# Exploring Many-Body Force Induction in the Similarity Renormalization Group in 1 Dimension

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Apr 11, 2018

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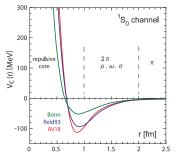
## Goals of Low-Energy Nuclear Theory

- Model atomic nuclei and calculate observables like ground state and excited state energies, nuclear radii, half-lives for different decays
- Model nuclear matter to better understand the structure of astrophysical systems like neutron stars
- Use accurate models of nuclear systems to aid in the search for beyond Standard Model physics
- Leverage improved understanding of nuclear physics in a variety of applications

#### Challenge: The Interaction

The first challenge of low-energy nuclear theory is due to the nature of inter-nucleon forces, which are given by the strong interaction:

- Underlying theory, quantum chromodynamics, cannot be solved in closed form at low energies
- Modern 2-body and 3-body potentials are determined from scattering experiments and few-body nuclei
- Hard repulsive core couples low and high-energy parts of Hamiltonian

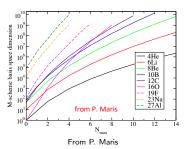


From R.J. Furnstahl, Nucl. Phys. Proc. Suppl. 228 (2012)

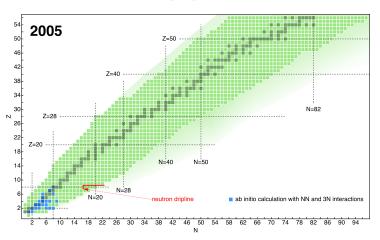
## Challenge: The Problem Size

The second challenge of low-energy nuclear theory is that naive approaches quickly become intractable because of the rapid growth of the problem size:

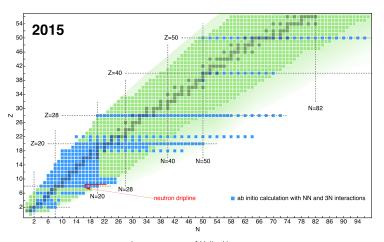
- Basis size grows combinatorially with respect to A, the number of particles
- Sets limit on what nuclei can be modeled, even when taking advantage of the most modern parallel computing clusters
- Strong off-diagonal couplings prevent significant basis truncation



Range of nuclei able to be modeled via ab-initio methods (starting from inter-nucleon forces) has rapidly expanded due to effective field theory (EFT) and renormalization group (RG) methods:

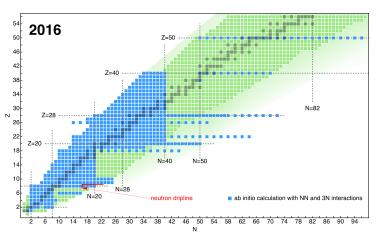


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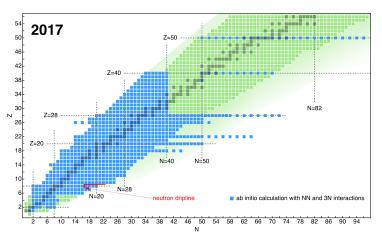
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- $\hat{G}_s$  is the flow operator, given by the user, which dictates the form to which the operator evolves
- s is the flow parameter, which goes from s=0 towards  $s=\infty$  over the course of an SRG evolution

#### Benefits of SRG

A typical example of an application of the SRG to a calculation looks like:

- Start with 2-body or 3-body operator in a large basis
- Apply SRG to drive operator towards the diagonal
- Truncate basis, keeping low-energy sector and discarding high-energy sector
- Embed 2-body or 3-body operator in A-body
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#### Problem Size Reduction

Truncating the basis in the 2-body or 3-body space greatly reduces the size of the resulting A-body space, thus making calculations of A-body observables feasible.

#### Limits of SRG

While the SRG evolution of an A-body operator is unitary when done in the A-body space, it induces many-body forces that are seen when embedding the A-body operator in the basis for a larger system. These many-body forces:

- Change the eigenvalues of the operator
- Are in general not possible to fully account for
- Show up as an error in the computed observables
- Scale with the size of the full system, limiting the application of SRG to small and medium systems

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#### Alternative Flow Operators

Currently, the kinetic energy,  $\hat{T}$ , is used as the flow operator,  $\hat{G}_s$ , for nearly all SRG calculations. Alternative choices for  $\hat{G}_s$  many induce fewer many-body forces, extending the range of SRG to heavier isotopes.

### 1-Dimensional Setting

SRG was originally explored in the context of low-energy nuclear physics in a 1-dimensional system:

- System of A bosons
- Factor out center-of-mass behavior and focus on behavior relative to center-of-mass
- 2-body local potential, V(p, p'), given in terms of relative Jacobi momenta
- Transform to discrete harmonic oscillator basis to embed 2-body potential in A-body system

#### Harmonic Oscillator Basis

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When working with relative Jacobi coordinates:

- Define harmonic oscillator states to be with respect to Jacobi momenta
- ullet Require A-1 Jacobi coordinates and thus product states of A-1 HO states

## Symmetrizing the HO Basis

#### Spin-statistics theorem

The state of any system of identical bosons must be symmetric under any permutation of the particles.

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We can create an A-body basis of HO product states that reflect this condition by:

- Diagonalizing the 2-body symmetrizer,  $S = (1 + P_{12})/2$ , and selecting the eigenstates with eigenvalue 1 to get our 2-body basis
- ullet Generating a set of A-body product states from the symmetrized (A-1)-body basis
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- Getting the A-body states by diagonalizing the A-body symmetrizer In this work, we do this to get symmetrized 2-body and 3-body bases.

