Measuring Hash Trustworthiness via Collision Utility Metrics: Logical Cryptanalysis of MD4

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***Abstract*—The discovery of fast collision attacks in cryptographic hash functions has traditionally resulted in the immediate deprecation of that hash function. In this paper we propose five scalable and practical metrics for evaluating the utility of collision classes based on boolean constraints and show that the published attacks by X. Wang, Y. Sasaki, P. Kasselman, H. Dobbertin, and M. Schlaffer in MD4 have high utility. We expand on existing attacks by developing a series of techniques based on logical cryptanalysis to find over 35,000 collisions in MD4 based on existing collisions, through the novel definition of a collision neighborhood. We demonstrate new techniques for inductively building full collisions from reduced round variants of MD4. We propose these techniques as a mechanism for measuring hash trustworthiness and discuss potential applications to real-world systems.**

I. Introduction

Cryptographic hash functions form the core of many protocols. From file integrity checks to verify long term storage of data, to cache invalidation techniques, and use as a building block in network protocols such as Kerberos and TLS, this class of functions necessarily has strong guarantees about properties of its members. There are three basic properties hash functions must have to be considered cryptographically secure:

- **Preimage Resistance**: It should be computationally hard to find the inverse of a hash function.
- **Second Preimage Resistance**: Given an input block, it should be computationally hard to find a second block which hashes to the same value as the first block.
- **Collision Resistance**: It should be computationally hard to find two blocks which hash to the same value.

Note that a second preimage is necessarily a collision and is also a stronger result in practice.

From an attacker's perspective, finding a preimage or second preimage has been difficult. To the author's knowledge, the current bound for a preimage attack in MD5 is $2^{123.4}$ by Y. Sasaki and K. Aoki [1]. Further, while J. Kelsey and B. Schneier proposed a method for finding second preimages in less than 2^n, we note that this requires rather long messages [2]. However, collision attacks are well within reach for adversaries, and well past only theoretical attacks. The work of H. Dobbertin [3], X. Wang [4], M. Schlaffer [5] and others demonstrate the ease with which collisions can be found.

From a cryptographic perspective, the existence of a feasible collision attack breaks the requisite properties, thus deprecating the function's use. In some instances however, there continues to be widespread use of deprecated hash functions. One widespread example is the continued use by Git of SHA-1, despite M. Stevens et al. publishing a full SHA-1 collision in February of 2017 [6]. However, this attack had a bound of $2^{63.1}$ – faster than the theoretical 2^{80} but still requiring significant compute resources.

We seek to address the problem of evaluating the use cases for a collision, to determine–or partially determine–whether or not a collision affects a particular system. This paper contributes several novel techniques to logical cryptanalysis and uses these techniques to derive metrics of the trustworthiness of hash functions. We claim the following results are novel in the field:

- We define the neighborhood of a class of collisions and show that it is frequently non-empty.
- We discuss techniques for finding useful collisions within a class of collisions.
- We propose additional techniques for building new classes of collisions and show that collisions in MD4 can be found inductively.
- We propose five universal metrics for evaluating the utility of a class of collisions.

TODO - Transition

We propose that collision classes of high utility are closer to a second preimage attack in the amount of flexibility they provide to an attacker, whereas classes of low utility may not affect all systems which use a particular hash function. Furthermore, we demonstrate new techniques for working with collisions under a logical cryptanalysis framework. Throughout this paper, we make use of the MD4 hash function due to the ease of generating new classes of collisions.

TODO - Finish Organization

The remainder of this paper is organized as follows. In Section II we discuss previous results and show how our research expands upon prior work. In Section III, we discuss the terminology and notations we use throughout the remainder of the paper. In Section IV, we give the intuition between measuring the utility of a collision and motivate why it is useful. In Section V, we present new techniques in logical cryptanalysis to analyze collisions in hash functions.

II. Related Work

Broad context... intersection... subsection introductions.

This paper draws upon the public cryptanalysis of MD4, the wide availability of high quality SAT solvers, and previous work in logical cryptanalysis.

A. On Differential Cryptanalysis

TODO - expand?

