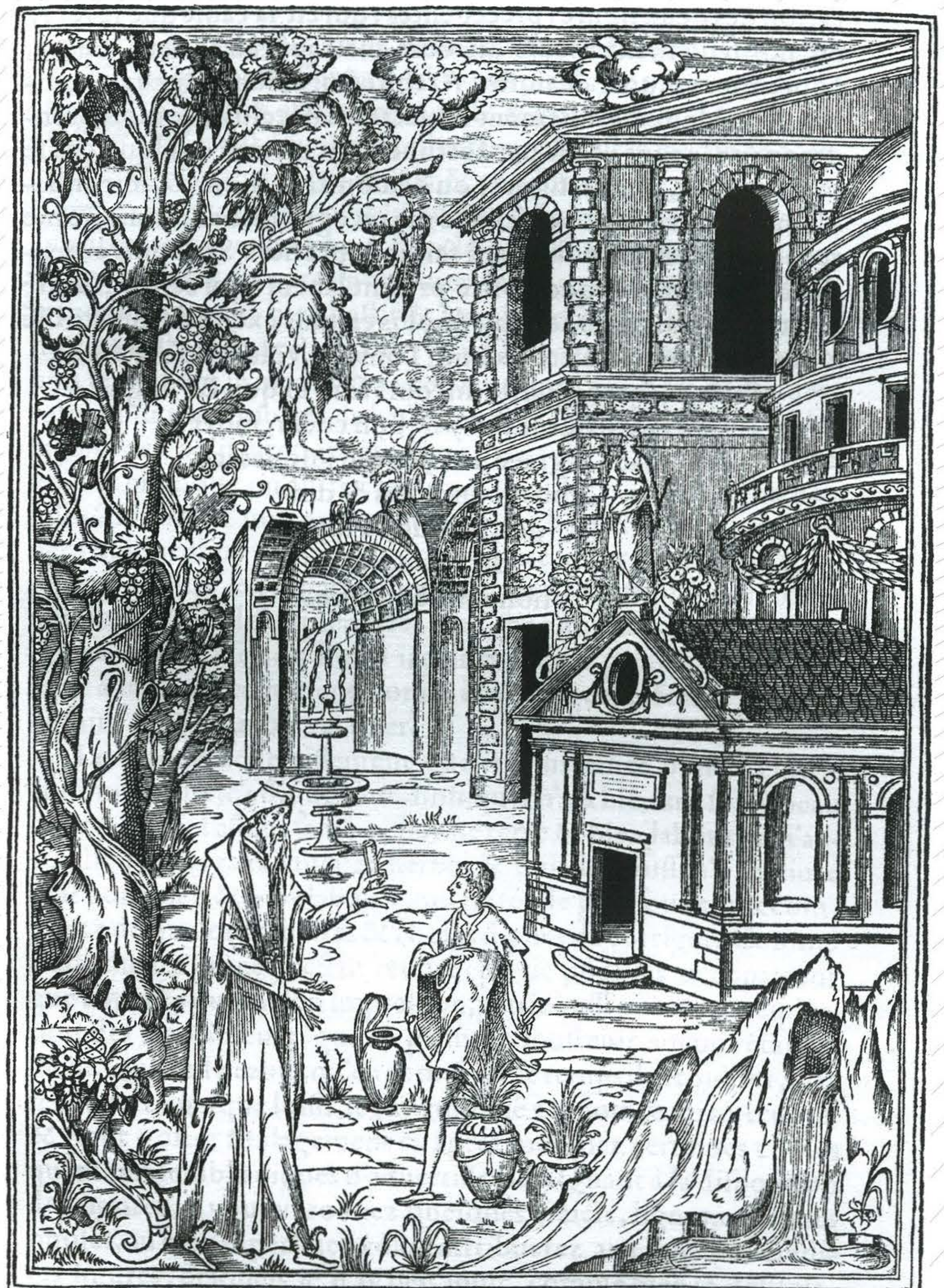

Figure 1. Philibert De L'Orme, *The Good Architect*, from *Le Premier Tome de l'Architecture de Philibert De L'Orme* (Paris), 1567
Restraint is clearly among the characteristics of the good architect.

