí y utilizar estas representaciones para análisis posteriores. Un buen criterio para el diseño del nuevo subespacio es restringir el porcentaje total de varianza explicada por el subespacio a un cierto umbral. En esta práctica, os quedareis con las L primeras componentes principales que expliquen, al menos, un 75% de la varianza inicial. Calcular el valor mínimo de L, es decir, el mínimo número de componentes principales necesarias para explicar un 75% de la varianza de nuestros datos.

Respuesta:

- $L = 3$
- Copiamos código R usado para conseguir la respuesta:

```
> #Calculamos la variabilidad acumuladada de las variables originales y de las
> #componentes principales

> library(data.table)
> variabilidad_org = 100/12
> variabilidad_org

[1] 8.333333


> com_principales <- round(porc_varianza, digits = 2)
> variabilidad_cp = data.frame(variables = 1:12, com_principales,
+                              variabilidad_org )
> dt <- as.data.table(variabilidad_cp)
> dt[, acumulado_com.prin:= cumsum(com_principales)]
> dt[,acumulado_original := cumsum(variabilidad_org)]
> dt
```

	variables	com_principales	variabilidad_org	acumulado_com.prin	acumulado_original
1:	1	43.49	8.333333	43.49	8.333333
2:	2	17.71	8.333333	61.20	16.666667
3:	3	15.30	8.333333	76.50	25.000000
4:	4	11.88	8.333333	88.38	33.333333
5:	5	6.58	8.333333	94.96	41.666667
6:	6	1.79	8.333333	96.75	50.000000
7:	7	1.37	8.333333	98.12	58.333333
8:	8	0.85	8.333333	98.97	66.666667
9:	9	0.49	8.333333	99.46	75.000000
10:	10	0.43	8.333333	99.89	83.333333
11:	11	0.12	8.333333	100.01	91.666667
12:	12	0.00	8.333333	100.01	100.000000

```
> #Podemos observar en la columna acumulado_com.prin que necesitamos como
> #mínimo de 3 componentes principales para explicar un 75% de la varianza
```

Pregunta 6. [10 %] Considerar la componente principal C e indicar que variables contribuyen en mayor y menor peso (en valor absoluto).

Respuesta:

- $C = 1$
- Variable que contribuye en mayor peso = 3
- Variable que contribuye en menor peso = 8
- Copiamos código R usado para conseguir la respuesta:

>#Obtenemos la carga de cada variable en la componente principal C = 1 y las
>#ordenamos en valores absolutos para detectar el valor máximo y el valor mínimo.

```
> loadings_cpC1 <- comp_prin$rotation[,1]
> loadings_cpC1_abs <- abs(loadings_cpC1)
> loadings_cpC1_ordmax <- sort(loadings_cpC1_abs, decreasing = TRUE)
> loadings_cpC1_ordmin <- sort(loadings_cpC1_abs, decreasing = FALSE)
> loadings_cpC1_ordmax[0:1]
 inc_ret
0.393225

> loadings_cpC1_ordmin[0:1]
  dist8020
0.07551688
```

#Comprobamos que sus posiciones son 3 y 8 respectivamente

```
> loadings_cpC1[3:3]
  inc_ret
-0.393225

> loadings_cpC1[8:8]
   dist8020
-0.07551688
```

Pregunta 7. [10 %] Calcular las nuevas variables proyectadas a las componentes principales. Para la componente principal C, anotar las secciones censales (relacionarlo con la variable id) con el valor máximo y mínimo.

<u>Respuesta:</u>

- ***C*** = 1
- Número de las dos regiones censales = {5 , 44 }
- Copiamos código R usado para conseguir la respuesta:

Cálculo de las nuevas variables proyectadas

```
> nuevas_vars <- predict(comp_prin, newdata = Xs)
> nuevas_vars
```

	PC1	PC2	PC3	PC4	PC5	PC6	PC7
1	-0.50794175	4.160920259	-1.358941167	1.377985127	-0.743106650	0.7389278499	0.26811024
2	-1.16093410	1.176711951	2.718741157	0.279414147	1.659629993	-0.8997296842	-0.23911466
3	-0.70326104	-0.008370836	-0.623620313	-0.120038497	-0.288280861	-0.4870146888	0.25049441
4	0.66376338	1.561242089	-0.522259943	-0.372580365	-0.171668850	-0.4806855247	0.15148039
5	**7.53212491**	-1.418681812	5.844886318	2.095023111	-2.229412686	-0.0219708550	0.33604441
6	2.85603905	-0.232678915	1.898477497	-1.460840712	-0.867420117	-0.0490803707	-0.47406645
7	-3.44837180	-0.793740884	0.701570102	0.853413972	-0.694867623	0.1437992747	-0.85390217
8	-1.65193465	1.106928360	0.634993608	0.976624710	-1.734932206	0.0579745415	0.25884832
9	-3.00719695	-0.435952235	1.181810561	-0.401342367	-0.388761024	-0.0369140157	-0.41558901
10	-2.92144558	0.721523613	0.864824177	-0.461269197	-0.086813947	-0.1827376379	-0.24715240
11	-2.40611561	-0.552045154	0.290695782	-0.636536564	-1.620055092	-0.0005384315	-0.33873119
12	-2.41155289	0.617334033	0.870641832	-0.361522916	0.828362048	-0.9548757813	-0.04624334
13	-1.83097026	0.275114641	0.882997097	1.244024296	-0.392906354	0.1181731956	0.20669120
14	-2.25081180	-0.272663553	-0.005282770	0.183468291	-0.297202653	0.4244824891	-0.13968576
15	-0.73101713	0.051583193	-1.122406227	-1.045611009	-0.194308554	0.4587193303	1.00883543
16	0.36727677	-1.062539284	-1.868893976	-0.210968717	-0.252516486	-0.6641057260	0.21596271

