Bias, Variance and Parsimony in Regression Analysis
ECS 256 Winter 2014

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Introduction

California Housing Data

- Derived from 1990 Census
- Response Variable: median house value
- Predictor Variables: median income, housing median age, total rooms, total bedrooms, population, households, latitude, and longitude

Parsimony

Method	Parsimony (k=0.01)	Parsimony (k=0.05)	Sig Test
Columns Deleted	Total Rooms Total Bedrooms	Total Rooms Total Bedrooms Median Age	None
Adjusted R^2	0.6321316	0.6218261	0.6369649

Regression Coefficients

```
Coefficients:
                 Estimate Std. Error t value Pr(>|t|)    
(Intercept)    -3.594e+06  6.254e+04 -57.468  < 2e-16 ***
Median.Income   4.025e+04  3.351e+02 120.123  < 2e-16 ***
Median.Age      1.156e+03  4.317e+01  26.787  < 2e-16 ***
Total.Rooms    -8.182e+00  7.881e-01 -10.381  < 2e-16 ***
Total.Bedrooms  1.134e+02  6.902e+00  16.432  < 2e-16 ***
Population     -3.854e+01  1.079e+00 -35.716  < 2e-16 ***
Households      4.831e+01  7.515e+00   6.429 1.32e-10 ***
Latitude       -4.258e+04  6.733e+02 -63.240  < 2e-16 ***
Longitude      -4.282e+04  7.130e+02 -60.061  < 2e-16 ***
```

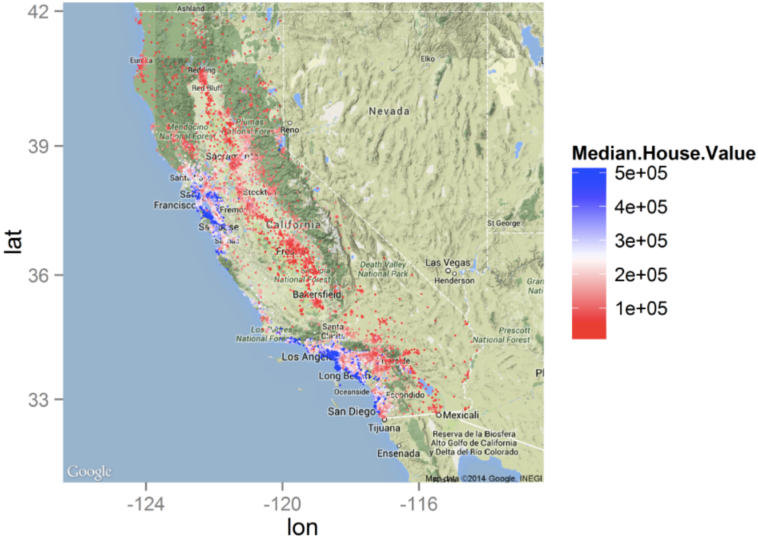
Latitude & Longitude

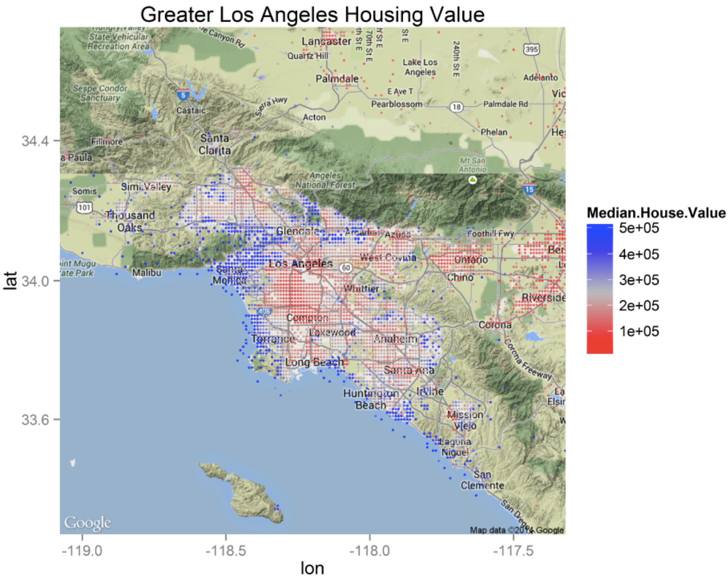
```
Latitude        -4.258e+04  6.733e+02 -63.240  < 2e-16 ***
Longitude       -4.282e+04  7.130e+02 -60.061  < 2e-16 ***
```

