Cell-centered Poisson Solvers

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Introduction

SAMRAI provides the classes for solving Poisson's equation on a single level (using hypre) or a hierarchy (using the fast adaptive composite, or FAC, algorithm). These classes solve the general equation of the form

$$C(\boldsymbol{x})+\nabla\cdot D(\boldsymbol{x})\nabla u(\boldsymbol{x})=f(\boldsymbol{x})$$

for $u(\boldsymbol{x})$, where $C(\boldsymbol{x})$ is a scalar field, $D(\boldsymbol{x})$ is the diffusion coefficient and $f(\boldsymbol{x})$ is the source term. The solver supports the Robin boundary condition, which is any that can be writen in the form

$$\alpha u+\beta\frac{\partial u}{\partial n}=\gamma$$

or (using two parameters)

$$a u+(1-a)\frac{\partial u}{\partial n}=g$$

where n is the coordinate in the direction of the outward normal on the boundary. Note that $a=\frac{\alpha}{\alpha+\beta}$ and $g=\frac{\gamma}{\alpha+\beta}$. This boundary condition is a generalization of the Dirichlet (a=1) and Neumann (a=0) boundary conditions.

The discretization is a standard central-difference, cell-centered finite-volume. $C(\boldsymbol{x})$, $f(\boldsymbol{x})$ and $u(\boldsymbol{x})$ are cell-centered quantities, and $D(\boldsymbol{x})$ is side-centered.

Simpler forms of the partial differential equation (PDE) can be solved, and different optimizations are made to take advantage of those cases. For example, $C(\boldsymbol{x})$ and $D(\boldsymbol{x})$ may be constants and $C(\boldsymbol{x})$ may be zero.

The class `CellPoissonHypreSolver` (in the `SAMRAI::solv` namespace) is used to solve a single-level problem, and the class `CellPoissonFACSolver` solves the problem on a hierarchy. This document shows the basics of using these classes. It covers specifying the PDE parameters, controlling the solver algorithm, calling the `solveSystem` methods to perform the solve, getting data on the solve, and setting up an input file. Examples are provided. The solver classes documented here references the FAC preconditioner class (`FACPreconditioner`) and the Robin boundary condition class (`RobinBcCoefStrategy`), which are documented separately.

Providing the Robin Boundary Condition Coefficients

Boundary condition implementations are required for both `CellPoissonHypreSolver` and `CellPoissonFACSolver`. Use member function `CellPoissonHypreSolver::setPhysicalBcCoefObject()` or `CellPoissonFACSolver::setBcObject()` to specify the implementation. You should decide whether to use a SAMRAI-provided implementation or your own implementation. In this document, we use the SAMRAI-provided `LocationIndexRobinBcCoefs`.

The `LocationIndexRobinBcCoefs` implementation is appropriate for problems where the

coefficients are determined completely by the location index of the boundary box. This covers, among others, the case of parallelpiped domains where each side of the parallelpiped is uniformly set to some boundary condition. For each location index, one may specify uniform Dirichlet boundary values, uniform Neumann boundary values or uniform values of a and g.

Using the Single-level Hypre-Poisson Solver

After setting up your boundary condition as described above, solving the Poisson equation is simple.

1. Initialize the solver object to an existing hierarchy using `CellPoissonHypreSolver::InitializeSolverState()`. You can specify the hierarchy and level number to solve on (defaults to level zero).
2. Then use `CellPoissonHypreSolver::setBcObject()` to tell it to use the Robin boundary condition object you have set up.
3. Make sure that ghost cells at the coarse-fine boundaries are initialized with the appropriate values. For example, refine the coarse grid values into the ghost cells using a refine schedule. The solver operates on a single level only, so it doesn't do anything requiring other levels.
4. Use `CellPoissonHypreSolver::setMatrixCoefficients()` to set up the matrix coefficients. You can specify the descriptor indices of $C(\boldsymbol{x})$ and $D(\boldsymbol{x})$.
5. Call `CellPoissonHypreSolver::solveSystem()` to solve the system. You have to specify the descriptor indices of the cell-centered solution and the cell-centered source function. The solution data will be modified.

