

HOW TIME PASSES BY

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Published in Vol.3 in the English edition of *Die Reihe* musical journal 1959

The original version dates from 1957

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Music consists of order-relationships in time; this presupposes that one has a conception of such time. We hear alterations in an acoustic field: silence – sound – silence, or sound – sound; and between the alterations we can distinguish time-intervals of varying magnitude. These time-intervals may be called *phases*.

In order to compare one group of phases with another, we make a distinction between ‘periodic’ and ‘aperiodic’ phase-groups, and, between these extremes, we distinguish a greater or smaller number of transitional stages (as deviations from either periodicity or aperiodicity, depending on which predominates).

To differentiate various phases, we compare one phase-duration with another. We measure time in phase-durations, or in durations of phase-groups. Our sense-perception measures shorter or longer phases. Proportions serve for more exact definition – one phase is twice, thrice as long as another. In order to fix proportions, one chooses a unit-quantum, and this is usually based on time as measured by the clock; we say one phase-duration lasts one second, two seconds, a tenth of a second.

Our sense-perception divides acoustically-perceptible phases into two groups; we speak of *durations* and *pitches*. This becomes clear if we steadily shorten the length of a phase (e.g., that between two impulses) from 1'' to $\frac{1}{2}$ '', to $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, etc. Until a phase-duration of approx. $\frac{1}{16}$ '', we can still just hear the impulses separately; until then, we speak of ‘duration’, if of one that becomes extremely short. Shorten the phase-duration gradually to $\frac{1}{32}$ '', and the impulses are no longer separately perceptible; one can no longer speak of the ‘duration’ of a phase. The latter process becomes perceptible, rather, in a different way: one perceives the phase-duration as the ‘pitch’ of the sound. $\frac{1}{32}$ '' phase-duration makes us, say, a “low” note’. If a musician has learned to hear ‘absolute’ pitches in the scale system as we have known it up to now, he will say that he hears approximately double-bass B (,,B). But to recognize a pitch, the ear requires at least two equal phase-durations, otherwise it cannot ‘tune in’ – the ‘note’ is too short. Our sense-perception cannot react to a single phase quickly enough to perceive it as ‘duration’, so it summarizes several quanta to give the sensory quality ‘pitch’. Steadily shorten the phase-duration still further, from $\frac{1}{32}$ '' to $\frac{1}{64}$ '' (B), $\frac{1}{128}$ '' (B), $\frac{1}{256}$ '' (b), etc., and the note ascends as a glissando from ‘low to high’, and we can still speak of clearly recognisable pitches with phase-durations up to approx. $\frac{1}{64}$ ''. We can perceive still shorter phase-durations up to approx. $\frac{1}{1024}$ '', but exact pitch-orientation gets lost in this time-sphere. Higher still, we do not ‘hear’ any more.

Thus one differentiates phase-durations up to approx. $\frac{1}{16}$ '' as durations, and, in music up to the present time, so-called ‘metre and rhythm’ (the time-ordering of durations) took place in the area between approx. 6'' and $\frac{1}{16}$ ''. The time-area in which phase-proportions were defined as pitch-relations – harmonic and melodic – extends from approx. $\frac{1}{16}$ '' to $\frac{1}{3200}$ '' phase-duration; instruments with higher notes have not been used.

Thus the transition from one time-area to another causes a change in our perception of phases. This observation could form the basis of a new morphology of musical time. The *notation of durations* has involved the use of signs that correspond to the system of whole numbers. Thus, different durations were only indicated in so far as each long duration was the whole-number multiple of the unit-quantum, of a defined shortest duration. The absolute duration of the shortest time-quantum either remained indefinite, or was metronomically defined. Thus, if the sign ♩ was selected as the smallest unit, then with a given metronome marking $\text{♩} = 60$, it meant $\frac{1}{60}$ ', 1''. And all the other signs – ♩ , ♩ , ♩ , ♩ – ♩ , etc., were whole-number multiples of this: 2'', 3'', 4'', 5'', etc.

A second type of indication took as its starting-point not a *smallest quantum* which was multiplied, but a *largest quantum* which it then divided. Thus if the sign ○ were selected as largest unit, then with a given metronome marking $\text{○} = 60$, it meant 1'', and the fractions of this duration were designated ♩ , ♩^3 , ♩ , ♩^5 , etc., as $\frac{1}{2}$ '', $\frac{1}{3}$ '', $\frac{1}{4}$ '', $\frac{1}{5}$ '', etc.

But our powers of discrimination cannot register the fact that, in themselves, two differences are of equal size; one has to take into account the absolute lengths of the durations involved. Thus if a first phase lasts 1'', and a second phase 2'', there is a difference of 1''. Two phases of 11'' and 12'' duration have the same difference of 1''. But we perceive the difference between 1'' and 2'' as relatively large, whereas the same difference between 11'' and 12'' is hardly perceptible. This means that we do not perceive differences, but rather proportions: 1:2 is the larger proportion, as compared with 11:12.

For a *scale of durations*, whose dissimilarities shall be perceived as equally large, one must use logarithmic relationships. The interval, i.e. the size-relationship, is thereby defined: a scale with the constant interval of perception 2/1 from duration to duration would be $2^1, 2^2, 2^3, \dots (2, 4, 8, \dots)$, hitherto designated as $\text{♩}, \text{♩}, \text{♩}, \dots$ Such a scale-interval may not, however, be too small, because our powers of discrimination impose limits; with relationships of approx. 15:16 we perceive durations as almost the same length. This interval 15:16 corresponds to the relationship which is decisive for the discrimination of ‘duration’ and ‘pitch’ (at the ‘threshold of hearing’¹). The same interval is also approached in the chromatic scale of pitches used up to now, where the ‘semitone’ (approx. 15/16) is defined as the smallest perceptible quantum.

In the time-area where phase-durations are designated as pitches, music has hitherto used a severely limited selection of phase-lengths. As a result of long development, we today find a chromatic system, at the basis of which the simplest phase-relationship 2:1 (the ‘octave’) is the main proportion, and where each octave is once again logarithmically divided into twelve intervals ($\sqrt[12]{2}$), perceived as equal. Most people who today write pitches in this system are not aware that they are giving form to time-proportions. This is primarily the fault of the one-sided development of instrumental music, and of instrument construction. Instruments with prepared scale-tuning, and an increasing mechanisation of note-production (tabular notation, etc.) have eradicated the consciousness of what really happens when the ‘pitch a’ is produced, with a phase-duration of 1/440”.

This should be discussed further. We are familiar with the pitch-keyboard, on which

¹ Trans. Note: ‘Threshold of hearing’ should not be confused with ‘threshold of audibility’ – a standard used for the phon-curve on a decibel-frequency graph.

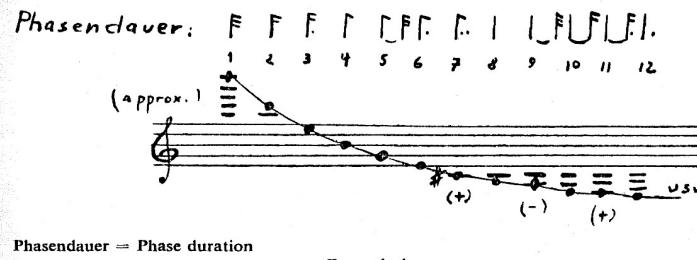
it is possible to present a chromatic scale of 88 pitches with a constant phase-relationship $\sqrt[13]{2}$; where every thirteenth chromatic step is perceived as 'twice as high, or low' as the first. If the development of the duration system were equally advanced, there would correspondingly be a keyboard having a scale of 88 'durations' with a constant phase-relationship $\sqrt[13]{2}$, where likewise every thirteenth chromatic step would be perceived as 'twice as short, or long'. If it should be objected that there is no sense in imagining a duration-keyboard unless we also imagine something that is to have duration, we must not get annoyed. It is a question of turning a familiar idea upside-down: one depresses a key, and the pitch is determined by which key is pressed down; on the piano, a string then vibrates periodically with a certain phase-duration, and this continues as long as the finger is held on the key. Now let us imagine the reverse: one depresses a key, and this releases a mechanism which measures a defined length of the note; and one determines the pitch – i.e., the time-duration of a single phase – by the variable pressure of the key (mechanically, this would mean that through the variable key-pressure, the vibrating string would be lengthened or shortened). It would then be irrelevant how long the finger remained on the key.

Up to now, the pitch of a note has mainly been produced mechanically (and we have mentioned the fact that this is true not only of keyboard instruments), while larger-scale phase-relationships (i.e., durations) have taken shape through the direct conversion of feeling into an action of some given length; as an adequate complement, one could imagine a system, working the other way round, for the representation of proportioned durations; the most useful thing would be an instrument bringing together both these scales. But this is not the way matters have developed so far, and such considerations bring us on to something else.

In *serial music* an attempt is made to put the time-proportions of the elements in order, by means of series. The beginnings of serial control had to do with that sphere of time-proportions which is perceived as pitch. The system of twelve notes in the octave was taken as given (in what follows, the relationship 2:1, or simply the number 2, will be substituted, when the context requires it, for the term 'octave', which has really become meaningless). The twelve notes were a reasonably limited number of magnitudes. From these magnitudes one could construct a series, and the distribution of the intervals between each pair of magnitudes would produce relationships peculiar to that particular series. Between the one extreme, where all eleven intervals are the same (chromatic series), and the other, where all eleven intervals are different (all-interval series), there was considerable latitude. The work for which the series was constructed would have a characteristically uniform pitch-structure, corresponding to the layout and distribution of the intervals in the series. Almost thirty years later, and after many detours, it occurred to composers to extend this principle into that sphere of time-proportions that we distinguish as durations. A *scale of twelve durations* was then added, which was intended to correspond to the chromatic scale of twelve pitches in 2. This scale, however, could neither be related to a system that already existed, nor could it be developed into one that would correspond. It was arrived at by the *multiplication of a smallest unit* from 1 $\frac{1}{2}$ to 12 $\frac{1}{2}$ /1 $\frac{1}{2}$ to 12 $\frac{1}{2}$ /1 $\frac{1}{2}$ to 12 $\frac{1}{2}$ /etc. What is such a scale?

We have already mentioned that durations are distinguished by the relationships, not

by the differences, of phases. In a series from $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, ..., $\frac{1}{11}$, $\frac{1}{12}$, ... we automatically perceive a hierarchy; the relationship $\frac{1}{2}:\frac{1}{3}(1:2)$ is the largest and simplest (the 2 of durations); the relationship $\frac{1}{11}:\frac{1}{12}(11:12)$ is the smallest and most complicated. Let us see what such a scale of durations really is, in the sphere of smaller phase-lengths: if the shortest phase-duration were $\frac{1}{2000}$ ", one twice as long would be $\frac{1}{1000}$ ", thrice would be $\frac{1}{666}$ " ... twelve times would be $\frac{1}{166}$ ". This is called a *subharmonic series of proportions*:

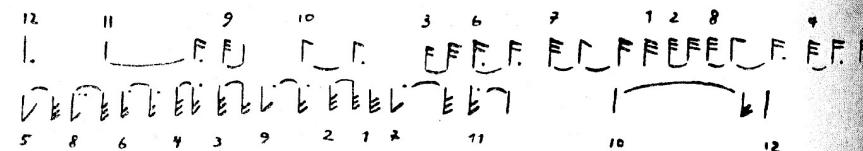


In comparison with a scale built on chromatic intervals, the subharmonic scale is a *mode*. The composer Messiaen was well aware of this when he characterised such scales as 'modes' in his fourth Study for piano.

