Exploring Many-Body Force Induction in the Similarity Renormalization Group in 1 Dimension

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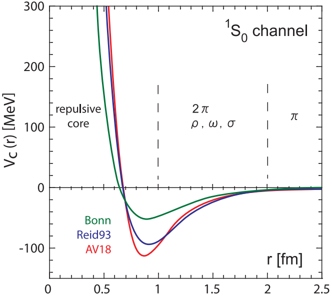
Goals of Low-Energy Nuclear Theory

- Model atomic nuclei and calculate observables like ground state and excited state energies, nuclear radii, half-lives for different decays
- Model nuclear matter to better understand the structure of astrophysical systems like neutron stars
- Use accurate models of nuclear systems to aid in the search for beyond Standard Model physics
- Leverage improved understanding of nuclear physics in a variety of applications

Challenge: The Interaction

The first challenge of low-energy nuclear theory is due to the nature of inter-nucleon forces, which are given by the strong interaction:

- Underlying theory, quantum chromodynamics, cannot be solved in closed form at low energies
- Modern 2-body and 3-body potentials are determined from scattering experiments and few-body nuclei
- Hard repulsive core couples low and high-energy parts of Hamiltonian

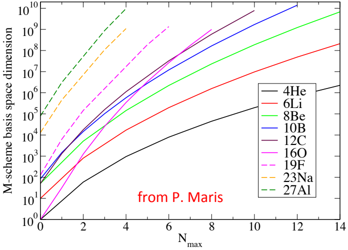


From R.J. Furnstahl, Nucl. Phys. Proc. Suppl. 228 (2012)

Challenge: The Problem Size

The second challenge of low-energy nuclear theory is that naive approaches quickly become intractable because of the rapid growth of the problem size:

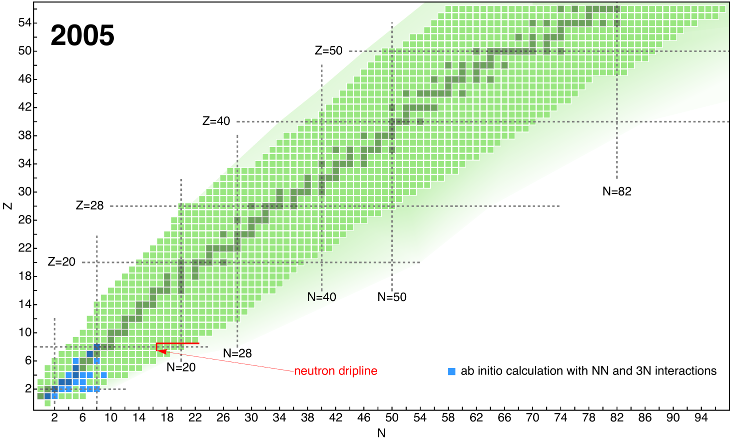
- Basis size grows combinatorially with respect to A, the number of particles
- Sets limit on what nuclei can be modeled, even when taking advantage of the most modern parallel computing clusters
- Strong off-diagonal couplings prevent significant basis truncation



From P. Maris

Recent Ab-Initio Explosion

Range of nuclei able to be modeled via ab-initio methods (starting from inter-nucleon forces) has rapidly expanded due to effective field theory (EFT) and renormalization group (RG) methods:



Images courtesy of Heiko Hergert

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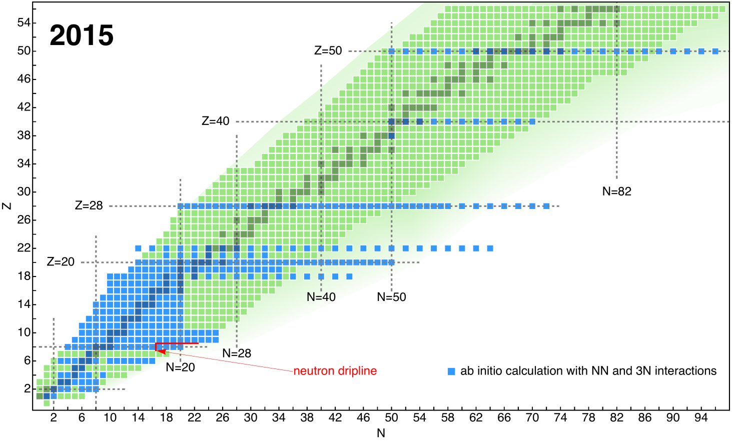