Antoine Picon

ARCHITECTURE AND MATHEMATICS

BETWEEN HUBRIS AND RESTRAINT

The introduction of calculus-based mathematics in the 18th century proved fatal for the relationship of mathematics and architecture. As **Antoine Picon**, Professor of History of Architecture and Technology at Harvard Graduate School of Design, highlights, when geometry was superseded by calculus it resulted in an ensuing estrangement from architecture, an alienation that has persisted even with the widespread introduction of computation. It is a liaison that Picon characterises as having shifted between hubris and restraint.

Until the 18th century, following the Renaissance's preoccupations with perspective and geometry, the relations between mathematics and architecture were both intense and ambiguous. Mathematics was sometimes envisaged as the true foundation of the architectural discipline, sometimes as a collection of useful instruments of design. Mathematics empowered the architect, but also reminded him of the limits of what he could reasonably aim at. Inseparably epistemological and practical, both about power and restraint, the references to mathematics, more specifically to arithmetic and geometry, were a pervasive aspect of architecture.

Towards the end of the 18th century, the diffusion of calculus gradually estranged architecture from mathematics. When they looked for foundations, 19th-century architects like Eugène-Emmanuel Viollet-le-Duc were more interested by the sciences of life than by the new calculus-based mathematics of their time. While arithmetic and geometry remained highly useful practical tools, they gradually lost their aura of cutting-edge design techniques.

This estrangement has lasted until today, even if the computer has enabled architects to put calculus to immediate use. Actually, today's situation is quite paradoxical insofar that under the influence of digital tools architecture has never used so many mathematical objects, from Bézier curves to algorithms, while remaining indifferent to the question of its relation to mathematics. A better understanding of the scope and meaning that mathematics used to have for architects might very well represent a necessary step in order to overcome this indifference. The purpose of this article is to contribute to such an understanding.

Mathematics as Foundation

From the Renaissance onwards, the use of mathematical proportions was widespread among architects who claimed to follow the teachings of Roman architect and engineer Vitruvius. This use was clearly related to the ambition to ground architecture on firm principles that seemed to possess a natural character. For the physical world was supposed to obey proportions, from the laws governing the resistance of materials and constructions to the harmonic relations perceived by the ear.

The interest in proportions was related to the intuitive content that arithmetic and geometry possessed. As we will see, the strong relation between mathematics and the intuitive understanding of space was later jeopardised by the development of calculus. But it bore also the mark of two discrepant points of view.

Proportion could be first interpreted as the very essence of the world. Envisaged in this light, proportion possessed a divine origin. The 17th-century French theologian and philosopher Jacques Bénigne Bossuet gave a striking expression of this conception when he declared in one of his treatises that God had created the world by establishing the principles of order and proportion.[1] In this perspective, proportion was about power, about the demiurgic power of creation, and the architect appeared as the surrogate of God when he mobilised, in his turn, this power to plan his buildings.

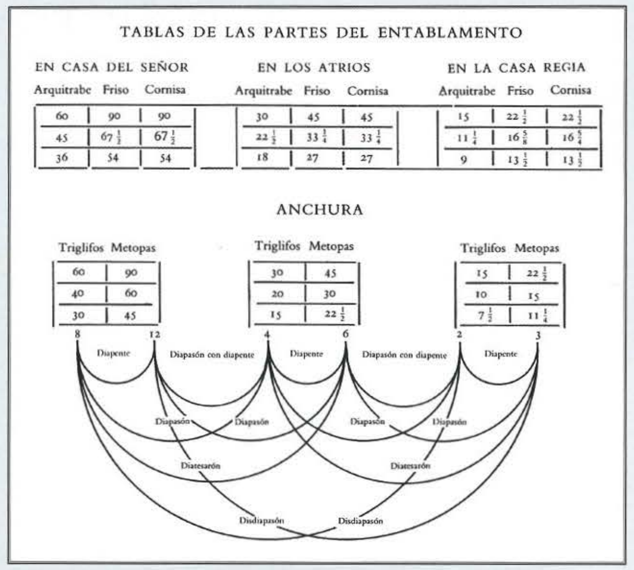

Figure 2. Juan Bautista Villalpandus, Chart of the proportions of the entablature of the Temple of Jerusalem, from *El Tratado de la Arquitectura Perfecta en La Última Visión del Profeta Ezequiel* (Rome), 1596–1604
The architectural discipline was supposed to emulate the creative power of the Divinity by following those very rules of proportion that were constitutive of the Creation and that had been dictated by God to the builders of the Temple of Jerusalem.

