

Lab Problem 14.5(a), Physics 430

[> **restart;**

Write down the Korteweg-deVries equation

[> **kdv:=diff(v(x,t),t)+v(x,t)*diff(v(x,t),x)+alpha*diff(v(x,t),x\$3)=0**
;

$$kdv := \left(\frac{\partial}{\partial t} v(x, t) \right) + v(x, t) \left(\frac{\partial}{\partial x} v(x, t) \right) + \alpha \left(\frac{\partial^3}{\partial x^3} v(x, t) \right) = 0$$

Write down the soliton solution

[> **soliton:=12*k^2*alpha/cosh(k*(x-x0-4*alpha*k^2*t))^2;**

$$soliton := 12 \frac{k^2 \alpha}{\cosh(k(x - x0 - 4 \alpha k^2 t))^2}$$

Substitute the soliton solution into the Korteweg-deVries equation

[> **subs(v(x,t)=soliton,kdv);**

$$\left(\frac{\partial}{\partial t} \left(12 \frac{k^2 \alpha}{\cosh(k(x - x0 - 4 \alpha k^2 t))^2} \right) \right) + \frac{12 k^2 \alpha \left(\frac{\partial}{\partial x} \left(12 \frac{k^2 \alpha}{\cosh(k(x - x0 - 4 \alpha k^2 t))^2} \right) \right)}{\cosh(k(x - x0 - 4 \alpha k^2 t))^2} + \alpha \left(\frac{\partial^3}{\partial x^3} \left(12 \frac{k^2 \alpha}{\cosh(k(x - x0 - 4 \alpha k^2 t))^2} \right) \right) = 0$$

Evaluate the derivatives

[> **value(%);**

$$96 \frac{k^5 \alpha^2 \sinh(k(x - x0 - 4 \alpha k^2 t))}{\cosh(k(x - x0 - 4 \alpha k^2 t))^3} - \frac{288 k^5 \alpha^2 \sinh(k(x - x0 - 4 \alpha k^2 t))}{\cosh(k(x - x0 - 4 \alpha k^2 t))^5} + \alpha \left(-288 \frac{k^5 \alpha \sinh(k(x - x0 - 4 \alpha k^2 t))^3}{\cosh(k(x - x0 - 4 \alpha k^2 t))^5} + \frac{192 k^5 \alpha \sinh(k(x - x0 - 4 \alpha k^2 t))}{\cosh(k(x - x0 - 4 \alpha k^2 t))^3} \right) = 0$$

And simplify to see if the left side vanishes, as it should for a solution

[> **simplify(%);**

$$0 = 0$$

[>