MAG 11/15/11 CS470

Modeling

Bayes ~~Net~~ Filter Examples: perfect observer

Prediction/Transition $x_t = x_{t-1} + n_t \qquad n_t \sim N(0, \sigma_x^2)$

$$\Rightarrow p(x_t \mid x_{t-1}) \sim N(x_{t-1}, \sigma_x^2)$$

Measurement $z_t = x_t + v_t \qquad v_t \sim N(0, \sigma_z^2)$

$$\Rightarrow p(z_t \mid x_t) \sim N(x_t, \sigma_z^2)$$

Initial Conditions $Bel(x_0) = p_{X_0}(x_0) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$

Assumptions $\sigma_x^2 = 2 \qquad \sigma_z^2 = 0$ // perfect observer //

Time t = 1

$z_1 = 3/4$ // This is what we observe //

$$\overline{Bel}(x_1) = \int_{-\infty}^{\infty} p(x_1 \mid x_0)\, Bel(x_0)\, dx_0$$

$$= \int_0^1 p(x_1 \mid x_0)\, dx_0$$ // Notice change in limits of integration //

$$= \int_0^1 \alpha \exp\left(-\frac{1}{2}\left(\frac{(x_1 - x_0)^2}{\sigma_x^2}\right)\right) dx_0$$

Bayes Filter Examples: Perfect observer continued...

$$\overline{Bel}(x_1) = \int_0^1 \alpha \exp\left(-\tfrac{1}{2}\frac{(x_1-x_0)^2}{\sigma_x^2}\right) dx_0 \quad \text{// from page 1 //}$$

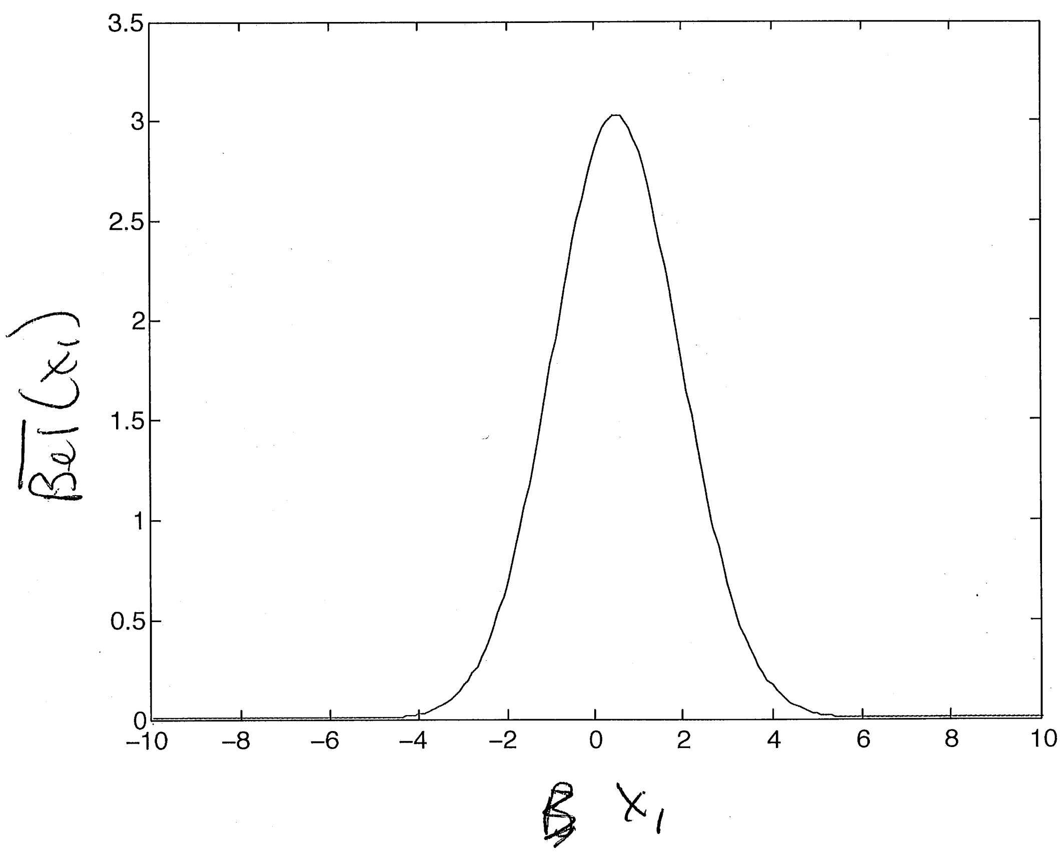
This generates a series of values, one for each possible value for x_1. $\alpha = \frac{1}{\sqrt{2\pi\sigma_x^2}}$. Unfortunately, the integral above doesn't have a known closed-form solution so we have to rely on the erf function, or compute the integral numerically. I did the latter and obtained the plot ~~below~~ on the next page.