17	1.79068248	0.433502859	-1.720314740	0.888885825	0.001468144	-0.4836690379	-0.10196197
18	-3.73303788	-0.252856886	1.469450114	-1.879798360	0.130590874	-0.3663400721	-0.13079873
19	-2.77633499	1.904522730	1.155564594	-0.249036129	0.781751335	-0.5739087256	-0.76059482
20	1.36494487	0.855527711	0.177098411	-0.448515334	-0.548006405	0.5598166915	-0.05513769
21	-1.58318379	-1.891240583	-0.776358592	1.486868392	0.340187356	0.3207802734	0.02346615
22	-4.88036789	-3.110858674	1.350679044	1.022277179	1.858903435	0.1707335990	0.94569094
23	-0.67123817	-0.943954176	-0.371341678	1.055805592	1.209474840	1.0015686521	-0.59543610
24	0.22750479	-1.658635139	-2.032573575	0.486574678	0.266937814	-0.5525423189	-0.69510498
25	1.46202412	-1.918865766	-1.641496323	1.826394337	-0.224572745	-0.6704005039	-0.41786845
26	-2.96483977	0.140045063	1.462419682	2.996109375	0.308887635	0.9424639041	0.01826190
27	-0.47878183	-0.369162542	0.035019802	-0.245828459	-0.235084202	-0.4973974281	0.12658472
28	3.95185691	-0.634462876	-1.043137224	2.249050127	-0.977221690	-0.8414931022	-0.33917673
29	-2.02109901	2.663297448	0.164933992	0.698632233	-0.126895871	-0.2396369692	-0.16020433
30	-3.23961250	0.971675702	-0.627586193	-0.157155661	-0.730065366	0.3762966735	-0.20618860
31	-0.63223387	1.245865280	0.040672816	0.907578953	-0.974034315	-0.2098520340	0.58758352
32	-1.38682197	-1.903982656	-0.580892859	0.334793617	-0.404088325	-0.4196169455	0.49287760
33	-1.33446967	-2.279825003	-0.342008989	0.841062625	-0.378342989	-0.7239472662	0.30371264
34	-0.53126893	-0.051946395	1.722240959	-0.819890083	-0.208103451	-0.4316601392	0.31131555
35	1.19099293	0.935144859	0.344764697	-0.642497904	1.139335222	-0.4690341696	0.51418931
36	1.95395328	0.553139396	-1.094632848	-0.133476254	0.770276040	0.0519168927	0.44011952
37	3.02290277	-0.212658036	0.836872233	-0.482024344	1.566080140	0.1441154435	-0.10696977
38	2.38589042	0.100517509	0.046984408	0.107994163	0.365334792	0.5810676069	0.19084341
39	2.77496509	0.230245959	-0.160954095	0.007208606	1.173979284	0.2879278600	-0.30123734
40	2.53309427	0.717192172	0.223078520	0.008091281	1.754173502	0.4653467621	-0.11105555
41	2.35791401	-0.403886734	1.115558697	-1.324670774	0.816518686	0.3736704289	-0.07437692
42	3.24977327	0.520598118	0.682296342	0.693252505	1.883465964	0.0046686206	0.45269993
43	0.14844835	1.076913444	0.446207524	-1.408803103	0.988219616	-0.4609375752	0.08012771
44	**-0.02298756**	0.336625432	-0.005377663	-1.499240714	1.051078028	0.2284079683	0.10350660
45	2.01525020	-0.048104574	-0.922663438	-0.220875423	0.313043001	-0.1386923455	0.30697305
46	3.13031957	0.656955584	-1.000795419	0.718049973	0.843956642	-0.1627779224	-0.18113491
47	0.18877462	-1.001959781	-1.228200990	0.809786569	-0.911003694	-0.1046766557	0.30145481
48	-1.10413083	-0.278702650	-0.284343326	-0.465956112	-0.435019741	0.3498028396	0.50535751
49	-0.33350198	4.048669925	-2.542005313	1.369831763	-0.084208625	0.2073324328	0.07299176
50	-0.12202331	3.267095911	0.042918061	0.905553260	-0.610201895	-0.0791005335	0.08033796
51	-3.45157212	-2.189301067	1.133516676	0.837474387	0.411285729	0.6556821617	0.01576882
52	0.55853126	1.271048166	-0.519566610	-1.353987937	-0.567473432	-0.1989154026	0.21119984
53	1.84920961	0.216173754	-0.755716552	-0.773764234	0.066353202	-0.0002145597	-0.30442385
54	-0.15857020	-1.174274029	-0.599231247	-0.903968582	0.148464617	-0.0726752709	-0.94526472
55	1.68281255	1.304433614	1.666902707	-0.975079101	-0.152538106	0.7644755525	-0.61078482
56	0.70805968	-2.057852596	-2.707888045	0.861912207	-0.060862002	0.4055729254	-0.26630616
57	0.77465011	-2.079506926	-0.900795004	-1.272611006	-0.350789145	0.1373955709	0.43980675
58	2.18067352	-1.361775046	-0.945819452	-0.936830785	-0.957188613	0.4867769627	-0.67983350
59	1.40982317	-1.700365175	-1.370563144	-1.577320271	0.203376907	0.3333050024	0.14550188
60	0.95281684	-0.830478916	-0.075503493	-0.873745771	-0.337210687	0.4383045848	0.21545445
61	-0.82744297	0.010780128	-0.255646203	-4.407354614	-1.645970437	0.2456356030	0.21605150