Following Spanish Jesuit Juan Bautista Villalpandus's detailed reconstruction of the Temple of Jerusalem based on an interpretation of Prophet Ezekiel's vision and published between 1596 and 1604, the large array of speculations regarding the proportions dictated by God to the builders of the temple stemmed from the belief that architectural design was ultimately an expression of demiurgic power. Historian Joseph Rykwert has shown how influential these speculations were on the development of the architectural discipline in the 16th and 17th centuries.[2] Through the use of proportions the architect experienced the exhilaration of empowerment.

But proportion could also be envisaged under a different point of view, a point of view adopted by Renaissance theorist Leon Battista Alberti for whom the purpose of architecture was to create a world commensurate with the finitude of man, a world in which he would be sheltered from the crude and destructive light of the divine. In this second perspective, proportion was no longer about the hubris brought by unlimited power, but about its reverse: moderation, restraint. Philosopher Pierre Caye summarises this second attitude by stating that the aim of architects like Alberti was to rebuild something akin to Noah's Ark rather than to emulate the Temple of Jerusalem.[3] It is worth noting that such a conception was to reappear much later with Le Corbusier and his Modulor, which was inseparable from the attempt to conceive architecture as the core of a totally designed environment that would reconcile man within his inherent finitude.[4]

Exhilaration of power on the one hand, and the restraint necessary to protect man from the unforgiving power of the divine on the other: the reference to mathematics in the Vitruvian tradition balanced between these two extremes. Such polarity was perhaps necessary to give proportion its full scope. Now, one may be tempted to generalise and to transform this tension into a condition for mathematics to play the role

of a basis for architecture. Another way to put it would be to say that in order to provide a truly enticing foundational model for architecture, mathematics must appear both as synonymous with power and with the refusal to abandon oneself to seduction of power. For this is what architecture is ultimately about: a practice, a form action that has to do both with asserting power and refusing to fully abandon oneself to it. On the one hand the indisputable presence of the built object is synonymous with the permanence of power; on the other the same built object opposes its opacity, and a certain form of instability, at least if we are to follow Peter Eisenman's analyses, to the sprawling domination of power.

To conclude on this point, one may observe that this polarity, or rather this balance, has been compromised today. For the mathematical procedures architects have to deal with, from calculus to algorithms, are decidedly on the side of power. Nature has replaced God, emergence the traditional process of creation, but its power expressed in mathematical terms conveys the same exhilaration, the same risk of unchecked hubris as in prior times. What we might want to recover is the possibility for mathematics to be also about restraint, about stepping aside in front of the power at work in the universe.

It is interesting to note how the quest for restraint echoes some of our present concerns with sustainability. The only thing that should probably not be forgotten is that just like the use of mathematics, sustainability is necessarily dual; it is as much about power as about restraint. Our contemporary approach to sustainability tends to be as simplistic as our reference to mathematics, albeit in the opposite direction.

Tools for Regulation and/or Invention

Let us turn now to mathematics as tools. From an architectural standpoint, the same mathematical principle can be simultaneously foundational and practical. Vitruvian proportion corresponded both to a way to ground architecture theoretically and to a method to produce buildings. In this domain also, a duality is at work.

This second duality has to do with the fact that tools may be seen as regulatory instruments enabling coordination and control, giving precedence to standardisation upon invention. Until the 18th century, most uses of mathematics and proportion had in practice to do with coordination and control rather than with the search for new solutions. But tools can be mobilised to explore the yet unknown; they can serve invention. From the Renaissance on, the geometry used for stone-cutting, also known as stereotomy, illustrates perfectly this ambivalence.

