Mixed-Integer Geometric Programming

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1 From GP to MIGP

gp standard form and convex form [1], example - aircraft wings [2]

gp with integer variables, example - gate sizing [1]

generalized GP:

$$
\begin{aligned}
&\min \quad \max_{S\in\mathcal{S}} \quad \sum_{i\in S} D_i : && && \text{(1a)}\\
g_i X_i^{-1} \sum_{j\in F_i} (a_j + b_j X_j) &= D_i && \forall i \in G\backslash O && \text{(1b)}\\
g_i c_i X_i^{-1} &= D_i && \forall i \in O && \text{(1c)}\\
X_i &\geq 1 && \forall i \in G && \text{(1d)}\\
\sum_{i\in G} h_i X_i &\leq \bar{p} && && \text{(1e)}\\
\sum_{i\in G} a_i X_i &\leq \bar{a} && && \text{(1f)}
\end{aligned}
$$

or GP, eliminating D_i:

$$
\begin{aligned}
&\min \quad D : && && \text{(2a)}\\
D^{-1} \sum_{i\in S\cap O} g_i c_i X_i^{-1} + D^{-1} \sum_{i\in S\backslash O} g_i X_i^{-1} \sum_{j\in F_i} (a_j + b_j X_j) &\leq 1 && \forall S \in \mathcal{S} && \text{(2b)}\\
X_i^{-1} &\leq 1 && \forall i \in G && \text{(2c)}\\
\bar{p}^{-1} \sum_{i\in G} h_i X_i &\leq 1 && && \text{(2d)}\\
\bar{a}^{-1} \sum_{i\in G} a_i X_i &\leq 1 && && \text{(2e)}
\end{aligned}
$$

now let $X_i = e^{\chi_i}, \forall i \in G$ and $D = e^{\delta}$. transform into convex form by taking log of both sides of each posynomial inequality and the objective.

$$\min \quad \delta : \tag{3a}$$

$$\log\left(\sum_{i\in S\cap O} g_i c_i e^{-\chi_i} + \sum_{i\in S\setminus O} g_i e^{-\chi_i} \sum_{j\in F_i}\left(a_j + b_j e^{\chi_j}\right)\right) \le \delta \qquad \forall S\in\mathcal{S} \tag{3b}$$

$$\chi_i \ge 0 \qquad \forall i \in G \tag{3c}$$

$$\log\sum_{i\in G} h_i e^{\chi_i} \le \log\bar{p} \tag{3d}$$

$$\log\sum_{i\in G} a_i e^{\chi_i} \le \log\bar{a} \tag{3e}$$

where in the example problem, $O=\{6,7\}$, $\mathcal{S}=\{\{1,4,6\},\{1,4,7\},\{2,4,6\},\{2,4,7\},\{2,5,7\},\{3,5,6\},\{3,7\}\}$, $F_i = \{...\}$ and $h_i = f_i e_i$, $\forall i\in G$.

can impose integer constraints on the X_i: a discrete number of transitors per gate: $X_i = e^{\chi_i}\in\mathbb{N}$, which implies $\chi_i \in \{\log n | n\in\mathbb{N}, n>0\}$. note $n\ge 1$ (valid for integer problem) implies the constraint $\chi_i\ge 0$.

2 MIDCP Formulation of MIGP

dcp [5], midcp [4]

the migp, in conic form, is... do we actually want to write with the expo cone?

selection of log-integer variables not MIP-representable

assuming bounds, disjunctive constraint formulations

3 Solution Algorithm

solve migp like other midcp problems with pajarito, an outer approximation alg [4]. can migp have unbounded feasible set? would cause problems for pajarito

find (and perhaps improve during iterations) bounds on the log-integer variables (tighter bounds led to significant speed up and avoids ECOS numerical issues, for gate sizing example)

4 Examples and Results

small gate-sizing problem

approximate nonconvex functions piecewise, extend Woody paper [3]

Woody aircraft example

5 Future Work

disjunctive formulations in the MILP - what encodings are best unbounded log-integer variables

cuts to the MILP or to the GP

mixtures of cones

References

[1] Boyd, Stephen, et al. "A tutorial on geometric programming." Optimization and engineering 8.1 (2007): 67-127.

[2] Hoburg, Warren, and Pieter Abbeel. "Geometric programming for aircraft design optimization." AIAA Journal 52.11 (2014): 2414-2426.

[3] Hoburg, Warren, and P. Abbeel. "Fitting Geometric Programming Models to Data." Optimization and Engineering (2014).

[4] Lubin, Miles, et al. "Extended Formulations in Mixed-integer Convex Programming." arXiv preprint arXiv:1511.06710 (2015).

[5] Grant, Michael, Stephen Boyd, and Yinyu Ye. *Disciplined convex programming.* Springer US, 2006.