Investigating stock market crash dynamics

Alex Janczewski, Zhiyu Zhang,
Charel Felten, James Kingsbury

Motivation

- Crashes in the financial markets can spill into the real economy
- Which market participant behaviour leads to emergence of crashes

Source: SP500 on Apple stocks, Data from Yahoo Finance

Source: Lehman Brothers Times Square by David Shankbone

Introduction and Research Question

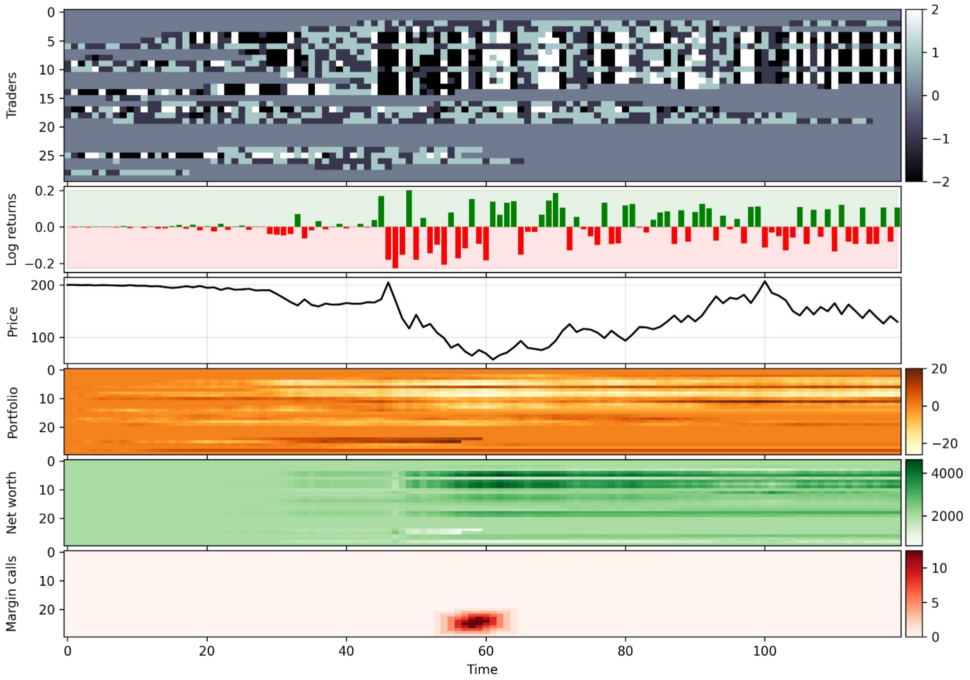
Do deterministic rules lead to different crash dynamics compared to stochastic trading rules?

Does our model accurately reproduce the stylised facts of a real stock market?

- Model
- Verification
- Crash dynamics
- Multifractality
- Conclusion

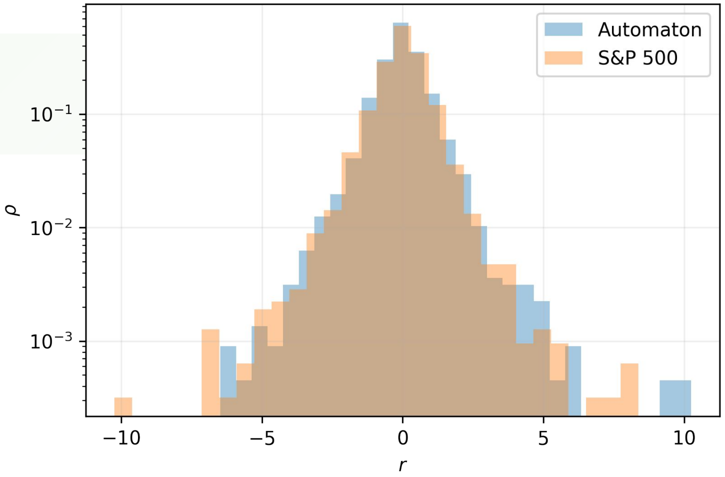
Model

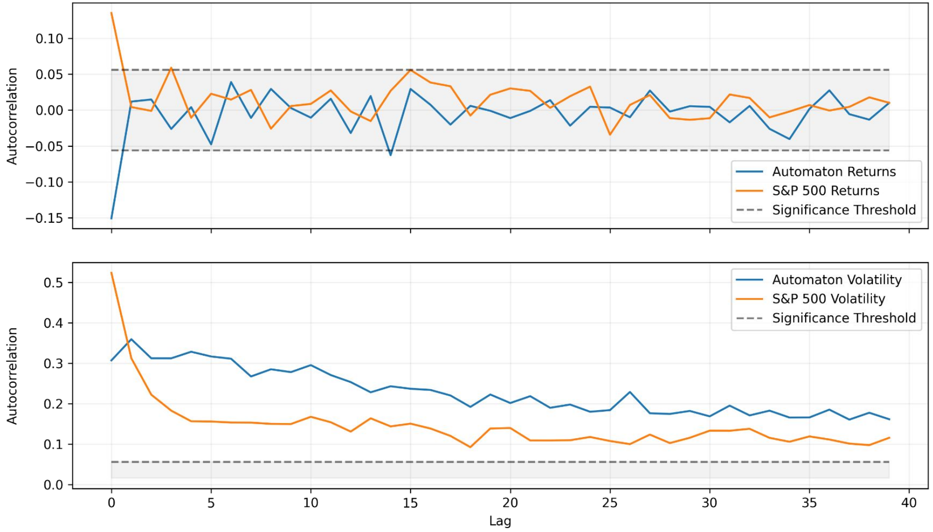
- Cellular automata / Agent-based model on 1D grid
- 3 types of traders
    - Stochastic traders with herding behaviour (Bartolozzi et al [1])
    - Deterministic momentum-based traders [3] [4]
    - Deterministic Moving-average traders [5] [6]
- Log returns
- Margin calls



Our Model vs Real Market

- How does the generated times series fit the stylised facts of a real index?