	PC8	PC9	PC10	PC11	PC12
1	0.316571377	0.05233203	0.104479128	-0.0022552549	1.571576e-04
2	-0.638730802	-0.46522267	-0.341499216	-0.0282859459	-5.344653e-04
3	0.340913758	0.33230971	-0.303947486	0.0917640209	-5.879721e-05
4	0.124763511	-0.10919148	-0.124803516	-0.0006297962	1.160190e-04
5	0.467811617	0.19388971	0.305492603	-0.0679023641	-1.619465e-03
6	-0.159311208	-0.49051711	-0.022957126	-0.0111954757	9.974688e-04
7	0.426330906	-0.30768299	-0.367395819	-0.0152434712	-1.208613e-04
8	-0.732790750	0.05317348	-0.161830900	-0.0431480228	2.612461e-04
9	0.599849027	0.06478591	0.066408706	-0.0885876705	3.938604e-04
10	-0.007193477	-0.29454916	-0.140275984	0.0487716264	-3.762498e-04
11	-0.383544074	-0.05202353	0.235074024	0.2756136765	-1.042364e-04
12	0.226977125	0.02889947	0.053375773	-0.0972329028	-2.295225e-05
13	-0.124391576	0.20406990	-0.165445136	-0.0287829626	5.935276e-04
14	0.573317420	0.01869240	0.061112657	-0.0955890657	2.864816e-04
15	-0.145832896	-0.02930164	0.123485708	-0.0886265555	-2.661810e-04
16	0.123360660	0.15569162	-0.065560658	-0.0326834838	-2.448013e-04
17	-0.129769817	0.22481351	0.180824318	0.1169239880	8.939872e-05
18	-0.234935062	0.26040959	0.010414538	0.0893126198	-3.188739e-04
19	-0.041622044	0.30310279	0.513039339	-0.0554725426	-6.537855e-05
20	-0.348151954	-0.13773034	0.008618919	-0.0369510874	3.558797e-04
21	-0.163301829	0.24496521	-0.133332557	0.1135206747	-3.022725e-04
22	-0.028373983	-0.10876730	0.637521986	-0.0549815336	2.993143e-04
23	-0.416118088	0.35854228	-0.369413905	0.1079080676	-9.171161e-04
24	-0.245306597	0.06256866	-0.047995880	-0.0275953210	-8.214082e-04
25	-0.097647223	0.00658328	-0.151865830	0.0101604968	4.406103e-05
26	0.324017582	-0.17021649	-0.391617784	0.0901763095	2.447617e-04
27	-0.174697078	0.30509248	-0.099321921	-0.0002994829	2.382381e-04
28	0.157438606	0.17449732	0.178895249	0.2454278895	1.199507e-03
29	-0.075850619	-0.24630258	0.426611907	0.0616358203	-2.308706e-04
30	0.562472764	-0.04939042	0.480329814	0.0370018458	5.397045e-05
31	-0.372021328	-0.43285774	-0.096218635	0.1311230251	-6.367735e-05
32	0.181752963	-0.49968297	-0.341308879	0.0486458767	-1.212569e-04
33	0.036217901	-0.17762844	0.173138909	-0.0741739499	3.335915e-04
34	-0.574836570	0.30745647	-0.010432309	-0.1171582548	4.558822e-04
35	0.430776460	0.19380745	-0.346352529	0.0090685961	2.836349e-05
36	0.164877654	-0.02419910	-0.089934431	0.0755228205	-3.711807e-04
37	0.342449897	-0.06021424	-0.159185560	0.1020413953	4.097408e-04
38	-0.124207794	0.16502463	0.010492709	0.1015444173	3.003465e-04
39	0.475395110	0.36824858	0.073533795	0.0721964234	4.493188e-04

```
40 -0.302672474 -0.11255004  0.183290780  0.0066737181 -1.637943e-04
41 -0.021404937 -0.21974411  0.029003795  0.1663277633  3.769272e-04
42  0.124340983 -0.16656441 -0.111461373  0.1409567783  3.050650e-04
43  0.065424393  0.16237847  0.111847932  0.1448615167 -1.458806e-04
44  0.067541300  0.44042193  0.112601532 -0.0342018313 -2.341120e-04
45  0.138794793 -0.19562218 -0.197529620 -0.2307154062  1.459368e-04
46 -0.257581890 -0.31531431  0.190081988 -0.0331638057 -1.076899e-04
47 -0.004119639  0.14350069  0.012658523  0.0346826603  3.748274e-04
48 -0.619072503  0.38361171  0.219717713  0.1146717264 -1.726807e-04
49  0.204568208 -0.34058054  0.194869307 -0.0259086731 -8.341841e-04
50  0.011636245  0.39661053 -0.328293086 -0.3116924321  2.318727e-04
51  0.214918791 -0.02215440 -0.060346315  0.0111730651  2.132221e-04
52  0.109563729  0.14487230 -0.254061010 -0.0653914073 -2.575271e-04
53 -0.081735680 -0.39052689  0.358801766 -0.0928449174 -1.260925e-04
54  0.385525805  0.10202168 -0.003967989 -0.2203050005 -2.646952e-04
55 -0.489334269  0.12083869 -0.141083522 -0.1346276346  5.339248e-04
56 -0.421229976  0.03897256 -0.003503670 -0.1200867684 -5.603405e-04
57  0.009493622 -0.02444772  0.016762984 -0.2547576679  2.387585e-04
58 -0.184185898 -0.07878713  0.144655242  0.0271711917  3.216706e-04
59  0.242619095 -0.32654287  0.082752906 -0.0681963827 -3.954628e-04
60 -0.212072882 -0.11569446 -0.021358650 -0.2069686398  2.768668e-04
61  0.362323616 -0.04817782 -0.247593254  0.2907777006 -5.047028e-04
```

```
#También podríamos calcularlas así
comp_prin$x

#Calculamos valores máximos y mínimos en C=1

> max <- max(abs(nuevas_vars[,1]))
> min <- min(abs(nuevas_vars[,1]))

> max
[1] 7.532125

> min
[1] 0.02298756

> ord <- sort(abs(nuevas_vars[,1]), decreasing = FALSE)
> ord

        44         50         43         54         47         24         49         16         27          1         34         52         31
0.02298756 0.12202331 0.14844835 0.15857020 0.18877462 0.22750479 0.33350198 0.36727677 0.47878183 0.50794175 0.53126893 0.55853126 0.63223387

         4         23          3         56         15         57         61         60         48          2         35         33         20
0.66376338 0.67123817 0.70326104 0.70805968 0.73101713 0.77465011 0.82744297 0.95281684 1.10413083 1.16093410 1.19099293 1.33446967 1.36494487

        32         59         25         21          8         55         17         13         53         36         45         29         58
1.38682197 1.40982317 1.46202412 1.58318379 1.65193465 1.68281255 1.79068248 1.83097026 1.84920961 1.95395328 2.01525020 2.02109901 2.18067352

        14         41         38         11         12         40         39         19          6         10         26          9         37
2.25081180 2.35791401 2.38589042 2.40611561 2.41155289 2.53309427 2.77496509 2.77633499 2.85603905 2.92144558 2.96483977 3.00719695 3.02290277

        46         30         42          7         51         18         28         22          5
3.13031957 3.23961250 3.24977327 3.44837180 3.45157212 3.73303788 3.95185691 4.88036789 7.53212491

> #comprobamos que el id 44 coincide con min y que el id 5 coincide con max

> nuevas_vars[44]
[1] -0.02298756

> nuevas_vars[5]
[1] 7.532125
```