- "Center of Gravity"
- Avoid Overfitting

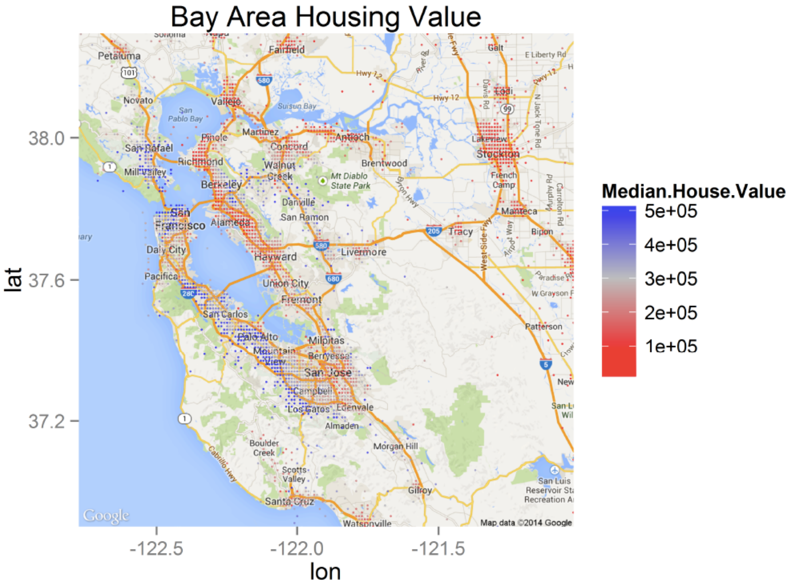
```
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)   -32165.268   2167.358  -14.84   <2e-16 ***
Median.Income  43094.918    284.263  151.60   <2e-16 ***
Median.Age      2000.544     45.080   44.38   <2e-16 ***
Population       -43.045      1.127  -38.20   <2e-16 ***
Households       152.700      3.344   45.66   <2e-16 ***
```

California Median House Value





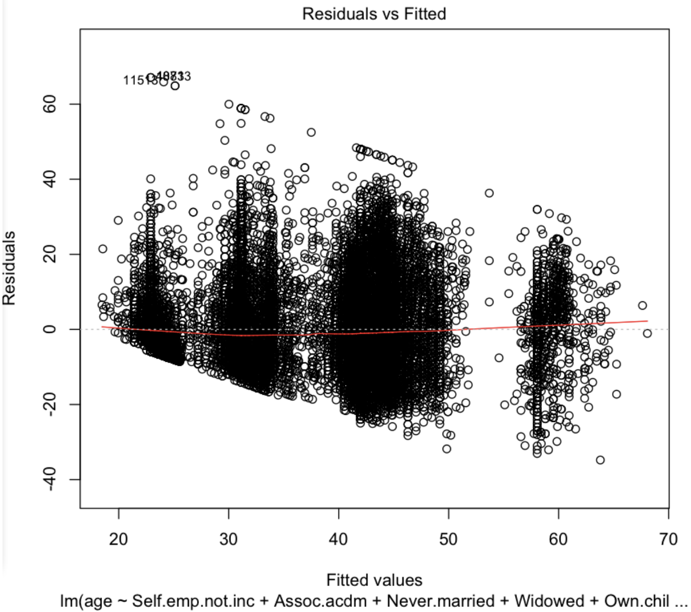
Greater Los Angeles Housing Value
Median.House.Value
5e+05
4e+05
3e+05
2e+05
1e+05
lat
34.4
34.0
33.6
lon
-119.0
-118.5
-118.0
-117.5
Lancaster
Palmdale
Santa Clarita
Simi Valley
Thousand Oaks
Glendale
Los Angeles
West Covina
Ontario
Chino
Whittier
Riverside
Compton
Torrance
Lakewood
Anaheim
Long Beach
Santa Ana
Huntington Beach
Irvine
Corona
Mission Viejo
Laguna Niguel
San Clemente
Malibu
Santa Monica
Google