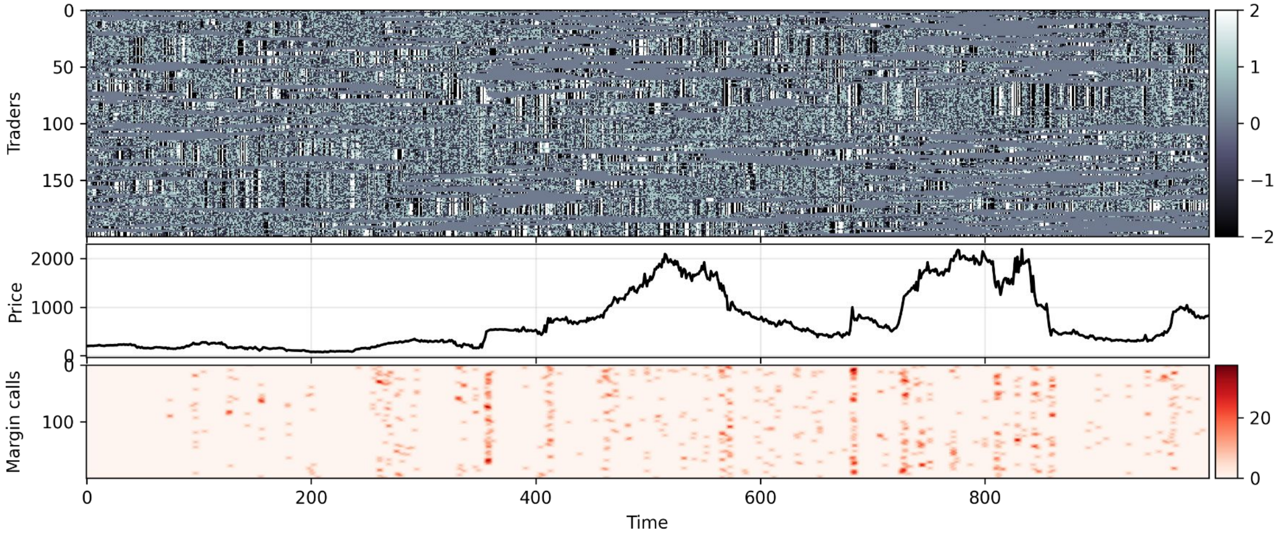


$$v(t) = |R(t)|$$

- Standardised return distribution
- Return clustering and volatility clustering behaviour

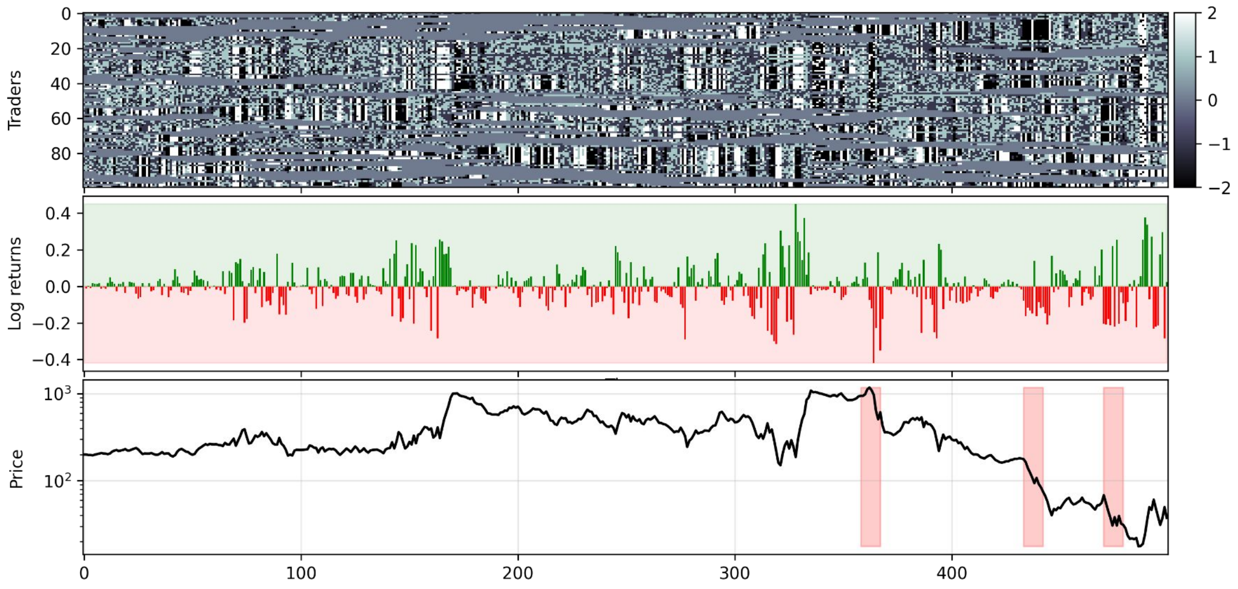
Limitations of Approach

- No order book / transactions / liquidity
- Limited data availability
- Only 3 types of agents



