CTF RESIDUAL FORMULATION OF SOLID LIQUID COUPLING

PENNSTATE 1855

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Objective: To integrate 1D solid conduction equations into the residual formulation CTF

1-D Single Phase Liquid + Solid

Partial Differential Equations

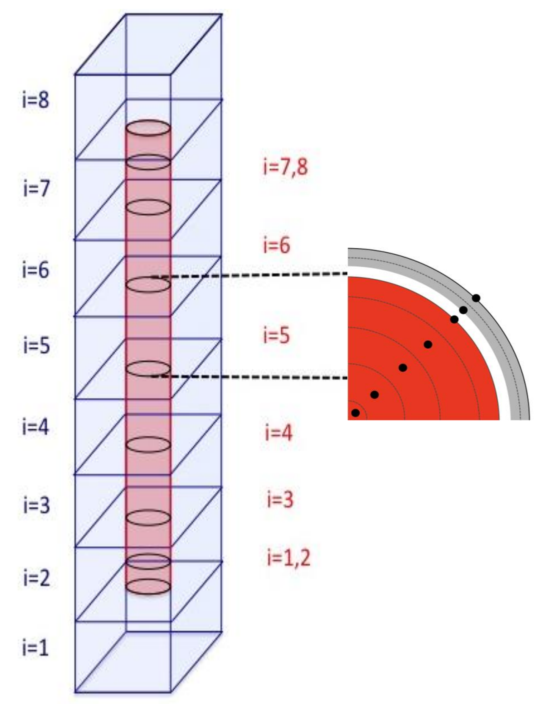
Liquid Mass
$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0$$

Liquid Momentum
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial P}{\partial x} + g = 0$$

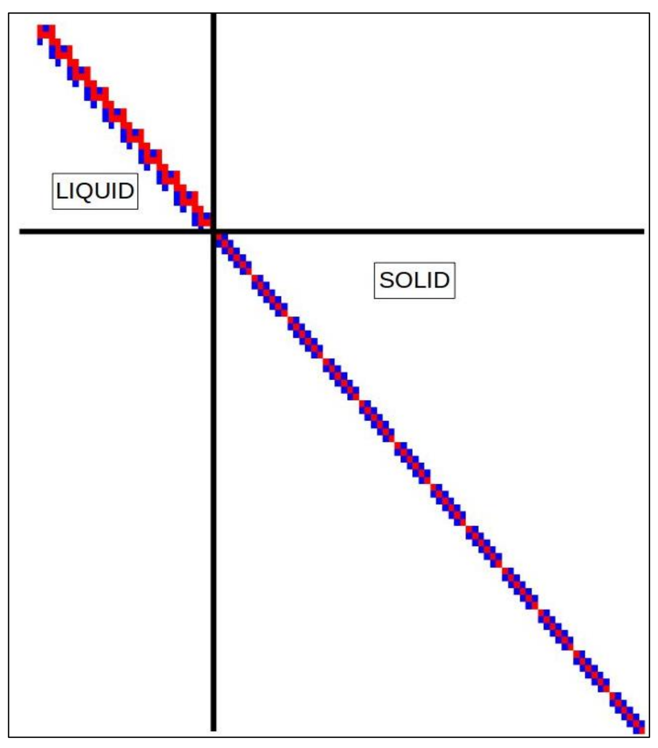
Liquid Energy
$$\rho\frac{\partial h}{\partial t} + h\frac{\partial \rho}{t} - \frac{\partial P}{\partial t} + \rho u\frac{\partial h}{\partial x} + h\frac{\partial \rho u}{\partial x} - \frac{q_{rod}}{\forall_{liq}} = 0$$

Solid Conduction
$$\rho c_p\frac{\partial T}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) - q''' = 0$$