Figure 3. Le Corbusier, Unité d'Habitation de Firminy, France, 1946–52
Le Corbusier's objective is to place man within a totally designed environment.

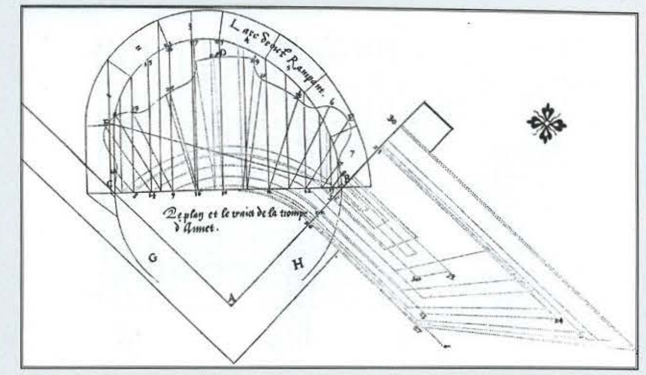

Figure 4. Philibert De L'Orme, geometrical construction of the squinch of Anet, from *Le Premier Tome de l'Architecture de Philibert De L'Orme* (Paris), 1567
For De L'Orme, mathematics, and geometry in particular, was the foundation of architectural invention.

On the one hand, from Philibert De L'Orme to Gaspard Monge, this geometry of a projective nature was seen by its promoters as a means to exert greater control on architectural production. But this ambition was accompanied with the somewhat contradictory desire to promote individual invention. De L'Orme epitomises this contradiction. Besides major architectural realisation like the castle of Anet or the Tuileries royal palace, his main legacy was the first comprehensive theoretical account of the geometric methods enabling designers to master the art of stereotomy. Until De L'Orme, this art was a secret transmitted from master mason to apprentice, a secret based on recipes and knowhow. De L'Orme was actually the first to understand some of the underlying projective principles at work in such a practice. For the architect, the aim was twofold. First, he wanted to achieve a better control of the building production. It is not fortuitous that De L'Orme was the first architect to be entrusted with major administrative responsibility by the crown. But the objective was also to invent. The best demonstration of this art of invention was in his eyes the variation of the Montpellier squinch that he designed for Anet. The identification of the true regulatory principles at work in a given practice could lead to truly innovative combinations.[5]

Control and invention, these two objectives were also on the agenda of mathematician Gaspard Monge, the inventor of descriptive geometry. Descriptive geometry actually derived from the geometry used for stereotomy that had been explored by De L'Orme at the dawn of the French Renaissance. For Monge, descriptive geometry was both about standardisation and invention.[6]

In a similar vein to the hypothesis concerning mathematics as a theoretical foundation for architecture, the following can be proposed: in order to provide truly enticing tools for designers, mathematics must be about both control and invention. Today, what might be often lacking is not so much

. . . calculus would also be instrumental in the development of economic theory by providing the means to study the circulation of goods and capitals.

