CAIN2.1(β)

Kaoru Yokoya, KEK

$\boxed{\textbf{ABEL}}$	1984. K. Yokoya (KEK) Disruption, Beamstrahlung.
$\boxed{\textbf{ABELMOD}}$	T. Tauchi (KEK) Incoherent pair creation.
$\boxed{\textbf{CAIN1.1}}$	A. Spitkovsky, P. Chen, (SLAC) T. Takahasi, T. Ohgaki (Hiroshima) Nonlinear laser interaction.
$\boxed{\textbf{CAIN2.1}(\beta)}$	M. Xie (LBL) New interface, coherent pair creation, polarization in most interactions.

Now in use for

- Beam-beam interaction in JLC/NLC.
- Gamma-gamma collider.
- Polarized positron production at KEK.
- IP of laser-plasma wakefield accelerator.

LC97, Zvenigorod, Sep.28-Oct.3, 1997

Constituents

Particles	Photon, Electron, Positron.
External Fields	Constant field, Laser.

Interactions

- Beam deformation due to the field (beam field + constant field).
- Beamstrahlung, Coherent pair creation.
- Incoherent (particle-particle) interaction. Breit-Wheeler, Bethe-Heitler, Landau-Lifshitz, bremsstrahlung. with finite beamsize effects, Landau-Pomeranchuk suppression....
- Linear and nonliner Compton scattering between electron/position and laser.
- Linear and nonlinear Breit-Wheeler process between photon and laser.

Outputs

- Particle distribution at any time
- Luminosity $\mathcal{L}$ and $d\mathcal{L}/dW$, $d\mathcal{L}/dE_1dE_2$ for any combination of (γ,e$^-$,e$^+$) for any polarization (16 components).

User Interface

- Any type of successive interactions can be defined.
- Simple arithmetics using user-defined variables.
- `DO-ENDDO` loop, `IF-ELSE-ENDIF` block.
- Easy histogram and scatter plot with arbitrary horizontal/vertical axes.
- A big users' manual.
  See
  `http://www-acc-theory.kek.jp/members/cain/`

CAIN Commands

Constituents	BEAM, LASER, EXTERNALFIELD
Interactions	BBFIELD, CFQED, LASERQED, PPINT, LUMINOSITY
Motion	PUSH, ENDPUSH, LORENTZ, DRIFT
Job Control	FLAG, SET, DO, ENDDO, IF, ELSE, ENDIF
Output	WRITE, PRINT, PLOT, FILE, STORE, RESTORE

```
SET   mm=1E-3, micron=1E-6, nm=1E-9,
  ee=250E9,  gamma=ee/Emass,  an=0.7E10,
  sigz=0.1*mm, betax=10*mm, betay=100*micron,
  emitx=3.3D-6/gamma,  emity=4.8D-8/gamma,
  sigx=Sqrt(emitx*betax), sigy=Sqrt(emity*betay),
  incpair=1, Smesh=sigz/2 ;
SET photon=1, electron=2, positron=3;
BEAM  RIGHT, KIND=electron, NP=5000, AN=an, E0=ee,
     BETA=(betax,betay), EMIT=(emitx,emity), SIGT=sigz ;
BEAM  LEFT,  KIND=positron, NP=5000, AN=an, E0=ee,
     BETA=(betax,betay), EMIT=(emitx,emity), SIGT=sigz ;
LUMINOSITY  KIND=(electron,positron), W=(0,2*1.001*ee,50),
     WX=8*sigx, WY=8*sigy,  FREP=85*150 ;
BBFIELD  NX=32, NY=32, WX=8*sigx, R=sigx/sigy/2;
CFQED    BEAMSTRAHLUNG;
IF incpair > 0;
  PPINT VIRTUALPHOTON, FIELDSUP, EMIN=10E6;
  PPINT BW; PPINT BH; PPINT LL;
ENDIF;
SET it=0;
PUSH  Time=(-2.5*sigz,2.5*sigz,200);
   IF Mod(it,20)=0;  ! plot (s,y) profile every 20th time step
     PLOT SCAT, KIND=electron, H=S/micron, V=Y/nm,
       HSCALE=(-250,250), VSCALE=(-12,12),
       HTITLE='s(Mm);  G  ;', VTITLE='y(nm);';
   ENDIF;
   SET it=it+1;
ENDPUSH;    DRIFT S=0;
PLOT  HIST, KIND=photon, H=En/1E9, HSCALE=(0,1.001*ee/1E9,50),
       VLOG, TITLE='Beamstrahlung Energy Spectrum;',
       HTITLE='E0G1  (GeV); XGX       ;';
IF incpair >0;
  PLOT  SCAT, INCP, H=En/1E9, V=Sqrt[(Px^2+Py^2)/Ps^2]/mm,
         HSCALE=(0,10), VSCALE=(0,50),
         TITLE='Incoherent Pair Energy-Angle Distribution;',
         HTITLE='E (GeV);', VTITLE='Q (mrad);G        ;';
ENDIF;
PLOT LUMINOSITY, KIND=(electron,positron), VLOG;
```

Beam Field

- Ultra-relativistic Collinear approximation is used for calculating the field.
  Interaction within each longitudinal slice only.
  (When the crossing angle is large, use `LORENTZ`.)