Differential Cryptanalysis has been the major technique behind the discovery of collisions in cryptographic hash functions. Its importance can be seen everywhere from X. Wang's attacks on MD4 [7] to more recent cryptanalysis of general purpose hash functions such as Murmur3 by J. Aumasson, D. Bernstein, and M. Boßlet [8]. However, while differential cryptanalysis plays an important role, techniques for automated analysis of hash functions are still an emerging area. In [4], X. Wang claimed to have found the MD4 collision "with hand calculation", though in [7], she specifies message modification techniques which can be implemented in code.

Our work removes the need for finding message modification techniques by specifying the differential path as part of the SAT formula and letting the solver find a pair of satisfying messages which follow the differential path and produce a collision. Further constraints can then be placed upon this model, such as a chosen prefix or desired start state. We believe performing these attacks by hand to be difficult, and note that message modification techniques may affect the chosen prefix.

B. On Logical Cryptanalysis

TODO - expand?

Logical Cryptanalysis and its encoding as SAT likely started under the work of F. Massacci in 1999 with his paper titled "Using walk-SAT and rel-sat for cryptographic key search". [9]. However, much of the early work by F. Massacci was focused on symmetric and asymmetric ciphers (in [9], [10], and [11]). It wasn't until the work of D. Jovanović and P. Janičić in [12] that exploring collisions in the context of SAT was introduced, and until the work of I. Mironov and L. Zhang in [13] that this was studied for a X. Wang's collision class.

We make use of the *bc2cnf* utility by T. Junttila (available here [14]) as the basis for our models. This allows us to encode hash functions as generic structures in the circuit description language and ease the algorithmic creation of models which build on top of them. We then use CryptoMiniSat5 by M. Soos ([15]) to run the models and check for a satisfying witness, often an example collision.

With the exception of the work by I. Mironov, the authors find that much of the previous work in the field has been focused on evaluating the performance of various SAT solvers. We deviate from this by instead focusing on evaluating the properties of the hash function–not the SAT solver. We have developed new techniques for using a SAT solver to inductively build full collisions in MD4 from collisions in reduced-round MD4 and to build addition examples of collision classes from a single instance of a collision in MD4.

C. On Trustworthy Computing

TODO - Assigned @erozier

III. Terminology & Notation

TODO.

$\mathcal{C}$ for collision class. Ordered tuple, indexable. $\mathcal{F}$ for family of collision classes. Ordered set, indexable. $[i]$ is an indexing function. R is number of rounds of F or C.

IV. Intuition on the Utility of a Collision

TODO - introductory paragraph

We propose the following metrics for evaluating the utility of a collision class:

1) The number of unique differentials a collision class has.
2) The number of unit-step neighbors a collision class has.
3) The maximum count of zeros in a the binary representation of a colliding block (and likewise with ones).
4) Whether there exists a block which collides under multiple initial values.
5) Whether or not zero, one, or both of the blocks in a collision may be of ASCII values under any input block difference.

Note that the first three are quantitative measures providing some measure of flexibility of a collision class, whereas the latter two are merely looking for a single witness for having the property. Depending on the scenario, specific properties of a collision may be of more interest than others.

The first metric evaluates the flexibility of the differential path. A differential path with more flexibility will have more differentials which produce blocks with the given differential path. More differentials implies a greater flexibility in choice of colliding block, and possibly allowing for multiple collisions for a given colliding block. Furthermore, a collision with more differentials is more likely to satisfy the last metric, having a pair of blocks–both ASCII–which produce a collision.

The second metric evaluates the density of the neighborhood of a collision class. If a collision resides in a dense neighborhood, it provides more possible collision classes to search for a second preimage, chosen prefix, or other structure desired in a collision. If however, a collision class has no neighbors, then it cannot be used to find other possible classes for other input blocks.

The maximum quantity of zeros (or ones) in the binary representation of a block serve as a measure of the extremes to which a collision can be pushed. This can additionally be extended to any suitable bit pattern in any base to provide a more relevant metric as desired by the system under study.

The fourth metric evaluates the utility of a collision when the internal state of the hash function is unknown. If a collision occurs under multiple initial values, this could be used to attack some systems where user provided input is appended to unknown data and then hashed. If a block collides under a suitably large number of initial values, the attack becomes highly likely to occur successfully. However, measuring the exact number of initial values a block collides under is beyond the scope of this work.