Bay Area Housing Value
lat
38.0
37.6
37.2
lon
-122.5
-122.0
-121.5
Median.House.Value
5e+05
4e+05
3e+05
2e+05
1e+05
Petaluma
Novato
Fairfield
Vallejo
San Pablo Bay
Suisun Bay
Pinole
Martinez
Antioch
Concord
San Rafael
Richmond
Walnut Creek
Brentwood
Mill Valley
Berkeley
Mt Diablo State Park
Danville
San Ramon
San Francisco
Alameda
Daly City
Hayward
Livermore
Tracy
Pacifica
Union City
Fremont
San Carlos
Palo Alto
Mountain View
Milpitas
Berryessa
San Jose
Campbell
Los Gatos
Edenvale
Almaden
Boulder Creek
Morgan Hill
Scotts Valley
Santa Cruz
Gilroy
Watsonville
Galt
Lodi
Lakeview
Stockton
French Camp
Manteca
Ripon
Patterson
San Luis Reservoir State Recreation Area
Google
Map data ©2014 Google

Census Based on 1994

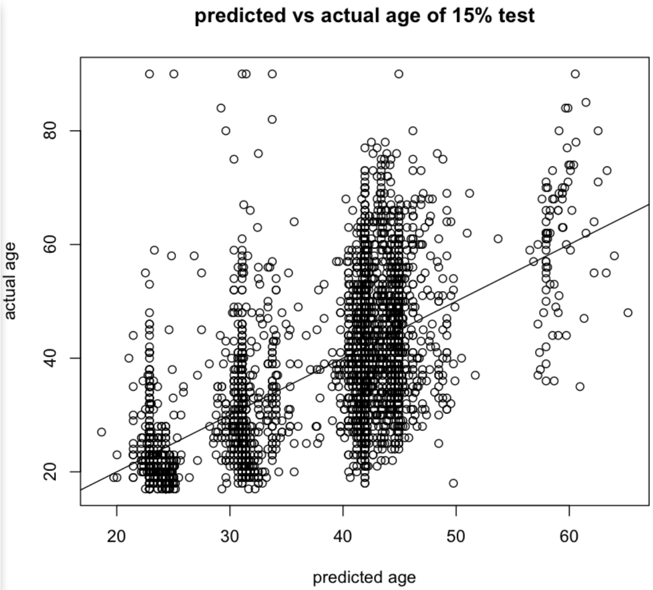
Age



Residuals vs Fitted
Residuals
Fitted values
lm(age ~ Self.emp.not.inc + Assoc.acdm + Never.married + Widowed + Own.chil ...
60
40
20
0
-20
-40
20
30
40
50
60
70

Census Based on 1994

Census Based on 1994


predicted vs actual age of 15% test
actual age
predicted age
20
40
60
80
20
30
40
50
60

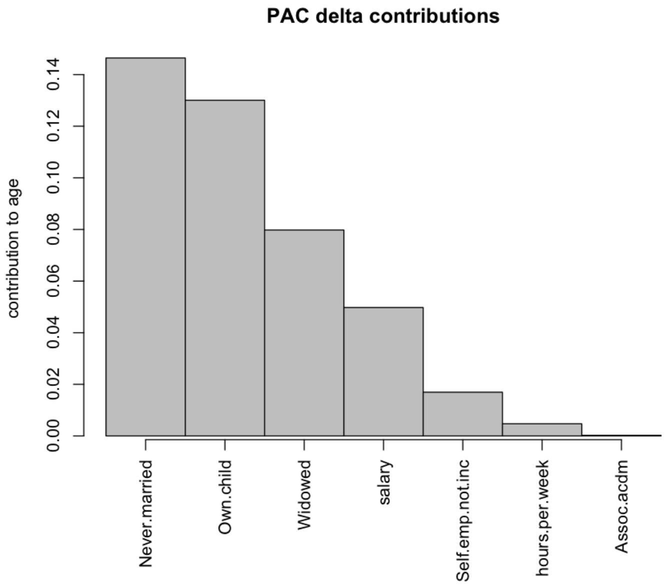

PAC delta contributions
contribution to age
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
Never.married
Own.child
Widowed
salary
Self.emp.not.inc
hours.per.week
Assoc.acdm

