10.3.2 Balanced Strain Condition

A balanced strain condition exists at a cross section when the maximum strain at the extreme compression fiber just reaches $\varepsilon_u = 0.003$ simultaneously with the first yield strain of $\varepsilon_s = \varepsilon_y = f_y/E_s$ in the tension reinforcement. This balanced strain condition is shown in Fig. 6-13.

The required reinforcement ratio, ρ_b, to produce a balanced strain condition in a rectangular section with tension reinforcement only may be obtained by applying equilibrium and strain compatibility conditions. Referring to Fig. 6-13, for the linear strain condition:

$$\frac{c_b}{d} = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$$

$$= \frac{0.003}{0.003 + f_y/29{,}000{,}000} = \frac{87{,}000}{87{,}000 + f_y}$$

For force equilibrium:

$$C_b = T_b \quad \text{or,} \quad 0.85f'_c\, ba_b = A_{sb}f_y \quad \text{or,} \quad 0.85f'_c b\,(\beta_1 c_b) = \rho_b b d f_y$$

thus,

$$\rho_b = \frac{0.85\,\beta_1 f'_c}{f_y} \times \frac{c_b}{d}$$

$$= \frac{0.85\,\beta_1 f'_c}{f_y} \times \frac{87{,}000}{87{,}000 + f_y} \qquad \text{Eq. (8-1)}$$

Values of ρ_b for various concrete and reinforcement strengths are listed in Table 6-1.

The balanced reinforcement ratio ρ_b for flanged sections and rectangular sections with compression reinforcement may be obtained by applying the equilibrium and compatibility conditions in a similar manner:

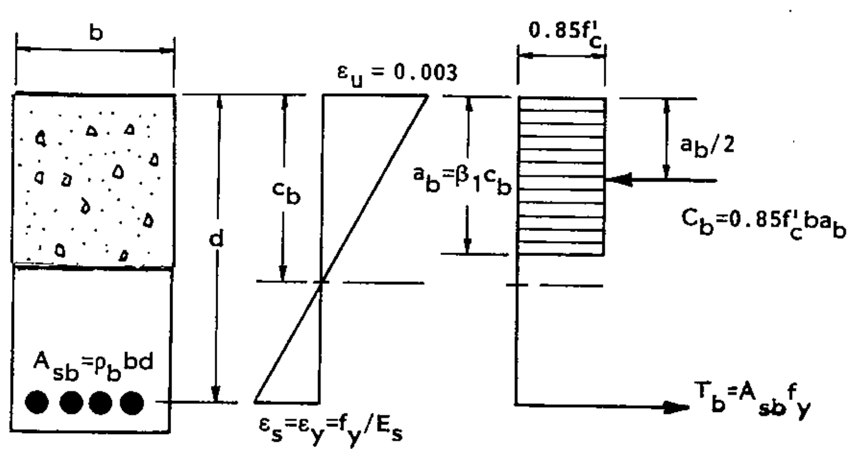

Fig. 6-13 Balanced Strain Condition in Flexure

Table 6-1—Balanced Ratio of Reinforcement ρ_b for Rectangular Sections with Tension Reinforcement Only

f_y	$f'_c = 3000$ $\beta_1 = 0.85$	$f'_c = 4000$ $\beta_1 = 0.85$	$f'_c = 5000$ $\beta_1 = 0.80$	$f'_c = 6000$ $\beta_1 = 0.75$	$f'_c = 8000$ $\beta_1 = 0.65$	$f'_c = 10{,}000$ $\beta_1 = 0.65$
40,000	0.0371	0.0495	0.0582	0.0655	0.0757	0.0946
60,000	0.0214	0.0285	0.0335	0.0377	0.0436	0.0545
75,000	0.0155	0.0207	0.0243	0.0274	0.0316	0.0396

For a flanged section with tension reinforcement only:

$$\rho_b = \frac{b_w}{b}\left(\overline{\rho_b} + \rho_f\right) \qquad (8)$$

where

$$\rho_f = \frac{A_{sf}}{b_w d} \text{ and } A_{sf} = 0.85\,\frac{f'_c}{f_y}(b - b_w)h_f$$

For a rectangular section with compression reinforcement:

$$\rho_b = \overline{\rho_b} + \rho'\,\frac{f'_{sb}}{f_y} \qquad (9)$$

where f'_{sb} = stress in compression reinforcement at balanced strain condition

$$= 87{,}000 - \frac{d'}{d}(87{,}000 + f_y) \le f_y$$

and $\overline{\rho_b}$ = balanced reinforcement ratio for a rectangular section with tension reinforcement only.

