A New Model for Layer Jamming-Based Continuum Robots

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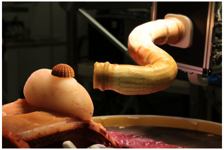
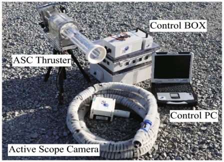
Background



Medicine

Manipulation



Search & Rescue

- Inherent flexibility and lightweight
- Allowing for flexible soft motion or rigid resistance
- Variable stiffness
  - Layer jamming[1]: utilise thin plastic or paper layers as its jamming flaps.
  - Rapid reversible responses

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[1] Kim *et al.*, IEEE TRO, 2013

Layer-Jamming Continuum Robots

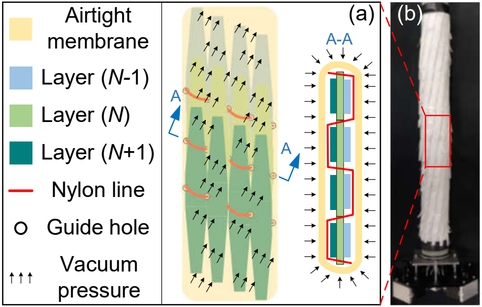


Figure: Schematic of a layer-jamming structure in continuum robots

- An airtight pneumatic chamber: overlapping layers covering the robot spine
- Exploit frictions between layers
- Controlled by external pressure via a vacuum

Problem Set

> *"Nevertheless, these studies have not yet provided analytical or computational models for laminar jamming beyond an initial deformation phase [...]"*
>
> — Narang, Vlassak and Howe, 2018

Aims:

1. Propose a new control-oriented dynamic model for LJ-based continuum robots
2. Theoretically analyse the model, showing its capability to interpret two phenomena
   - Shape locking
   - Adjustable stiffness

New Dynamical Model

Jamming-free model (port-Hamiltonian)[2]:

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0_{n\times n} & I_n \\ -I_n & -D(q) \end{bmatrix} \begin{bmatrix} \nabla_q H \\ \nabla_p H \end{bmatrix} + \begin{bmatrix} 0_n \\ G(q)u \end{bmatrix} \tag{1}$$

- Configuration variable $q \in \mathcal{X} \subset \mathbb{R}^n$
- Generalized momentum $p \in \mathbb{R}^n$
- Tension input $u \in \mathbb{R}^m$
- Total energy (i.e. Hamiltonian): $H = \frac{1}{2}p^\top M^{-1}(q)p + U(q)$
- Potential energy function $U(q)$: gravitational $U_{\mathtt{G}}$ + elastic $U_{\mathtt{E}}$

Question: How can we characterise the effects from layer jamming?

[2] Yi, Fan, Liu and Romero, ArXiv Preprint, 2023

New Dynamical Model (cont'd)

Our Solution: Negatively cascade a LuGre model

Variable Stiffness Model Using Layer Jamming

$$\dot{\chi} = [\mathcal{J} - \mathcal{R}]\nabla\mathcal{H} + \mathcal{G}(\chi)u \qquad (2)$$

- State: $\chi := \mathrm{col}(q, p, z) \in \mathbb{R}^{3n}$ (Internal state z in LuGre)
- Input:

$$u_\chi = \begin{bmatrix} u \\ u_{\mathrm{P}} \end{bmatrix}, \qquad (u_{\mathrm{P}} \text{ negative pressure})$$

- Total Hamiltonian:

$$\mathcal{H}(\chi, u_{\mathrm{P}}) := \underbrace{\frac{1}{2}p^\top M^{-1}(q)p}_{\text{kinematic energy}} + \underbrace{\frac{1}{2}\sigma_0 u_{\mathrm{P}}|z|^2 + U(q)}_{\text{potential energy}} \qquad (3)$$

