Mixed-Integer Geometric Programming

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1 From GP to MIGP

gp standard form and convex form [1], example - aircraft wings [2]

gp with integer variables, example - gate sizing [1]

generalized GP:

$$\min \quad \max_{S\in\mathcal{S}} \quad \sum_{i\in S} D_i : \tag{1a}$$

$$\gamma_i X_i^{-1} \sum_{j\in F_i} (\alpha_j + \beta_j X_j) = D_i \qquad \forall i \in G\backslash O \tag{1b}$$

$$\gamma_i c_i X_i^{-1} = D_i \qquad \forall i \in O \tag{1c}$$

$$X_i \geq 1 \qquad \forall i \in G \tag{1d}$$

$$\sum_{i\in G} f_i e_i X_i \leq \bar{p} \tag{1e}$$

$$\sum_{i\in G} a_i X_i \leq \bar{a} \tag{1f}$$

or GP:

$$\min \quad D : \tag{2a}$$

$$D^{-1} \sum_{i\in S\cap O} \gamma_i c_i X_i^{-1} + D^{-1} \sum_{i\in S\backslash O} \gamma_i X_i^{-1} \sum_{j\in F_i} (\alpha_j + \beta_j X_j) \leq 1 \qquad \forall S \in \mathcal{S} \tag{2b}$$

$$X_i^{-1} \leq 1 \qquad \forall i \in G \tag{2c}$$

$$\bar{p}^{-1} \sum_{i\in G} f_i e_i X_i \leq 1 \tag{2d}$$

$$\bar{a}^{-1} \sum_{i\in G} a_i X_i \leq 1 \tag{2e}$$

now let $X_i = e^{Y_i}, \forall i \in G$ and χ. transform into convex form by taking log of both sides of each posynomial inequality.

$$\min \quad D: \tag{3a}$$

$$D^{-1} \sum_{i \in S \cap O} \gamma_i c_i X_i^{-1} + D^{-1} \sum_{i \in S \setminus O} \gamma_i X_i^{-1} \sum_{j \in F_i} (\alpha_j + \beta_j X_j) \leq 1 \qquad \forall S \in \mathcal{S} \tag{3b}$$

$$X_i^{-1} \leq 1 \qquad \forall i \in G \tag{3c}$$

$$\bar{p}^{-1} \sum_{i \in G} f_i e_i X_i \leq 1 \tag{3d}$$

$$\bar{a}^{-1} \sum_{i \in G} a_i X_i \leq 1 \tag{3e}$$

Where in the example problem, $O = \{6, 7\}, \mathcal{S} = \{\{1, 4, 6\}, \{1, 4, 7\}, \{2, 4, 6\}, \{2, 4, 7\}, \{2, 5, 7\}, \{3, 5, 6\}, \{3, 7\}\}, F_i = \{...\} \forall i$.

2 MIDCP Formulation of MIGP

dcp [5], midcp [4]

migp in conic form

$$\min \quad D: \tag{4a}$$

$$D^{-1} \sum_{i \in S \cap O} \gamma_i c_i X_i^{-1} + D^{-1} \sum_{i \in S \setminus O} \gamma_i X_i^{-1} \sum_{j \in F_i} (\alpha_j + \beta_j X_j) \leq 1 \qquad \forall S \in \mathcal{S} \tag{4b}$$

$$X_i^{-1} \leq 1 \qquad \forall i \in G \tag{4c}$$

$$\bar{p}^{-1} \sum_{i \in G} f_i e_i X_i \leq 1 \tag{4d}$$

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selection of log-integer variables not MIP-representable

assuming bounds, disjunctive constraint formulations

3 Solution Algorithm

solve migp like other midcp problems with pajarito, an outer approximation alg [4]. can migp have unbounded feasible set? would cause problems for pajarito

find (and perhaps improve during iterations) bounds on the log-integer variables (tighter bounds led to significant speed up and avoids ECOS numerical issues, for gate sizing example)

4 Examples and Results

small gate-sizing problem

approximate nonconvex functions piecewise, extend Woody paper [3]

Woody aircraft example

5 Future Work

disjunctive formulations in the MILP - what encodings are best

unbounded log-integer variables

cuts to the MILP or to the GP

mixtures of cones

References

[1] Boyd, Stephen, et al. “A tutorial on geometric programming.” Optimization and engineering 8.1 (2007): 67-127.

[2] Hoburg, Warren, and Pieter Abbeel. “Geometric programming for aircraft design optimization.” AIAA Journal 52.11 (2014): 2414-2426.

[3] Hoburg, Warren, and P. Abbeel. ”Fitting Geometric Programming Models to Data.” Optimization and Engineering (2014).

[4] Lubin, Miles, et al. “Extended Formulations in Mixed-integer Convex Programming.” arXiv preprint arXiv:1511.06710 (2015).

[5] Grant, Michael, Stephen Boyd, and Yinyu Ye. *Disciplined convex programming.* Springer US, 2006.