
Classifying Numerical Values of Handwritten Digits

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Abstract

Greyscale data of 60,000 handwritten digits from the MNIST database were used to train classifiers to predict the numerical value of a handwritten digit from test data of 10,000 labeled digits. Classifiers were based on regression, support vector machines, and neural networks. Accuracy of the classifiers ranged roughly between 80 to 98 percent.

1 The Data

The present project focused on the MNIST database of handwritten digits, which may be found at <http://yann.lecun.com/exdb/mnist/>. The data consists of a training set of 60,000 labeled, fixed-sized, greyscale images of handwritten digits and a test set of 10,000 handwritten digits with the same properties. The images are saved in a special file format called IDX, which can be converted to a NumPy array of the greyscale values of the pixels with the library `idx2numpy` found at <https://pypi.org/project/idx2numpy/>.

Further, the website hosting the data describes that the data from the training set is comprised of 30,000 handwritten digits from employees of the Census Bureau and 30,000 digits recorded from high school students. The test data is also a fair mix of handwritten digits from these two categories. There are at least 250 different writers represented in the training data. Also, note some greyscale pixels appear due to anti-aliasing of the data.

2 The Algorithms Used

The data is labelled according to the value of a digit the handwriter was instructed to write, making the dataset prime for analysis by supervised learning techniques. The question that was proposed for study was: Can the computer be trained to learn the (label) value of a handwritten digit from its IDX representation? The three algorithms that were used to approach this question were: 1. linear/ridge regression. 2. support vector machines. 3. neural networks. A brief overview of each method is given in the following subsections.

2.1 Regression

The idea of linear regression for binary classification is as follows: Given a set of training data X with labels y , interpret the features as inputs and the labels as output. Then, compute the hyperplane represented by w in the feature-label space that minimizes the sum of squares of the difference between the actual labels y and the value of the hyperplane evaluated at each point of training data. Namely, $\|Xw - y\|_2^2$ is minimized. Now, each side of the hyperplane in the feature space corresponds to a predicted label for any input data. The hope is that because the squared-error is minimized that there are few misclassifications.

However, the model is biased because it assumes that labels can be predicted by a linear function. One way to try to reduce the bias is to introduce a regularization parameter λ that tells the regression model to prefer w with smaller coefficients. Namely, instead of minimizing $\|Xw - y\|_2^2$, we seek to minimize $\|Xw - y\|_2^2 + \lambda\|w\|_2^2$. Both regularized (ridge) regression and standard linear regression have multiple algorithmic solutions, say, such as gradient descent, but they also have closed-form solutions, which were the solutions used in this project. Namely, for linear regression, the solution is $w = (X^T X)^{-1} X^T y$, and for ridge regression, we have $w = (X^T X + \lambda I)^{-1} X^T y$.

3 Headings: first level

All headings should be lower case (except for first word and proper nouns), flush left, and bold.

First-level headings should be in 12-point type.

3.1 Headings: second level

Second-level headings should be in 10-point type.

3.1.1 Headings: third level

Third-level headings should be in 10-point type.

Paragraphs There is also a `\paragraph` command available, which sets the heading in bold, flush left, and inline with the text, with the heading followed by 1 em of space.

4 Citations, figures, tables, references

These instructions apply to everyone.

4.1 Citations within the text

The `natbib` package will be loaded for you by default. Citations may be author/year or numeric, as long as you maintain internal consistency. As to the format of the references themselves, any style is acceptable as long as it is used consistently.

The documentation for `natbib` may be found at

<http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf>

Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

```
\citet{hasselmo} investigated\dots
```

produces

Hasselmo, et al. (1995) investigated...

If you wish to load the `natbib` package with options, you may add the following before loading the `neurips_2020` package:

```
\PassOptionsToPackage{options}{natbib}
```

If `natbib` clashes with another package you load, you can add the optional argument `nonatbib` when loading the style file:

```
\usepackage[nonatbib]{neurips_2020}
```

As submission is double blind, refer to your own published work in the third person. That is, use “In the previous work of Jones et al. [4],” not “In our previous work [4].” If you cite your other papers that are not widely available (e.g., a journal paper under review), use anonymous author names in the citation, e.g., an author of the form “A. Anonymous.”



Figure 1: Sample figure caption.

Table 1: Sample table title

Part		
Name	Description	Size (μm)
Dendrite	Input terminal	~ 100
Axon	Output terminal	~ 10
Soma	Cell body	up to 10^6

4.2 Footnotes

Footnotes should be used sparingly. If you do require a footnote, indicate footnotes with a number¹ in the text. Place the footnotes at the bottom of the page on which they appear. Precede the footnote with a horizontal rule of 2 inches (12 picas).

Note that footnotes are properly typeset *after* punctuation marks.²

4.3 Figures

All artwork must be neat, clean, and legible. Lines should be dark enough for purposes of reproduction. The figure number and caption always appear after the figure. Place one line space before the figure caption and one line space after the figure. The figure caption should be lower case (except for first word and proper nouns); figures are numbered consecutively.

You may use color figures. However, it is best for the figure captions and the paper body to be legible if the paper is printed in either black/white or in color.

