

A quick overview on Floating Point and Monte Carlo arithmetics litterature - Study of random rounding scheme applied to Deep Neural Networks

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Abstract

Numerous research fields eventually come up with mathematical models, corresponding to a given physical reality. Most of these are too hard to handle *by hand* for decades. In this context, numerical simulations offered an essential tool to handle these complex schemes. However, in order to cope with evident hardware limitations, the *computer arithmetic*, meaning the way computers handle mathematical operations is distinct of our *elementary*¹ arithmetic. In fact, it consist in a whole very standardised set of rules, format and operations : floating point arithmetic. But as one may have already experienced in sometimes even the simplest cases ($0.1+0.9$), the use of this arithmetic always comes with a price. Numerical precision and therefore the very reliability of simulation results are the keystones in this everlasting struggle.

In this paper, we propose to explore the vast literature addressing these concerns since the late XXth century, and we introduce the reader to Monte Carlo Arithmetic (MCA)^[1], a promising extension of floating point arithmetic, which exploit randomness in floating point computations in order to analyze and estimate quality of numerical results.

Lastly, we extend this work in applying the `round_random_nearness` rounding method, giving an insightful trail on its use and performance on various gradient descent methods such as Krylov's conjugate gradient methods. We also experiment its properties during stochastic gradient method, in the context of Deep Neural Network (DNN) training, for image classification.

This document aim to sum up my work during my thesis *Improving trust in coastal erosion numerical simulations* and to keep track of my progress.

¹find a more accurate term

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1 State of the art

Reference : Goldberg^[2] work.

We also couldn't not steeped in MCA^[1] and Verificarlo^[3], which were the initial motivation of this study.

We can also cite Cadna^[4].

2 Monte Carlo Arithmetic and stochastic rounding

2.1 Gradient descent methods

Conjuguate gradient method^[5].

Present the context of the method, why we chose it, and which properties interested us.

Also introduce the ‘matrice_grad’ program.

2.2 Sensibility to rounding mode changes

2.2.1 Stochastic rounding

Our implementation of stochastic rounding : mathematical definition, proof, and benchmarks.

2.2.2 Experimenting sensibility of rounding mode changes

Experimentations ; method.

Varying the rounding mode : results.

2.3 Other experimented sensibilities

Experiments method : changing

- *matrix size,*
- *matrix type,*
- *number of iterations,*
- *and floating point numbers precision*

Results.

2.4 Random rounding in stochastic gradient method - Efficiency on Deep Neural Networks

Extending Gupta&Al^[6] and Courbariaux^[7] works.

3 Conclusion and future works

Cite other initiatives, such as Precimonious^[8].

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