Lab Problem 16.1, Physics 430

```
> restart;
```

Write down the two equations, with Lx(Thalf) written so as to not explicitly depend on Thalf

```
> eq1:=(Thalf-Tn)/(tau/2)=D*(LxofThalf+Ly(Tn));
```

$$eq1 := 2\,\frac{Thalf - Tn}{\tau} = \mathrm{D}\,(LxofThalf + \mathrm{Ly}(Tn))$$

```
> eq2:=(Tnew-Thalf)/(tau/2)=D*(LxofThalf+Ly(Tnew));
```

$$eq2 := 2\,\frac{Tnew - Thalf}{\tau} = \mathrm{D}\,(LxofThalf + \mathrm{Ly}(Tnew))$$

Subtract the two equations to find a formal expression for Thalf

```
> eq3:=simplify(eq1-eq2);
```

$$eq3 := -2\,\frac{-2\,Thalf + Tn + Tnew}{\tau} = \mathrm{D}\,\mathrm{Ly}(Tn) - \mathrm{D}\,\mathrm{Ly}(Tnew)$$

```
> Thalf:=solve(eq3,Thalf);
```

$$Thalf := \frac{1}{2}\,Tn + \frac{1}{2}\,Tnew + \frac{1}{4}\,\mathrm{D}\,\mathrm{Ly}(Tn)\,\tau - \frac{1}{4}\,\mathrm{D}\,\mathrm{Ly}(Tnew)\,\tau$$

Equation 2 now looks like this

```
> eq2;
```

$$2\,\frac{\frac{1}{2}\,Tnew - \frac{1}{2}\,Tn - \frac{1}{4}\,\mathrm{D}\,\mathrm{Ly}(Tn)\,\tau + \frac{1}{4}\,\mathrm{D}\,\mathrm{Ly}(Tnew)\,\tau}{\tau} = \mathrm{D}\,(LxofThalf + \mathrm{Ly}(Tnew))$$

Now use the expression for Thalf to convert Lx(Thalf) into an expanded expression

```
> LxofThalf:=1/2*(Lx(Tn)+Lx(Tnew))+1/4*D*tau*Lx(Ly(Tn))-1/4*D*tau*Lx
  (Ly(Tnew));
```

$$LxofThalf := \frac{1}{2}\,\mathrm{Lx}(Tn) + \frac{1}{2}\,\mathrm{Lx}(Tnew) + \frac{1}{4}\,\mathrm{D}\,\tau\,\mathrm{Lx}(\mathrm{Ly}(Tn)) - \frac{1}{4}\,\mathrm{D}\,\tau\,\mathrm{Lx}(\mathrm{Ly}(Tnew))$$

which now puts eq2 into this form

```
> eq2;
```

$$2\,\frac{\frac{1}{2}\,Tnew - \frac{1}{2}\,Tn - \frac{1}{4}\,\mathrm{D}\,\mathrm{Ly}(Tn)\,\tau + \frac{1}{4}\,\mathrm{D}\,\mathrm{Ly}(Tnew)\,\tau}{\tau} =$$

$$\mathrm{D}\left(\frac{1}{2}\,\mathrm{Lx}(Tn) + \frac{1}{2}\,\mathrm{Lx}(Tnew) + \frac{1}{4}\,\mathrm{D}\,\tau\,\mathrm{Lx}(\mathrm{Ly}(Tn)) - \frac{1}{4}\,\mathrm{D}\,\tau\,\mathrm{Lx}(\mathrm{Ly}(Tnew)) + \mathrm{Ly}(Tnew)\right)$$

Now rearrange terms so that it looks as much like Crank-Nicholson as possible

```
> lhsnew:=simplify(lhs(eq2)+1/2*D*Ly(Tn)-1/2*D*Ly(Tnew));
```

$$lhsnew := -\frac{-Tnew + Tn}{\tau}$$

```
> rhsnew:=simplify(rhs(eq2)+1/2*D*Ly(Tn)-1/2*D*Ly(Tnew));
```

$$rhsnew := \frac{1}{2}\,\mathrm{D}\,\mathrm{Lx}(Tn) + \frac{1}{2}\,\mathrm{D}\,\mathrm{Lx}(Tnew) + \frac{1}{4}\,\mathrm{D}^2\,\tau\,\mathrm{Lx}(\mathrm{Ly}(Tn)) - \frac{1}{4}\,\mathrm{D}^2\,\tau\,\mathrm{Lx}(\mathrm{Ly}(Tnew)) + \frac{1}{2}\,\mathrm{D}\,\mathrm{Ly}(Tnew) + \frac{1}{2}\,\mathrm{D}\,\mathrm{Ly}(Tn)$$

On the left we recognize the forward time difference of Crank-Nicholson, and on the right we also see the time-averaged spatial operators. The leftover term on the right is

```
> leftover:=-D^2*tau/4*Lx(Ly(Tnew-Tn));
```

$$leftover := -\frac{1}{4}\,\mathrm{D}^2\,\tau\,\mathrm{Lx}(\mathrm{Ly}(Tnew - Tn))$$

which becomes, if we interpret Tnew-Tn as a time derivative by putting τ underneath it

```
> leftover:=-D^2*tau^2/4*Diff(Diff(Diff(T(x,y,t),t),y$2),x$2);
```

$$leftover := -\frac{1}{4}\,\mathrm{D}^2\,\tau^2\left(\frac{\partial^5}{\partial x^2\,\partial y^2\,\partial t}\mathrm{T}(x, y, t)\right)$$

```
>
```