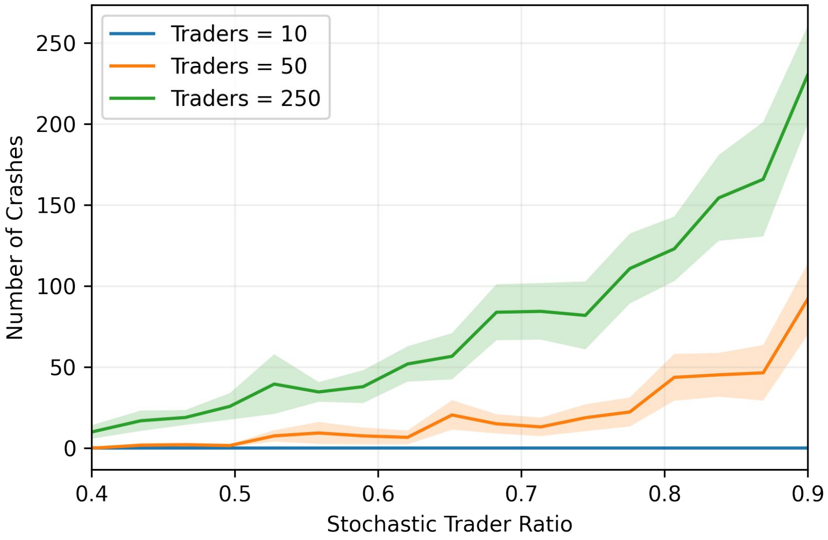
Stock market crash

- Rolling geometric mean of log returns falls below a particular threshold.
- Emerging phenomena



Number of crashes

- Two Sample t-test ➡ 1000 simulations
  - **H0:** #crashes without momentum traders = #crashes with momentum traders
  - **H1:** #crashes without momentum traders > #crashes with momentum traders
  - Two-sample t-test, p-value: 0.002, **H0 rejected**



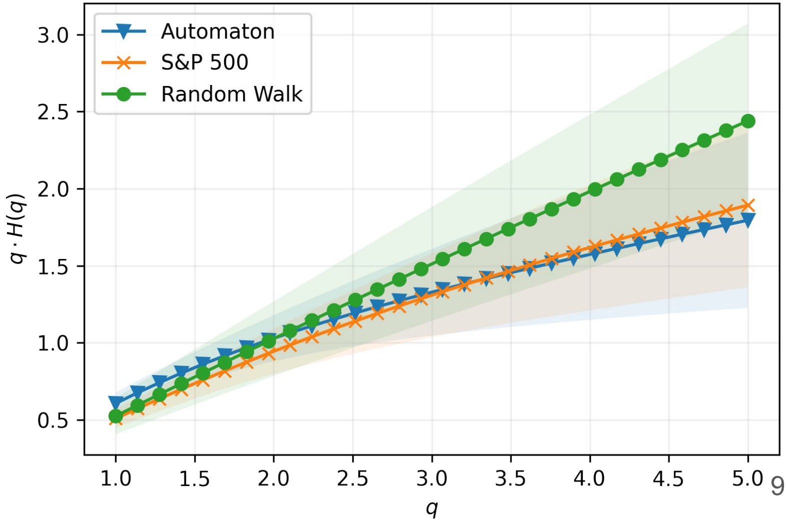
Fractal analysis of time-series

- Monofractal: 1 exponent enough to describe fractal system
- Multifractal: spectrum of fractal exponents needed to describe fractal system

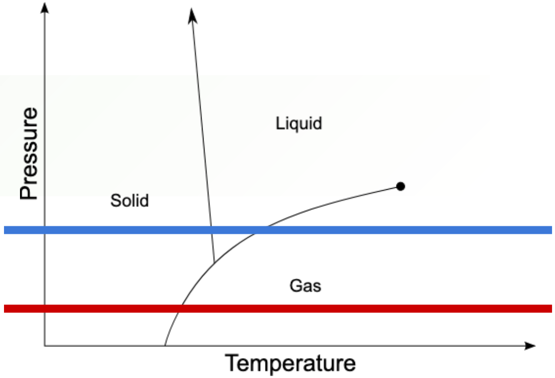
- Hurst exponent H(q) related to fractal dimension
  - H(q) = H for every q ➡ Monofractal
  - H(q) not constant ➡ Multifractal

- Monofractality ➡ random walk
- Multifractality ➡ phase transition

$$S_q(\tau) = \langle | \, x(t+\tau) - x(t) \, |^q \rangle_T \propto \tau^{qH(q)}$$



Multifractality to Thermodynamics



- q: scaling exponent ⟺ T: temperature in thermodynamics

$$\mu(\tau)_i = \frac{\mid x(t+\tau) - x(t) \mid}{\sum_{n=1}^{\mathcal{N}} \mid x(t+\tau) - x(t) \mid}$$

$$p(T)_i \propto e^{-\frac{E_i}{k_B T}}$$

$$Z(q, \mathcal{N}) = \sum_{i=1}^{\mathcal{N}} \mu_i(\tau)^q \propto \mathcal{N}^{-\chi_q}$$