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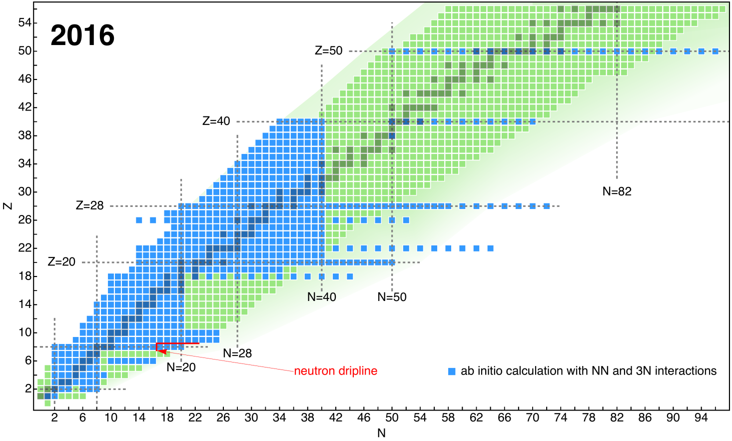

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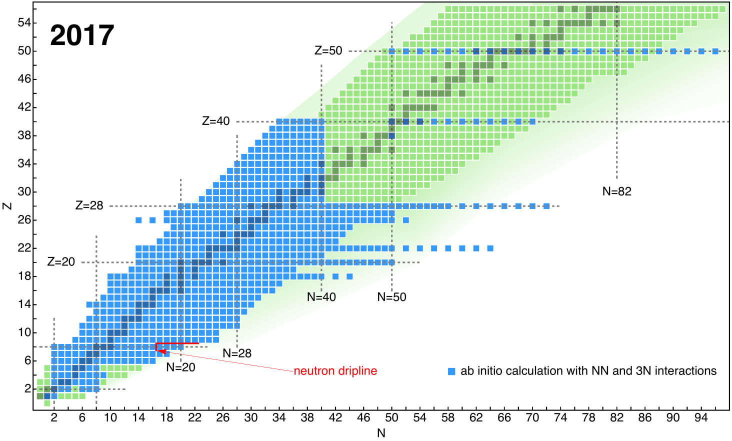

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The Similarity Renormalization Group

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- Get rid of coupling between low and high energies
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- $\hat{G}_s$ is the flow operator, given by the user, which dictates the form to which the operator evolves
- s is the flow parameter, which goes from $s = 0$ towards $s = \infty$ over the course of an SRG evolution

Benefits of SRG

A typical example of an application of the SRG to a calculation looks like:

1. Start with 2-body or 3-body operator in a large basis
2. Apply SRG to drive operator towards the diagonal
3. Truncate basis, keeping low-energy sector and discarding high-energy sector
4. Embed 2-body or 3-body operator in A-body
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Problem Size Reduction

Truncating the basis in the 2-body or 3-body space greatly reduces the size of the resulting A-body space, thus making calculations of A-body observables feasible.

Limits of SRG

While the SRG evolution of an A-body operator is unitary when done in the A-body space, it induces many-body forces that are seen when embedding the A-body operator in the basis for a larger system. These many-body forces:

- Change the eigenvalues of the operator
- Are in general not possible to fully account for
- Show up as an error in the computed observables
- Scale with the size of the full system, limiting the application of SRG to small and medium systems

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Alternative Flow Operators

Currently, the kinetic energy, $\hat{T}$, is used as the flow operator, $\hat{G}_s$, for nearly all SRG calculations. Alternative choices for $\hat{G}_s$ many induce fewer many-body forces, extending the range of SRG to heavier isotopes.

1-Dimensional Setting

SRG was originally explored in the context of low-energy nuclear physics in a 1-dimensional system:

- System of A bosons
- Factor out center-of-mass behavior and focus on behavior relative to center-of-mass
- 2-body local potential, $V(p, p')$, given in terms of relative Jacobi momenta
- Transform to discrete harmonic oscillator basis to embed 2-body potential in A-body system

Harmonic Oscillator Basis

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When working with relative Jacobi coordinates:

- Define harmonic oscillator states to be with respect to Jacobi momenta
- Require $A - 1$ Jacobi coordinates and thus product states of $A - 1$ HO states

Symmetrizing the HO Basis

Spin-statistics theorem

The state of any system of identical bosons must be symmetric under any permutation of the particles.

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We can create an A-body basis of HO product states that reflect this condition by:

- Diagonalizing the 2-body symmetrizer, $S = (1 + P_{12})/2$, and selecting the eigenstates with eigenvalue 1 to get our 2-body basis
- Generating a set of A-body product states from the symmetrized $(A - 1)$-body basis
- Getting the A-body states by diagonalizing the A-body symmetrizer

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In this work, we do this to get symmetrized 2-body and 3-body bases.