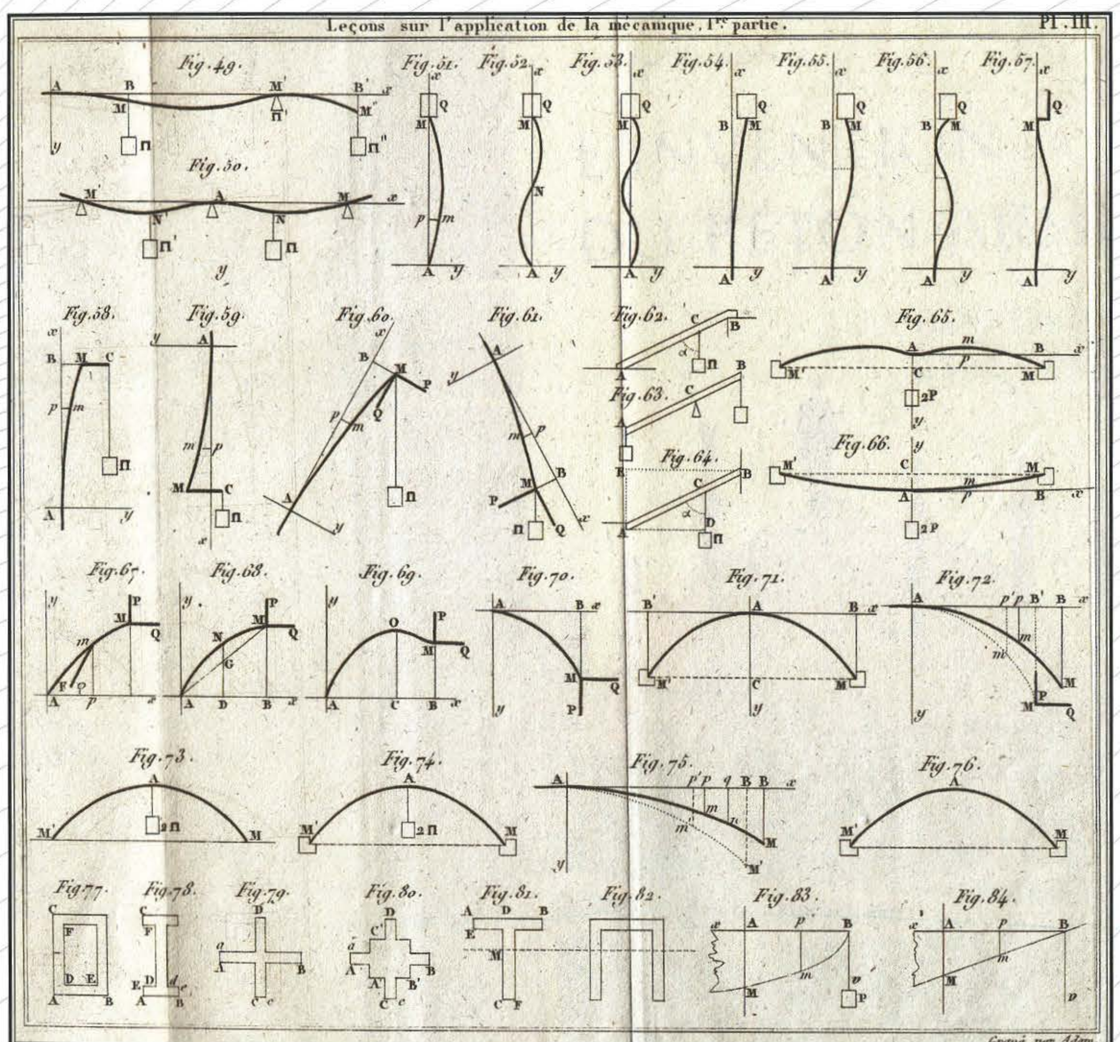

Figure 5. Claude Navier, Study of the bending of elastic curves, from *Résumé des Leçons (...) sur l'Application de la Mécanique à l'Etablissement des Constructions et des Machines* (Paris), 1826
Navier's work marks the triumph of calculus in the science of constructions.

the capacity of mathematics to be on the side of invention, but rather its contribution to the framing and standardisation of design problems. In contemporary cutting-edge digital practices, mathematical entities and models are most of the time mobilised in a perspective that has to do with emergence, with the capacity to amaze, to thwart received schemes.

One of the best examples of this orientation lies in the way topology has been generally understood these days. What has most of the time retained the imagination of designers are topological singularities, what mathematician René Thom has dubbed as 'catastrophes'. This explains the fascination exerted on architects by topological entities like the Möbius strip or the Klein bottle. This interpretation of topology is at odds with what mathematicians consider as its principal objective, namely the study of invariance. Whereas architects are usually interested in extreme cases that allow surprising effects to emerge, the mathematicians' perspective is almost opposite. It has to do with conservation rather than sheer emergence. It might be necessary to reconcile, or at least articulate these two discrepant takes on the role of mathematics to fully restore their status.

The Calculus Breaking Point

The end of the 18th century clearly marks a breaking point in the relationship between architecture and mathematics. Until that time, arithmetic and geometry had been a constant reference for the architect, as foundational knowledge as well as practical tools, as empowerment and as incentive for restraint, as a means of control and standardisation as well as a guideline for surprising invention.