Hypre-Poisson Solver Example

Here is an example following the steps outlined in the previous section, on using `CellPoissonHypreSolver`.

```
/*
  Initialize the solver.
*/
d_poisson_hypre.initializeSolverState( hierarchy, ln );

/*
  LocationIndexRobinBcCoefs is an implementation of the
  Robin boundary condition coefficient strategy class.  It allows
  one to specify a uniform boundary condition for each location
index.
  We set Dirichlet values of 0.0 and 10.0 on the min x and max x
sides
  and zero slope on the min y and max y sides.
*/
LocationIndexRobinBcCoef bc_coefs(tbox::Dimension(2), “bccoefs”);
bc_coefs.setBoundaryValue(0, 0.0);
bc_coefs.setBoundaryValue(1, 10.0);
bc_coefs.setBoundarySlope(2, 0.0);
bc_coefs.setBoundarySlope(3, 0.0);
d_poisson_hypre.setBcObject( bc_object );

/*
  The solver can now set up the matrix.
  The PoissonSpecifications object holds the values of C and D
  in Poisson's equation.  By default, C=0 and D=1, leading to
```

```
Laplace's
  equation.  These values can be changed through the public
interfaces
  of PoissonSpecifications.
*/
PoissonSpecifications spec("Laplace");
d_poisson_hypre.setMatrixCoefficients(spec);

/*
  Solve the system.
  solution is the patch data index of the solution u.
  source is the patch data index of the source f.
*/
int solver_ret = d_poisson_hypre.solveSystem( solution, source );
```

Multi-level FAC-Poisson Solver Settings

The `CellPoissonFACSolver` class uses the following methods for setting the parameters C and D:

```
setDConstant(double value)
setDPatchDataId(int id)
setCConstant(double value)
setCPatchDataId(int id)
```

To set the diffusion coefficient $D(\boldsymbol{x})$, use `setDConstant(double value)` or `setDPatchDataId(int id)`, depending on whether it is a constant or spatially varying and stored on the mesh. Similarly, use `setCConstant(double value)` or `setCPatchDataId(int id)`, to set $C(\boldsymbol{x})$.

The methods specifying the algorithm parameters, corresponding input parameter name and the default settings are:

Methods	Input name	Default setting
setPresmoothingSweeps(int num_pre_sweeps)	num_pre_sweeps	1
setPostsmoothingSweeps(int num_post_sweeps)	num_post_sweeps	1
setMaxCycles(int max_cycles)	max_cycles	10
setResidualTolerance(double residual_tol)	residual_tol	1.00E-006
setCoarseFineDiscretization(const string &coarsefine_method)	coarse_fine_discretization	"Ewing"
setCoarsestLevelSolverChoice(const string &choice)	coarsest_level_solver_choice	"hypre" or "redblack"
setCoarsestLevelSolverTolerance(double tol)	coarsest_level_solver_tolerance	1e-10 or 1e-8
setCoarsestLevelSolverMaxIterations(int max_iterations)	coarsest_level_solver_max_iterations	20 or 500
setUseSMG(bool use_smg)	use_smg	FALSE
setProlongationMethod(const string &prolongation_method)	prolongation_method	"CONSTANT_REFINE"