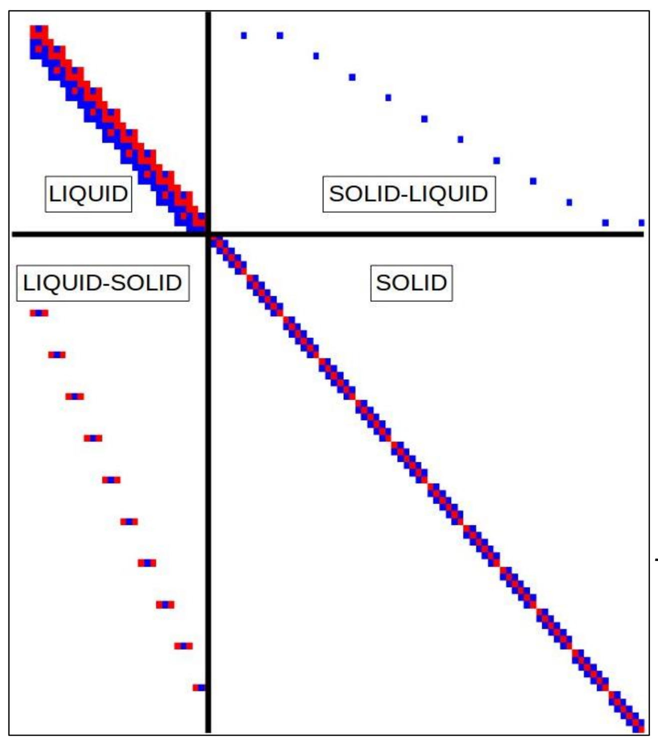
Finite Difference Mesh



Semi-Implicit Jacobian



Implicit Jacobian



Matrix Construction

$$J_{i,j} = \frac{\delta F_j(X_i)}{\delta X_i}$$

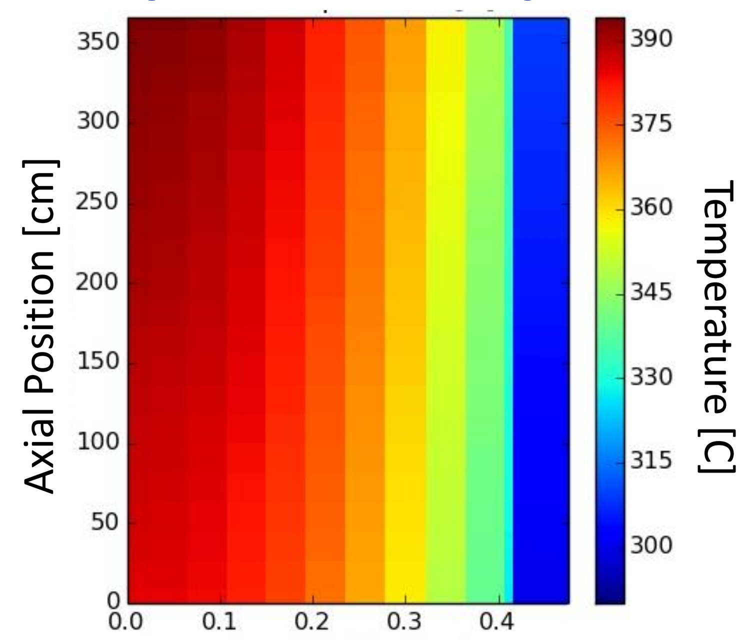
$$J_{i,j} \approx \frac{F_j(X_i + \epsilon) - F_j(X_i)}{\epsilon}$$

Verification Problem Setup

- A 1-D single phase channel coupled to a solid cylindrical rod with only axial conduction
- Constant mass flow rate
- Constant heat generation rate
- Constant material properties
- Temperature changes axially due to liquid advection
- Analytical solution for relative rod temperature in solid volume for each radial region

Parameter	Symbol	Value	Unit
Mass Flow Rate	$\dot{m}$	0.300	kg/sec
Reference Pressure	P_{ref}	16.50	MPa
Liquid Inlet Temperature	T_{inlet}	290.0	°C
Rate of Heat Generation	q'	4.0	W/m
Active Fuel Length	L	3.658	m
Fuel Radius	r_{fuel}	0.4096	cm
Outer Cladding Radius	r_{co}	0.475	cm
Inner Cladding Radius	r_{ci}	0.4174	cm
Rod Pitch	p	12.60	cm
Clad Specific Heat Capacity	$C_{p,clad}$	0.431	kJ/kg-K
Clad Density	ρ_{clad}	8470.57	kg/m^3
Clad Thermal Conductivity	k_{clad}	14.83	W/m-k
Fuel Specific Heat Capacity	$C_{p,fuel}$	0.289	kJ/kg-K
Gap Heat Transfer Coefficient	ρ_{fuel}	10970.40	kg/m^3