Pregunta 8. [20 %] Cuando reducimos la dimensión del subespacio generado por los datos iniciales a ***L***, se produce una pérdida de información. Una manera de medir el error cometido en esta aproximación es calculando el error residual, tal y como se indica en la Sección 2.5.1 de los apuntes del módulo. Considerando el valor de ***L*** calculado en el apartado 5, calcular la desviación típica del error residual cuando se consideran solo las ***L*** primeras componentes principales.

<u>Respuesta:</u>

- ***L*** = 3
- Desviación típica = 4.801111
- Copiamos código R usado para conseguir la respuesta:

> #Tomamos solo las tres primeras componentes principales

```
> L <- comp_prin$x[, 1:3]
> L

           PC1          PC2          PC3
1  -0.50794175  4.160920259 -1.358941167
2  -1.16093410  1.176711951  2.718741157
3  -0.70326104 -0.008370836 -0.623620313
4   0.66376338  1.561242089 -0.522259943
5   7.53212491 -1.418681812  5.844886318
6   2.85603905 -0.232678915  1.898477497
7  -3.44837180 -0.793740884  0.701570102
8  -1.65193465  1.106928360  0.634993608
9  -3.00719695 -0.435952235  1.181810561
10 -2.92144558  0.721523613  0.864824177
11 -2.40611561 -0.552045154  0.290695782
12 -2.41155289  0.617334033  0.870641832
13 -1.83097026  0.275114641  0.882997097
14 -2.25081180 -0.272663553 -0.005282770
15 -0.73101713  0.051583193 -1.122406227
16  0.36727677 -1.062539284 -1.868893976
17  1.79068248  0.433502859 -1.720314740
18 -3.73303788 -0.252856886  1.469450114
19 -2.77633499  1.904522730  1.155564594
20  1.36494487  0.855527711  0.177098411
21 -1.58318379 -1.891240583 -0.776358592
22 -4.88036789 -3.110858674  1.350679044
23 -0.67123817 -0.943954176 -0.371341678
24  0.22750479 -1.658635139 -2.032573575
25  1.46202412 -1.918865766 -1.641496323
26 -2.96483977  0.140045063  1.462419682
27 -0.47878183 -0.369162542  0.035019802
28  3.95185691 -0.634462876 -1.043137224
29 -2.02109901  2.663297448  0.164933992
30 -3.23961250  0.971675702 -0.627586193
31 -0.63223387  1.245865280  0.040672816
32 -1.38682197 -1.903982656 -0.580892859
33 -1.33446967 -2.279825003 -0.342008989
34 -0.53126893 -0.051946395  1.722240959
35  1.19099293  0.935144859  0.344764697
36  1.95395328  0.553139396 -1.094632848
37  3.02290277 -0.212658036  0.836872233
38  2.38589042  0.100517509  0.046984408
39  2.77496509  0.230245959 -0.160954095
40  2.53309427  0.717192172  0.223078520
41  2.35791401 -0.403886734  1.115558697
42  3.24977327  0.520598118  0.682296342
43  0.14844835  1.076913444  0.446207524
44 -0.02298756  0.336625432 -0.005377663
45  2.01525020 -0.048104574 -0.922663438
46  3.13031957  0.656955584 -1.000795419
47  0.18877462 -1.001959781 -1.228200990
48 -1.10413083 -0.278702650 -0.284343326
49 -0.33350198  4.048669925 -2.542005313
50 -0.12202331  3.267095911  0.042918061
51 -3.45157212 -2.189301067  1.133516676
52  0.55853126  1.271048166 -0.519566610
53  1.84920961  0.216173754 -0.755716552
54 -0.15857020 -1.174274029 -0.599231247
55  1.68281255  1.304433614  1.666902707
56  0.70805968 -2.057852596 -2.707888045
57  0.77465011 -2.079506926 -0.900795004
58  2.18067352 -1.361775046 -0.945819452
59  1.40982317 -1.700365175 -1.370563144
60  0.95281684 -0.830478916 -0.075503493
61 -0.82744297  0.010780128 -0.255646203
```

```
#Calculamos los datos originales con pérdida de información
dtorg_lossinf <- nuevas_vars[,1:3]%*%t(L)
org_lossinf
```

```
1             2           3           4           5           6
1   19.41798332  1.79128221  1.16984858  6.8687712 -17.6717592  -4.9987791
2    1.79128221 10.12397247 -0.88887255 -0.3533429   5.4770525   1.5719997
3    1.16984858 -0.88887255  0.88354846 -0.1541759  -8.9301643  -3.1905224
4    6.86877124 -0.35334291 -0.15417591  3.1508141  -0.2679071   0.5409673
5  -17.67175924  5.47705252 -8.93016429 -0.2679071  92.9082598  32.9385254
6   -4.99877911  1.57199973 -3.19052241  0.5409673  32.9385254  11.8153153
7   -2.50451301  4.97671553  1.99423645 -3.8945265 -20.7469039  -8.3320827
8    4.58200826  4.94670635  0.75648045  0.3000577 -10.3014918  -3.7700277
9   -1.89249263  6.19120428  1.38149267 -3.2939065 -15.1245572  -6.2435943