A scale from 1 $\frac{1}{2}$ to 12 $\frac{1}{2}$ has eleven intervals that we perceive as being variously large, and that correspond to those of the subharmonic series. What scale of durations would correspond to the chromatic scale in the hitherto-existing pitch-system? It could by no means be presented with the traditional signs for durations, since these are built on a system of whole-number relationships. We should have to divide the 2 of durations ('time-octave') into eleven apparently equal intervals, i.e., to fix twelve logarithmic values between $\frac{1}{2}$ and $\frac{1}{12}$, between $\frac{1}{12}$ and $\frac{1}{1}$, and so on.

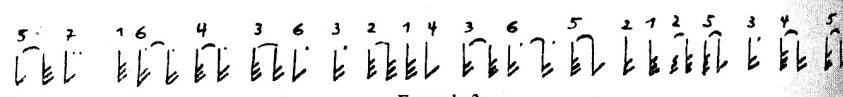
The attempt to use a subharmonic scale of twelve durations as if it corresponded to the chromatic scale of twelve pitches leads to major contradictions, particularly when such a scale is adopted and used uncritically. It was not surprising that in a time-structure formed with such a series of durations, the long values devoured the short ones; and that, even where the smallest unit was extremely short, the result was a slow average speed. Furthermore, the proportions were completely uneven, and hierarchically preordained. It was recognised as an immutable stipulation, if these proportions were to be at all perceptible, that the absolute duration of the shortest phase of such a series of proportions should be fixed, i.e., that the tempo should if possible be fixed metronomically, or at least kept constant. When structural changes occurred, one willingly altered the tempo; that is, the whole subharmonic scale slid 'up or down', so that one did not continually travel around in the same mode: the scale was *transposed*. If a 2-relationship were required from scale to scale (an 'octave-transposition'), the sign for the smallest phase was changed; instead of $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ etc., one now wrote $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}$ etc. (if the transposition were upwards), or $\frac{1}{1}, \frac{1}{2}, \frac{1}{4}$ etc. (if it were downwards). Or one made a 2:3 ('fifth') transposition, where the smallest interval was 'dotted', thus: $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ etc. This much was possible with the means of traditional notation.

Composers were more or less conscious that within the subharmonic duration scales, from which series arose, proportions were uneven. They found a truly drastic way to escape from the difficulties into which such irrevocably slow time-structures had led them. Various such subharmonic series of proportions were simply piled on top of one another, so that, through the correspondingly multiplied number of alterations, greater average speed would be achieved. These series either had the same smaller phase (.) and were merely arranged in different proportions:



Example 2

or series in which the shortest note-values were different were superposed. In the latter case something similar to polymodality arose, on a basis of subharmonic scales, but the process the original intention of forming phase-relationships serially was nullified. The result, apart from the stylistic inappropriateness of using modal and polymodal time-structures, is that the intervals stemming from such a superposition give anything but serial proportions. We see this in the intervals resulting from the superposition of the two series above:



Example 3

Such a procedure corresponds exactly to the treatment of pitch-series that was criticised in the popular 'twelve-tone' method: it is the remains of the stylistic practice of thinking in parts, while handling very questionably the effect they make together.

Nor is there any sense in attempting to conceal such superposition of 'parts' by making the parts cross a great deal, and by making great differences between the phase-durations and the real 'note-durations'. (The latter is a real distinction; a note can independently of the interval of entry of the next note, have reached its minimum intensity earlier - \downarrow - or can still continue after the next note has begun - \uparrow ; thus the effective note-duration is determined by the number of phases of the same duration if the note remains at the same pitch - that succeed each other. We can therefore describe a note-duration as a *phase-group*.) The density of a simultaneous superposition of series was varied; this was justified on the ground that one could thereby regulate the changing average speeds.

A much more significant consequence, however, was that not all the elements were used at every moment of a work - that the same series was not reeled off continually; supra-ordered series were introduced - series of series - that made *selections of elements* for the respective *structural phases*. In a first structural phase, only the elements 1-5, for example, were used; in a second, only 1-7; in a third, 1-11, etc., so that the relationship of long to short values was always differently handled, and the overall result was an

organically perceived time-structure. Correspondingly, series were sub-grouped, as had already become the practice with pitches; i.e., supra-ordered series selected elements in groups from the element-series. If, for example, the *element-series* ran thus: 12, 11, 9, 10, 3, 6, 7, 1, 2, 8, 4, 5/11, 10, 8, 9, 2, 5, 6, 12, 1 . . . and 5, 8, 6, 4, 3, 9 . . . was selected as the supra-ordered *group-series*, this meant that five elements would be used in a first structure, namely 12, 11, 9, 10, 3, then eight, namely 6, 7, 1, 2, 8, 4, 5, 11, then six, namely 10, 8, 9, 2, 5, 6, etc. Either the phase-durations of the structures, in which such groups of elements would be used, were pre-selected (e.g., 30'', 2', 60'', etc.), or a *multiplication-series* was taken in addition, to regulate how often each group of elements should be permuted. If the multiplication-series was 8, 11, 9, 7, . . . , this meant that the first group of five elements would have eight permutations, the second group of eight elements would have eleven permutations, the third group of six elements would have nine permutations, etc. These series with various functions could either be identical, or be derived from a common original series. It was quite natural that phase-proportions, at first composed only between the single elements (in the 'pointillist' style), should be applied to all the phases of the supra-ordered structural sequences; and that all the micro-time processes should be made to accord with those in macro-time ('group-composition').

To compose separate parts as *polyrhythm* is a stylistic error; and this criticism led to a result which is certainly the most relevant for further developments. One said to oneself: when, in an initial phase of the structure, three such subharmonic time-series overlap, and in a second one, five, etc., there results an overall impression of varying density, and the average speeds are a complex result of serial density-relationships. It is also possible, in the process, to take the parts so far from their original function as 'voices' - i.e., their 'register' - that they become merely inextricable threads in a network, and this network must be audible only as such, and not as a superposition of parts. If in the end one carries such polyrhythmic complexes so far that 'pointillist' hearing of the individual duration-relationships turns into structural hearing, then serial method will be concerned, above all, with such *statistical form-criteria*, with average relationships.

A corresponding procedure was followed in the sphere of pitch; there were structures in which single notes and the intervals between them could not be heard 'pointillistically', but for which the average properties of the groups - of the 'flocks of notes' in particular pitch-fields - were decisive. This could lead at times to a complete suspension of recognizable phase-relationships - to structured 'noise'. Reports on the methodic consequences of this conception of structure and structural hearing can be found elsewhere (in the programme book of the Bayerische Rundfunk, 6th Year, Series 23, and in the late-night programme on 'Webern and Debussy', WDR/NDR). So we will go no further into the details of such experiments in method.

Our musical perception reiterated that something was not in order in the work being done on time-structures, and the mistake was sought in the compositional method: it did not occur to anyone to return to the elements, to duration-proportions themselves - to ask whether perhaps the contradiction lay in the basic tenets, in primary scale-relationships. The outcome was indeed an extraordinarily rich expansion of the serial method, many of whose results will remain valid. The real question, however, remained unanswered.

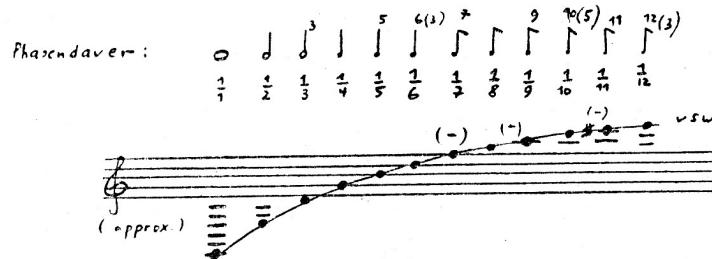
Let us examine the second possible way, mentioned above, of matching the chromatic pitch-scale by a corresponding scale in the sphere of durations; the process of *dividing a largest time-quantum*, instead of multiplying a smaller one, in order to arrive at a scale of durations.

In what follows, a sub-divisible time-quantum will be called the *fundamental phase*. Let a fundamental phase of 1'' duration be σ ; divide it by whole numbers in the order 2, 3, 4 . . . , and the following scale of durations results: $\sigma, \frac{1}{2}\sigma, \frac{1}{3}\sigma, \frac{1}{4}\sigma, \frac{1}{5}\sigma, \frac{1}{6}\sigma, \frac{1}{7}\sigma, \frac{1}{8}\sigma, \frac{1}{9}\sigma, \frac{1}{10}\sigma, \frac{1}{11}\sigma, \frac{1}{12}\sigma$. The notational symbols for the uneven values (1/3, 1/5, 1/7, 1/9, 1/11) have hitherto been called *irrational values*. It can be seen from this, how exclusive the dominance of the 2-proportion has been in the music of the past. Even today, it is quite impossible to make a musician play a single 1/3 or 1/5 of a fundamental phase ('counted value'); this would mean, for example, that he would have to play first $1'' = \sigma$, then $\frac{1}{3}'' = \frac{1}{3}\sigma$, then $\frac{1}{5}'' = \frac{1}{5}\sigma$ etc.: $\sigma, \frac{1}{3}\sigma, \frac{1}{5}\sigma$. Still less would it be possible to combine multiples of these values (2/3, 3/5, 5/7, etc.): $\sigma, \frac{2}{3}\sigma, \frac{3}{5}\sigma, \frac{5}{7}\sigma$ etc.

The reason for this is that we did not perceive a particular duration as a whole, but automatically quantified it, and attempted to bring it, together with the neighbouring durations, into line with a common smallest or largest counting unit. Thus 1/3, 1/5, 1/7 were described, in comparison with 1/2, 1/4, 1/8, as 'irrational' values, because no common smallest counting-value could be found. But this only means that this common counting-value has slipped out of the sphere of perceptibility as a duration, and is so small that it can be described as 'the irrational of perception'.

The dominating 2-relationship seems to rest on a fundamental principle of our sense-perception, to be the acoustical 'golden section'. In the sphere of micro- and macro-phases, of pitch and duration, all proportions based on the 2 are felt to be the 'simplest', to be regulative. 'Twice or half as high (a pitch-octave) or long (a duration-octave)' appears to us as the purest proportion, to which all others are related.