The Negele Potential

Negele Potential

$$\hat{V}(p, p') = \frac{V_1}{2\pi\sqrt{2}} e^{-(p-p')^2\sigma_1^2/8} + \frac{V_2}{2\pi\sqrt{2}} e^{-(p-p')^2\sigma_2^2/8} \quad (4)$$

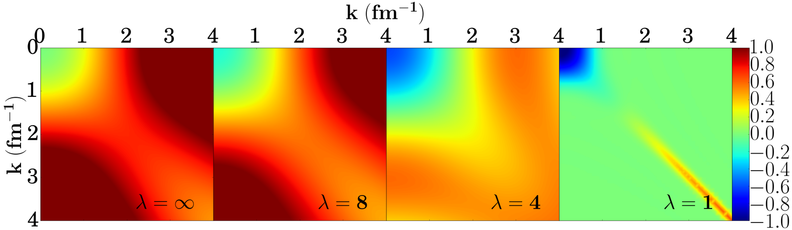
We use two parametrizations of the Negele potential:

- $\hat{V}_\alpha$ is nuclear-like, with a mid-range attractive part and a short-range repulsive part
- $\hat{V}_\beta$ is purely attractive

2-Body Momentum Space SRG

With both $\hat{T}$ as the flow operator and an alternative block diagonal flow operator, we see SRG achieve the desired decoupling of low and high momenta:

With $\hat{G}_s = \hat{T}$:



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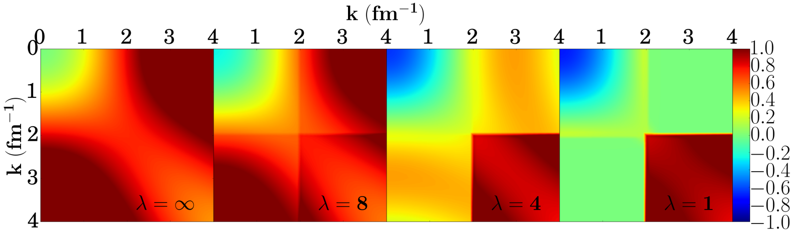
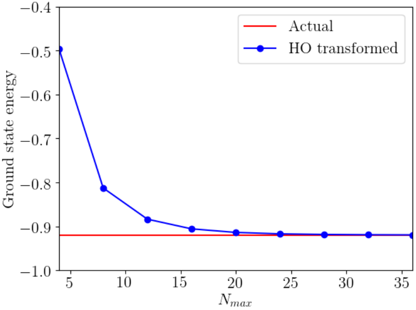
With $\hat{G}_s = \hat{H}_{BD}$:

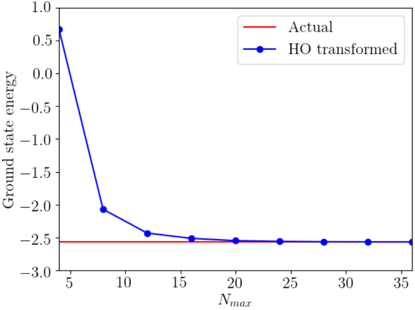


Transformation to HO Space

When transforming operators to HO space, we must pick a value of N_{max} such that our low-energy observables are converged to their true values. These figures focus on $\hat{V}_\alpha$ since $\hat{V}_\beta$ converges much faster. We find $N_{max} = 28$ to be sufficient for the 2-body and 3-body ground state energies.



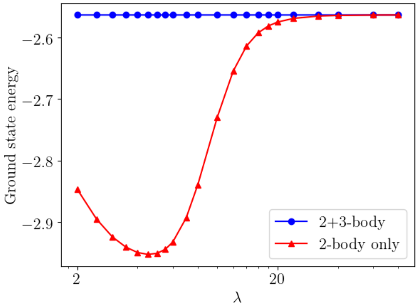
(a) 2-body



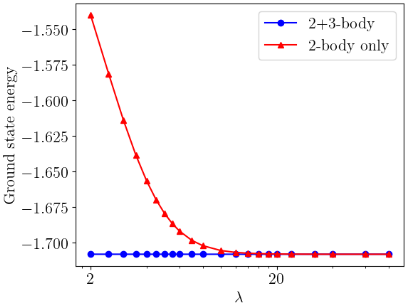
(b) 3-body

Induced 3-Body Force with $G_s = T$

We find a large 3-body force induced by the 2-body SRG evolution with $\hat{G}_s = \hat{T}$.



(a) $\hat{V}_\alpha$



(b) $\hat{V}_\beta$

Induced 3-Body Force with Alternative Flow Operator

We define an alternative flow operator, $\hat{H}_{BDHO}$, that is block-diagonal in HO space:

$$\hat{G}_s = \hat{H}_{BDHO} = \hat{T} + \hat{V}\Theta(N - \Lambda)\Theta(N' - \Lambda), \tag{5}$$

where Λ is the harmonic oscillator cutoff for the low-energy sector.