10   3.31088122  6.59186432  1.50917718 -1.2643386 -17.9735064  -6.8697967
11  -1.46988776  2.93407010  1.51546465 -2.6107863 -15.6409031  -6.1916315
12   2.61045505  5.89312809  1.14783366 -1.0915940 -13.9511156  -5.3782359
13   0.87481521  4.85000703  0.73469218 -1.2469645  -9.0203792  -3.6169858
14   0.01592895  2.27783521  1.58848511 -1.9169413 -16.5974500  -6.3749925
15   2.11123170 -2.14217074  1.21361939  0.1814993 -12.1396292  -4.2306787
16  -2.06797947 -6.75772577  0.91608315 -0.4390477  -6.6496932  -2.2518659
            7          8           9          10          11          12
1   -2.5045130  4.58200826 -1.89249263  3.3108812  -1.4698878   2.6104551
2    4.9767155  4.94670635  6.19120428  6.5918643   2.9340701   5.8931281
3    1.9942365  0.75648045  1.38149267  1.5091772   1.5154646   1.1478337
4   -3.8945265  0.30005770 -3.29390649 -1.2643386  -2.6107863  -1.0915940
5  -20.7469039 -10.30149179 -15.12455717 -17.9735064 -15.6409031 -13.9511156
6   -8.3320827 -3.77002768  -6.24359427  -6.8697967  -6.1916315  -5.3782359
7   13.0134933  5.26336310  11.54508924 10.1082626   8.9393055   8.4367440
8    5.2633631  4.35739536   5.23556710  6.1738700   3.5482613   5.2199243
9   11.5450892  5.23556710  10.62996405  9.4928707   7.8198762   8.0118200
10  10.1082626  6.17386995   9.49287075  9.8033614   6.8824229   8.2435937
11   8.9393055  3.54826126   7.81987617  6.8824229   6.1786502   5.7147707
12   8.4367440  5.21992432   8.01182004  8.2435937   5.7147707   6.9547058
13   6.7149808  3.88987292   6.42968662  6.3112189   4.5103339   5.3540934
14   7.9743539  3.41302045   6.88125942  6.3743223   5.5647003   5.2550278
15   1.6924285  0.55197064   0.84935313  1.2021612   1.4041567   0.8175167
16  -1.7342861 -2.96960781  -2.84993584 -3.4558910  -0.8404203  -3.1687863
           13          14          15         16          17           18
1   0.8748152  0.01592895  2.11123170 -2.0679795  3.23201496  -1.152847797
2   4.8500070  2.27783521 -2.14217074 -6.7577258 -6.24584684   8.031325748
3   0.7346922  1.58848511  1.21361939  0.9160831 -0.19012279   1.711037791
4  -1.2469645 -1.91694127  0.18149927 -0.4390477  2.76384383  -3.640059572
5  -9.0203792 -16.59744996 -12.13962924 -6.6496932  2.81759739 -19.170215305
6  -3.6169858 -6.37499254 -4.23067874 -2.2518659  1.74741329  -7.813149531
7   6.7149808  7.97435391  1.69242851 -1.7342861 -7.72594930  14.104527702
8   3.8898729  3.41302045  0.55197064 -2.9696078 -3.57062269   6.819931599
9   6.4296866  6.88125942  0.84935313 -2.8499358 -7.60700756  13.072825333
10  6.3112189  6.37432231  1.20216120 -3.4558910 -6.40636864  11.994240794
11  4.5103339  5.56470030  1.40415671 -0.8404203 -5.04799045   9.548872077
12  5.3540934  5.25502779  0.81751671 -3.1687863 -5.54848741  10.125685861
13  4.2078240  4.04149104  0.36158047 -2.6150209 -4.67845629   8.063036885
14  4.0414910  5.14052706  1.63724653 -0.5270822 -4.13960165   8.463547796
15  0.3615805  1.63724653  1.79684260  1.7743635  0.64423388   1.066551502
16 -2.6150209 -0.52708221  1.77436346  4.7566466  3.41214811  -3.848634178
           19          20          21          22         23          24
1    7.7644394  2.62580378 -6.01011046 -12.3005856 -3.0821367 -4.25485983
2    8.6058976 -0.09641662 -2.49819141   5.6773476 -1.3410808 -7.74189529
3    1.2159122 -1.07751620  1.61337574   2.6159022  0.7115336  1.12144308
4    0.5270865  2.14919488 -3.59808282  -8.8016185 -1.7253474 -1.37698988
5  -16.8594700 10.10233375 -13.77939718 -24.4516564 -5.8871290 -7.81349149
6   -6.1786501  4.03548994 -5.55548224 -10.6504563 -2.4024280 -2.82310321
7    8.8728473 -5.26165778  6.41589135  20.2461348  2.8034116 -0.89398743
8    7.4282703 -1.19533548  0.02886558   5.4762237 -0.1718477 -3.50248455
9    8.8843636 -4.26833050  4.66792725  17.6286600  1.9912088 -2.36318293
10  10.4844299 -3.21716973  2.58919687  13.1812711  0.9587553 -3.61920605
11   5.9647182 -3.70502332  4.62768926  13.8527005  2.0282345 -0.22262190
12   8.8770882 -2.60930109  1.97447400  11.0247840  0.7126857 -3.34221533
13   6.6277091 -2.10742788  1.69293208   9.2726113  0.6414279 -2.66762588