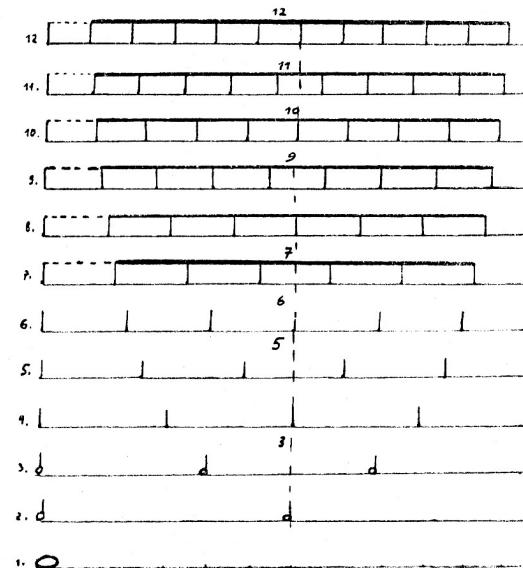
What is such a series of proportions, 1/1, 1/2, 1/3, . . . 1/12 . . . , when applied to time-phases? Let us once again take a helpful example from the sphere of micro-phases, because here the musician has much more conscious experience. If the fundamental phase is $\sigma = \frac{1}{100}$ '', then the half-phase is $\frac{1}{2}\sigma = \frac{1}{200}$ '', a third of a phase as $\frac{1}{3}\sigma = \frac{1}{300}$ '', and a twelfth of a phase is $\frac{1}{12}\sigma = \frac{1}{1200}$ '' etc. But that is nothing more nor less than a *harmonic or overtone series*:



It is well known how little there is in common between a harmonic series of proportions and a chromatic series as actually perceived. Consequently, in practice, metrical notation

excluded almost all single proportions other than the 2-relationships: $\sigma, \frac{1}{2}\sigma, \frac{1}{3}\sigma, \frac{1}{4}\sigma$, the 'duration-octaves'. But what was the function of the irrational values, when they were used?

Because the fundamental phase serves as the unit of perception, the divided values are always referred to it. Thus the fractions must always repeat themselves until they reach the total fundamental value. There are two halves, three thirds, etc., to one fundamental phase. We define such a formation as a *harmonic phase-spectrum*, both when it applies to micro-phases (pitch) and macro-phases (durations).



Example 5

In the harmonic spectra of pitch, the fundamental phase is also described as the *fundamental tone*². We choose the term *formants* for the single 'harmonic' divisions. Thus the first formant is the fundamental phase itself, the second formant is the fundamental phase divided by two, etc. When not all the formants are contained in the spectrum, the special expression *formant-spectrum* will be used instead of 'phase-spectrum'. The duration of a phase-spectrum is defined by the fundamental phase (e.g., MM. $\sigma = 60 = 1''$), and the individual durations in the formants are the result of dividing the duration of the fundamental phase by the ordinal number of the formant.

It is important, for what follows, that a single formant (such as $\frac{1}{2}\sigma, \frac{1}{3}\sigma, \frac{1}{4}\sigma$) remains unrelated; it is heard as a repetition of the same phase. But two formants are already heard, automatically, as related to a common fundamental phase. We begin to perceive

² Trans. Note: Or simply 'fundamental'. The full expression is used here to avoid confusion with other 'fundamentals'.

proportionally again, and orientate ourselves to the largest common unit. The sequence of formants  is thus related to the fundamental phase $\phi : 3/3 : 4/4 = 1$. The same holds good for simultaneous superposition. A completed phase with two simultaneous formants  defines the fundamental phase, even if the latter is not itself included in the formant-spectrum as a duration. We can make this clear as follows: every beginning of a fundamental phase is marked by a synchronization of the formants, and is thus experienced as a corresponding increase in intensity: 

The musician is aware of all this when he remembers the earlier definition of a 'bar'. Thus it is easy to see that the more formants are contained in a spectrum, the clearer the fundamental phase will become.

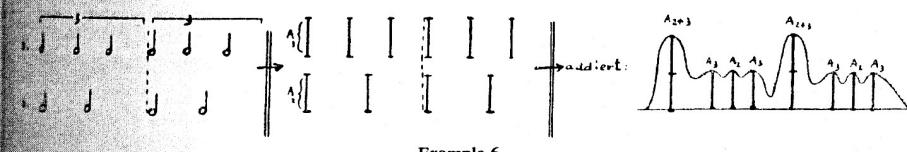
One can now also define as harmonic phase-spectra all sounds used in music up to now (but not noises). The duration of the fundamental phase defines the pitch of the fundamental tone. The number and combination of formants defines what is commonly called the *tone-colour* of the spectrum. Very few musicians are conscious of the fact that tone-colour is the result, in the first analysis, of micro-phase structure.

Since we have now come up against the direct connections between macro- and micro-acoustical time-relationships (even if at present only in the very limited sphere of 'harmonic' phase-spectra), we must remember this: when we speak of recognizable 'pitch', this means there must be at least two complete phases. Normally, we hear many more equal phases in succession, if a note continues at the same pitch. Thus we arrive at the duration of a note or a sound. The shorter the note-duration, the more difficult it is to recognize pitch: if, finally, the note-duration goes below $\frac{1}{16}$ sec., the perception of pitch gradually disappears. Here we again meet the 'dissolving interval', already mentioned more than once (for a more exact evaluation of these threshold quantities, we must first define what sort of note we are dealing with, or to be more exact, whether it is a 'pure' tone, a 'sound' or a 'noise', etc.). In our divergent researches (separating pitch and duration), which take sense-perception as their point of departure, we always presuppose that this threshold must be crossed neither in the one direction, upwards, nor the other, downwards.

Let us return to the harmonic spectrum. As far as pitch is concerned, a single formant defines nothing but its own phase, which repeats itself more or less often when the pitch is constant. Thus the formant is itself a fundamental tone; in this case, to say 'formant' is unnecessary, and it is called a *pure tone*.

A phase-duration was described in the opening paragraph as the time-interval perceived from one alteration in an acoustic field to the next. Let us imagine these alterations as perceptible alterations of sound-pressure. The phase-duration would then be the time-interval between two intensity-maxima. A 'pure tone' – a series of simple time-phases – has only one intensity-maximum in each phase:  Two or more formants, however, define not only themselves, but their common fundamental tone, as they do this through the phase-interval between the *main maxima*, which result from simultaneous superposition of the formants, where two or more phase-maxima coincide simultaneously:  But further *subsidiary maxima* come into existence.

within the fundamental phase, through the summation of the intensities of the formant-phases. A periodic sound-process which has several intensity maxima (of various sizes) per fundamental phase, is no longer called a 'tone' but, more precisely, a 'sound', a 'spectrum'. We obtain a resultant *intensity-curve*, that returns periodically when the same phase is repeated. If we suppose that each phase-duration is the time-interval between equally large intensities ('amplitude values', A), then we can obtain the following intensity-curve from the superposition of a second and a third formant (such a curve is also called an 'envelope curve'):



Example

Just as the combination of duration-formants makes us perceive their common fundamental phase (which can also be described as the 'combination-duration', the 'tactus' or 'bar' of earlier times), the combination of pitch-formants similarly makes us perceive their common fundamental or *combination-tone*.

The important thing for the musician is that his perception of tone-colour is aroused by the intensity-curves of phase-spectra, and that these intensity-curves result from the superposition of formants: that 'tone-colour' is the result of time-structure: and that he can intervene compositionally among these complex connections – as is the case today in electronic music.

In the sphere of traditional metric-rhythmic relationships, the correspondences with harmonic relations are quite familiar. The whole allusive richness of pitches in cadential music resulted from the intervals of the harmonic series, and the same is the case in the sphere of durations. The bar corresponds (as a metrical fundamental unit) to the fundamental phase of a time-spectrum. All definitions of 'accented and unaccented parts of the bar' (resulting from 'main and subsidiary maxima'), of syncopations and their resolution (phase-displacement and restoration of phase-periodicity), etc., originated in the practice of 'part-writing' (which gave rise to the problems of harmonic time-spectra). In it, the bar, as fundamental phase, was rendered (through *time-formants*) in various ways, if mainly through the 'consonant' formants – the octave (duplet), fifth (triplet), later the third (quintuplet), and at most the seventh (septuplet); i.e., with up to seven formants.

The difference between *metre* and *rhythm* is exactly that which we discern between the 'fundamental tone' and the 'tone-colour' of sound-spectra: the fundamental phase (metric fundamental) is defined by the periodic main intensity-maxima (the heaviest accents), and these result from the formant-structure. The relationships of the subsidiary to the main maxima (subsidiary to main accents) define the 'tone-colour', i.e. the rhythm. 'Tone-colour' is a confusing idea, that could well be replaced by 'sound-rhythm', and one should use the general term 'formant-rhythm'.

Let us call to mind a few well-known theories of acoustics and acoustical perception. How can these help to answer the question whether a serial duration-structure can be added to serial pitch-structure, without the two contradicting each other?

Exactly defined, a twelve-tone series is a sequence of twelve fundamental tones, so long as the question is one of harmonic formant-spectra. Such spectra characterize almost all the sounds (not 'tones', which are hardly ever found 'pure' in nature) used in music up to now. The same harmonic series of proportions (also 'overtone-series') was the standard in 'tonal' music, both for the formant-spectra of the sounds used, and for the intervals that connected such sounds; spectral proportions were identical with the proportions of the fundamental tones, both simultaneous ('harmony') and successive ('melody'). With the introduction of the chromatic scale system, this identity steadily disintegrated. Finally, in the twelve-tone system, harmonic-melodic 'laws' were formulated that totally contradicted the spectral structure of the instrumental sounds used: the instrumental sounds' *harmonic scale of perception* was irreconcilably opposed to the *chromatic scale of perception* of the twelve fundamental tones in the octave, whose steps were serially composed. Here the earlier identity of material and composition fell completely apart. This is what is really meant when the 'emancipation of the fundamental tone' is mentioned. (Schönberg's occasional regression to tonal harmony and melody could be explained not least by the contradiction demonstrated above. His metric-rhythmic composition was always 'tonal', a classical cadential rhythm with merely a lot more unresolved syncopations, equivalent to a tonal harmony with lots of 'wrong notes'.)

'Rhythm', however, developed in such a way that no-one thought at first of doing anything that would correspond, in the sphere of macro-phases (durations), to twelve-tone composition. This would have meant 'tempering' the fundamental phases (the 'bars') of the duration-spectra to an equivalent of the chromatic scale of twelve fundamental phases per time-octave, and composing them serially, while the formant-spectra of the durations still remained harmonic. Once again we are at the basic problem of our investigation. What would a scale of fundamental durations, corresponding to the scale of fundamental tones, look like? Furthermore: how would the duration-spectra have to be structured over these fundamental durations, in order to achieve a complete correspondence (befitting the present state of instrumental composition) between the chromatic system of fundamental tones and the harmonic formant-system, in the realm of both micro- and macro-time perception?

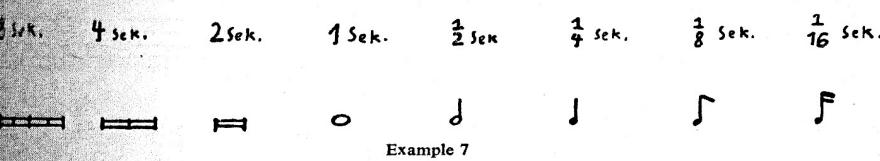
In tonal music, the total duration of a work was divided into *tonal fields*: one fundamental tone – central note, tonic, dominated for a while, then another, and so on. By and large, the transposition of such fields was called *modulation*. These relations were ruled by a system of hierarchical functions. Modulations came to predominate more and more, and finally, in twelve-tone music, there was 'modulation' from each fundamental tone to the next. No note was more important than any other, over a time-phase that was at all long. Similarly, a *fundamental bar-length* – the fundamental phase in macro-time – continued, in tonal composition, to dominate for particular *metrical fields* (e.g., a $\frac{4}{4}$ bar). Subtleties of cadence were used as details, just as on a larger scale there was modulation from one metrical field to the next, and finally, from one movement to the next, etc. Modulations were made to the 'dominant' (3:2 – triplets), or to the 'subdominant' (2:3 – dotted values). This modulation, too, proceeded ever more

rapidly, and finally, via Debussy to Stravinsky, there are fundamental phase-relationships in which the phase-duration changes from bar to bar. Now, this could not correspond to the twelve-tone chromatic connections of fundamental tones, because evolution had produced no 'tempered system' for durations. So, in serial composition of fundamental phases ('bars'), one lapsed into sub-harmonic modality, as has been demonstrated above for durations in general. We meet the scale of 1 $\frac{1}{1}$ to 12 $\frac{1}{1}$ not only from one individual duration to the next (in the 'formants'), but also, in certain serial works, in written-out series of bars (e.g., bars from $\frac{1}{8}$ to $\frac{1}{12}$).