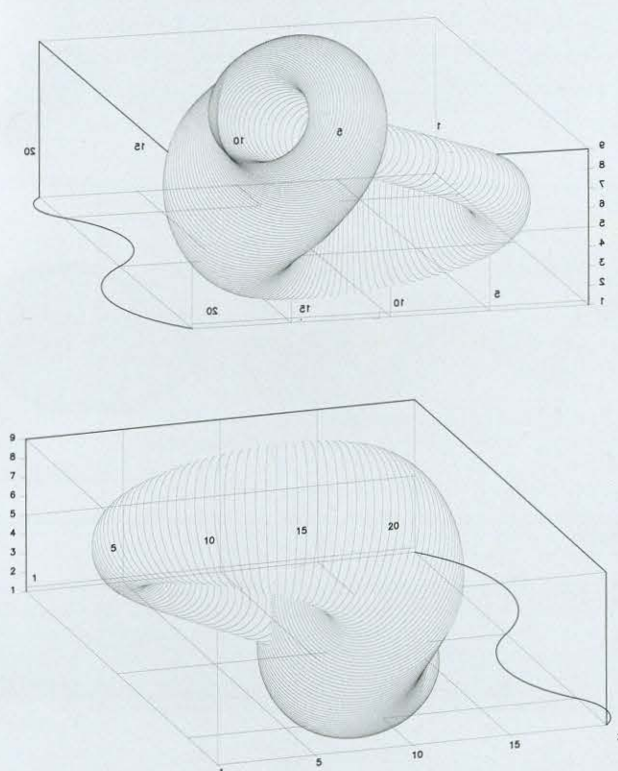

Figure 6. IJP, Klein Bottle
An intriguing topological singularity.

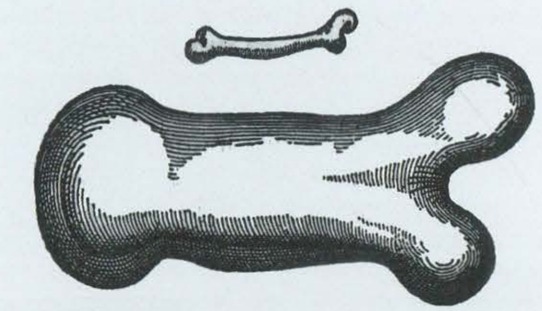

Figure 7. Galileo Galilei, Animal bones, from *Discorsi e dimostrazioni matematiche intorno a due nuove scienze*, 1638
The example is used by Galileo to illustrate how strength is not proportional to size, contrary to what proportion-based theories assumed.

In all these roles, mathematics had a strong link with spatial intuition. Arithmetic and geometry were in accordance with the understanding of space. This connivance was brought to an end with the development of calculus and its application to domains like strength of materials. First, calculus revealed the existence of a world that was definitely not following the rules of proportionality that architects had dwelt upon for centuries. Galileo, for sure, had already pointed out the discrepancy between the sphere of arithmetic and geometry and domains like strength of materials in his *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* (Discourses and Mathematical Proofs Regarding Two New Sciences) published in 1638. But such discrepancy became conspicuous to architects and engineers only at the end of the 18th century.

The fact that some of the operations involved in calculus had no intuitive meaning was even more problematic. It meant that the new mathematics were like machines that possessed a certain degree of autonomy from intuitive experience. A century later, nascent phenomenology would return to this gap and explore its possible philosophical signification.

Among the reasons that explain such a gap, the most fundamental lies in the fact that calculus has generally to do with the consideration of time instead of dealing with purely spatial dimensions. What was the most puzzling, like the so-called infinitely small, were actually elementary dynamic processes rather than static beings. Calculus's most striking results were by the same token related to dynamic phenomena like hydraulics and the study of flows. Later on, calculus would also be instrumental in the development of economic theory by providing the means to study the circulation of goods and capitals.