Figure:

Testing Parsimony on Simulated Data

Predictors: $X = X_1, ..., X_10$
Response: Y drawn from $U(m_{Y;X}(t) - 1, m_{Y;X}(t) + 1)$

where $m_{Y,X}(t) = t_1 + t_2 + t_3 + 0.1t_4 + 0.01t_5$

Testing Parsimony on Simulated Data

		prsm(k=0.01)	prsm(k=0.05)	sig test
n=100	Run 1	X_1, X_2, X_3, X_9	X_1, X_2, X_3	X_1, X_2, X_3
	Run 2	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3
	Run 3	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3
n=1000	Run 1	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3, X_4
	Run 2	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3
	Run 3	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3
n=10K	Run 1	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3, X_4
	Run 2	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3, X_4
	Run 3	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3, X_4, X_9
n=100K	Run 1	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3, X_4
	Run 2	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3, X_4, X_9
	Run 3	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3, X_4, X_9

Testing Parsimony on Simulated Data

k=0.01	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
N = 100	1	1	1	0.24	0.11	0.14	0.21	0.22	0.26	0.28
N = 1000	1	1	1	0.08	0	0	0	0	0	0
N = 10K	1	1	1	0	0	0	0	0	0	0
N = 100K	1	1	1	0	0	0	0	0	0	0
N = 1M	1	1	1	0	0	0	0	0	0	0
k=0.05	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
N = 100	1	1	0.99	0.1	0.02	0.05	0.04	0.03	0.07	0.02
N = 1000	1	1	1	0	0	0	0	0	0	0
N = 10K	1	1	1	0	0	0	0	0	0	0
N = 100K	1	1	1	0	0	0	0	0	0	0
N = 1M	1	1	1	0	0	0	0	0	0	0

Testing Parsimony on Simulated Data

Sig Test	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
N = 100	1	1	1	0.14	0.03	0.05	0.05	0.03	0.09	0.04
N = 1000	1	1	1	0.31	0.02	0.05	0.05	0.05	0.02	0.04
N = 10K	1	1	1	1	0.04	0.01	0.07	0.07	0.03	0.06
N = 100K	1	1	1	1	0.35	0.06	0.09	0.03	0.05	0.03
N = 1M	1	1	1	1	1	0.05	0.03	0.08	0.02	0.03

Small N, Large P

Automobile Data Set:

- UCI Machine Learning Repository
- 195 automobiles,
- 25 attributes per entry.

Goals:

- Determine accurate predictors of vehicle price.
- Gauge characteristics of safe automobiles.

Parsimony: Automobile Prices

- What factors best predict a vehicle's price?
- What are traits that increase price?
- What are the ones that decrease it?

Method	Parsimony (k = 0.01)	Parsimony (k = 0.05)	Significance Testing
Columns Retained	ohcv, twelve-cylinders, engine.size, stroke, compression.ratio, peak.rpm	engine.size	bmw, dodge, 'mercedes-benz', mitsubishi, plymouth, porsche, saab, std, front, wheel.base, length, width, height, curb.weight, dohc, ohc, engine.size, peak.rpm
AIC	0.8676842	0.7888274	0.9308