14   5.7236090 -3.30644082  4.08322246  11.8258721  1.7701744 -0.04908349
15   0.8307769 -1.15244359  1.93116796   1.8911539  0.8587906  2.02950564
16  -5.2029413 -0.73869742  2.87898269  -1.0113121  1.4504564  5.64458673
           25          26          27          28           29          30
1  -6.49617360   0.1013399 -1.34045247  -3.2297004  11.88423328   6.5414523
2  -8.41806954   7.5827169  0.21664596  -8.1704456   5.92870952   3.1981146
3   0.01154836   1.1718894  0.31795975  -2.1233545   1.29620998   2.6615350
4  -1.16808816  -2.5130710 -0.91243938   2.1773365   2.73038215  -0.3055520
5   4.14004885 -13.9825461 -2.87873363  24.5689623 -18.03752142 -29.4478346
6   1.50573375  -5.7239128 -1.21503897   9.4539112  -6.07890739 -10.6700065
7  -4.67014529  11.1387003  1.96860603 -13.8557067   4.97124553   9.9598340
8  -5.58155491   5.9813686  0.40451716  -7.8928998   6.39153499   6.0286903
9  -5.49999835  10.5831072  1.64211528 -12.8402071   5.11169304   8.5768606
10 -7.07533657  10.0273998  1.16266155 -12.9050451   7.96880167   9.6226869
11 -2.93567456   7.4815553  1.36597894  -9.4616080   3.44068303   7.0760367
12 -6.13948499   8.5095662  0.95720082 -10.8299864   6.66172974   7.8659426
13 -4.65426723   6.7583742  0.80599566  -8.3313696   4.57892053   5.6447995
14 -2.75886473   6.6273855  1.17811997  -8.7163806   3.82205804   7.0301329
15  0.67467980   0.5331437  0.29164869  -1.7507790   1.42971644   3.1227410
16  5.64362034  -3.9708275  0.15095596   4.0750798  -3.88040502  -1.0493860
          31          32         33          34          35          36
1   5.4498121  -6.4284960 -8.3435671 -2.286715279  2.81759322   2.7966161
2   2.3105853  -2.2097276 -2.0632999  5.237969644  0.65505780  -4.5935486
3   0.4088321   1.3534924  1.1708483 -0.699968873 -1.06040914  -0.6961342
4   1.5042018  -3.5897224 -4.2665132 -1.333195219  2.07046821   2.7322300
5  -6.2918229 -11.1398234 -8.8160497  6.138414105  9.65914499   7.5346868
6  -2.0183548  -4.6206131 -3.9301267  1.764397734  3.83846185   3.3737272
7   1.2198181   5.8860096  6.1713946  3.081517531 -4.60737255  -7.9449684
8   2.4493196  -0.1854964 -0.5363198  1.913732611 -0.71338075  -3.3106023
9   1.4061816   4.3139870  4.6027181  3.655638999 -3.58178225  -7.4107134
10  2.7811329   2.1753863  1.9578553  3.004028326 -2.50653109  -6.2559300
11  0.8452773   4.2190753  4.3700341  1.807619393 -3.28168723  -5.3250006
12  2.3291919   1.6632516  1.5129633  2.748569860 -1.99467913  -5.3236231
```

```
13  1.5362712   1.5024896  1.5141681  2.479180156 -1.61897437  -4.3920112
14  1.0831225   3.6436906  3.6270720  1.200852053 -2.93750217  -4.5430194
15  0.4807881   1.5675749  1.2417926 -1.547366854 -1.20936452  -0.1712179
16 -1.6319988   2.5993360  2.5714625 -3.358613402 -1.20053278   2.1756620
            37          38          39          40         41          42
1   -3.5575718 -0.85749706 -0.23275840   1.3943645 -4.3942021  -0.41172886
2   -1.4843892 -2.52384294 -3.38821094  -1.4903342 -0.1797258  -1.30519143
3   -2.6460001 -1.70804562 -1.85307794  -1.9265463 -2.3505333  -2.71529064
4    1.2374166  1.71606077  2.28544975   2.6845808  0.3519204   2.61352412
5   27.9619984 18.10284080 19.63397953  19.3659836 24.8534034  27.72707970
6   10.2717925  6.88000673  7.56626755   7.4912502  8.9461335  10.45567142
7   -9.6681728 -8.27425924 -9.86478758  -9.1478104 -7.0277401 -11.14096783
8   -4.6976265 -3.80023457 -4.43140002  -3.2489724 -3.6338209  -4.35889444
9   -8.0087308 -7.16313655 -8.63546004  -7.6665384 -5.5962574  -9.19331917
10  -8.2609364 -6.85709000 -8.07997859  -6.6899023 -6.2151691  -8.52834556
11  -6.9127815 -5.78256020 -6.85078165  -6.4259921 -5.1261618  -7.90838319
12  -6.6925550 -5.65074145 -6.68996977  -5.4717222 -4.9643153  -6.92158144
13  -4.8543947 -4.29935345 -5.15965650  -4.2437325 -3.4433505  -5.20454835
14  -6.7504221 -5.39784596 -6.30785355  -5.8982491 -5.2029887  -7.46018057
15  -3.1600739 -1.79167733 -1.83601430  -2.0651250 -2.9966193  -3.11459947
16  -0.2278260  0.68166944  1.07534097  -0.2486083 -0.7897084  -0.63472925