⟺

$$Z(T) = \sum_i e^{-\frac{E_i}{k_B T}}$$

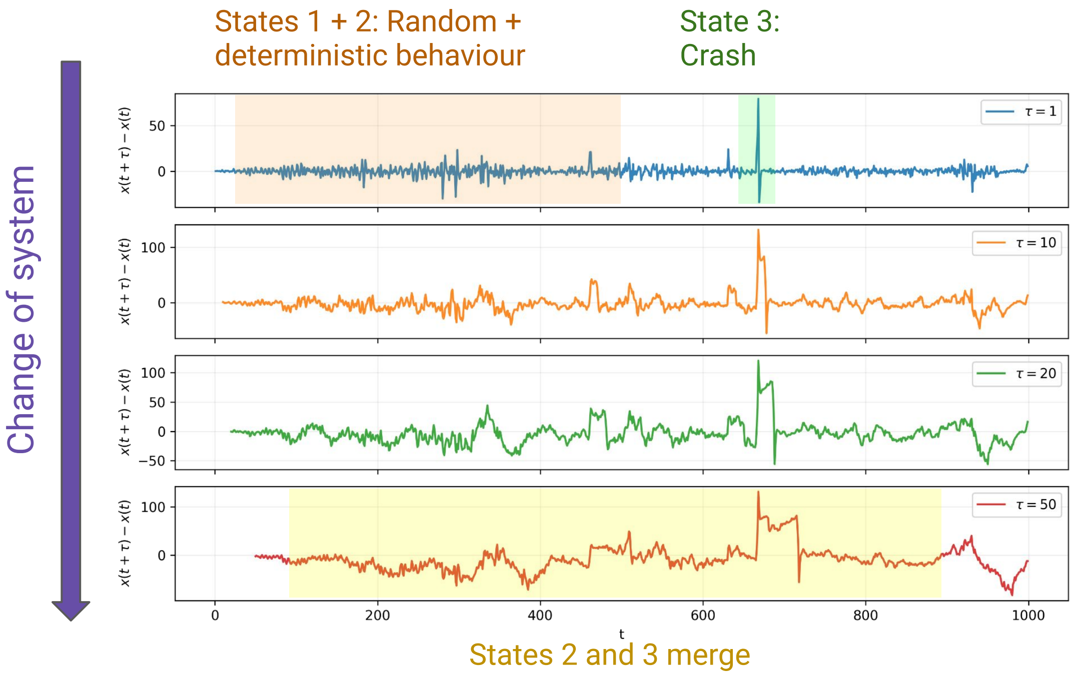
Z - partition function
μ_i - normalized probability measure of state i
q - scaling exponent
x - stock price time series
τ - time delay
$\mathcal{N}$ - number of equal states
χ_q - free energy of the system

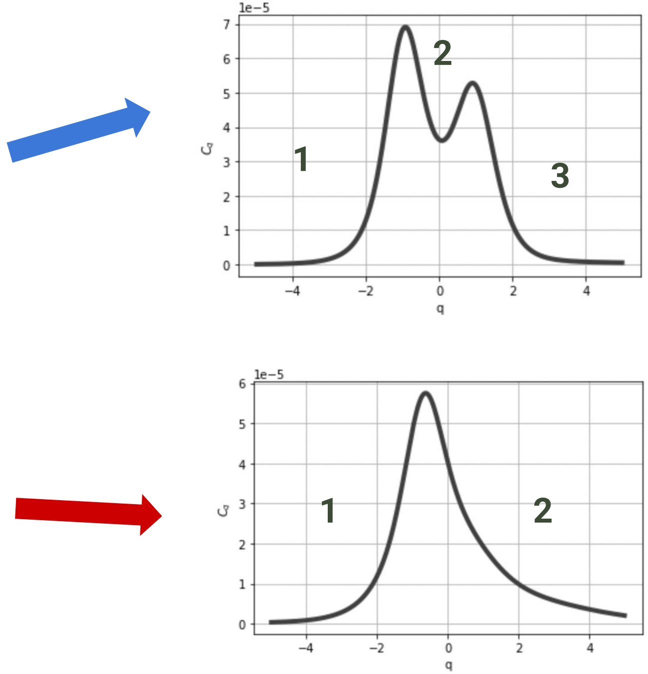
Z - partition function
p_i - probability of state i
T - temperature

Phase transitions from time series

$$Z(q, \mathcal{N}) = \sum_{i=1}^{\mathcal{N}} \mu_i(\tau)^q \propto \mathcal{N}^{-\chi_q}$$

$$C_q = -\frac{\partial^2 \chi_q}{\partial q^2} \approx \chi_{q+1} - 2\chi_q + \chi_{q-1}$$