Another reason explaining the growing gap between architecture and mathematics was the new relation between theory and practice involved in the transition from arithmetic and geometry to calculus. In the past, mathematical formulae were seen as approximate expressions of a higher reality deprived, as rough estimates, of absolute prescriptive power. One could always play with proportions for they pointed towards an average ratio. The art of the designer was all about tampering with them in order to achieve a better result. As indicators of a higher reality, formulae were an

Figure 8. Jenny Sabin + Jones LabStudio, Branching Morphogenesis, 2008
Complex organisation is today found at every level of living organism, from macro- to microstructures. Computer simulation is instrumental in the exploration of such complex organisational patterns.

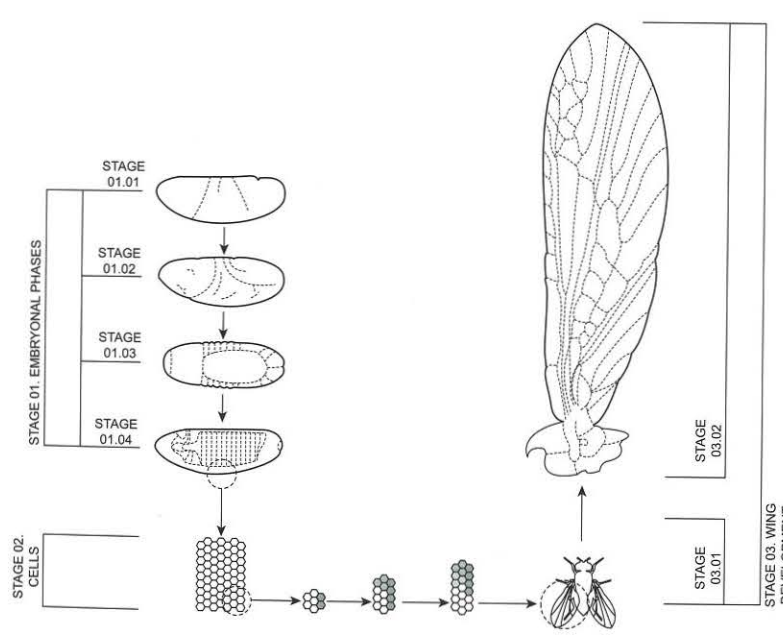

Figure 9. Michael Weinstock, Drosophilia Wing Development, 2010
The emergence of small complex anatomical organisations makes possible the emergence of ever larger and even more complex organisations. Complexity builds over time by a sequence of modifications to existing forms.

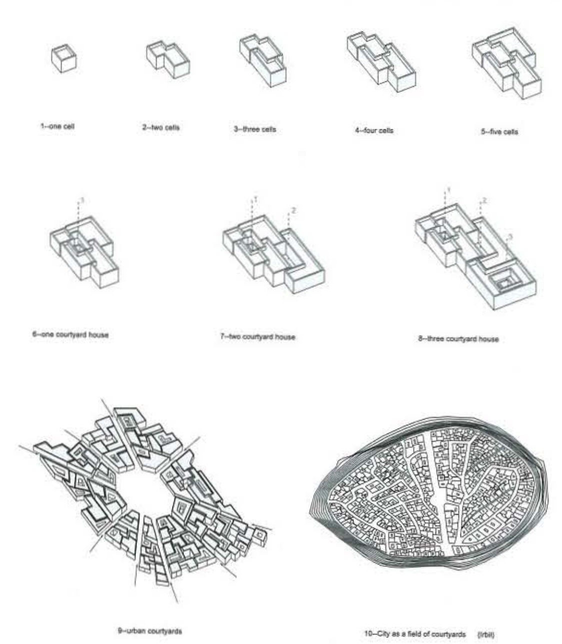

Figure 10. Michael Weinstock, Architectural and urban forms in Mesopotamia
The organic property of emergence is supposed to apply to both the natural and the human realms.

expression of power, as average values that could be tampered with; they went along with the notion of restraint.

In the new world of calculus applied to domains like strength of materials or hydraulics, mathematics no longer provided averages but firm boundaries that could not be tampered with. From that moment onwards, mathematics was about setting limits to phenomena like elasticity, then modelling them with laws of behaviour. Design was no longer involved. Theory set limits to design regardless of its fundamental intuitions.[7]

As a consequence, 19th-century architects became far less interested in the new mathematics of their time than in history, anthropology and the biological sciences. Theorists like Viollet-le-Duc or Gottfried Semper are typical of this reorientation. Despite claims to the contrary made by architects like Le Corbusier, this indifference to mathematics was to remain globally true of modern architecture.