            43          44          45          46         47          48
1   3.79917808 1.419655847  0.03005632   2.5035419 -2.5959084 -0.21241951
2   2.30800093 0.408177733 -4.90466096  -5.5819509 -4.7373335  0.18081448
3  -0.39167668 0.016702037 -0.84145262  -1.5828147  0.6415605  0.95614744
4   1.54682086 0.513104032  1.74441654   3.6261335 -0.7975599 -1.01950278
5   2.19836624 -0.682141359  9.85449841  16.7964116 -4.3353590 -9.58301533
6   1.02051417 -0.154188373  4.01517041   6.8874676 -1.5594293 -3.62841193
7  -1.05364947 -0.191696533 -7.55846248 -12.0180864 -0.7173377  3.82918452
8   1.23017799 0.407179397 -3.96819533  -5.0793793 -2.2008408  1.33489191
9  -0.38856348 -0.083979878 -7.12969635 -10.8826393 -1.5823768  3.10579995
10  0.72922576 0.305389361 -6.72009402  -9.5365614 -2.3366103  2.77866059
11 -0.82197809 -0.132084983 -5.09058342  -8.1855069 -0.2581194  2.72786545
12  0.69531121 0.258564026 -5.69288841  -8.0147045 -2.1431070  2.24305605
13  0.41847009 0.129951657 -4.51780658  -6.4344834 -1.7057924  1.69388119
14 -0.63012155 -0.040016413 -4.51795834  -7.2196011 -0.1452099  2.56268486
15 -0.55379375 0.040204435 -0.44006061  -1.1311303  1.1888587  1.11191089
16 -1.92365576 -0.356070258  2.51562772   2.3220331  3.4293316  0.42201844
             49          50          51          52          53           54
1   20.47002796 13.59778318  -8.8966920  5.71108917  0.98716532 -3.991196162
2   -1.75976236  4.10277493   4.5126095 -0.56532755 -3.94703394 -2.826847386
3    1.78589435  0.03170134   1.7387985 -0.07942072 -0.83100644  0.495038677
4    7.42717504  4.99731866  -6.3010465  2.62649532  1.95961766 -1.625624652
5  -23.11348501 -5.30321318 -16.2664747 -0.63309346  9.20471867 -3.030887859
6   -6.72047468 -1.02720870  -7.1964647  0.31305547  3.79640492 -1.317280912
7   -3.84695096 -2.14233582  14.4352832 -3.29941875 -7.07853637  1.058475584
8    3.41835391  3.84526835   3.9981480  0.15438064 -3.29535975 -1.418397631
9   -3.76628930 -1.00662862  12.6735898 -2.84775910 -6.54829274  0.280601391
10   1.69713118  2.75088789   9.4842403 -1.16396119 -5.89995272 -0.902241903
11  -2.17155452 -1.49750617   9.8429831 -2.19660259 -4.78843339  0.855596523
12   1.09046323  2.34852142   7.9590057 -1.01462281 -4.98397380 -0.864234692
13  -0.52010274  1.16014349   6.7183091 -1.13174598 -3.99367075 -0.561842111
14  -0.33984571 -0.61639320   8.3597938 -1.60097251 -4.21717323  0.680259005
15   3.30580157  0.20955687   1.1379610  0.24043360 -0.49243200  0.727925611
16   0.32638005 -3.59644338  -1.0598864 -0.17438814  1.86183273  2.309372804
            55           56          57          58          59         60
1    2.3076508  -5.24235309 -7.82201221 -5.48857951 -5.92867727 -3.8369272
2    4.1131851 -10.60555704 -5.79532682 -6.70547349 -7.36375824 -2.2886665
3   -2.2338901   1.20796915  0.03438003 -0.93235131 -0.12252922 -0.6160416
4    2.2829695  -1.32860055 -2.26198022 -0.18464569 -1.00310246 -0.6247013
5   20.5674349  -7.57466581  3.51987561 12.82882368  5.02045560  7.9136114
6    7.6672414  -2.63979952  0.98614932  4.74932813  1.82016587  2.7711753
7   -5.6688965  -2.70802457 -1.65266276 -7.10243521 -4.47349124 -2.6794526
8   -0.2775092  -5.16705531 -4.15353561 -5.71030688 -5.08141702 -2.5412160
9   -3.6592563  -4.43236015 -2.48752879 -7.08182531 -5.11808394 -2.5924896
10  -2.5334879  -5.89519410 -4.54254077 -8.17123940 -6.53087144 -3.4481099
11  -4.2845862  -1.35481751 -0.97777329 -4.77013700 -2.85193610 -1.8560741
12  -1.8016450  -5.33550640 -3.93612991 -6.92294959 -5.64282603 -2.8761876
13  -1.2504407  -4.25363886 -2.78586348 -5.20254844 -4.25934292 -2.0397256
14  -4.1521717  -1.01830252 -1.17182715 -4.53198272 -2.70237864 -1.9177712
15  -3.0338199   2.41559605  0.33750782 -0.60276076  0.42001346 -0.6546186
16  -3.8832184   7.50735875  4.17755914  4.01548648  4.88593729  1.3734720
             61
1   0.812556234
2   0.278256012
3   0.741244334
4  -0.398882181
5  -7.741920403
6  -2.851056318
7   2.665420660
8   1.216480842
9   2.181458984
10  2.204018717
11  1.910657062
12  1.779501152
13  1.292254389
14  1.860829579
15  0.892369949
16  0.162420760
```

