# Marvok-Chain Model for Unbounded Key Propagation

January 29, 2013

## 1 Three Children Model

In this case we let the number of children $m = 3$. Following the approach for the $m = 2$ case, we now define the following sets $S_1$, $S_2$, $S_3$, $S_4$, and $S_5$.

$$\begin{aligned}
S_1 &= \text{The number of nodes with 3 children} \\
S_2 &= \text{The number of nodes with 2 children} \\
S_3 &= \text{The number of nodes with 1 children} \\
S_4 &= \text{The number of nodes with 0 children} \\
S_5 &= \text{The number of nodes that are not connected}
\end{aligned}$$

In a network with $n$ nodes, we can see that $\sum_{i=1}^{5} S_i = n$.

Now we examine the change of the network state at each epoch, where a node is assumed to only obtain one new child node connection in an epoch. To capture this behavior, we define the following variables $D_2$, $D_3$, and $D_4$ to be the number of new nodes connected from nodes in sets $S_2$, $S_3$, and $S_4$, respectively. Using this information, the transfer equations clearly generalize to:

$$\begin{aligned}
S_1 &\to S_1 + D_2 \\
S_2 &\to S_2 - D_2 + D_3 \\
S_3 &\to S_3 - D_3 + D_4 \\
S_4 &\to S_4 + D_2 + D_3 \\
S_5 &\to S_5 - D_2 - D_3 - D_4
\end{aligned}$$

Clearly, the initial state of the network is $S* = (S_1, S_2, S_3, S_4, S_5) = (0, 0, 0, 1, n - 1)$. Using the aforementioned transfer equations we can represent this state as $S* = (D_2, D_3 - D_2, D_4 - D_3, 1 + D_2 + D_3, n - 1 - D_2 - D_3 - D_4)$. Therefore, we can represent the state of the network using a three-dimensional vector $D_k = (D_1, D_2, D_3)$.

If we now consider transitions in the state of the network by some vector $\bar{h} = (i, j, k)$, where the transition is defined as $D + \bar{h} = (D_2 + i, D_3 + j, D_4 + k)$, as well as the transfer equations used to define the network state evolution, we come up with the following constraints for $\bar{h}$

Based on the transfer equations, we can also define the following constraints for the network state.

$$\begin{aligned}
0 \le i &\le D_3 - D_2 \\
0 \le j &\le D_4 - D_3 \\
0 \le k &\le 1 + D_2 + D_3 \\
i + j + k &\le n - 1 - D_2 - D_3 - D_4
\end{aligned}$$

## 2 Generalized Model