Scaling Schroedinger's Equation, Lab 5.3 and 5.5, Physics 430

```
> restart;
```

(5.3)

```
> eq:=-hbar^2/2/m*diff(psi(x),x$2)+1/2*k*x^2*psi(x)=E*psi(x);
```

$$-\frac{1}{2}\frac{\mathrm{hbar}^2\left(\frac{d}{dx}\left(\frac{d}{dx}\psi(x)\right)\right)}{m}+\frac{1}{2}k\,x^2\,\psi(x)=E\,\psi(x)$$

Rescale it with $x = a\,\xi$.

```
> eqnew:=-hbar^2/2/m*diff(psi(xi),xi$2)/a^2+1/2*k*a^2*xi^2*psi(x)=E*
  psi(x);
```

$$-\frac{1}{2}\frac{\mathrm{hbar}^2\left(\frac{d}{d\xi}\left(\frac{d}{d\xi}\psi(\xi)\right)\right)}{m\,a^2}+\frac{1}{2}k\,a^2\,\xi^2\,\psi(x)=E\,\psi(x)$$

Divide by $k\,a^2$ so that the potential term is dimensionless

```
> eqnew:=expand(lhs(eqnew)/k/a^2=rhs(eqnew)/k/a^2);
```

$$-\frac{1}{2}\frac{\mathrm{hbar}^2\left(\frac{d}{d\xi}\left(\frac{d}{d\xi}\psi(\xi)\right)\right)}{k\,a^4\,m}+\frac{1}{2}\xi^2\,\psi(x)=\frac{E\,\psi(x)}{k\,a^2}$$

We now choose a so that

```
> eqa:=hbar^2/k/a^4/m=1;
```

$$\frac{\mathrm{hbar}^2}{k\,a^4\,m}=1$$

```
> s:=solve(eqa,a);
```

$$\frac{(\mathrm{hbar}^2\,k^3\,m^3)^{(1/4)}}{k\,m},\ \frac{(\mathrm{hbar}^2\,k^3\,m^3)^{(1/4)}\,I}{k\,m},\ -\frac{(\mathrm{hbar}^2\,k^3\,m^3)^{(1/4)}}{k\,m},$$
$$\frac{-I\,(\mathrm{hbar}^2\,k^3\,m^3)^{(1/4)}}{k\,m}$$

```
> a:=s[1];
```

$$\frac{(\mathrm{hbar}^2\,k^3\,m^3)^{(1/4)}}{k\,m}$$

```
> a:=simplify(a,symbolic);
```

$$\frac{\sqrt{\mathrm{hbar}}}{k^{(1/4)}\,m^{(1/4)}}$$

```
> eqnew;
```

$$-\frac{1}{2}\left(\frac{d}{d\xi}\left(\frac{d}{d\xi}\psi(\xi)\right)\right)+\frac{1}{2}\xi^2\,\psi(x)=\frac{E\,\psi(x)\sqrt{m}}{\sqrt{k}\;hbar}$$

And now we can make the right side dimensionless by making the scaled energy be $\varepsilon = \frac{E\sqrt{\frac{m}{k}}}{hbar}$, or $E = \varepsilon\;Ebar$ with $Ebar = hbar\sqrt{\frac{k}{m}}$.

```
> E:=epsilon*hbar*sqrt(k/m);
```

$$\varepsilon\;hbar\sqrt{\frac{k}{m}}$$

```
> simplify(eqnew,symbolic);
```

$$-\frac{1}{2}\left(\frac{d}{d\xi}\left(\frac{d}{d\xi}\psi(\xi)\right)\right)+\frac{1}{2}\xi^2\,\psi(x)=\varepsilon\,\psi(x)$$

(5.5) Now do the same thing for the 4th power potential

```
> restart;
> eq:=-hbar^2/2/m*diff(psi(x),x$2)+mu*x^4*psi(x)=E*psi(x);
```

$$-\frac{1}{2}\,\frac{hbar^2\left(\frac{d}{dx}\left(\frac{d}{dx}\psi(x)\right)\right)}{m}+\mu\,x^4\,\psi(x)=E\,\psi(x)$$