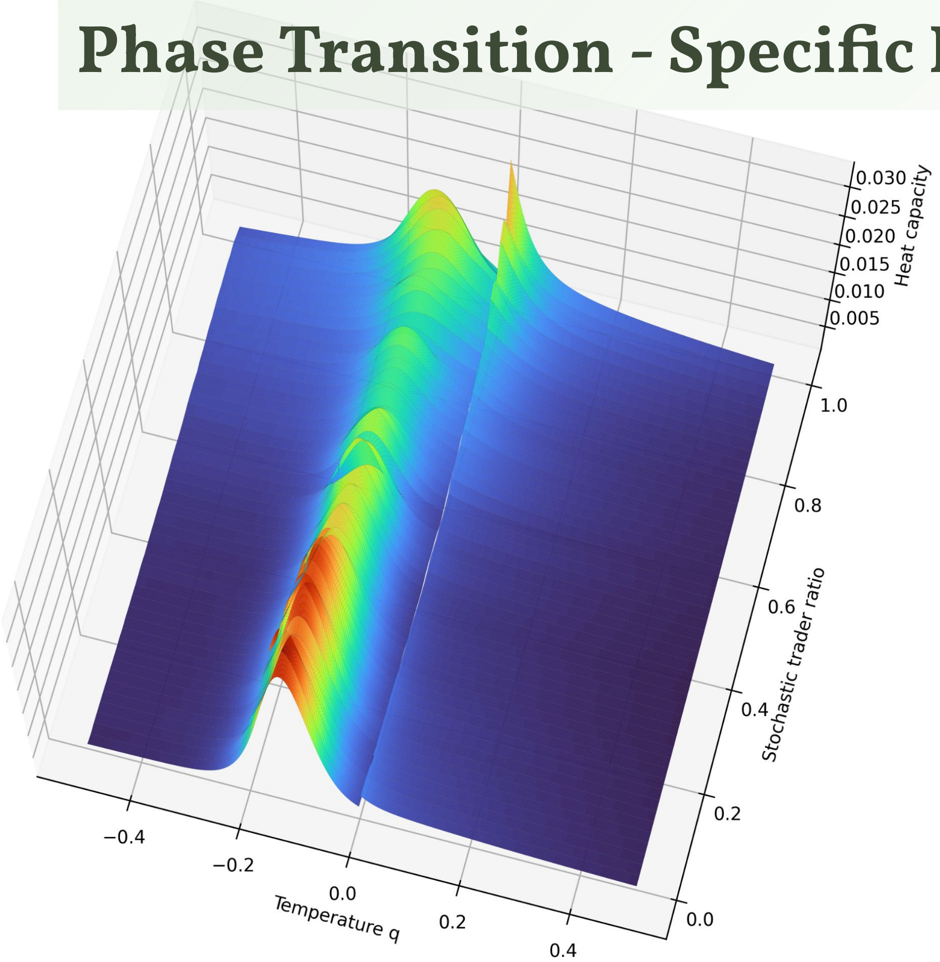


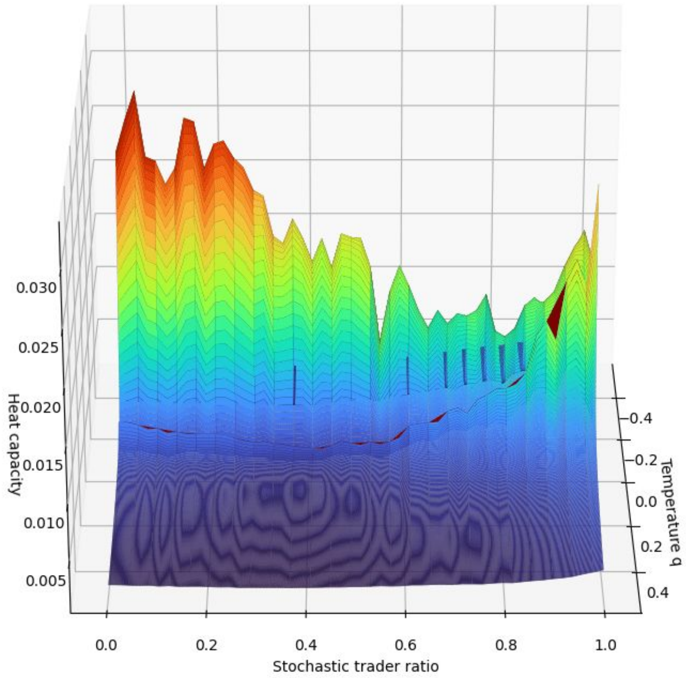


[2]

Phase Transition - Specific Heat

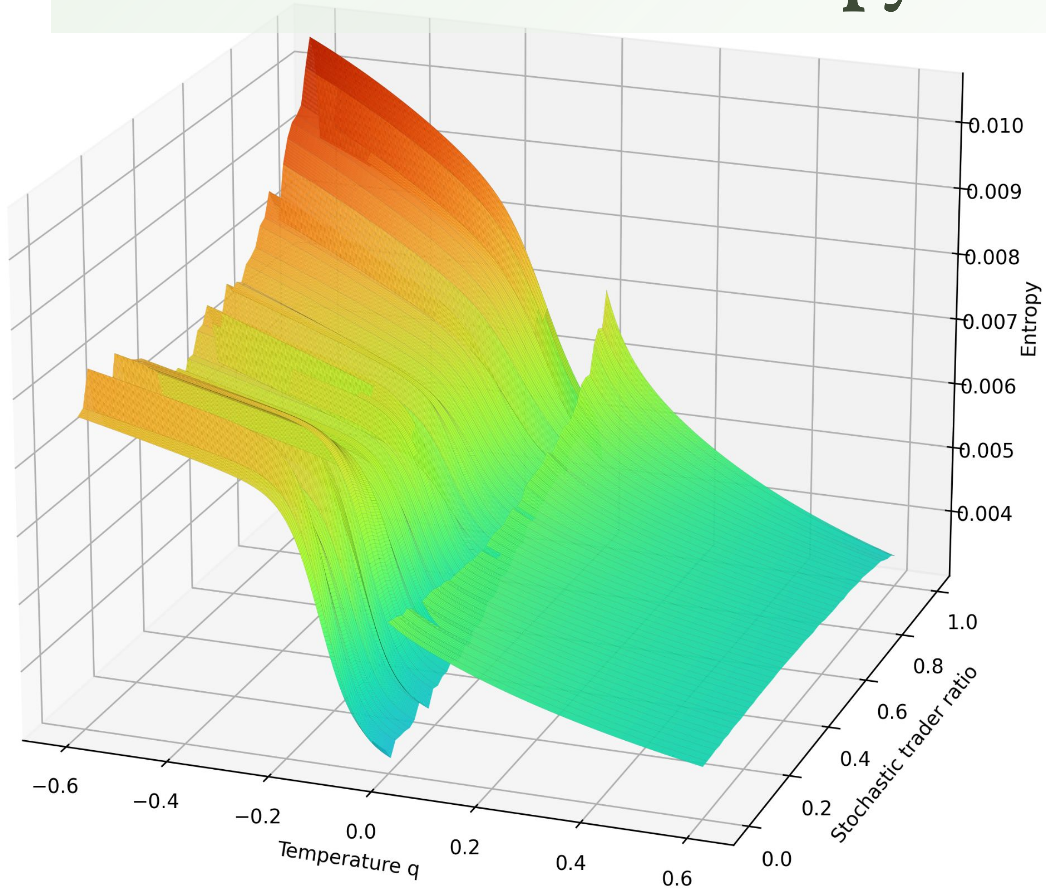
$$C_q = -\frac{\partial^2 \chi_q}{\partial q^2} \approx \chi_{q+1} - 2\chi_q + \chi_{q-1}$$