Table 10-1—Flexural Strength $M_u/\varphi f'_c bd^2$ or $M_n/f'_c bd^2$ of Rectangular Sections with Tension Reinforcement only

ω	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
0.0	0	.0010	.0020	.0030	.0040	.0050	.0060	.0070	.0080	.0090
0.01	.0099	.0109	.0119	.0129	.0139	.0149	.0159	.0168	.0178	.0188
0.02	.0197	.0207	.0217	.0226	.0236	.0246	.0256	.0266	.0275	.0285
0.03	.0295	.0304	.0314	.0324	.0333	.0343	.0352	.0362	.0372	.0381
0.04	.0391	.0400	.0410	.0420	.0429	.0438	.0448	.0457	.0467	.0476
0.05	.0485	.0495	.0504	.0513	.0523	.0532	.0541	.0551	.0560	.0569
0.06	.0579	.0588	.0597	.0607	.0616	.0625	.0634	.0643	.0653	.0662
0.07	.0671	.0680	.0689	.0699	.0708	.0717	.0726	.0735	.0744	.0753
0.08	.0762	.0771	.0780	.0789	.0798	.0807	.0816	.0825	.0834	.0843
0.09	.0852	.0861	.0870	.0879	.0888	.0897	.0906	.0915	.0923	.0932
0.10	.0941	.0950	.0959	.0967	.0976	.0985	.0994	.1002	.1011	.1020
0.11	.1029	.1037	.1046	.1055	.1063	.1072	.1081	.1089	.1098	.1106
0.12	.1115	.1124	.1133	.1141	.1149	.1158	.1166	.1175	.1183	.1192
0.13	.1200	.1209	.1217	.1226	.1234	.1243	.1251	.1259	.1268	.1276
0.14	.1284	.1293	.1301	.1309	.1318	.1326	.1334	.1342	.1351	.1359
0.15	.1367	.1375	.1384	.1392	.1400	.1408	.1416	.1425	.1433	.1441
0.16	.1449	.1457	.1465	.1473	.1481	.1489	.1497	.1506	.1514	.1522
0.17	.1529	.1537	.1545	.1553	.1561	.1569	.1577	.1585	.1593	.1601
0.18	.1609	.1617	.1624	.1632	.1640	.1648	.1656	.1664	.1671	.1679
0.19	.1687	.1695	.1703	.1710	.1718	.1726	.1733	.1741	.1749	.1756
0.20	.1764	.1772	.1779	.1787	.1794	.1802	.1810	.1817	.1825	.1832
0.21	.1840	.1847	.1855	.1862	.1870	.1877	.1885	.1892	.1900	.1907
0.22	.1914	.1922	.1929	.1937	.1944	.1951	.1959	.1966	.1973	.1981
0.23	.1988	.1995	.2002	.2010	.2017	.2024	.2031	.2039	.2046	.2053
0.24	.2060	.2067	.2075	.2082	.2089	.2096	.2103	.2110	.2117	.2124
0.25	.2131	.2138	.2145	.2152	.2159	.2166	.2173	.2180	.2187	.2194
0.26	.2201	.2208	.2215	.2222	.2229	.2236	.2243	.2249	.2256	.2263
0.27	.2270	.2277	.2284	.2290	.2297	.2304	.2311	.2317	.2324	.2331
0.28	.2337	.2344	.2351	.2357	.2364	.2371	.2377	.2384	.2391	.2397
0.29	.2404	.2410	.2417	.2423	.2430	.2437	.2443	.2450	.2456	.2463
0.30	.2469	.2475	.2482	.2488	.2495	.2501	.2508	.2514	.2520	.2527
0.31	.2533	.2539	.2546	.2552	.2558	.2565	.2571	.2577	.2583	.2590
0.32	.2596	.2602	.2608	.2614	.2621	.2627	.2633	.2639	.2645	.2651
0.33	.2657	.2664	.2670	.2676	.2682	.2688	.2694	.2700	.2706	.2712
0.34	.2718	.2724	.2730	.2736	.2742	.2748	.2754	.2760	.2766	.2771
0.35	.2777	.2783	.2789	.2795	.2801	.2807	.2812	.2818	.2824	.2830
0.36	.2835	.2841	.2847	.2853	.2858	.2864	.2870	.2875	.2881	.2887
0.37	.2892	.2898	.2904	.2909	.2915	.2920	.2926	.2931	.2937	.2943
0.38	.2948	.2954	.2959	.2965	.2970	.2975	.2981	.2986	.2992	.2997
0.39	.3003	.3008	.3013	.3019	.3024	.3029	.3035	.3040	.3045	.3051

$M_n/f'_c bd^2 = A_s f_y (d-a/2)\ f'_c bd^2 = \omega(1-0.59\omega)$, where $\omega = \rho f_y/f'_c$ and $a = A_s f_y/0.85 f'_c b$.

