Mixed-Integer Geometric Programming

Miles & Chris

1 From GP to MIGP

gp standard form and convex form [1], example - aircraft wings [2]

gp with integer variables, example - gate sizing [1]

generalized GP:

$$\min \quad \max_{S \in \mathcal{S}} \quad \sum_{i \in S} D_i : \tag{1a}$$

$$g_i X_i^{-1} \sum_{j \in F_i} (a_j + b_j X_j) = D_i \qquad \forall i \in G \backslash O \tag{1b}$$

$$g_i c_i X_i^{-1} = D_i \qquad \forall i \in O \tag{1c}$$

$$X_i \geq 1 \qquad \forall i \in G \tag{1d}$$

$$\sum_{i \in G} h_i X_i \leq \bar{h} \tag{1e}$$

$$\sum_{i \in G} a_i X_i \leq \bar{a} \tag{1f}$$

or GP, eliminating D_i:

$$\min \quad D : \tag{2a}$$

$$D^{-1} \sum_{i \in S \cap O} g_i c_i X_i^{-1} + D^{-1} \sum_{i \in S \backslash O} g_i X_i^{-1} \sum_{j \in F_i} (a_j + b_j X_j) \leq 1 \qquad \forall S \in \mathcal{S} \tag{2b}$$

$$X_i^{-1} \leq 1 \qquad \forall i \in G \tag{2c}$$

$$\sum_{i \in G} h_i \bar{h}^{-1} X_i \leq 1 \tag{2d}$$

$$\sum_{i \in G} a_i \bar{a}^{-1} X_i \leq 1 \tag{2e}$$

now let $X_i = e^{\chi_i}, \forall i \in G$ and $D = e^{\delta}$. transform into convex form by taking log of both sides of each posynomial inequality and the objective.

$$
\begin{aligned}
&\min \quad \delta : && && \text{(3a)}\\
&\log\left(\sum_{i\in S\cap O} g_i c_i e^{-\chi_i} + \sum_{i\in S\setminus O} g_i e^{-\chi_i} \sum_{j\in F_i}\left(a_j + b_j e^{\chi_j}\right)\right) \le \delta && \forall S \in \mathcal{S} && \text{(3b)}\\
&\chi_i \ge 0 && \forall i \in G && \text{(3c)}\\
&\log \sum_{i\in G} \frac{h_i}{\bar{h}} e^{\chi_i} \le 0 && && \text{(3d)}\\
&\log \sum_{i\in G} \frac{a_i}{\bar{a}} e^{\chi_i} \le 0 && && \text{(3e)}
\end{aligned}
$$

which we can write as

$$
\begin{aligned}
&\min \quad \delta : && && \text{(4a)}\\
&\log\left(\sum_{i\in S\cap O} e^{-\chi_i + \log g_i c_i} + \sum_{i\in S\setminus O}\sum_{j\in F_i}\left(e^{-\chi_i + \log g_i a_j} + e^{-\chi_i + \chi_j + \log g_i b_j}\right)\right) \le \delta && \forall S \in \mathcal{S} && \text{(4b)}\\
&\chi_i \ge 0 && \forall i \in G && \text{(4c)}\\
&\log \sum_{i\in G} e^{\chi_i + \log \frac{h_i}{\bar{h}}} \le 0 && && \text{(4d)}\\
&\log \sum_{i\in G} e^{\chi_i + \log \frac{a_i}{\bar{a}}} \le 0 && && \text{(4e)}
\end{aligned}
$$

substituting constants

$$
\begin{aligned}
&\min \quad \delta : && && \text{(5a)}\\
&\log\left(\sum_{i\in S\cap O} e^{-\chi_i + r_i} + \sum_{i\in S\setminus O}\sum_{j\in F_i}\left(e^{-\chi_i + s_{ij}} + e^{-\chi_i + \chi_j + t_{ij}}\right)\right) \le \delta && \forall S \in \mathcal{S} && \text{(5b)}\\
&\chi_i \ge 0 && \forall i \in G && \text{(5c)}\\
&\log \sum_{i\in G} e^{\chi_i + u_i} \le 0 && && \text{(5d)}\\
&\log \sum_{i\in G} e^{\chi_i + v_i} \le 0 && && \text{(5e)}
\end{aligned}
$$

where in the example problem, $O = \{6, 7\}, \mathcal{S} = \{\{1, 4, 6\}, \{1, 4, 7\}, \{2, 4, 6\}, \{2, 4, 7\}, \{2, 5, 7\}, \{3, 5, 6\}, \{3, 7\}\}, F_i = \{...\}$ and $h_i = f_i e_i$, $\forall i \in G$.

can impose integer constraints on the X_i: a discrete number of transitors per gate: $X_i = e^{\chi_i} \in \mathbb{N}$, which implies $\chi_i \in \{\log n | n \in \mathbb{N}, n > 0\}$. note $n \ge 1$ (valid for integer problem) implies the constraint $\chi_i \ge 0$.

2 MIDCP Formulation of MIGP

dcp [5], midcp [4]

the migp, in conic form, is... do we actually want to write with the expo cone?

selection of log-integer variables not MIP-representable

assuming bounds, disjunctive constraint formulations

3 Solution Algorithm

solve migp like other midcp problems with pajarito, an outer approximation alg [4]. can migp have unbounded feasible set? would cause problems for pajarito

find (and perhaps improve during iterations) bounds on the log-integer variables (tighter bounds led to significant speed up and avoids ECOS numerical issues, for gate sizing example)

4 Examples and Results

small gate-sizing problem

approximate nonconvex functions piecewise, extend Woody paper [3]

Woody aircraft example

5 Future Work

disjunctive formulations in the MILP - what encodings are best

unbounded log-integer variables

cuts to the MILP or to the GP

mixtures of cones

References

[1] Boyd, Stephen, et al. “A tutorial on geometric programming.” Optimization and engineering 8.1 (2007): 67-127.

[2] Hoburg, Warren, and Pieter Abbeel. “Geometric programming for aircraft design optimization.” AIAA Journal 52.11 (2014): 2414-2426.

[3] Hoburg, Warren, and P. Abbeel. ”Fitting Geometric Programming Models to Data.” Optimization and Engineering (2014).

[4] Lubin, Miles, et al. “Extended Formulations in Mixed-integer Convex Programming.” arXiv preprint arXiv:1511.06710 (2015).

[5] Grant, Michael, Stephen Boyd, and Yinyu Ye. *Disciplined convex programming.* Springer US, 2006.