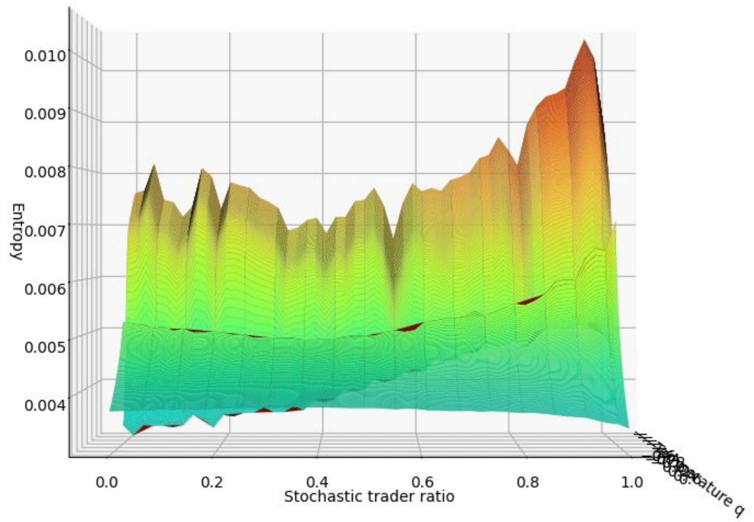




Phase Transition - Entropy

$$S_q = -\frac{\partial \chi_q}{\partial q} \approx \chi_{q+1} - \chi_{q-1}$$





Conclusion

- Our model is able to reproduce the investigated stylised facts of stock market dynamics.
- Deterministic agents counteract emergence of crashes and the associated phase transition.
- Number of crashes decreases with more deterministic traders

Questions?

References

[1] Marco Bartolozzi and Anthony William Thomas. "Stochastic cellular automata model for stock market dynamics". In: *Physical review E* 69.4 (2004), p. 046112.

[2] Enrique Canessa. "Multifractality in time series". In: *Journal of Physics A: Mathematical and General* 33.19 (2000), pp. 3637–3651.

[3] Kalok Chan, Allaudeen Hameed, and Wilson Tong. "Profitability of momentum strategies in the international equity markets". In: *Journal of financial and quantitative analysis* (2000), pp. 153–172.

[4] John M Griffin, Xiuqing Ji, and J Spencer Martin. "Global momentum strategies". In: *The journal of portfolio management* 31.2 (2005), pp. 23–39.

[5] Abeyratna Gunasekarage and David M Power. "The profitability of moving average trading rules in South Asian stock markets". In: *Emerging Markets Review* 2.1 (2001), pp. 17–33.

[6] Massoud Metghalchi, Juri Marcucci, and Yung-Ho Chang. "Are moving average trading rules profitable? Evidence from the European stock markets". In: *Applied Economics* 44.12 (2012), pp. 1539–1559.

Value & Limitations

Value

- Framework for studying crashes
- Emergence of realistic market behaviour
- 3 basic types of traders with simple rules

Limitations

- No order book / transactions / liquidity
- Biased, naive agents
- Short period of investigation
- Only tested against S&P 500

Model & Theory

-

$$x(t) = \beta \sum_{k=1}^{N_{cl}(t)} \sum_{i-1}^{N^k(t)} N^k(t)\sigma_i^k(t)$$

$$I_i^k(t) = \frac{1}{N^k(t)} \sum_{j=1}^{N^k(t)} A_{ij}^k \sigma_j^k(t) + h_i^k$$

$$p_i^k(t) = \frac{1}{1 + e^{-2I_i^k(t)}}$$

Research Question

- Do deterministic trading rules lead to different crash dynamics compared to stochastic trading rules

Introduce the initial model

- Cellular automata introduction
- Stochastic trading behaviour
- Power law of cluster size
- Autocorrelation of returns/volatility
- Probability distributions

Deterministic Trading Rules

- Why bother investigating deterministic rules
- Introduce/justification for the rules
  - portfolio management
  - moving average values
- Margin call mechanics
- Ising model