2D Map of Rod Temperature

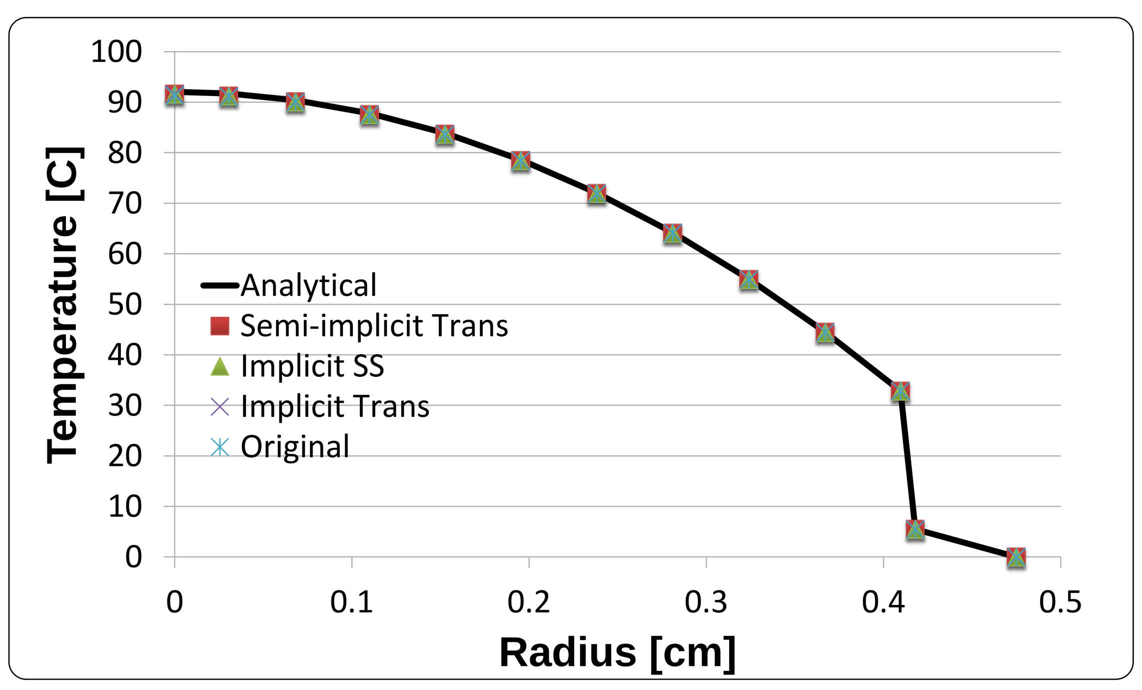


Analytical Result for any Axial Level

$$T(r,z) - T(r_{fuel}, z) = \frac{q'''\, r_{fuel}^2}{4\, k_{fuel}}\left(1 - \frac{r^2}{r_{fuel}^2}\right)$$

Steady State Results

Relative Rod Temperature



Convergence of Error for Finite Difference Solution

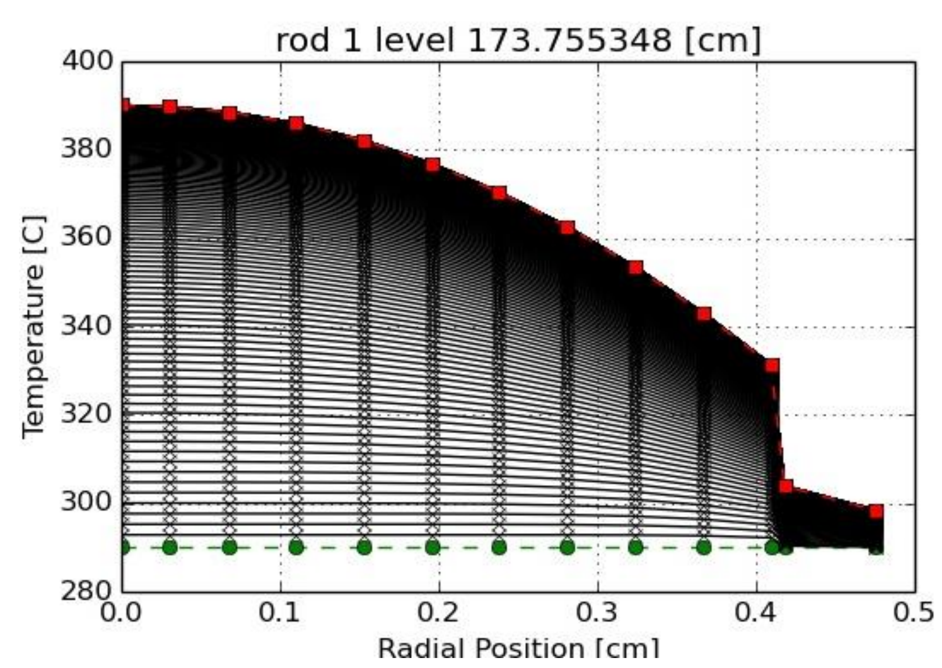
Radial Nodes in Fuel	Semi-Implicit Transient	Implicit Transient	Implicit Steady State	Original Steady State
5	1.33%	1.35%	1.32%	2.15%
10	0.45%	0.48%	0.45%	0.78%
20	0.15%	0.18%	0.14%	0.20%