4.4 Tables

All tables must be centered, neat, clean and legible. The table number and title always appear before the table. See Table 1.

Place one line space before the table title, one line space after the table title, and one line space after the table. The table title must be lower case (except for first word and proper nouns); tables are numbered consecutively.

Note that publication-quality tables *do not contain vertical rules*. We strongly suggest the use of the booktabs package, which allows for typesetting high-quality, professional tables:

`https://www.ctan.org/pkg/booktabs`

This package was used to typeset Table 1.

¹Sample of the first footnote.

²As in this example.

5 Final instructions

Do not change any aspects of the formatting parameters in the style files. In particular, do not modify the width or length of the rectangle the text should fit into, and do not change font sizes (except perhaps in the **References** section; see below). Please note that pages should be numbered.

6 Preparing PDF files

Please prepare submission files with paper size “US Letter,” and not, for example, “A4.”

Fonts were the main cause of problems in the past years. Your PDF file must only contain Type 1 or Embedded TrueType fonts. Here are a few instructions to achieve this.

- You should directly generate PDF files using `pdflatex`.
- You can check which fonts a PDF files uses. In Acrobat Reader, select the menu Files>Document Properties>Fonts and select Show All Fonts. You can also use the program `pdffonts` which comes with `xpdf` and is available out-of-the-box on most Linux machines.
- The IEEE has recommendations for generating PDF files whose fonts are also acceptable for NeurIPS. Please see <http://www.emfield.org/icuwb2010/downloads/IEEE-PDF-SpecV32.pdf>
- `xfig` "patterned" shapes are implemented with bitmap fonts. Use "solid" shapes instead.
- The `\bbold` package almost always uses bitmap fonts. You should use the equivalent AMS Fonts:

```
\usepackage{amsfonts}
```

followed by, e.g., `\mathbb{R}`, `\mathbb{N}`, or `\mathbb{C}` for \mathbb{R} , \mathbb{N} or \mathbb{C} . You can also use the following workaround for reals, natural and complex:

```
\newcommand{\RR}{I\!\!R} %real numbers
\newcommand{\Nat}{I\!\!N} %natural numbers
\newcommand{\CC}{I\!\!C} %complex numbers
```

Note that `amsfonts` is automatically loaded by the `amssymb` package.

If your file contains type 3 fonts or non embedded TrueType fonts, we will ask you to fix it.

6.1 Margins in L^AT_EX

Most of the margin problems come from figures positioned by hand using `\special` or other commands. We suggest using the command `\includegraphics` from the `graphicx` package. Always specify the figure width as a multiple of the line width as in the example below:

```
\usepackage[pdftex]{graphicx} ...
\includegraphics[width=0.8\linewidth]{myfile.pdf}
```

See Section 4.4 in the `graphics` bundle documentation (<http://mirrors.ctan.org/macros/latex/required/graphics/grfguide.pdf>)

A number of width problems arise when L^AT_EX cannot properly hyphenate a line. Please give LaTeX hyphenation hints using the `\-` command when necessary.

Broader Impact

Authors are required to include a statement of the broader impact of their work, including its ethical aspects and future societal consequences. Authors should discuss both positive and negative outcomes, if any. For instance, authors should discuss a) who may benefit from this research, b) who may be put at disadvantage from this research, c) what are the consequences of failure of the system, and d) whether the task/method leverages biases in the data. If authors believe this is not applicable to them, authors can simply state this.

Use unnumbered first level headings for this section, which should go at the end of the paper. **Note that this section does not count towards the eight pages of content that are allowed.**

Acknowledgments and Disclosure of Funding

Use unnumbered first level headings for the acknowledgments. All acknowledgments go at the end of the paper before the list of references. Moreover, you are required to declare funding (financial activities supporting the submitted work) and competing interests (related financial activities outside the submitted work). More information about this disclosure can be found at: <https://neurips.cc/Conferences/2020/PaperInformation/FundingDisclosure>.

Do **not** include this section in the anonymized submission, only in the final paper. You can use the ack environment provided in the style file to automatically hide this section in the anonymized submission.

References

References follow the acknowledgments. Use unnumbered first-level heading for the references. Any choice of citation style is acceptable as long as you are consistent. It is permissible to reduce the font size to small (9 point) when listing the references. **Note that the Reference section does not count towards the eight pages of content that are allowed.**

[1] Alexander, J.A. & Mozer, M.C. (1995) Template-based algorithms for connectionist rule extraction. In G. Tesauro, D.S. Touretzky and T.K. Leen (eds.), *Advances in Neural Information Processing Systems 7*, pp. 609–616. Cambridge, MA: MIT Press.

[2] Bower, J.M. & Beeman, D. (1995) *The Book of GENESIS: Exploring Realistic Neural Models with the GENeral NEural Simulation System*. New York: TELOS/Springer-Verlag.

[3] Hasselmo, M.E., Schnell, E. & Barkai, E. (1995) Dynamics of learning and recall at excitatory recurrent synapses and cholinergic modulation in rat hippocampal region CA3. *Journal of Neuroscience* **15**(7):5249-5262.