- Ultra-relativistic Collinear approximation is not used for calculating the motion.
  (Low energy particles can be treated if they do not create significant fields.)

- Use equal-size rectangular 2D mesh.
  2D FFT employed in the mesh region.
  Circular/elliptic harmonic expansion outside the region.
  (Non-equal-size mesh desired but not ready.)

- The classical precession of $e^{\pm}$ spin in the field is computed with the anomalous magnetic moment as a function of the field strength.

Beamstrahlung, Coherent Pair Creation

- Final angle distribution is not computed. (Will be included if needed in practice.)
- All polarization effects are included (final photon polarization, $e^{\pm}$ spin-flip, etc.) except for the polarization correlation between final particles.

Incoherent Pair Creation

Breit-Wheeler	$\gamma + \gamma \to e^- + e^+$
Bethe-Heitler	$\gamma + e^\pm \to e^\pm + e^- + e^+$
Landau-Lifshitz	$e + e \to e + e + e^- + e^+$
Bremsstrahlung	$e + e \to e + e + \gamma$

- Weizäcker-Williams approximation is used (except for Breit-Wheeler).
  The adopted virtual photon spectrum is not accurate for high energy final pairs.
- The finite beam size effect is included in the virtual photon spectrum.
- The Landau-Pomeranchuk effect (due to the strong field but not due to the random Coulomb scattering) is approximately included in the virtual photon spectrum.
  Probably, this is not accurate enough for very large Υ.
  Should be improved.
- Polarization effects are not included (except for Breit-Wheeler).

Laser Field

Gaussian laser field is approximated by

$$\mathbf{E} = \Re \mathbf{E}_0 e^{ik(z-t)} \sqrt{A} e^{i\Phi},$$

$$A = \frac{1}{\sqrt{1+(z/\beta_1)^2}} \frac{1}{\sqrt{1+(z/\beta_2)^2}} \exp\left[-\frac{(z-t)^2}{2\sigma_t^2}\right] \\ \times \exp\left[-\frac{x^2}{\lambda\beta_1(1+(z/\beta_1)^2)} - \frac{y^2}{\lambda\beta_2(1+(z/\beta_2)^2)}\right]$$

λ	Wavelength. $k = 2\pi/\lambda$.
β_i	Rayleigh length.
σ_t	r.m.s. pulse length.

The wave front is given by the contour of $kz + \Phi$. At particle points, the laser field is approximated by a plane wave along the direction $\nabla(kz + \Phi)$.

Linear Laser Interaction

- Throughout CAIN, the particle polarization is represented by the density matrix, i.e., the polarization 3-vector for electrons/positrons and the Stokes parameter for photons.

$$\langle(\boldsymbol{\epsilon}\cdot\mathbf{e}_i)(\boldsymbol{\epsilon}^*\cdot\mathbf{e}_j)\rangle = \tfrac{1}{2}(1+\boldsymbol{\xi}\cdot\boldsymbol{\sigma})_{ij}$$
$$\langle\phi_i\phi_j^*\rangle = \tfrac{1}{2}(1+\boldsymbol{\zeta}\cdot\boldsymbol{\sigma})_{ij}$$

- Compton and Breit-Wheeler ($\gamma+\gamma\to e^+e^-$) processes involve all four polarizations.
  CAIN2.1β includes the terms involving upto two polarizations only.
  This will be extended to three polarizations soon.
  (Correlation between final particle polarizations will never been taken into account.
  So, 4-polarization terms are not needed.)

- Linear laser polarization is included.

- Polarization density matrix changes even when an event is not generated.
  This is taken into account in CAIN2.1β.
  But the prescription seems to be not enough according to the recent paper by Kotkin-Perlt-Serbo (N.I.M. **A**, P.L. **B**).
  Must be corrected soon.

Nonlinear Laser Interaction

- Only $\pm 100\%$ laser polarization is allowed. Should be extended to arbitrary polarization for the demand of linearly polarized γ-γ collider. But not easy.
- Only the helicity of γ, e^-, e^+ is taken into account.
- Expansion w.r.t. the field intensity parameter ξ using Bessel functions is employed. Valid for not too large ξ.
- The computing time is comparable to the linear interaction.
- It is possible to artificially enhance a part of spectrum for better statistics of rare events.

What is included and what is not can be seen from

$$W_{Compt} = \frac{\alpha m^2 \xi^2}{4E^2} \sum_{n=1}^{\infty} \int_0^{\omega_n} d\omega \, [(1 + h_e \bar{h}_{e'}) F_{1n} \\ + h_L (h_e + \bar{h}_{e'}) F_{2n} + h_e \bar{h}_{e'} F_{5n} + \bar{h}_\gamma (h_L F_{3n} + h_e F_{4n})].$$

$$W_{BW} = \frac{\alpha m^2 \xi^2}{2\omega^2} \sum_{n>(1+\xi^2)/\eta} \int_{E_n}^{\omega - E_n} dE \, [G_{1n} + h_L h_\gamma G_{3n} \\ + \bar{h}_e (h_L G_{2n} + h_\gamma G_{4n})]$$

Future Developments

Near future

- Improvement of linear laser interaction (polarization).
- Check everything for large Υ.

Far future

- Linear polarization in nonlinear laser interaction.
- Plasma interaction.
- Muons? Probably not.