So durations have now been included in the serial system: the latter should be extended to take in metric-rhythmic relationships, despite the contradiction between fundamental-tone composition and the nature of sounds. For this purpose a *tempered chromatic scale of durations* would be necessary. How could this be represented? We can only approximate to a chromatic time-scale, so long as we have to rely on feeling, and are not assisted by a duration-keyboard. We take a pocket metronome, which can be quickly altered while still in motion. We fix eleven duration-intervals per 2, in such a way that they are felt to be equal. As long as we use the traditional signs for duration, the only possibility is to take the same sign (e.g., \circ) for all twelve chromatic time-values, and to differentiate its duration metronomically. If we choose a logarithmic scale of 12 ($12\sqrt[12]{2}$), within a 2 from, for instance $\circ = 1''$ to $\circ = \frac{1}{2}''$ (2:1), we get:

M.M. $\circ = 60, 63.6, 67.4, 71.4, 75.6, 80.1, 84.9, 89.9, 95.2, 100.9, 106.9, 113.3, 120$. For the last value $\circ = 120$ we can also write J with $\circ = 60$, and the same chromatic scale sets out, with this value, into the next 2. Thus we obtain the 2-transpositions of the scale ('octave-transpositions') by altering the sign for the fundamental duration: $\circ = 60-113, \text{J} = 120-226, \text{J} = 240-452$, etc.

The sphere of *duration-composition* has not hitherto exceeded 2⁷ (seven duration-octaves); fundamental phases longer than 8'' or shorter than $\frac{1}{16}''$ are seldom required ('playability' sets a bound here, and we have seen above that the perception of duration passes over into the perception of pitch at this point; equally, our powers of recollection impose limits on the length of time-phases, ruling out fundamental phases that are much more than 8"). Thus, the composition of durations has at its disposal a *chromatic scale of durations over approx. seven octaves, between 8'' and $\frac{1}{16}''$* :



and in every 2:1 relationship, the chromatic scale of twelve durations, fixed by metronome markings, repeats itself.

Together with the seven or eight pitch-octaves, *musical time* would thus be circumscribed in fourteen or fifteen *time-octaves*, in which the composer proportions phase-relationships both in the sphere of duration and in that of pitch.

With the aid of the metronome, we have used normal note-values to fix a scale of durations that corresponds to the twelve-note scale. What is to be done with it?

The notion of a fundamental phase – and, in a narrower sense, of a fundamental duration and a fundamental tone – was derived from the concept of a time-spectrum. It corresponded to the earlier ‘metre’. Moreover, we now know the nature of time-formants in the harmonic spectrum. We have substituted formant-rhythm, in pitch and duration, for tone-colour and rhythm; this was to maintain the contradiction between harmonic spectra and the chromatic relationships of fundamental phases. And now it is time to discuss the question of applying the serial system to *fundamental durations*.

No fundamental tone holds sway over the others; none of the twelve pitches per 2 is repeated before all have had their turn. Correspondingly, no fundamental duration – no metre – will be repeated before all twelve per 2 have had their turn.

Let us choose the simplest case, when a series of fundamental durations is directly proportional to one of fundamental tones. The metronome markings apply to d :

Skaleneinheiten: -4 1+5 1-3 1-1 1+2 1+6 1-2 1+1 1+3 1-5 1+4 1-6

MM $\text{d} \sim 75.6 / 60 / 80.1 / 67.4 / 63.6 / 71.4 / 100.9 / 89.9 / 95.2 / 113.3 / 84.9 / 106.9 /$

Skaleneinheiten = Degrees of the scale

Example 8

These would be fundamental durations, just as in pitch we must think of fundamental tones.

In the composition of fundamental tones, it is exceptional to confine the series to one octave, and this is equally true of fundamental durations. The fundamental durations of our series must be distributed among the duration ‘octaves’; their *time-register* must be defined. The pitches are once again given as a visual aid. Let us select a disposition of the registers:

MM $\text{d} =$

75.6	60	80.1	67.4	63.6	71.4	100.9	89.9	95.2	113.3	84.9	106.9
d	o	o	d	o	d	o	d	o	o	d	

Einheit = Unit

Example 9

If this were barred normally the result would be the following metrical series:

$\text{d} \cdot 76 / \text{d} \cdot 60 / \text{d} \cdot 80 / \text{d} \cdot 67.4 / \text{d} \cdot 63.6 / \text{d} \cdot 71.4 / \text{d} \cdot 100.9 / \text{d} \cdot 89.9 / \text{d} \cdot 95.2 / \text{d} \cdot 113.3 / \text{d} \cdot 84.9 / \text{d} \cdot 106.9$

$\frac{1}{4} \frac{2}{2}$	$\frac{2}{2}$	$\frac{1}{2} \frac{2}{2}$	$\frac{2}{2}$	$\frac{1}{2}$						
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Example 10

A harmonic proportion that can be expressed in familiar note-values ($\text{o} : \text{d} : \text{d} : \text{o}$) only appears twice in our series (between the second and third, and between the tenth and eleventh durations), and in these two cases we can alter the note-value instead of the tempo.

$\text{d} \cdot 76 / \text{d} \cdot 60$ $\text{d} \cdot 67$ $\text{d} \cdot 95 / \text{d} \cdot 85$ $\text{d} \cdot 106.9$

$\frac{1}{4} \frac{2}{2}$	$\frac{3}{4}$	$\frac{1}{2}$	$\text{o} \text{---} \dots \dots$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{2}{2}$	$\frac{1}{2}$
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Example 11

In a series where the proportions 3:4 or 2:3 appeared more often, several metres could be comprehended in one tempo, by using bars of varying length.

In our example of a duration-series, we started by including all the twelve chromatic steps of a 2. We were not concerned with particular interval-proportions. Next we fixed the register of the durations. This is the normal method in the twelve-tone technique. One is misled into doing things in this order because the chromatic scale with the unit-interval $\sqrt[12]{2}$ (Ex. 9) is simply presupposed. In reality, it is less interesting, when listening to series, that at some time or other all the chromatic steps should appear (this is true of every series), than which proportions are chosen between durations or notes, and how these proportions are distributed, how they are composed in relation to each other. If, however, a *proportion-series* of intervals is taken as the point of departure, the register of each duration is already chosen with every step. We should examine the above series from the point of view of its interval proportions. We shall then recognise eleven intervals, of various sizes, *from the harmonic series*. The comparison with pitch is now only approximately accurate, for we proceed with pure intervals that deviate to a greater or lesser extent from the chromatic tempered system. This will be seen if the following metronome markings are compared with those in Example 9:

$\text{d} \cdot 76 / \text{d} \cdot 60 / \text{d} \cdot 80 / \text{d} \cdot 67.4 / \text{d} \cdot 63.6 / \text{d} \cdot 71.4 / \text{d} \cdot 100.9 / \text{d} \cdot 89.9 / \text{d} \cdot 95.2 / \text{d} \cdot 113.3 / \text{d} \cdot 84.9 / \text{d} \cdot 106.9$

Einheit: Proportionen: 2:10:12:7:5:9:10:2:3:5:10:4
4:3:6:13:8:11:13:6:9:12

Example 12

The relationship of one fundamental duration to the next is expressed in the following eleven harmonic proportions: 2:10, 4:3, 12:7, 6:13, 5:9, 8:11, 7:2, 13:6, 3:5, 9:12, 10:4. Measured in seconds, the fundamental-durations are approximately:

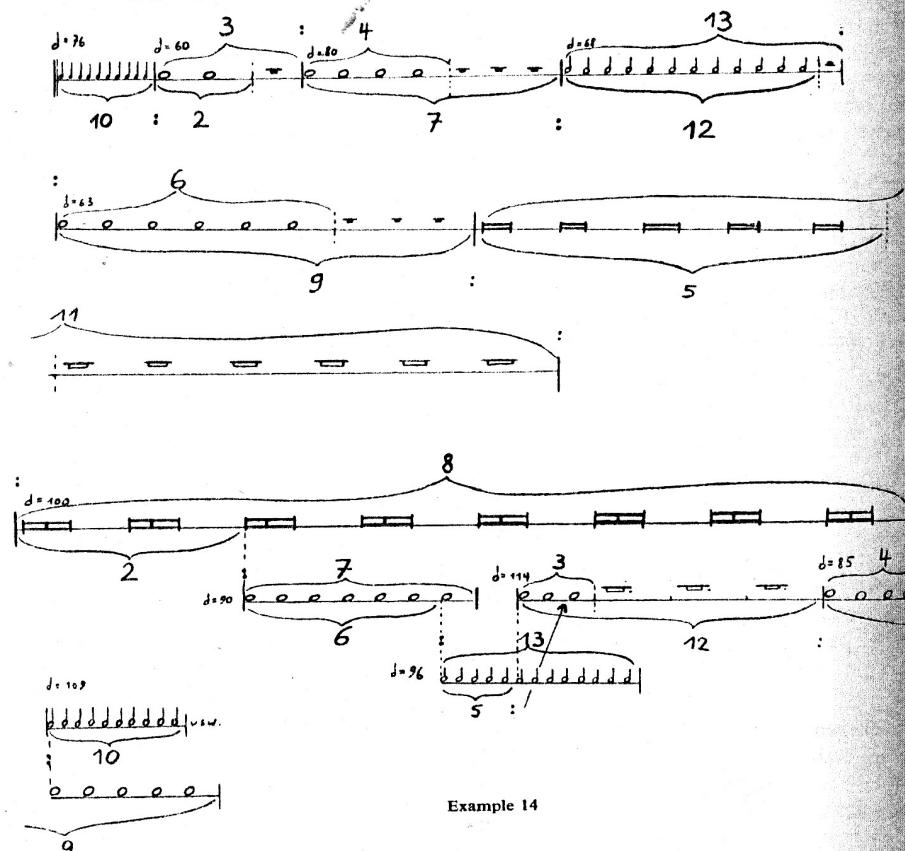
$\frac{1}{5} \text{---} | 2 \text{---} | 1\frac{1}{2} \text{---} | \frac{2}{3} \text{---} | 1\frac{1}{8} \text{---} | 3\frac{1}{2} \text{---} | 4\frac{1}{16} \text{---} | 1\frac{1}{3} \text{---} | \frac{1}{5} \text{---} | 1\frac{1}{9} \text{---} | 1\frac{1}{11} \text{---} | \frac{5}{9} \text{---} |$

Example 13

It seems sensible to retain the metronome values of the chromatic series ($\sqrt[12]{2}$), because further derivations from the series of harmonic intervals will produce further deviations from the tempered scale, and thus more and more new tempi, instead of which we could always operate with the same twelve tempi, even when a series is transposed.