### The Negele Potential

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$$\hat{V}(p,p') = \frac{V_1}{2\pi\sqrt{2}}e^{-(p-p')^2\sigma_1^2/8} + \frac{V_2}{2\pi\sqrt{2}}e^{-(p-p')^2\sigma_2^2/8}$$
(4)

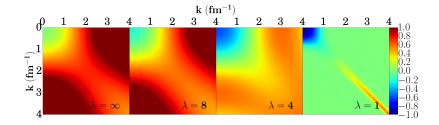
We use two parametrizations of the Negele potential:

- $oldsymbol{\hat{V}}_{lpha}$  is nuclear-like, with a mid-range attractive part and a short-range repulsive part
- ullet  $\hat{V}_eta$  is purely attractive

## 2-Body Momentum Space SRG

With both  $\hat{T}$  as the flow operator and an alternative block diagonal flow operator, we see SRG achieve the desired decoupling of low and high momenta:

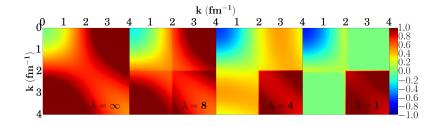
With  $\hat{G}_s = \hat{T}$ :



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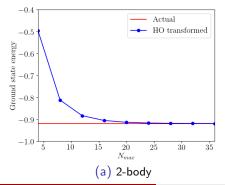
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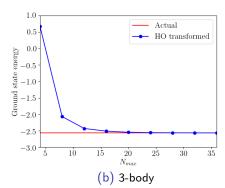
With  $\hat{G}_s = \hat{H}_{BD}$ :



#### Transformation to HO Space

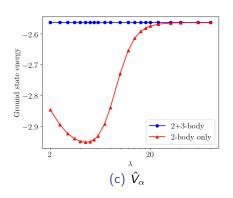
When transforming operators to HO space, we must pick a value of  $N_{max}$  such that our low-energy observables are converged to their true values. These figures focus on  $\hat{V}_{\alpha}$  since  $\hat{V}_{\beta}$  converges much faster. We find  $N_{max}=28$  to be sufficient for the 2-body and 3-body ground state energies.

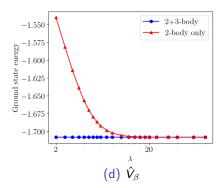




## Induced 3-Body Force with $G_s = T$

We find a large 3-body force induced by the 2-body SRG evolution with  $\hat{G}_s = \hat{T}$ .





## Induced 3-Body Force with Alternative Flow Operator

We define an alternative flow operator,  $\hat{H}_{BDHO}$ , that is block-diagonal in HO space:

$$\hat{G}_{s} = \hat{H}_{BDHO} = \hat{T} + \hat{V}\Theta(N - \Lambda)\Theta(N' - \Lambda), \tag{5}$$

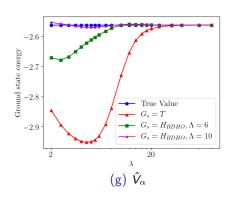
where  $\Lambda$  is the harmonic oscillator cutoff for the low-energy sector.

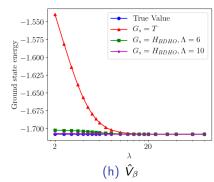
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#### srg1d

srg1d is the Python framework behind this work.

- Validated against previous results for 2-body and 3-body cases
- Abstracts technical details (basis construction and symmetrization, operator embedding, etc.) from user
- Simplifies writing SRG calculations
- Testing a new flow operator requires user to change 2 lines of code in an existing script

With srg1d, further exploration of this 1-dimensional setting is made easy.

#### Conclusions and Future Work

#### Conclusions:

- Validated implementation against previous results
- Alternative flow operator results are preliminary
  - Need to compare rate of decoupling between different flow operators
  - Need to explore 4-body and 5-body forces induced to see general trends
- srg1d framework makes additional testing easy

#### Conclusions and Future Work

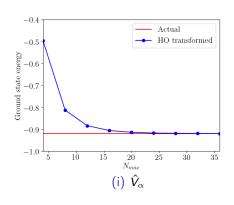
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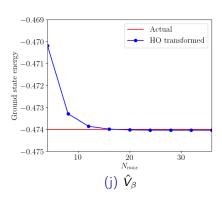
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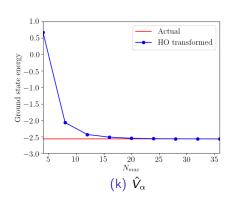
- Test proposed alternative flow operators further
- Formulate new flow operators with different features
- Seek to understand what features of flow operators are desirable

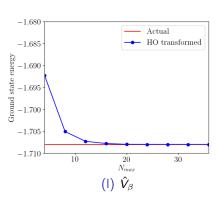
# 2-Body $V_{\alpha}$ vs. $V_{\beta}$



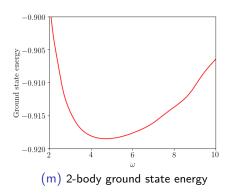


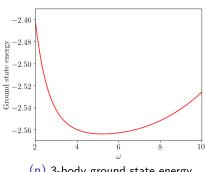
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## $\omega$ Optimization





(n) 3-body ground state energy