Results of Significance Testing (Auto Price):

```
(Intercept)      -4.234e+04  1.125e+04  -3.764 0.000229 ***
bmw               9.290e+03  8.611e+02  10.788  < 2e-16 ***
dodge            -1.504e+03  8.532e+02  -1.762 0.079785 .
`mercedes-benz`   6.644e+03  1.003e+03   6.625 4.17e-10 ***
mitsubishi       -2.628e+03  7.331e+02  -3.585 0.000438 ***
plymouth         -1.628e+03  8.881e+02  -1.833 0.068485 .
porsche           4.053e+03  2.238e+03   1.811 0.071936 .
saab              2.413e+03  1.028e+03   2.347 0.020043 *
std              -1.109e+03  5.129e+02  -2.162 0.031973 *
front            -1.275e+04  2.663e+03  -4.785 3.63e-06 ***
wheel.base        1.141e+02  7.390e+01   1.544 0.124355
length           -7.918e+01  4.225e+01  -1.874 0.062586 .
width             7.652e+02  2.029e+02   3.772 0.000222 ***
height           -1.377e+02  1.164e+02  -1.183 0.238332
curb.weight       3.781e+00  1.118e+00   3.381 0.000890 ***
dohc              1.569e+03  8.067e+02   1.944 0.053451 .
ohc               8.531e+02  4.575e+02   1.865 0.063911 .
engine.size       7.733e+01  1.035e+01   7.470 3.74e-12 ***
peak.rpm          1.522e+00  3.938e-01   3.864 0.000157 ***
---
Multiple R-squared:  0.9373, Adjusted R-squared:  0.9308
F-statistic: 144.5 on 18 and 174 DF,  p-value: < 2.2e-16
```

Top Predictors - Price

- Engine specifications, machinery
- Adds Value: Luxury Brands (BMW, Porsche)
- Reduces Value: Front-based Engine (Found in lower-end vehicles), economy brands (Mitsubishi, Plymouth)

Parsimony: Auto Safety

- Each auto is rated from -3 to 3 by insurers. -3 is safest, 3 is least safe.
- Use logistic regression to determine attributes of safe vehicles

Method	Parsimony (k = 0.01)	Parsimony (k = 0.05)	Significance Testing
Columns Retained	saab, toyota, volkswagen, turbo, two-doors, hatchback, sedan, 4wd, rwd, rear, wheel.base, length, width, height, curb.weight, l, ohc, ohcf ,ohcv, five-cylinders, four-cylinders, three-cylinders, twelve-cylinders, engine.size, 2bbl, idi, mfi, mpfi, spdi, bore, stroke, compression.ratio, horsepower, peak.rpm, city.mpg, highway.mpg	saab, toyota, volkswagen, turbo, two-doors, hatchback, sedan, 4wd, rwd, rear, wheel.base, length, width, height, curb.weight, l, ohc, ohcf ,ohcv, five-cylinders, four-cylinders, three-cylinders, twelve-cylinders, engine.size, 2bbl, idi, mfi, mpfi, spdi, bore, stroke, compression.ratio, horsepower, peak.rpm, city.mpg, highway.mpg	audi, saab, volkswagen, diesel, std, four-doors, 4wd, fwd, 1bbl
AIC	74	74	130.24

Results of Significance Testing (Auto Safety):

```
Coefficients:
               stimate Std. Error z value Pr(>|z|)
(Intercept)     E 2.5122     1.1216    2.240  0.02510 *
audi            20.3574  2027.3521    0.010  0.99199
saab            17.7446  1985.9220    0.009  0.99287
volkswagen       1.8112     0.9634    1.880  0.06011 .
diesel          -2.0155     1.2716   -1.585  0.11297
std             -0.4196     1.0765   -0.390  0.69668
`four-doors`    -5.9725     1.1293   -5.288 1.23e-07 ***
`4wd`           -0.1377     2.1849   -0.063  0.94976
fwd              3.3028     1.1093    2.977  0.00291 **
`1bbl`          -4.4965     1.4035   -3.204  0.00136 **
---
Null deviance: 266.06  on 192  degrees of freedom
Residual deviance: 110.24  on 183  degrees of freedom
AIC: 130.24
```

Top Predictors - Safety

- A negative z is a safer vehicle.
- The larger four-doored vehicles tend to be safer than two-doored ones.
- Sporty, rear-wheel drive vehicles tend to be more risky.
- prsm() unsuited for dimension reduction in this case - not enough data points. Plymouth)

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