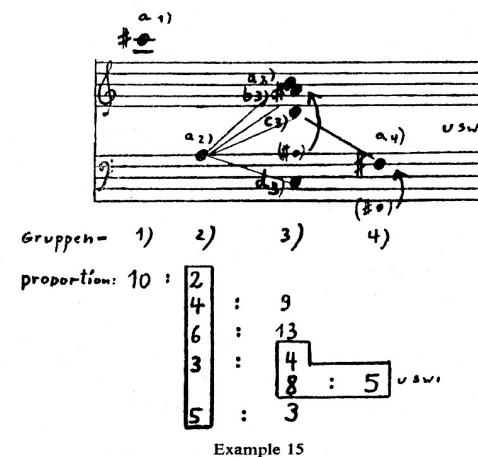
However, the presentation of intervals in harmonic proportions becomes essential as

soon as we relate single fundamental durations to *groups of fundamental durations*. This could happen in the following way, among others. The proportion 2:10 means that the duration of a first phase is two-tenths of the duration of a second fundamental-phase, i.e., five times as short. But it also means that two phases of a second fundamental duration are equal to ten phases of a first. We can therefore reverse the proportion (10:2/3:4, etc.), because now we refer not to the duration-relationships of the single fundamental durations, but to the relationships between the numbers of fundamental durations in each group. If such *formant-proportions* are presented *successively*, varying numbers of equal durations add up to groups: these are the same length from one group to the next (the first ten are the same length as the following two, etc.). But each group, with the exception of the first and last, is ambiguous; each is the hind-limb of a first interval (10:2) and the fore-limb of a following interval (3:4 - $\frac{1}{3}$:4). The result of this ambiguity is either a rest or an overlap. The selected series of proportions gives the following groups of fundamental values:



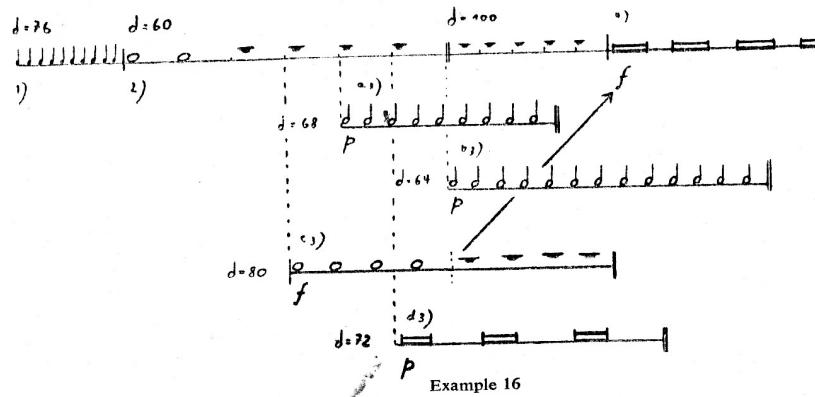
In the groups of fundamental durations, changes of tempi extend over much longer time-phases, and such groups meet normal playing requirements better than a series of single fundamental durations. Let us remember that it is a matter of fundamental durations, of metres, each of which will have a formant-spectrum; and therefore that the realization of our example requires three chordal instruments, or, better, three orchestras, that would at times play independently of each other, at differing tempi, orientating themselves to the others only at the points where they entered. Before each entry, each group – or each conductor – could prepare for the next tempo without difficulty (with a metronome). Spatial separation would result naturally from the need to make various time-strata appreciable.

Each *proportion* value, however, can not merely be ambiguous but can have many meanings: i.e., different intervals can spring simultaneously from the 'limbs' of the proportions. Here there are new proportions that must be taken into account. The numbers again refer to the number of fundamental values in a group. For the sake of comparison, the same thing is also presented in terms of pitch.



One can settle freely or serially the question of whether all four intervals, or a selection, or only one, should be operative for the proportion from the third to the fourth group. If we choose that there shall be only one link, there is room here, perhaps, for a function of intensity – e.g., to make clear the linking groups c³) and a⁴). (Ex. 16)

We described the durations in each group as fundamental durations. Now we have to ask what sort of *formant-spectra* these fundamental durations will receive (just as in the case of fundamental tones we had to ask which instrument – which 'tone-colour', or, better, which *formant-rhythm* – should be linked to the fundamental tones, or which instrument to which fundamental tone). If it were still a question of single fundamental durations, as in Exs. 8 and 9, then in the simplest case, each fundamental duration would be given its own formant-spectrum (cf. Ex. 5). But the fact that there are *groups* of fundamental durations means that a formant-spectrum must be related to the



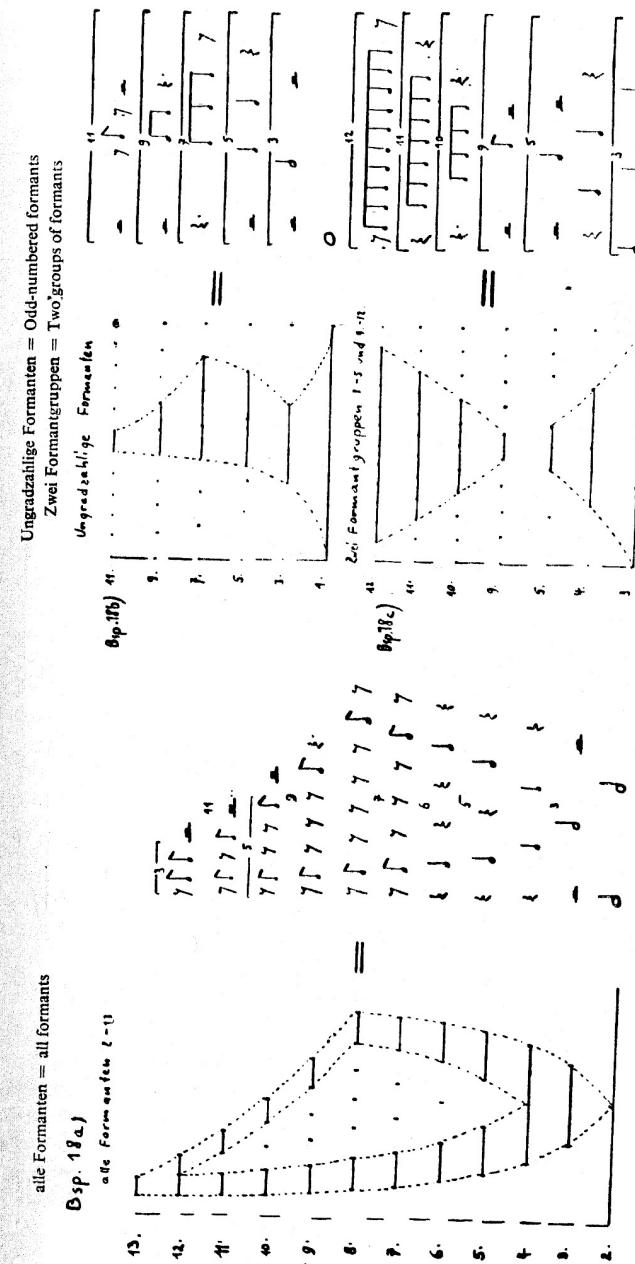
Example 16

supra-ordered duration of the whole group, i.e., to the *group-phase*. The spectrum to be composed is then called a *group-spectrum*. It may help if one pictures a group of fundamental durations as the formants of a virtual fundamental phase that lasts, in most cases, so long that it lies far beyond the boundaries of musical time.

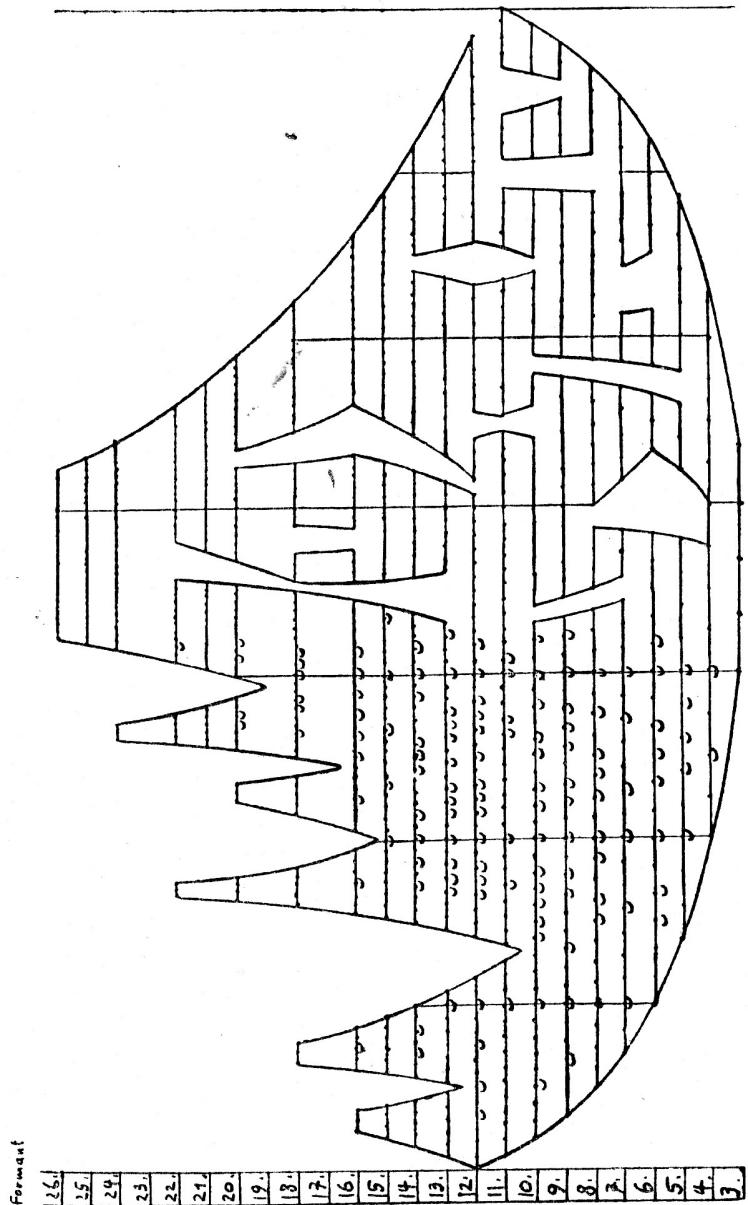
Finally, an entire piece can be imagined as the *one* time-spectrum of a single fundamental duration, of a 'tonality', as was in effect the case with tonal music. (Naturally this must not be confused with serial compositions which are notated in even metre to make performance easier, but in which the function of a constant fundamental metre is nullified; the mental filling-in of bar accents, too, is here senseless and a distortion, but basically, we find the principle of 'tonal' metre and rhythm in any work where a constant fundamental duration perceptibly defines sections of the whole work, however well such a metre may be effaced by emancipated syncopations.)