Multifractality

- Hurst exponent, does it meet multifractality criteria? Does the data match that of the S&P500?
- Thermodynamic entropy and C_k, discontinuity of entropy
- How does entropy depend on the probability of different rule sets

Crash Dynamics

- Crash methodology
- How do number of crashes vary with the probability of individual parameters
- How do number of crashes change with the probability of individual rule sets

Introduction

Close Price to
Log Return

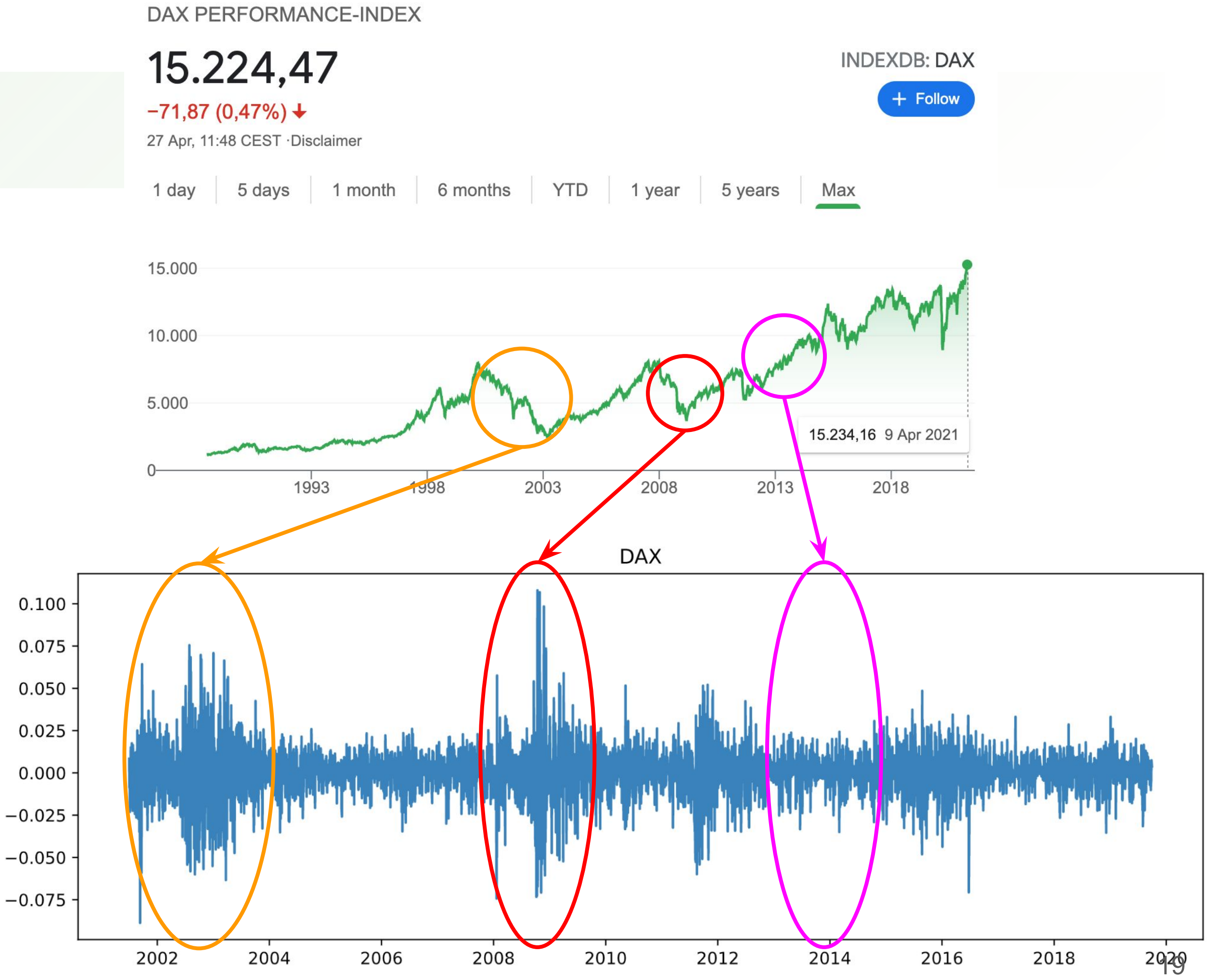


Image source: Kingsbury et al 2021

Multifractality

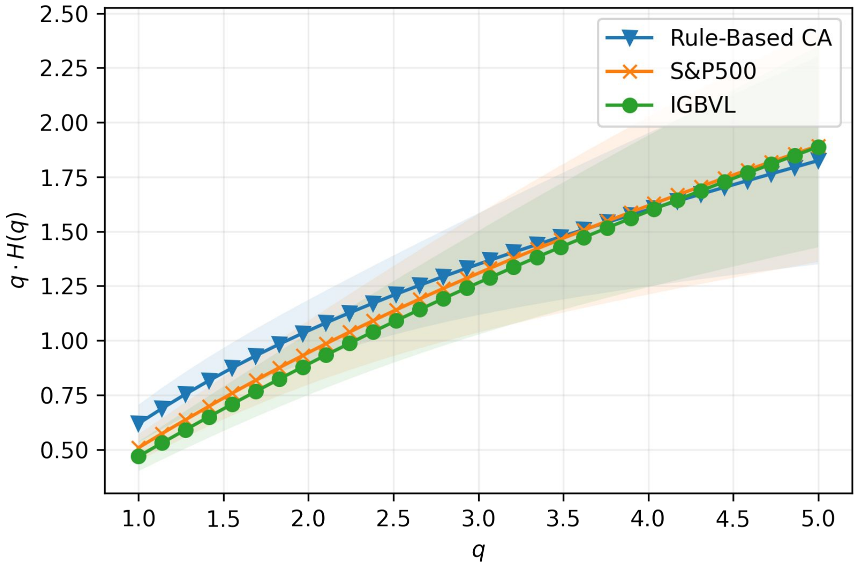
$$S_q(\tau) = \langle \mid x(t+\tau) - x(t) \mid^q \rangle_T \propto \tau^{qH(q)}$$