- The pre- and post-smoothing sweeps refer to the amount of smoothing used in the FAC cycle.
- The max cycles refer to the maximum number of FAC cycles to take.
- The coarse-fine discretization refers to how the PDE is discretized at the coarse-fine boundary. Other than one specific exception, the argument must be the name of a refinement operator is used (i.e., "`LINEAR_REFINE`", "`CONSTANT_REFINE`", etc.). The coarse-fine discretization results from using the specified refinement to get the fine-grid ghost cell, followed by a normal stencil applied across the fine patch boundary. This may seem reasonable, but it results in a discretization that is specified implicitly by the refinement operator. These discretizations may have unanticipated, though usually subtle, numerical behaviors. In the exceptional case, the string "`Ewing`" specifies the coarse-fine discretization of Ewing, Lazarov and Vassilevski. This discretization tends to give better accuracy at the coarse-fine boundaries and is the default.
- Methods beginning with "`setCoarsestLevelSolver...`" refer to the coarsest level solver. By default, hypre is used to solve the coarsest level if it is available, otherwise, red-black Gauss-Seidel iterations are used.
- The different default tolerance and max iterations for the coarsest level reflect the fact that the hypre solver converges much faster than the Gauss-Seidel algorithm.
- The usage of hypre's SMG (semicoarsening multigrid) algorithm is set by the function `setUseSMG`. This setting has effect only when hypre is chosen as the coarsest level solver. If SMG usage is false, hypre's PFMG (parallel semicoarsening multigrid) algorithm is used.
- The prolongation method should be the name of a refine operator, such as "`CONSTANT_REFINE`" or "`LINEAR_REFINE`". Be aware that linear refinement (or any refinement that involves the creation and filling of temporary levels) requires an `RobinBcCoefStrategy` implementation that can fill non-hierarchy data, such as data in temporary levels). The default implementation used (`SimpleCellRobinBcCoefs`) does not satisfy this requirement.

Multi-level FAC-Poisson Usage

Once the solver object is set up using the above methods, the method

```
bool solveSystem( int solution,
                  int rhs,
                  tbox_Pointer<hier_PatchHierarchyX> hierarchy,
                  int coarse_ln=-1,
                  int fine_ln=-1 )
```

performs the solve on the hierarchy and level range specified in the arguments. The integers `solution` and `rhs` are patch data indices for the solution and the right hand side, respectively. The solution data must have a ghost cell width of at least one. Since the solver is for a scalar equation, only the first depth is used. The solver returns true if convergence to the specified level is reached.

The above function call is the simplest way to perform a solve. It initialize the solver state, which is dependent on the hierarchy configuration, level range, boundary condition types, etc. After performing the solve, it deallocates the solver state. When performing multiple solves with different right-hand-side values, efficiency is improved if you take steps to preserve the solver state between solves. This is done with two functions:

```
void initializeSolverState( const int solution,
                            const int rhs,
                            tbox_Pointer<hier_PatchHierarchyX>
hierarchy
                            const int coarse_level=-1,
                            const int fine_level=-1 )
void deallocateSolverState()
```

In between these two function calls, you can perform any number of solves using

```
bool solveSystem(const int solution, const int rhs)
```

The solver state initializer uses the patch data indices to set up matching temporary memory. The `solution` and `rhs` used in the solve may be different from those used to initialize the solver state and different each time the solver is called.

Note that it is an error to mix up the two `solveSystem` methods, as one expects an uninitialized state and one expects an initialized state.

After a solve, the number of FAC iterations, the residual norm and the convergence factors can be retrieved by the functions

```
int getNumberIterations() const
void getConvergenceFactors(double &avg_factor,
                           double &final_factor) const
double getResidualNorm() const
```

The convergence factor is the factor by which the residual is reduced by one FAC iteration. The average factor is that which, when applied the number of iterations used gives the same overall reduction, while the final factor is that of the last iteration taken. The residual norm is the RMS norm of the final residual.