The most diverse methods of serial composition can be used in deciding the number (the shorter the fundamental duration, the fewer formants!), the combination, the register (fixed or movable formants), the changes of register (!), the intensity-relationships, etc., of the formants. A formant-spectrum will then be seen as a unitary time-complex, characterised by its total duration, envelope-curve (see Ex. 6), average speed, speed tendency, average intensity, density, density progression, sonority (which group or combination of instruments), sound-form, movement in pitch, harmonic field, and so on. The resultant of all these compositional details is what we have described generally as the formant-rhythm (instead of the notion of 'tone-colour'), and this will be heard either the *rhythm of the sound* or the *rhythm of the bar*, depending on whether it has to do with pitch or duration. That is why we represented formant-rhythm as a single intensity-curve (Ex. 6).

Special reference must be made to one of the formal criteria most essential to serial composition of such envelope curves. The form of the entry and exit processes of formant-spectra is highly important in the presentation of extended time-structures, in which one has to recognise particular forms of the formant-spectra when they return. There follow three examples of formant-spectra on single fundamental durations (Ex. 17), and one of a group-spectrum over seven fundamental durations (Ex. 18): it is superfluous to translate Ex. 18 into normal note-values.

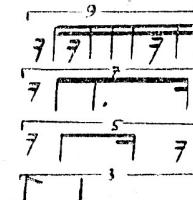


Example 17



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It is now obvious, as regards the varied serial structuring of the formant-spectra, that not all durations are used all the time in the individual formant-spectra; differences between particular families of spectra are produced by either omitting durations or 'tying' them. Here, successive series of sub-harmonic intervals can arise; since these can be reduced neither to whole-number relationships nor to any perceptible smallest unit, they make listeners feel the formant-rhythm to be 'irrational'. This means that, in any formant, one can do away with periodicity, and thus with the 'harmonic' effect of the whole formant-spectrum; one composes the time-counterpart of 'noise'.



Example 19

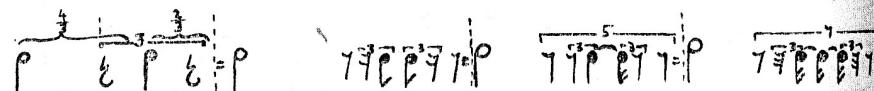
This enquiry started by developing the idea of a new morphology of musical time. On this basis, we tried to see how far it is possible to reconcile accepted methods of structuring time, in the sphere of durations, with methods used in the serial composition of pitches. We came up against a contradiction between pitch- and duration-composition. A second fundamental contradiction appears, between material and method, i.e. (ultimately) between instrumental music and serial music. To surmount this second contradiction, we have attempted to bring the method of duration-composition to a state commensurate with that of pitch-composition. Here we had to take into account the conditions imposed by nature on the representation of time by instrumentalists. The result was the idea of manifold simultaneity, whose tempo-strata could in practice be represented by orchestral groups under separate conductors, or by smaller, independent instrumental groups. There remains the question: how does the composer of serial instrumental music stand with regard to the second unresolvable contradiction – will he return to 'tonal' composition? Or will he accept the contradiction, and take just this dialectical relationship as his point of departure, since it often seems more fruitful to work from a contradiction than from the definition that 'two times two make four'? Or will he completely renounce instrumental music, and compose only electronic music? Or will he seek a completely different path in composing for instruments, through a conception of musical time that is absolutely new? The last seems the most likely candidate for the mainstream.

Up to now, serial composition has presupposed regular scales of time-magnitudes (phases of musical micro- and macro-time as pitches and durations). All differences in magnitude were defined by a constant unit-interval (arithmetical or logarithmic). A series defined the proportional relationships between the magnitudes, so that every individual magnitude had to be exactly measured, and fixed by a discrete value in each dimension (one pitch, one duration, one loudness). Thus, time and intensity were

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presented discontinuously. Each time-value was either a counting-value or a multiple of a smallest time-quantum, and each time-relationship could be represented by two discrete values (e.g., 2:3). As regards the notation and measurement of individual magnitudes, however, this was quantitative only in the case of durations (e.g., J , J) since for pitch one simply named a degree of a given scale, and for the description of micro-time, phase spectra, one simply named an instrument.

In some recent scores, the notation of duration-relationships has become extremely differentiated. The result has been that, with an increase of metric-rhythmic complexity, the degree of precision in playing correspondingly decreased. To put it differently, the more complicated the way in which a time-value was indicated, the less sure the performer was about when it should begin and end. A simple example will explain this. Not only can various time-values be expressed variously, but one and the same value can be notated in quite different ways, since it can be related to any of a number of smallest time-quanta. If one defines $\text{o} = 30$ as the metronomic unit, the relationship of 4/3:2/3' can be notated in triplets, quintuplets, septuplets, etc., simply by writing the beginning and end of the second value in various ways.



Example 20

In an attempt to play each of these examples as exactly as possible, the first will come closest to the proportion 4:2, and the last will show the greatest deviation from it, compared with mechanically-measured time. If one enquires more minutely into such factors of dubiety, one can ascertain the different sizes of the zones in which the accuracy of performance is subject to such 'scatter'. These zones may be described as *time-fields*, and the sizes of the zones as *field-sizes*. Thus, from the mere relationship between various methods of notation and the resulting degrees of precision in performance there arises a fluctuation in one's conception of time, a fluctuation that cannot be described simply in terms of discrete values. For the moment, the main point about our example is that one and the same time-proportion fluctuates to a varying degree, and furthermore, that the factors of dubiety depend progressively on the method of notation.

How can one determine such field-sizes more closely? It cannot simply be said that an increasing complexity of notation effects a corresponding increase in the size of the field. The best thing to do is to have several good instrumentalists play – each one as often as possible at various times – a sequence of equal and unequal time-proportions, notated with varying degrees of complexity. The results are recorded on tape. One then makes a note of the deviations between the time-proportions as notated and as played, and measures the order of magnitude of each deviation. One does in fact obtain a more or less typical scale of deviation for each instrumentalist, and these scales can be compared with the scale of degrees of complexity in the notation; but, among the various instrumentalists' scales, there also appear common factors in the relationship between field-sizes and the different ways of notating durations. It is best to pursue such researches, however, with tape-recordings of compositions that already include such

field-proportions. For one thing, the player no longer feels that he is a guinea-pig, under artificial conditions; and for another, the musical context, which is of the greatest importance for field-proportions, can be taken into account in determining the size of the fields – one and the same degree of notational complexity can produce different field-sizes, according to the context in which, and the frequency with which, different or similar fields follow each other or are superposed. The more experience a composer has accumulated from such research, the clearer his composition of time-fields will be. And it is a secondary question, whether, perhaps, in the course of time, the instrumentalists achieve, in playing such music, a generally higher degree of precision: secondary, because a scale of field-sizes is regulated, not by the absolute sizes of the time-fields, but by the proportions of one time-field to another.

One could now choose a series of field-sizes, starting from a field-value scale that would have, in each case, to be defined. This series would be valid for a time-structure, instead of, as hitherto, for single 'pointillist' values. The field-sizes would not be 'incidental' (like the latitude left for 'interpretation' in music up to now), but rather functionally included in the time-composition, and proportioned. This actually occurred at a relatively early stage in the serial composition of time. It does not, however, apply to all compositions that exhibit, at a first glance, a complicated rhythmic texture. Each case would have to be examined to see if relations of dubiety have in fact been structurally composed, or if they result, as 'chance-criteria,' from the interpretation of a consistently complex notation of time-proportions. Anyway, it is easy to see what idea of time underlies those serial compositions in which the composer has 'simplified' the originally complex notation of time-relationships to make it easier to play.

At first, the relationship between the degree of notational complexity and the exactness of performance appeared unimportant, but after observing it, we have drawn conclusions that are decisive for the further development of instrumental music, and that open up a new path, separate from that of electronic music.

Let us now see what are the further possibilities of presenting field-proportions with the aid of the existing system of metric-rhythmic notation. Phase-relationships are regular as long as the metronomic definition of the fundamental-phase remains constant. When discussing the development of the time-spectrum in the sphere of serial fundamental durations, we have already met the problem of what would happen when several groups of instruments – if possible, separated in space – played simultaneously in different constant tempi. Here there is obviously a relation between the durations of the *tempo-strata* and the definition of the field-proportions; the longer two orchestras play in different tempi, the more probable it is that the time-strata will get out of step, be displaced. Even apart from the fact that such displacements require a corresponding control of field-harmony, field-intensity, field-density, etc., the method of time-composition must aim at regulating such *field-times*. Clearly, the flow of time can no longer be imagined as 'quantified'; displacement can come about gradually and continuously within particular time-fields, and the associated field-sizes can not be thought of as a sort of discrete succession (the time-alterations 'flow', as it were, continuously past an 'acoustical window', like a motion-picture).

If the metronome markings for the relationships of fundamental durations (the *tempo*), are composed flexibly, much more complicated field-proportions arise than in the previous example. We have evolved the term formant-spectra for durations; each

single formant is quantified into regular durations, that are related to a fundamental phase. Now, if the single time-quanta of the formants are no longer in a constant relation to each other, but speed up or slow down, moreover in various degrees, then the formant-rhythm becomes more or less diffuse. Different field-sizes result, according to the number of *variable tempi in the formants*, and according to the degree of their alterations, in which the original harmonic phase-relationships can no longer be traced back to a scale of discrete time-quanta. For example: a first duration-formant has a constant tempo, a second is 'as fast as possible', a third speeds up and a fourth slows down, and all are to be played simultaneously; and only the fundamental duration of such a time-spectrum is exactly measured as a single value. Time-structures will result in which one composes simultaneously with both a series of discrete fundamental durations and another series of proportions for the simultaneous field-sizes. Each measured fundamental duration would be associated with a *simultaneous time-field* (e.g. field-size 0 = all formants have constant tempi; 1 = four formants with constant, one with variable tempo; 2 = three formants with constant, two with different variable tempi, etc.).

Combinations of the aforementioned possibilities result automatically if, finally, *successive and simultaneous* proportions of the field-sizes coincide, i.e., when both the fundamental phases and the formant-spectra are composed, and suitably realised, as smaller time-fields in a time-continuum (phase-displacement within defined limits).

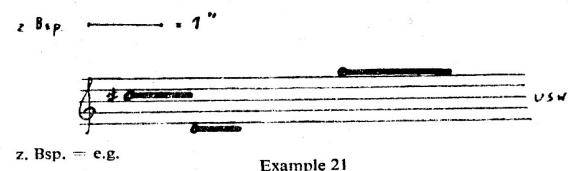
A move toward this sort of time-composition was already recognizable when, in connection with the composition of formant-spectra, we spoke of 'statistical' criteria of form. By this we meant that, with a particular chronometric density of singly-defined phase-relationships, a formant-spectrum would be presented and perceived as a 'complex', i.e., it could no longer be broken down into single proportional connections. We spoke of group-spectra whose fundamental phases were 'perceived irrationally'; time-complexes that would be heard not as the sum of the single durations (be they simultaneous or successive), but rather through their structure as a whole. The necessary condition was that a certain number or mass of details were crowded into a short time. However, as soon as one can hear through such processes, or follow them slowly enough, they are, and remain, nothing but 'periodic, harmonic, sub-harmonic, temper-chromatic and other' relationships of regular, individually-defined facts. Here, the mass-structure is a special case of a unitary structure that is, at bottom, individual determined. 'Complex' time-processes are thus the result of heaping up exactly defined time-points more or less densely in time. Mass-structure means, then, merely the momentary opacity of a group.

