Algorithms. Assignment 1

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1 Combinatorics

1. How many different ways are there to order a list containing 99 distinct elements?

We have our original list X containing elements $\{x_1, x_2, \cdots, x_{99}\}$ and want to count the way to put these elements into a new list $Y = \{y_1, y_2, \cdots, y_{99}\}$. For the first element of Y we have 99 options, all of the elements in X could be our first element. For the second element, however, we only have 98 options as we have already taken one of the elements from X and placed it in Y. Using the same reasoning for elements y_1 through y_{99} gives us $100 - n$ options for element y_n and a total of

$$(100-1)(100-2)\cdots(100-98)(100-99) = 99 \cdot 98 \cdots 2 \cdot 1 = 99!$$

options for the entire list.

2. A graph is complete if any pair of its vertices is connected by an edge. How many edges are there in a complete graph with 4 vertices? What about 40 or, more generally, n vertices? We make some ordering V of the vertices in our graph. E.g. for a graph with 4 vertices this would be denoted

$$V = [v_1, v_2, v_3, v_4]$$

. For every vertex in V we can count how many new, meaning not yet counted, edges it adds to a running total by going through V and noticing that the number of new edges added by an element v_n is equal to the number of elements to the right of v_n in V. For the complete graph with 4 vertices we would then have

$$3 + 2 + 1 + 0 = 6 \quad \text{(compare with } [v_1, v_2, v_3, v_4] \text{ above)}$$

edges. For the graph with 40 vertices we would have

$$39 + 38 + 37 + \cdots + 1 + 0$$

and in general for n vertices we would have

$$n - 1 + n - 2 + n - 3 + \cdots + 1 + 0$$

. This is a simple arithmetic series for which we know the sum to be

$$\frac{n(n-1)}{2}$$

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3. How many different ways are there to order the letters contained in the word "engineering"?

This is quite similar to question 1. However, if we take the naïve approach of 11! we count for example the following reorderings:

	e	n	g	i	n	e	e	r	i	n	g
x	1	2	11	4	10	7	6	8	9	5	3

	e	n	g	i	n	e	e	r	i	n	g
x	7	5	11	9	10	1	6	8	4	2	3

where x denotes the letter's placement in the original word. Clearly these should not count as new ways to order the letters and so we are over-counting by some amount. This amount is the number of ways to arrange the letters within the word while still keeping it the same word. The number of ways to do this is determined by the number of duplicate letters. We have 3 e's, 3 n's, 2 g's, and 2 i's.

x	#
e	3
n	3
g	2
i	2

The g's and i's both make us count twice as many words as if they wouldn't have a duplicate. For e and n each word is counted six times as many as it should. For example here are six words that look the same but have their e's in different places.

e	e	g	n	g	i	r	n	i	e	n
e	e	g	n	g	i	r	n	i	e	n
e	e	g	n	g	i	r	n	i	e	n
e	e	g	n	g	i	r	n	i	e	n
e	e	g	n	g	i	r	n	i	e	n
e	e	g	n	g	i	r	n	i	e	n

Thus the total number of ways to order the letters contained in the word "engineering" is

$$\frac{11!}{2 \cdot 2 \cdot 6 \cdot 6} = 277200$$

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