> #Calculamos el error residual

```
> Error_res <- Xs[, 1:3] - dtorg_lossinf[,1:3]
> Error_res <- as.matrix(Error_res)
> Error_res
```

```
           rent      inc_sal       inc_ret
1  -19.30583235  -1.79524107 -2.100988298
2   -1.87601524 -10.67939461  1.534284813
3   -0.95447078   1.33018772 -0.891455846
4   -7.51795735  -0.21261498 -0.557978043
5   15.39227952  -8.14101076  7.154365202
6    3.46165403  -3.38616663  2.350323620
7    4.22844578  -3.73690149 -1.011251458
8   -3.59258614  -3.82444908 -0.498112794
9    2.86842310  -5.92456075  0.107123901
10  -2.53293291  -6.31607132 -0.482540537
11   2.54339739  -2.01892348 -0.407344934
12  -2.19034928  -5.92897145  0.089058416
13   0.15508187  -3.91462069 -0.106740582
14   0.94415667  -1.73504415 -0.724814618
15  -2.03673186   2.36722579 -0.915965254
16   2.14875451   7.41807361 -1.227369482
17  -3.57356002   6.50555896 -0.797036545
18   1.91134303  -7.80238910  0.398159693
19  -7.29209266  -8.86964588  0.384743564
20  -3.29381545  -0.54994568  0.387915594
21   7.49119349   4.33687961 -0.989061772
22  14.69894413  -3.89383347  0.720634089
23   3.88800957   2.40733223 -0.571025341
24   4.76736773   8.89659157 -1.366524414
25   6.98248260   9.71555295 -0.911404397
26   1.81445669  -6.14605067 -0.450814276
27   1.51237459   0.16173197 -0.005027534
28   2.67746596   8.38607391  0.281350370
29 -11.33940817  -5.86363150 -0.739556069
30  -5.40315133  -2.66003690 -1.467567063
31  -5.01746971  -1.76363537 -0.710660601
32   7.39940633   3.39242675 -1.076936531
33   9.51434212   3.41817541 -0.328276128
34   2.15005508  -5.45513879  1.503982116
35  -3.93553580  -1.77081544  0.510493805
36  -3.72049853   3.98101163 -0.349954877
37   1.99565971  -0.17673212  1.693762133
38  -0.02402785   1.84892322  0.834380574
39  -1.01664935   2.17153191  0.914662951
40  -2.62039720   0.24897930  1.169285651
41   2.99779809  -1.42677597  1.856637532
42  -1.02091436  -0.05288844  1.498960157
43  -4.68117363  -3.41654990  0.640586482
44  -2.05519342  -1.18568238  0.428640174
45  -0.88553920   4.37252836 -0.230099902
46  -3.57222426   4.87459219  0.149681055
47   3.14559676   5.78140443 -0.911377762
48   0.72320173   0.55078830 -0.197240900
49 -20.40494089   1.88694585 -2.942567573
50 -13.80205852  -4.27391949 -0.765681123
51  10.74204969  -3.11891805  0.062654913
52  -6.51464488  -0.11624633 -0.511963681
53  -1.89065338   3.05391331  0.170506930
54   3.97564707   2.71170865 -0.276685029
55  -3.46042066  -5.53032066  1.681792176
56   5.93684142  12.14480816 -1.746971578
57   7.58432150   5.86733626 -0.021189127
58   4.88912427   6.40956348  0.133621589
59   5.26427384   6.87543218 -0.284063215
60   3.43404218   1.99857885  0.403699045
61  -1.70694528  -1.42672993 -0.485059259
```

```
> #Calculamos la desviación típica del error residual.

> sd(Error_res)

[1] 4.801111

> round(sd(Error_res), digits = 2)

[1] 4.8
```