Such a switch, from 'pointillist' to 'statistical' perception of time has become further occasion for the *statistical composition of fields*. But this means that the elements themselves are no longer presented as discrete degrees of some scale or other (whether as discrete pitches or durations, i.e., as a measured duration and number of micro-time phases – phase-duration = $\frac{1}{440}$ "", and number of phases = 220, giving a time-lapse = $\frac{1}{2}$ "'). Rather, a field-size, in the sense described above, is substituted for each discrete value (field-duration from $\frac{1}{440}$ "" to $\frac{1}{550}$ "" – i.e., a possible pitch between a and c'',) and fundamental-duration between $\frac{1}{2}$ " and 1", etc). Such field-sizes are no longer the 'elements', and composition thus includes the *statistical character of mass-structure among the elements*. A 'pointillistic' time-structure can now be presented, vice versa,

a special case of mass-structure – the case when field-size equals zero, and each time-process is fixed in the time-continuum by a *point* instead of by a *field*.

Let us again recall the example in which differing field-sizes resulted from degrees of notational complexity. It would be more reasonable to *describe such field-sizes directly*, by choosing a suitable notation. This is possible neither for duration nor for pitch, if one uses the signs used hitherto, because we have only discrete values in discontinuous scales. Actually, to fix any event in time (e.g., the beginning of a duration), one would need not one datum but at least two, to define the limits of a time-field. If, for example, two time-phases follow one another, the point in time where the second begins has hitherto been fixed by the note-value of the first phase (J J); the first phase could, of course, also be a rest (J J). If the beginning of the second phase had to be defined as a time-field, rather than as a time-point, the duration of the first phase would have to be more or less indefinite, according to the size of the field; one would have to state the limits between which it was indefinite, but this does not mean that the indefinite value would once more have to be made quantitative or countable.

Something else must still be mentioned, before we go into the possibilities of a notation for field-proportions. The composer John Cage, in some of his more recent works, has used time-proportions that are evidently influenced by statistical ideas. Thus, among other things, time-durations are drawn to scale, so that no attempt can be made to quantify them.



Example 21

Here, the beginning and end of each duration are played with much less certainty than before. Instead of 'counting' – dividing up the durations into quanta – the eye measures the time-proportions, and converts them into the action of playing. Optical size-relationships must be translated into acoustical relationships of durations. Each event in time does indeed receive a field-size that is psychologically determined, but this field-size is the same for all time-proportions, and is thus not proportioned. It has always been customary for interpretation to include zones within which time-values as realised deviate from those notated (in so far as the latter are measured in metronomic or 'clock' time); all Cage has done is to extend these zones slightly. He makes *all* proportions less distinct than ever before (logically enough, he is in fact not at all interested in proportional time-relationships), and the result is a continual disorientation in time, as a result of which the duration of a time-lapse is felt unusually strongly. Instead of the suspension of time-consciousness, perhaps intended, time is bound to one plane, and is therefore equally strongly present at each moment.

However, the concept of field-composition will only become meaningful if one does something more than merely substitute a crude system for a more differentiated one, in the hope of giving the time-structure greater 'vitality'. If, however, a point could be seen to be the shortest line, and a line were thought of as an extended point, and if this

could be communicated to the performer – i.e., if a series of field-sizes served to present a time-structure in which the composed fields mediated between the pointillist and statistical extremes –, then we should really be dealing with a new musical time-continuum: time as a discontinuum – and time as a continuum would then merge in a supra-ordered concept of serial *field-time*.

What are the possibilities of *notating field-sizes* in the sphere of durations? Like all the reflections described, those that follow have been triggered off by ‘purely coincidental’ processes in earlier compositions. One wrote something or other, and was then startled by the way certain things hung together. Differentiated field-sizes had, in fact, already been presented, even with the normal signs for notation.

There was the small note, for one, written independently of the other, measured time-values – the ‘grace-note’. If its tempo was ‘as fast as possible’, and if it were not only single but came in groups of various sizes, either before, over or after a measured time-duration, then these groups of grace-notes would take over the function of a second time-stratum ‘fading in’ to the measured durations. Here, each individual grace-note in the group received its own field-value in time, determined in the following way: the pitches of a group were distributed on the piano in such a way that the player’s hand had to make movements of very different magnitudes over the keyboard. The larger the pitch-interval, the larger the time-interval from note to note, for everything was to be played ‘as fast as possible’. Besides this, the instructions mentioned the fact that each note should be distinctly recognizable in pitch, thus automatically making the lower notes somewhat longer than the higher ones. Thus, instead of notating all the various durations, one used performing indications of a quite different kind, in order to produce a proportional series of field-sizes within the groups of grace-notes. The size-relationships of such a series depend, of course, on the time it takes for respective performers to react, and also on the instrument, and on space (the more resonant the room, the more slowly the grace-notes must be played, if they are not to become indistinct); but just because of this, the composed proportions continue to exist. Again, this example is a special case for series of relatively short field-durations.

The search continues from here. The decisive factors in determining field-sizes were as follows: the duration of each grace-note can vary within certain limits; their duration is not quantified; their proportions must be directly experienced through the degree to which they fluctuate, rather than being compared with a beat (counting-measure) that is either actually available or read into the flow of time. The limits of the field are set by the movement of the pianist’s arm from a low to a high register, which take a variable amount of time; i.e., by action-durations. And for each note, or small group of notes in a larger group, such an action-duration is different. So, in order to obtain field-proportions of a larger order, more time would have to be taken by the action of preparing the sound that was to be played. A series of field-sizes would correspond to a series of actions taking various lengths of time. This would depend, for example, on the number of *preparations*, etc.

But what, then, are *rests*? The field-size of a rest would result from the fact that it is the nature of a struck note to stop sounding, while the preparation of the next sound takes more time. Such preparation can be either mental (where the musical notation is more or less esoteric), or practical (as when movements are necessary in the course of the various preparations of resonating bodies, mechanical ‘registrations’, etc.).

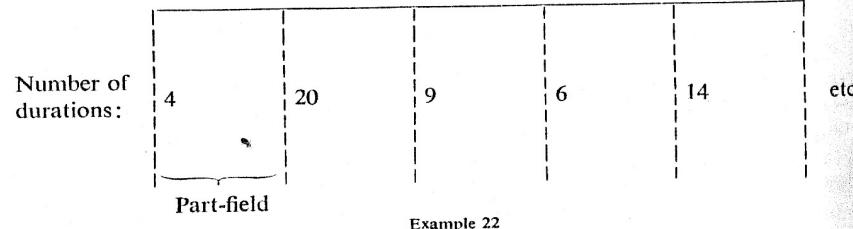
Such proportioning of larger field-values can already be seen in some serial scores – not to mention Cage, in whose compositions there appear many such procedures, which are, however, ‘exposed’ rather than composed, and are left to chance. We find them in places where, besides the durations that are notated normally, there are single unmeasured durations, that have particular instructions about the mode of attack. Sometimes the preparation and execution of such an attack takes a relatively long time (e.g., ‘engage the right-hand pedal, attack the note staccato and immediately allow the pedal to spring just so far back that the note goes on sounding softly as an echo’). Depending on the pitch of the note and the intensity of attack, there is considerable room for the composer to differentiate still further the field-sizes of such modes of attack. Other modes of attack again give quite different field-sizes, and one can merge a second, third, fourth, etc., series of field-sizes in the time-structure, which is already doubly determined: 1, by metronomically measured durations; 2, by first-order field-sizes, applying to grace-notes; 3, by a second (and further) order of field-sizes applying to the modes of attack, etc.

In any work, we can arrive quite naturally at a large number of different series of field-proportions, if we observe the requirements of the available instruments, and of their technique, from such a point of view.

The concept of a group has cropped up again in the example of grace-notes and modes of attack. A particular number of single field-sizes gives a *group-field*. Here the size of the group-fields depends on the number and size of the single fields. Similarly, it is possible to start from group-fields of various sizes, and from these to arrive at the magnitude-proportions of the single fields. Let us again give an example of this. We take as our point of departure the fact that a woodwind player can not play as long as he likes with one breath. If we ignore physiological factors, the duration of a breath depends on the register, density and loudness of the notes to be played. The lower and louder the notes, and the fewer sustained notes there are to play, the shorter the duration of the breath is. When the tempo indication ‘as slow as possible’ is given, this means that a group of single durations must be distributed over as long a total duration as possible, depending on the duration of the breath. The length of the breath would determine the group-field; but the different magnitude-proportions of such group-fields would result from the way the register, density and intensity of the groups were composed. And the proportions of the single durations in the groups could either be relatively measured, or could again be determined in detail as field-sizes. But if here the detailed field-proportions depend on the field-size of the group, then they can no longer simply be given by the action-durations. The different field-sizes would have to result, rather, from a completely different notation of durations. By selecting the size of a group-field of blown notes, one has fixed, as we said, the number of durations in the group. Now the group-field can be split up into a number of *part-fields*. To take the simplest case, the part-fields can be of equal length. Further, the number of durations must be determined for each part-field, and these durations can be accommodated with relative freedom in such a part-field (for example, with the restriction that no periodicity may occur). The fewer durations per part-field, the more possible it is for the player to distribute the durations in various ways; the more durations per part-field, the narrower the scatter of the field-size of each single duration.

Thus the single durations are not notated at all, only the number of durations to be

Group-field (length of breath)



distributed over each part-field. If one desires, generally, to keep the field-proportions very small, one chooses many short part-fields; but if the field-proportions are to increase in size, one chooses fewer part-fields. If the number of part-fields is also determined statistically, the process would run something like this: 3-5 (in the first part-field); 15-20 (in the second), etc. Either the size of the part-fields is constant and the number variable, or vice versa, or both can be variable.

Finally, a longer time-structure, and even the total *overall structure* of a piece, can be composed in *field-proportions*, when a correspondence is sought between the individual time-structures and the whole. Again, let us give an example of this. It is best to describe the formal concept. A particular number of note-groups have, at first, no fixed total durations, intensity-curves or modes of attack for each group; these group-properties are, rather, to be variable within chosen limits, and the field-sizes of a group are to result from the last group preceding that to be played. Consequently, the structure of the piece is not presented as a sequence of development in time, but rather as a *directionless time-field*, in which the individual groups also have no particular direction in time (as to which follows which). Thus all groups are composed simultaneously, each beginning of a group being a possible continuation of each end of a group. Within a group, several series of field-proportions (using very varied indications) fix the relative time-proportions, and thus also the number and succession of pitches (individual durations and pitches are *not* interchangeable, but have 'direction'). But no tempo is prescribed for the durations measured in note-values (some of which are normally notated), nor is the intensity-curve or the mode of attack given. Only *after* each group come the instructions how the following group – any one of the others – is to be played: for example, 'from $\frac{1}{4}$ = ca. 40 get faster and then slow down again'; 'make a diminuendo from *mf*'; 'r.h. *legato*, l.h. *staccato*' (or any other indication). Then the groups are irregularly distributed on a piece of paper of a particular size, and the general instructions are: 'play any group, selected *at random*, quite freely as the first, then, equally at random, look the paper over and play the group next seen, but observing the directions given at the end of the group played first, etc. When any group is seen for the third time, the piece is over'. As it is probable that several groups will be played twice, the pitch-structure of some groups is, in parts, notated twice; when a group is played for the second time, some single notes drop out, others are interchanged (the ones in brackets), whole groups are shifted one or more octaves (by ignoring octave signs), etc. The field-structure of a large form like this will become clearer, naturally, if it can be

compared with that of other pieces in a cycle, or, above all, when it is played several times in succession.