Rescale it with $x = a\,\xi$.

```
> eqnew:=-hbar^2/2/m*diff(psi(xi),xi$2)/a^2+mu*a^4*xi^4*psi(x)=E*psi
  (x);
```

$$-\frac{1}{2}\,\frac{hbar^2\left(\frac{d}{d\xi}\left(\frac{d}{d\xi}\psi(\xi)\right)\right)}{m\,a^2}+\mu\,a^4\,\xi^4\,\psi(x)=E\,\psi(x)$$

Divide by $\mu\,a^4$ so that the potential term is dimensionless

```
> eqnew:=expand(lhs(eqnew)/mu/a^4=rhs(eqnew)/mu/a^4);
```

$$-\frac{1}{2}\,\frac{hbar^2\left(\frac{d}{d\xi}\left(\frac{d}{d\xi}\psi(\xi)\right)\right)}{\mu\,a^6\,m}+\xi^4\,\psi(x)=\frac{E\,\psi(x)}{\mu\,a^4}$$

We now choose a so that

```
> eqa:=hbar^2/mu/a^6/m=1;
```

$$\frac{hbar^2}{\mu\,a^6\,m}=1$$

```
> s:=solve(eqa,a);
```

$$\frac{(\mathrm{hbar}^2\,\mu^5\,m^5)^{(1/6)}}{\mu\,m},\ \frac{\left(\frac{1}{2}+\frac{1}{2}\,I\,\sqrt{3}\right)(\mathrm{hbar}^2\,\mu^5\,m^5)^{(1/6)}}{\mu\,m},$$

$$\frac{\left(-\frac{1}{2}+\frac{1}{2}\,I\,\sqrt{3}\right)(\mathrm{hbar}^2\,\mu^5\,m^5)^{(1/6)}}{\mu\,m},\ -\frac{(\mathrm{hbar}^2\,\mu^5\,m^5)^{(1/6)}}{\mu\,m},$$

$$\frac{\left(-\frac{1}{2}-\frac{1}{2}\,I\,\sqrt{3}\right)(\mathrm{hbar}^2\,\mu^5\,m^5)^{(1/6)}}{\mu\,m},\ \frac{\left(\frac{1}{2}-\frac{1}{2}\,I\,\sqrt{3}\right)(\mathrm{hbar}^2\,\mu^5\,m^5)^{(1/6)}}{\mu\,m}$$

```
> a:=simplify(s[1],symbolic);
```

$$\frac{\mathrm{hbar}^{(1/3)}}{\mu^{(1/6)}\,m^{(1/6)}}$$

```
> eqnew;
```

$$-\frac{1}{2}\left(\frac{d}{d\xi}\left(\frac{d}{d\xi}\,\psi(\xi)\right)\right)+\xi^4\,\psi(x)=\frac{E\,\psi(x)\,m^{(2/3)}}{\mu^{(1/3)}\,\mathrm{hbar}^{(4/3)}}$$

And now we can make the right side dimensionless by making the scaled energy be

$\varepsilon = E\left(\frac{m^2}{\mu\,\mathrm{hbar}^4}\right)^{\left(\frac{1}{3}\right)}$ which is in the form $E = \varepsilon$ Ebar with Ebar $=\left(\frac{\mathrm{hbar}^4\,\mu}{m^2}\right)^{\left(\frac{1}{3}\right)}$.

```
> E:=epsilon*(mu*hbar^4/m^2)^(1/3);
```

$$\varepsilon\left(\frac{\mu\,\mathrm{hbar}^4}{m^2}\right)^{(1/3)}$$

```
> simplify(eqnew,symbolic);
```

$$-\frac{1}{2}\left(\frac{d}{d\xi}\left(\frac{d}{d\xi}\,\psi(\xi)\right)\right)+\xi^4\,\psi(x)=\varepsilon\,\psi(x)$$

which is the dimensionless form we want