Multi-level FAC-Poisson Example

```
int ln;
for ( ln=0; ln<=hierarchy->getFinestLevelNumber(); ++ln ) {
  /*
    Fill in the initial guess and Dirichlet boundary condition
    data.  For this example, we want u=0 on all boundaries.
    The easiest way to do this is to just write 0 everywhere,
```

```
    simultaneous setting the boundary values and initial guess.
  */
  Pointer<PatchLevel> level = hierarchy->getPatchLevel(ln);
  PatchLevel::Iterator ip(level);
  for ( ; ip; ip++ ) {
    Pointer<Patch> patch = level->getPatch(*ip);
    Pointer<CellData<double> > data = patch-
>getPatchData(comp_soln_id);
    data->fill(0.0);
  }
}
solver.setBoundaries( "Dirichlet" );

/*
  Set up solver object.
  The problem specification is set using the
  PoissonSpecifications object then passed to the solver
  for setting the coefficients.
*/
solver.setCConstant(0.0);
solver.setDConstant(1.0);
solver.initializeSolverState( comp_soln_id,
                     rhs_id,
                     hierarchy,
                     0,
                     hierarchy->getFinestLevelNumber() );

/*
  Solve the system.
*/
pout << "solving..." << endl;
bool solver_ret;
solver_ret = solver.solveSystem( comp_soln_id ,
                         rhs_id );

/*
  Present data on the solve.
*/
double avg_factor, final_factor;
solver.getConvergenceFactors(&avg_factor, &final_factor);
if ( solver_ret ) pout << " converged\n";
else pout << " NOT converged\n";
pout << "   iterations: " << solver.getNumberIterations() << "\n"
     << "   residual: " << solver.getResidualNorm() << "\n"
     << "   average convergence: " << avg_factor << "\n"
     << "   final convergence: " << final_factor << "\n"
     << flush;

/*
  Deallocate state.
*/
solver.deallocateSolverState();
```

Boundary Condition Options

As stated above, the general Robin boundary conditions are supported. These boundary conditions are specified by two coefficients, a and g. One provides an implementation of `RobinBcCoefStrategy` through which the solvers obtain the coefficients. But in most cases, one of the library-provided implementations would suffice. Once you have chosen an implementation, use the method `setBcObject()`, described above, to specifiy it.

For `CellPoissonHypreSolver` and `CellPoissonFACSolver`, there is one additional choice that is aimed at providing compatibility to codes that were written for SAMRAI's older Poisson solvers: using the internal boundary condition implementation. The internal implementation is the library-provided `SimpleCellRobinBcCoefs` class, whose primary interface for specifying the boundary conditions is

```
setBoundaries( const string &boundary_type,
           const int fluxes=-1,
           const int flags =-1,
           int *bdry_types=NULL );
```

The boundary conditions specified as the string argument "boundary_type." The boundary type argument can be `"Dirichlet"`, `"Neumann"`, or `"Mixed"`.

If using Dirichlet boundary conditions, then before the solver is called, the storage for the unknown must have a layer of ghost cells at least one cell wide that includes the Dirichlet boundary values.

If using Neumann boundary conditions, then before the solver is called, the outerface boundary flux data must be set for the Neumann conditions. The `fluxes` argument gives the patch data index of this flux data.

The mixed boundary type is for a mixture of Dirichlet and Neumann boundary conditions at the physical domain boundary. The `fluxes` argument gives the patch data index of the outerface data that specifies the flux data for the Neumann conditions. The `flags` array is an outerface data array of integer flags that specifies whether Dirichlet (`flag == zero`) or Neumann (`flag == one`) conditions are to be used at a particular cell boundary face. Note that the flag data must be set before the solver state is initialized. The `bdry_types` argument can be used if the boundary conditions are mixed but one or more of the faces of the physical boundary are entirely either Dirichlet or Neumann boundaries. The `bdry_types` argument should be an array of 2*NDIM integers, specifying the boundary conditions on each side of the physical domain. It should be ordered {x_lo, x_hi, y_lo, y_hi, z_lo, z_hi}, with the values for each face being 0 for Dirichlet conditions, 1 for Neumann conditions, and 2 for mixed boundary conditions. The `bdry_type` argument is never required, but if used it can sometimes make the solver more efficient.

As stated above, the default boundary condition implementation does not allow linear refinement in the prolongation of error in its most general usage. If you chose to use an other implementation of `RobinBcCoefStategy`, set it using `setBcObject(const solv::RobinBcCoefStrategy *bc_object)`. Some relatively easy to use implementations are available in the library. For example, problems where the coefficients are strictly a function of the boundary box's location index (see `hier::BoundaryBox::getLocationIndex()`), is supported by the class `LocationIndexRobinBcCoefs`.

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