Other concepts combine structures that 'flow' and time-structures that are undirected. Thus, one can now imagine not only the synchronisation of simultaneous *proper times* – i.e., time as an organism of spatially graduated strata – but also a serialisation of successive proper times – i.e., time as a developed unfolding, *and* as a statistical, undirected condition of continuous time-fields. (Here, 'simultaneous' and 'successive', are useful analogies with spatial presentation, and, as such, may be exchanged or identified at will; what is really meant here is the difference between 'present' and 'absent'.)

Thus it is possible for the musician to establish, between *time-quanta* as measured and *field-time* as experienced, a connection whose closeness will vary; by means of variously graded field-proportions, he will mediate between the two principles, those of indicating the duration of sounds and the actions to be carried out in order to produce them.

Degrees of freedom for the instrumentalist are apparently linked with field-composition. It might be thought that, with fields of varying size, the composer had left the instrumentalist a share of 'improvisation' – doses of various potency. If one examines this more closely an unmeasured time-field is no occasion freely to invent something in addition to the composed structure.

It is rather that rationally guided measurement of time – counting – is reinforced by a spontaneously reacting utterance of time – agitation of time. Whereas up to now the action of playing (and of hearing) had to orientate itself to time-relationships that were measured in durations, there are now, to some extent, cases in which time-proportions arise *only* through actions. In other words: up till now, one could see from the score the time-relationships composed in a piece of music, quite independently of its realisation in sound, and the 'rightness' of a realisation in sound could be checked against the time-notation in the score; but in a field-composition, the parts of the score in which actions are notated give no information at all about the measurement of time-proportions – the latter come into existence only at the moment when they are realised in sound, when they are played. In this case, the 'rightness' of a realisation is checked against itself; tested, that is, in order to find out whether the action-times in the moment of playing stand in an organic relationship to the sound-times to be produced. Of what, then, would the degree of freedom of an instrumentalist in a field consist? When, for example, he must play grace-notes 'as fast as possible', and pay attention to the fact that all notes in all registers must be distinctly recognisable as pitches, then he must decide – according to his musical feeling – what he considers to be 'sufficiently distinct'; and this depends, as we have said, on many factors. The same applies to the notation of attacks, preparations and to the precept 'if possible, only aperiodic time-relationships to be played in the part-fields', etc. Finally, there is the injunction to 'look the sheet of music over at random and play the group next seen'.

Whether one leaves the instrumentalist to use his musical judgment to decide that a certain sound has lasted exactly the right amount of time, and that the next must now sound; whether one attaches a certain degree of freedom to this 'weighing' instead of 'counting' of durations, is of no consequence. One might just as well say that, in his actions, the musician answers the 'proper time' of the sound, and, instead of mechanically quantifying durations that conflict with the regularity of metronomic time, he now

measures 'sensory quanta'; he feels, discovers the time of the sounds; he lets them take 'their' time.

That is obviously what is meant by 'freedom'. For how is it that the instrumentalists who have played the first compositions of this kind feel much freer than hitherto, although they are more engaged?

It can be a profitable working method for a composer to select a *series of degrees of freedom* for a work. The act of composition itself moves in 'action-fields' of various sizes. The popular conception of one person's working 'freely', and another's working 'more strictly', and a third 'schematically', springs really from a time when freedom itself was schematized. Thus there were 'free' and 'strict' forms, 'free' and 'strict' interpretations, etc. If a composer experiences musical time as multi-dimensional time, his composition has itself become multi-dimensional; for him measured and perceived proportions, time-fields and -quantities, systematic and 'chance' determinations, are extremes, between which there are many stages. Thus, in a work, one can successively compose part-structures, in which one can freely choose from a larger or smaller number of possible configurations. Such *fields of choice* – of any proportions – can mediate between totally predeterminate and indeterminate structures. Choice itself, distributing the weight in various ways, combines rational measuring with weighing according to perception. A proportion-series of fields of choice would then be peculiar to one work. How could it happen that a musical conception of time at last exists, if it did not spring from the act of composition itself?

At the outset of this investigation we outlined a morphology of time, and in view of this, one could apply to the sphere of pitch all the foregoing reflections about field-composition in the sphere of durations. But it seems superfluous to treat each detail all over again, and to introduce, instead of the field-definition of duration, the *field-definition of pitch*. However, certain consequences should be specially mentioned.

As soon as a pitch is thought of as a field-size, rather than as a discrete magnitude, no instruments with fixed scales can be used (these would include the piano), but only instruments on which *pitch* can be presented *continuously*. A discrete pitch would then, like a measured duration, be the special case in which field-size was zero. Here again, there are the two possibilities: to arrive at the group from the individual field-sizes, or to arrive at the individual pitches from the group-field. Either an interval in pitch results from a prescribed action – e.g., the size of a movement over the keyboard, and here the spatial spread of the keyboard could be built in various sizes – or the graphic notation marks pitch-fields with more or less exactly defined ambit, and indicates the group-number of pitches per part-field. Here again, fields of individual pitches result from the relationship of number and ambit in a part-field (see Ex. 22). The direction in pitch of a group of notes can be either indeterminate or indicated by a directional diagram (rising, falling, combined).

Not every pitch, however, is simply a periodic sequence of equal fundamental phases. This would mean once more that the fundamental tones are treated not as measured discrete sizes in a scale, but as fields of fundamental tones; that, however, the fundamental tone, once reached, would automatically be constant in pitch – i.e., a group-sequence of equally-long fundamental phases. Here, the individual fundamental duration (in micro-time) would be a field-size, but one fundamental phase in a group would be exactly the same as another. It would then be an exception, in

field-composition, for pitch to remain constant in the course of a duration. That would only be the case where no field-proportions came into play in a phase-group, but where all the phases were equally long, and no room was available for phase-displacement. What follows from all this?

A new instrument would have to be built, on which, for example, different pressures on a continuous band produce oscillations with phases that are more or less constant. The location of the pressure determines the pitch. If one presses only very lightly, the oscillation keeps a constant phase – the pitch remains the same. The heavier the pressure, the more irregular become the phase-relationships and the more indeterminate the pitch. This means, in fact, that this kind of continuous *phase-modulation* gradually transforms a *tone* into a *noise*. The strength of the pressure then corresponds to the spectral width (the field-size) of the noise. Maximum pressure finally obliterates any perception of pitch, and 'white noise' is produced. Minimum pressure produces a 'pure tone'. This would make possible field-regulation of fundamental phases.

Just as in field-composition with the formant-spectra of macro-time durations, we require field-sizes for simultaneous micro-time phase-relationships. Thus it must be possible to alter the register and number of the formants in a time-spectrum continuously, and therewith the formant-rhythm ('tone-colour') of a fundamental tone. Thus the minimum pressure on the band not only produces a 'pure' oscillation with a constant phase-length, but also a *continuum of 'tone-colour'* on the same tone. This is easy to imagine. The secondary oscillations would be added to the fundamental oscillation by pressing near the front or near the back (of the continuous band-keyboard), and when the sounds have several formants, these are controlled by the number and distance apart of fingers exerting pressure simultaneously. These secondary oscillations, too, would vary in the constancy of the phase-lengths, according to the amount of pressure exerted by the individual fingers. Lastly, the intensity of the oscillations could be continuously altered by the use of a pedal.

Such an ideal instrument would meet all the requirements for continuous alteration of time-proportions: for fundamental macro-time phases (durations), by the duration of the action; for fundamental micro-time phases (pitches), by the location on the continuous band-keyboard; for micro-time phase-relationships (the transition from 'tone' to 'noise'), by the amount of pressure on the band; for micro-time formant-rhythm ('tone-colour'), by the location of the pressure-point, and the number and distance apart of several pressure-points across the keyboard; for the amplitude of the oscillation (loudness) by pressure on the pedal. Formant-spectra of durations (superposition of various processes in time and in sound) would be achieved by several of such instruments playing together.

The builder of instruments can not be expected to have any idea of what sort of instrument the musician needs; the musician must tell him. It is impossible to say how long we shall have to wait; but one may expect that some day such an instrument will exist.

For it does not seem very fruitful to founder on a contradiction between, on the one hand, a material that has become useless – instruments that have become useless – and, on the other, our compositional conception. Some prefer to reconcile their craft with a new musical time-concept, rather than tilt at windmills, or devote all their efforts to building on a compromise. They need fear no rules, then, nor prohibitions, no system,

no theory, no constraint, for they inhabit this time-order, and their music gives answer to the nature of sound, if it reveals itself to them; if they realise . . .

(The article . . . *How time passes . . .* was written in September and October 1956, and refers particularly to the compositions *Zeitmasse* (*Time-measures*), *Gruppen für drei Orchester* (*Groups for three orchestras*) and *Klavierstück XI*).

TO DESCRIBE THE PROCESS OF COMPOSITION USED IN
'MUSIC FOR PIANO 21-52'

JOHN CAGE

1. Given ink, pen, and sheets of transparent paper of determined dimensions, a master-page (without notations) is made, having four total systems. 'Total' here means having enough space above and below each staff to permit either being bass or treble. Thus, there being the conventional two staves (one for each hand), each has enough space above it to accommodate 9 ledger lines (as equidistant as those of the staves) and below it to accommodate 6 ledger lines plus (leaving room for the extreme low piano key and string). Between the two there is a narrow space, bisected by a line, allowing for the notation of noises produced by hand or beater upon the interior (above the line) or exterior (below the line) piano construction. Measurements are such that the entire sheet (within margins) is potentially useful.

2. Laying the master-page aside, chance operations channelled within certain limits (1-128 for 21-36; 1-32 for 37-52) which are established in relation to relative difficulty of performance) – derived from the *I-Ching*¹ are employed to determine the number of sounds per page.

3. A blank sheet of transparent paper is then placed so that its pointal imperfections may readily be observed. That number of imperfections is intensified with pencil corresponding to the determined number of sounds.

4. Placing the pencilled sheet in a registered way upon the master-page, first the staves and inter-line and then the ledger lines where necessary are inscribed in ink. Secondly, conventional whole notes are written in ink wherever a pencilled point falls within the area of staves or ledger lines, inked-in notes (crotchets without stems) being written wherever such a point falls within the space between the two staves. This operation is done roughly, since, through the use of conventional lines and spaces, points falling in the latter are in the majority. Thus it is determined that a point, though not on a line, is actually more nearly so than it is at the centre of the adjacent space.

5. Eight single coin tosses are made determining the clefs, bass or treble, and inscribed in ink.

6. The 64 possibilities of the *I-Ching* are divided by chance operations into three groups relative to three categories: normal (played on the keyboard); muted; and plucked (the two latter played on the strings). For example, having tossed numbers 6 and 44, a number 1 through 5 will produce a normal; 6 through 43 a muted; 44 through 64 a plucked piano tone. A certain weight of probability exists in favour of the second and third categories. Though this has not appeared to be of consequence, it indicates a possible change in 'technique'. The categories having been determined, notations (M and P) are conveniently placed in reference to the notes.

A similar procedure is followed to determine whether a tone is natural, sharp, or flat,

¹ The *I-Ching*, an ancient Chinese book, has been translated into German by Richard Wilhelm (into English by Cary F. Baynes) accompanied by an introduction written by C. G. Jung.