- **Hurst exponent** *H(q)* is related to fractal dimension
- If *H(q)* = *H* for every *q*
  ⇨ Monofractal
- If *H(q)* is not constant with *q*
  ⇨ Multifractal

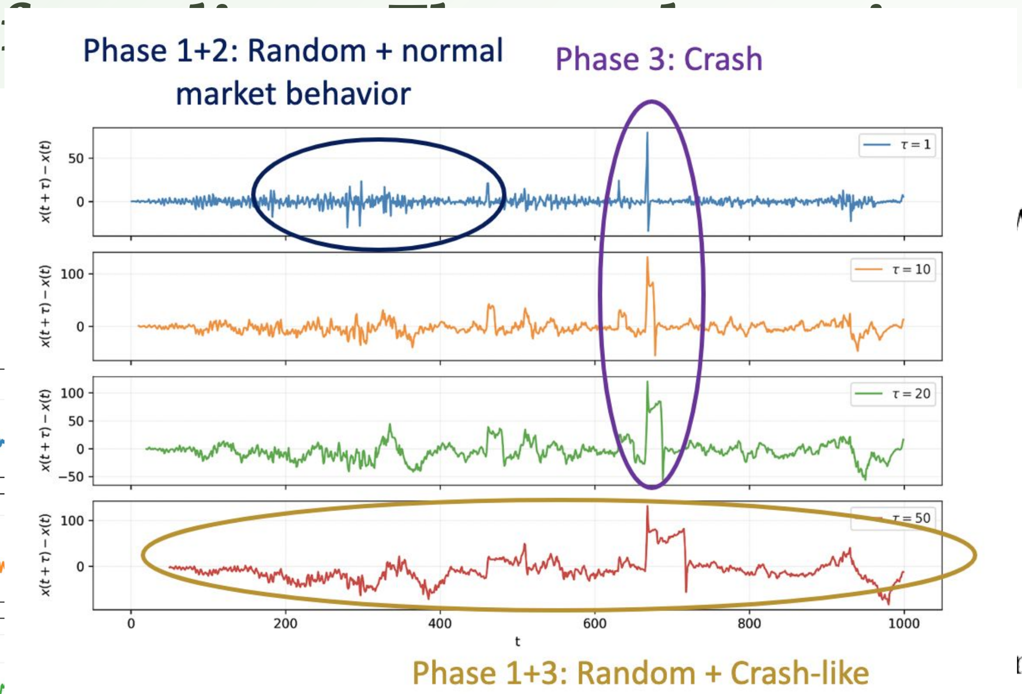
- Monofractality implies random walk behaviour

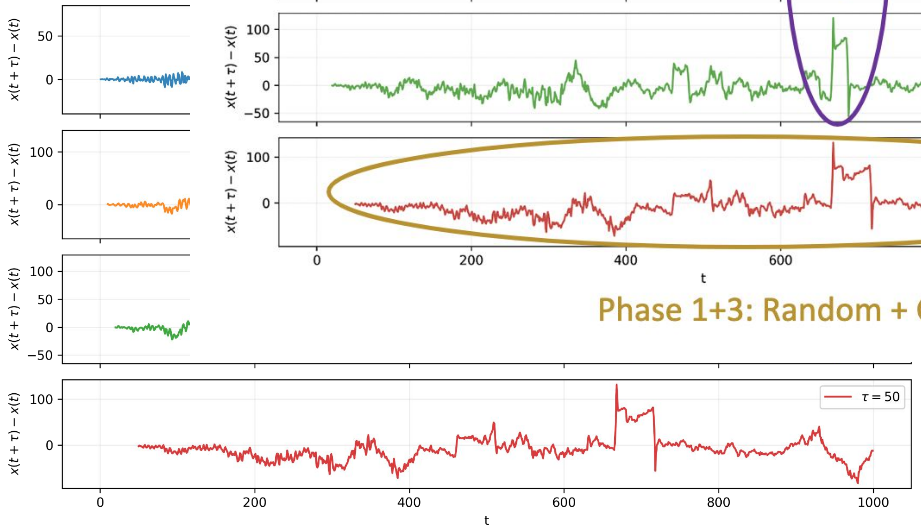
- **Scaling properties** of volatility for Brownian motion:

$$\sigma_T = \sqrt{T}\sigma_1$$



Multi





$$\mathcal{N}) = \sum_{i=1}^{\mathcal{N}} \mu_i(\tau)^q \propto \mathcal{N}^{-\chi_q}$$

$$= \frac{|\, x(t+\tau) - x(t) \,|}{\sum_{n=1}^{\mathcal{N}} |\, x(t+\tau) - x(t) \,|}$$

- normalized probability measure

me delay

umber of equal subintervals

i - index of the subinterval

q - temperature

χ_q - free energy of the system

Model

- Large-scale