TABLE I
DISTANCE BETWEEN EXISTING COLLISION CLASSES

	X. W.	Y. S.	P. K.	H. D.	M. S.	Absolute
X. Wang's	0	21	26	27	12	18
Y. Sasaki's		0	25	26	2	16
P. Kasselman's			0	1	25	11
H. Dobbertin's				0	26	12
M. Schlaffer's					0	17

The last metric is similar to the third in that it looks for specific bit patterns in a collision. ASCII is one example of a widely used constraint system. Further examples, such as JSON, XML, etc., may likewise be supplemented based on the specifics of the system.

If a collision class satisfies many of these properties, then it is more flexible and thus more likely to be used to target deployed systems. If, however, a collision class does not satisfy these properties, it impact is likely severely limited in scope, and may not provide useful information to find other collision classes which have higher utility.

V. TECHNIQUES FOR LOGICAL CRYPTANALYSIS

The following techniques have been extensively tested on MD4 and partially tested on MD5 and believe to apply fully to MD5. They may or may not apply to any later hash function, such as SHA-1, SHA-2, or SHA-3, and have not been tested yet.

In the following sections, we use the collisions of X. Wang [7], Y. Sasaki [16], M. Schlaffer [5], H. Dobbertin [3], and P. Kasselman [17] for examples.

A. Distance Metrics

We introduce a distance function, δ between collision classes by the number of differences in intermediate rounds deltas. That is, given two collision classes, $C_1, C_2 \in \mathcal{C}$:

$$\delta(C_1, C_2) = \left|\{i : C_1[i] \neq C_2[i]\}\right| \tag{1}$$

We find justification for this metric in the existing literature on MD4: H. Dobbertin's [3] and P. Kasselman's [17] collision classes have distance 1 under this metric. We extend this distance function to include an *absolute* distance, measuring the number of intermediate rounds with non-zero differences. We denote this as $\delta(C_1)$.

Refer to Table I for the distances between existing collisions in MD4.

We can define a similar distance function, Δ, between families of collision classes by cardinality of the symmetric difference in the two collision families. That is, given two families of collision classes, $F_1, F_2 \in \mathcal{F}$:

$$\Delta(F_1, F_2) = \left|(F_1 \cup F_2) \setminus (F_1 \cap F_2)\right| \tag{2}$$

This is convenient for when the specifics of the differential path do not matter, merely that there exist at least one collision class of the specified form.

B. Neighborhoods

We define the neighborhood of a collision class, $C \in \mathcal{C}$, to be the set of all other collisions at a fixed distance, $d \in \mathbb{N}$, from C. That is:

$$N(C, d) = \{C_j \in \mathcal{C} : \delta(C, C_j) = d\} \tag{3}$$

$$N(C) = N(C, 1) \tag{4}$$

For instance, $C_{Wang} \in N(C_{Sasaki}, 21)$. The distance parameter, d, may optionally be omitted, in which case the unit distance is implied. Thus, $C_{Kasselman} \in N(C_{Dobbertin})$.

The implications of collision class neighborhoods are discussed further in section VI-B.

We can similarly define the neighborhood of a family of collision classes, $F \in \mathcal{F}$, to be the set of all other families of collision classes at a fixed distance, $d \in \mathbb{N}$, from F. That is:

$$\mathcal{N}(F, d) = \{F_j \in \mathcal{F} : \delta(F, F_j) = d\} \tag{5}$$

$$\mathcal{N}(F) = N(F, 1) \tag{6}$$

The distance parameter, d, may optionally be omitted, in which case the unit distance is implied.