- Matrices: $\mathcal{G}(\chi) := \begin{bmatrix} G_r^\top & 0_{n\times m}^\top \end{bmatrix}^\top$

$$\mathcal{J}(\chi, u_{\mathrm{P}}) := \begin{bmatrix} J & -G_f\mathcal{N}^\top \\ \mathcal{N}G_f^\top & 0_{n\times n} \end{bmatrix}, \quad \mathcal{R}(\chi, u_{\mathrm{P}}) := \begin{bmatrix} G_f S(v) G_f^\top & G_f\mathcal{P}^\top \\ \mathcal{P}^\top G_f^\top & R_z \end{bmatrix}.$$

Theoretical Property 1: Shape Locking

Proposition. Consider the LJ-based continuum robot model without external input, i.e., $u = 0_m$. For arbitrary configuration $q_a \in \mathbb{R}^n$ and a *constant* pressure $u_{\mathrm{P}} > 0$,

(a) $\exists z_a \in \mathbb{R}^n$ such that $(q_a, 0_n, z_a)$ is an equilibrium;

(b) The equilibria manifold $\mathcal{M} := \{(q, p, z) : p = 0,\ \nabla U(q) = \sigma_0 u_{\mathrm{P}} z\}$ is locally asymptotically stable.

- If $\chi(0)$ starts from $\forall q_a$ and $p(0) = 0$, $\exists$ a virtual bristle vector z_a s.t. $\chi(t)$ maintains at the initial values over time, and $\mathcal{M} \subset \mathcal{E}_{\mathrm{SL}}$. Shape Locking!
- $z_a = \frac{1}{\sigma_0 u_{\mathrm{P}}} \nabla U(q_a)$. *Physically, a large pressure value* u_{P} *is capable of achieving shape locking.*

Experimental Results: Shape Locking

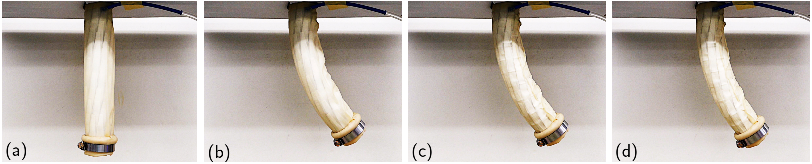
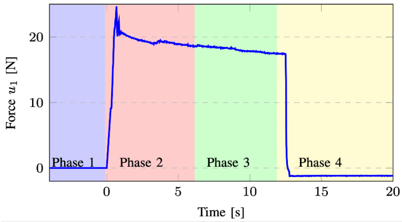
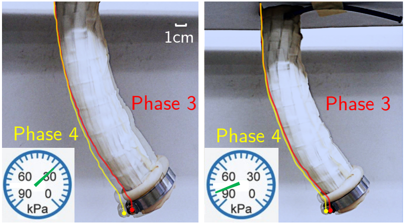


Figure: Photos showing sequence of the shape-locking experiments: (a) Phase 1: Initial configuration without u; (b) Phase 2: Drive to the bending configuration 60° via tendon force u; (c) Phase 3: Vacuum to $u_{\mathrm{P}} = 30$ kPa with motor-driven retained; (d) Phase 4: Vacuum retained and tendon released $u = 0$. (Photos were taken in the steady state of each phase.)





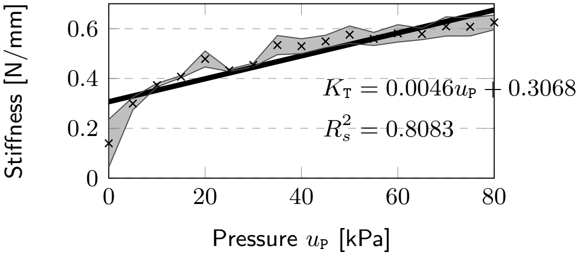
Theoretical Property 2: Adjustable Open-loop Stiffness

Proposition. Consider the LJ-based continuum robotic model. Its overall stiffness at the open-loop equilibrium $\chi_\star$ is given by

$$K = \alpha_1 \mathbf{1}_{n\times n} + [\alpha_2 + \sigma_0 u_{\mathrm{P}}] I_n, \tag{4}$$

with $\mathbf{1}_{n\times n} \in \mathbb{R}^{n\times n}$ an all elements ones.

Experimental Results:



THANKS

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