The Ambiguities of the Present

Today the computer has reconciled architecture with calculus. For the first time, architects can really play as much with time as with space. They can generate geometric flows in ways that transform architectural forms into sections or freezes of these flows. But this has not led so far to a new mathematical imaginary. To put it in the historical perspective adopted here, mathematics appear neither as foundational nor as tools.

Various reasons may account for this situation. First, the mathematical principles are very often hidden behind their effects on the screen. In many cases the computer veils the presence of mathematics. This is a real issue that should be overcome in the perspective of truly mastering what is at stake in computer-aided design.

Second, one has the disturbing nature of the underlying mathematical principles mobilised by design. In computer-aided design, one no longer deals with objects but with theoretically unlimited series of objects. One deals also with relations. This is what parametric design is about: considering relations that can be far more abstract than what the design of objects usually entails. Scripting and algorithmics reinforce this trend. With algorithmics, one sees the return of the old question of the lack of intuitive content of some operations.[8]

However, the main reason may have to do with the pervasiveness of a new kind of organicism, vitalism or, rather, materialism, a materialism postulating an animated matter, a matter marked by phenomena like emergence, a matter also in which change is as much qualitative as quantitative.[9] Mathematics serves this new materialism, but is not seen as the most profound layer of it. This might result from the fact that the polarities evoked earlier have not been reconstituted.

In architecture, today's mathematics is about power and invention. Restraint and control through the establishment of standards have been lost so far. The reconstitution of this polarity might enable something like the restoration of an essential vibration, something akin to music. Architects need mathematics to embrace the contradictory longing for power and for restraint, for standardisation and for invention.

To achieve that goal, one could perhaps follow a couple of paths. One has to do with parametricism, but parametricism understood as restraint and not only as power, and also parametricism as having to do with the quest for standards and not only of invention.

Another path worth exploring is simulation. Simulation goes with the new importance given to scenarios and events. In this perspective, architecture becomes something that happens, a production comparable to a form of action, an evolution that lies at the core of today's performalist orientation.[10] Mathematics and architecture might meet again in the name of action, under the aegis of an ethics prescribing when to use power and when to adopt restraint. ⌀

Notes

1. Jacques Bénigne Bossuet, *Introduction à la Philosophie, ou de la Connaissance de Dieu, et de Soi-Mesme*, R-M d'Espilly (Paris), 1722, pp 37–8.
2. Joseph Rykwert, *On Adam's House in Paradise: The Idea of the Primitive Hut in Architectural History*, Museum of Modern Art (New York), 1972.
3. Pierre Caye, *Empire et Décor: Le Vitruvianisme et la Question de la Technique à l'Age Humaniste et Classique*, J Vrin (Paris), 1999.
4. See Christopher Hight, *Architectural Principles in the Age of Cybernetics*, Routledge (New York), 2008, pp 55–69.
5. Philippe Potié, *Philibert De L'Orme: Figures de la Pensée Constructive*, Parenthèses (Marseilles), 1996.
6. Joël Sakarovitch, *Epures d'Architecture: De la Coupe des Pierres à la Géométrie Descriptive XVIe-XIXe Siècles*, Birkhäuser (Basel), 1998.
7. See Antoine Picon, *L'Invention de l'Ingénieur Moderne: L'Ecole des Ponts et Chaussées 1747–1851*, Presses de l'ENPC (Paris), 1992, pp 498–505.
8. Antoine Picon, *Digital Culture in Architecture: An Introduction for the Design Professions*, Birkhäuser (Basel), 2010.
9. Michael Weinstock, *The Architecture of Emergence: The Evolution of Form in Nature and Civilisation*, John Wiley & Sons (Chichester), 2010.
10. Yasha Grobman and Eran Neuman (eds), *Performalism: Form and Performance in Digital Architecture*, Tel Aviv Museum of Art (Tel Aviv), 2008.