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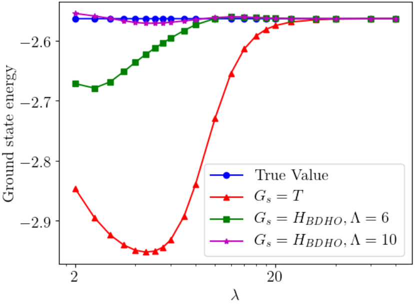

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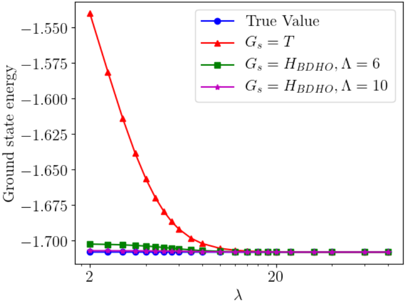

(b) $\hat{V}_\beta$

`srg1d`

`srg1d` is the Python framework behind this work.

- Validated against previous results for 2-body and 3-body cases
- Abstracts technical details (basis construction and symmetrization, operator embedding, etc.) from user
- Simplifies writing SRG calculations
- Testing a new flow operator requires user to change 2 lines of code in an existing script

With `srg1d`, further exploration of this 1-dimensional setting is made easy.

Conclusions and Future Work

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- Validated implementation against previous results
- Alternative flow operator results are preliminary
  - Need to compare rate of decoupling between different flow operators
  - Need to explore 4-body and 5-body forces induced to see general trends
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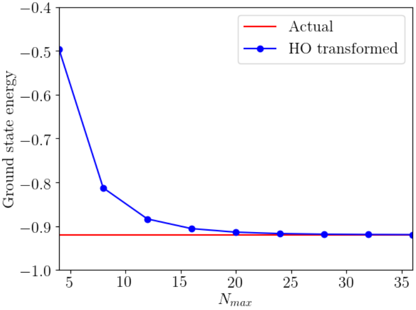
Future Work:

- Test proposed alternative flow operators further
- Formulate new flow operators with different features
- Seek to understand what features of flow operators are desirable

2-Body V_α vs. V_β



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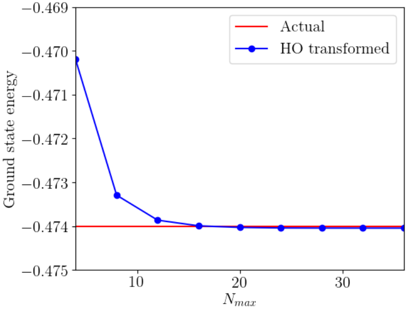
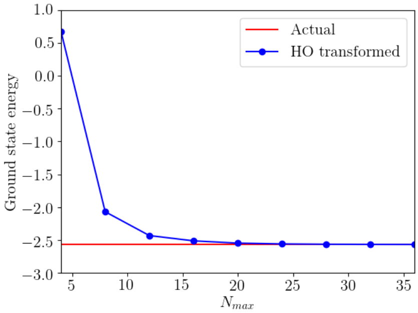


(b) $\hat{V}_\beta$

3-Body V_α vs. V_β



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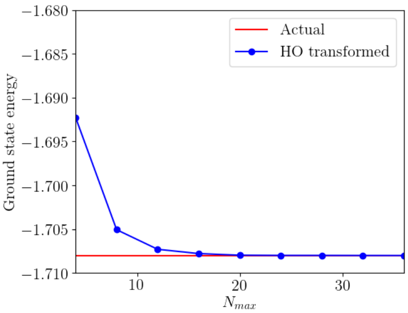
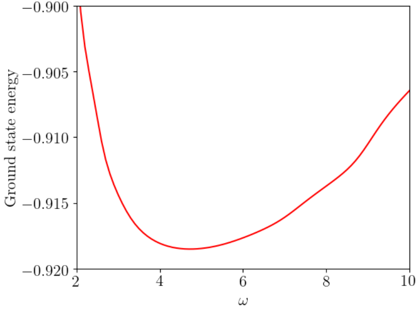
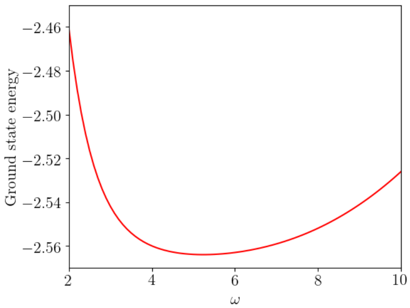


(b) $\hat{V}_\beta$

ω Optimization



(a) 2-body ground state energy



(b) 3-body ground state energy