$$Bel(x_1) = \beta\, p(z_1 = 3/4 \mid x_1)\, \overline{Bel}(x_1)$$

Since $\sigma_z = 0$, $p(z_1 = 3/4 \mid x_1) = \begin{cases} 1 & x_1 = 3/4 \\ 0 & \text{otherwise} \end{cases}$

so $Bel(x_1) = \begin{cases} 1 & x_1 = 3/4 \\ 0 & \text{otherwise.} \end{cases}$

Bayes Filter Example: perfect observer continued...



$$\boxed{\text{Bayes Filter Example: Perfect Initial Knowledge}}$$

Prediction / Transition

$$x_t = x_{t-1} + 1 + n_t \qquad n_t \sim N(0, \sigma_x^2)$$

Measurement

$$z_t = 2x_{t-1} + v_t \qquad v_t \sim N(0, \sigma_z^2)$$

Initial Conditions

$$Bel(x_0) = \begin{cases} 1 & x_0 = -1 \\ 0 & \text{otherwise} \end{cases}$$

Assumptions

$\sigma_x^2 = 0$ // perfect model //

$\sigma_z^2 = 4$ // noisy observer //

Time $t = 1$

$z_1 = -2$ // This is what we observe //

$$\overline{Bel}(x_1) = \int_{-\infty}^{\infty} p(x_1 | x_0) \, Bel(x_0) \, dx_0$$

$$= \int_{-\infty}^{\infty} \begin{cases} p(x_1 | x_0) & x_0 = -1 \\ 0 & \text{otherwise} \end{cases} dx_0$$

$$= \begin{cases} 1 & x_1 = x_0 + 1 = 0 \\ 0 & \text{otherwise} \end{cases}$$

Bayes Filter Examples: Perfect Initial Knowledge continued...

$$Bel(x_1) = \alpha \; p(z_1|x_1)\, Bel(x_1)$$

$$= \begin{cases} \alpha\, p(z_1=-2|x_1) & x_1 = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 1 & x_1 = 0 \\ 0 & \text{otherwise.} \end{cases}$$

Discrete States

Suppose $x \in \{red, green, blue\}$ are the only states.
Suppose $z \in \{yes, no\}$ are the only observations.

Prediction

$$p(x_{t+1}|x_t) = \begin{cases} 0.7 & x_{t+1} = x_t \\ 0.15 & x_{t+1} \neq x_t \end{cases}$$

// You tend to stay in the same state, and change to ~~another~~ each other state with equal probability //

Measurement

x	red	red	green	green	blue	blue
z	yes	no	yes	no	yes	no
$p(z\|x)$	1	0	0.6	0.4	0	1

Bayes Filter Examples:
~~Perfect Initial Knowledge~~
Discrete States
continued...

Initial Conditions

$$\text{Bel}(x_0) = \begin{cases} 1/2 & x_0 = \text{red} \\ 1/4 & x_0 = \text{green} \\ 1/4 & x_0 = \text{blue} \end{cases}$$

Time t=1

$z_1 = \text{yes}$ // observation at t=1 //

$$\overline{\text{Bel}}(x_1) = \sum_{x_0} p(x_1|x_0)\,\text{Bel}(x_0)$$ // change to sum since discrete //

$$\overline{\text{Bel}}(x_1=\text{red}) = p(\text{red}|\text{red})\,\text{Bel}(\text{red}) + p(\text{red}|\text{green})\,\text{Bel}(\text{green}) + p(\text{red}|\text{blue})\,\text{Bel}(\text{blue})$$

$= \cancel{0.7(1/2)+}$

$= 0.7(1/2) + 0.15(1/4) + 0.15(1/4)$

$= 0.425$

$$\overline{\text{Bel}}(x_1=\text{green}) = p(\text{green}|\text{red})\,\text{Bel}(\text{red}) + p(\text{green}|\text{green})\,\text{Bel}(\text{green}) + p(\text{green}|\text{blue})\,\text{Bel}(\text{blue})$$

$= 0.15(1/2) + 0.7(1/4) + 0.15(1/4)$

$= 0.2875$

Bayes Filter Examples:
Discrete States
continued...

$\overline{Bel}(x_1 = blue) = 0.2875$

$Bel(x_1) = \alpha \, p(z_1 = yes \mid x_1) \, \overline{Bel}(x_1)$

$Bel(x_1 = red) = \alpha \, p(yes \mid red) \, \overline{Bel}(red)$
$= \alpha \, 1 \, (0.425)$

$Bel(x_1 = green) = \alpha \, p(yes \mid green) \, \overline{Bel}(green)$
$= \alpha \, 0.6 \, (0.2875)$

$Bel(x_1 = blue) = \alpha \, p(yes \mid blue) \, \overline{Bel}(blue)$
$= \alpha \, 0 \, (0.2875)$
$= 0.$

$\sum_{x_1} Bel(x_1) = 1 = 0.425\alpha + 0.6(0.2875)\alpha$

$= 0.5975\alpha$

$\Rightarrow \alpha = 1/0.5975 = 1.6736$

$\Rightarrow$

$Bel(red) = \alpha(0.425) = .7113$
$Bel(green) = \alpha(.6)(.2875) = .2887$
$Bel(blue) = 0$