Neighborhoods can be classified into three types: *expansion*, *internal*, and *mixed*. Let d be fixed. An *expansion* neighborhood of a collision class, $C \in \mathcal{C}$, is the neighborhood restricted only to those collision classes which only differ in rounds external to the collision family of C. An *internal* neighborhood of C is the neighborhood restricted only to those collision classes which differ in rounds internal to the collision family of C. An *mixed* neighborhood of C is the neighborhood restricted only to collisions which differ in a round internal and a round external to the collision family of C. That is:

$$N_{exp}(C, d) = \{C_j \in N(C, d) : \forall i \in F(C), C_j[i] = C[i]\}$$

$$N_{int}(C, d) = \{C_j \in N(C, d) : \forall i \notin F(C), C_j[i] = C[i]\}$$

$$N_{mix}(C, d) = \{C_j \in N(C, d) : \exists i \in F(C), C_j[i] \neq C[i] \text{ and } \exists k \notin F(C), C_j[k] \neq C[k]\}$$

C. Family Similarity

We define a relation among families of collisions across rounds. Let F_1 and F_2 be two different families of collisions. Then we say that F_1 and F_2 are *similar*, and notate it $F_1 \lesssim F_2$, if $R(F_1) \leq R(F_2)$ and $F_1 \subseteq F_2$. Note that, when $R(F_1) = R(F_2)$, this is equivalent to saying that F_2 is in some expansion neighborhood of F_1. Further, when $R(F_1) < R(F_2)$ and $F_1 = F_2$, then we say that F_2 is the *trivial extension* of F_1.

We claim that the following statements are true:

1) For every $F \in \mathcal{F}$, there exists F' such that $F' \lesssim F$ and $R(F') + 4 = R(F)$.
2) For every $F \in \mathcal{F}$, there exists F' such that $F \lesssim F'$.

TODO - Describe justification? Later sections?

TABLE II
NUMBER OF DIFFERENTIALS FOR EXISTING COLLISION CLASSES

Attack	Size	Attack	Size
X. Wang's	64	Y. Sasaki's	4
H. Dobbertin's	32	P. Kasselman's	32
M. Schlaffer's	64		

TABLE III
NEIGHBORHOOD SIZES FOR EXISTING COLLISION CLASSES

Attack	Size	Attack	Size
X. Wang's	54	Y. Sasaki's	157
H. Dobbertin's	55	P. Kasselman's	60
M. Schlaffer's	100		

D. Class Similarity

We claim that the aforementioned statements hold when considering individual collision classes instead of families of collision classes, but with less likelihood.

TODO - Describe technique applied neighborhood extensions

Give some reasoning for why it works, example graphs.

E. Miscellaneous

TODO - Describe chosen prefix, second preimage, ASCII, and other tips and tricks for faster model runs.

VI. EMPIRICAL RESULTS

A. Unique Differentials

TODO Refer to Table II for the number of differential paths...

B. Unit-Step Neighborhood

TODO

Refer to Table III for the sizes of neighborhoods of existing collisions. In particular, note that Y. Sasaki's attack–which claimed to improve upon X. Wang's attack–has a larger neighborhood size, and likewise with P. Kasselman's attack which improved upon H. Dobbertin's attack.

C. Zeroes & Ones

TODO

Include graphic of count of max number of zeros/ones for all 35k collision classes.

Separate table for known attacks as data points. Examples as appendix.

D. Multicollisions

TODO

Include table detailing how many have multicollisions and how many don't. Hypothesis: all do since all existing datapoints do.

Separate table for known attacks as data points. Examples as appendix.

TABLE IV
ASCII BLOCKS IN EXISTING COLLISION CLASSES

Attack	Single Block	Both Blocks
X. Wang's	true	false
Y. Sasaki's	true	false
P. Kasselman's	true	**true**
H. Dobbertin's	true	**true**
M. Schlaffer's	true	false

E. ASCII Blocks

TODO

Include table detailing how many have ASCII blocks and how many don't.

See Table IV for results. Note that the validation of P. Kasselman's and H. Dobbertin's results do not hold for the latter attacks by X. Wang, Y. Sasaki, or M. Schlaffer. Thus, while the latter attacks have been viewed as being of better quality, under this particular metric, P. Kasselman's and H. Dobbertin's collision classes are better.

VII. NEW COLLISION

VIII. DISCUSSION OF IMPACTS

TODO - Discuss impacts on real-world systems like MD5, SHA-1, etc.

IX. CONCLUSION

The conclusion goes here.

X. FUTURE WORK

The framework can be seen here **TODO**. Complete data set available upon request.

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