Problem Run Times

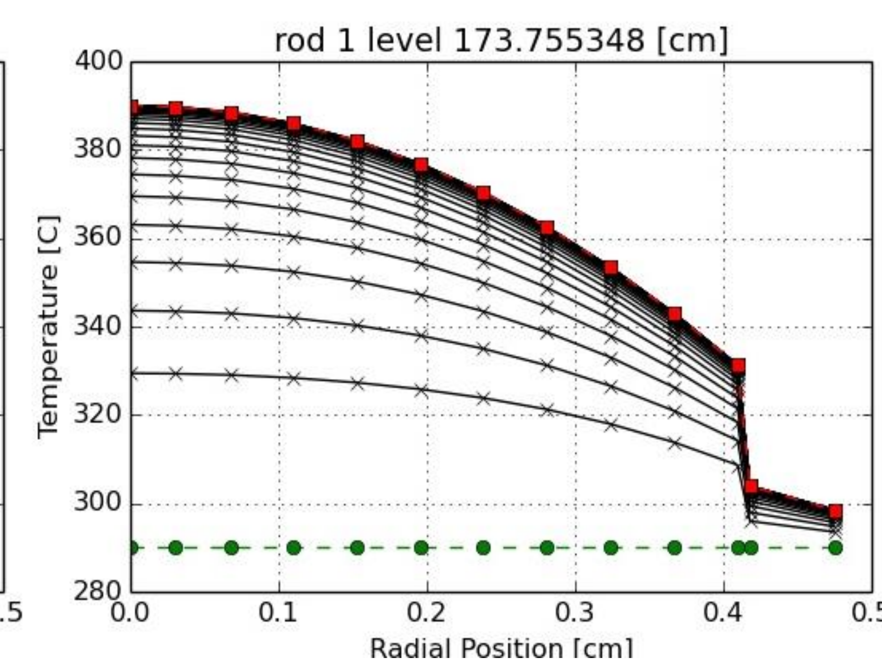
Code Version	Solution Method	Time Step Size [sec]	Wall Clock Time [sec]
Original	Original Semi-Implicit	0.05	2.226
Residual	Semi-Implicit	0.05	8.591
Residual	Implicit	0.05	36.709
Residual	Implicit	1.00	2.377
Residual	Implicit	10.00	0.622

Transient Results

Semi-Implicit



Implicit



Conclusions and Future Work

The residual formulation of the one-dimensional single-phase liquid and solid residual formulations were listed. Combining the liquid and solid equations into a single Jacobian matrix allowed for easy explicit or implicit coupling. This solution method was tested against the analytical solution for a single rod with uniform heat generation. Similar results were obtained between the two solutions, and the ability to exceed the time step limitations of the semi-implicit method was demonstrated. Future work will involve performing a more in depth verification analysis of the steady state and transient solutions. Further work will also include examining more challenging test problems that can properly demonstrate the advantages of the implicitly coupled fluid solid Jacobian matrix. The effect of temperature dependent material properties and dynamic gap conductance will also be considered. A homogenous energy equation can now be easily implemented by adding the liquid and solid conservation equations. Future work will be analyzing the homogeneous energy approximation over a state space to see when the approximation is valid. The conduction equations will be extended into the azimuthal and axial directions for more realistic heat transfer.