Design: Using factored moment M_u enter table with $M_u/\varphi f'_c bd^2$; find ω and compute steel percentage ρ from $\rho = \omega f'_c/f_y$

Investigation: Enter table with ω from $\omega = \rho f_y/f'_c$; find value of $M_n/f'_c bd^2$ and solve for nominal strength, M_n.

Example 10.1—Design of Rectangular Beam with Tension Reinforcement Only

Select a rectangular beam size and required reinforcement A_s to carry service load moments of: M_d = 55 ft-kips and M_ℓ = 36 ft-kips. Select reinforcement to control flexural cracking for exterior exposure.

f'_c = 4000 psi
f_y = 60,000 psi
z = 145 kips/in. (exterior exposure)

Calculations and Discussion	Code Reference

1. To illustrate a complete design procedure for rectangular sections with tension reinforcement only, a minimum beam depth will be computed using the maximum reinforcement permitted for flexural members, $0.75\rho_b$. The design procedure will follow the method outlined on the preceding pages. — 10.3.3

Step 1. Determine maximum reinforcement ratio for material strengths f'_c = 4000 psi and f_y = 60,000 psi.

ρ_b = 0.0285, from Table 6-1   p 6-15

$\rho_{max} = 0.75\rho_b = 0.75(0.0285) = 0.0214$ — 10.3.3

Step 2. Compute bd^2 required.

Required moment strength:

$M_u = 1.4 \times 55 + 1.7 \times 36 = 138$ ft-kips — Eq. (9-1)

$$R_n = \rho f_y\left(1 - 0.5\frac{\rho f_y}{0.85 f'_c}\right)$$

$$= 0.0214 \times 60{,}000\left(1 - \frac{0.5 \times 0.0214 \times 60{,}000}{0.85 \times 4000}\right) = 1042 \text{ psi}$$

$$bd^2 \text{ (required)} = \frac{M_u}{\varphi R_n} = \frac{138 \times 12 \times 1000}{0.90 \times 1042} = 1766 \text{ in.}^3$$

Step 3. Size member so that

bd^2 required ≤ bd^2 provided

Set b = 10 in. (column width)

Calculations and Discussion	Code Reference

$d = \sqrt{1766/10} = 13.3$ in.

Minimum beam depth $\approx 13.3 + 2.5 = 15.8$ in.

For moment strength, a 10 in. x 16 in. beam size is adequate. However, deflection is an essential consideration in designing beams by the Strength Design Method. Control of deflection is discussed in Part 8.

Step 4. Using the 16 in. beam depth, compute a revised value of ρ. For illustration, ρ will be computed by all four methods outlined earlier.

$d = 16 - 2.5 = 13.5$ in.

(1) by Eq. (4) (exact method):

$$R_n = \frac{M_u}{\varphi(bd^2 \text{ provided})} = \frac{138 \times 12 \times 1000}{0.90(10 \times 13.5^2)} = 1010 \text{ psi}$$

$$\rho = \frac{0.85f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85f'_c}}\right)$$

$$= \frac{0.85 \times 4}{60}\left(1 - \sqrt{1 - \frac{2 \times 1010}{0.85 \times 4000}}\right) = 0.0206$$

(2) by strength curves such as that shown in Fig. 10-2:

for $R_n = 1010$ psi, $\rho \approx 0.0205$

(3) by strength tables such as Table 10-1:

$$\text{for } \frac{M_u}{\varphi f'_c bd^2} = \frac{138 \times 12 \times 1000}{0.90 \times 4000 \times 10 \times 13.5^2} = 0.252$$

$\omega = 0.308$

$\rho = \omega f'_c / f_y = 0.308 \times 4/60 = 0.0205$

(4) by approximate proportion:

$$\rho \approx (\text{original } \rho)\frac{(\text{revised } R_n)}{(\text{original } R_n)}$$

Calculations and Discussion	Code Reference

$$\rho \approx 0.0214 \times \frac{1010}{1042} = 0.0207$$

Step 5. Compute A_s required.

$$A_s = (\text{revised}\,\rho)\,(\text{bd provided})$$

$$= 0.0206 \times 10 \times 13.5 = 2.78\ \text{in.}^2$$

THIS CAN BE SIZED AS
(3) X #9 BARS = 3 X 1.0 = 3in²