ECS256 - Homework II

Olga Prilepova, Christopher Patton, Alexander Rumbaugh,
John Chen, Thomas Provan

March 10, 2014

Problem 1

As suggested by the hint, in order to find the bias at $t = 0.5$, we're going to be examining the value of β as n goes to ∞. The idea behind this is that the proposed model will converge to some β as the number of samples increases, to the point that it fits as closely as possible to the actual function. Of course, it isn't possible for the model to fit precisely, because the actual relationship isn't linear like the model is, so there will be some bias in the resulting model.

The first thing to consider is model construction. The simplest way to construct the model would be to use a least-squares method, so in the construction we would minimize the following with respect to $g(X)$, which is the set of all possible functions for our model.

$$E[(EY - g(X))^2] \tag{1}$$

For this problem, $g(X) = \beta X$ for all possible β, and the actual distribution of EY is $X^{0.75}$, so we can rewrite this to minimizing the following with respect to β

$$E[(X^{0.75} - \beta X)^2] \tag{2}$$

To actually find this value, we will first make a new variable $Q = (X^{0.75} - \beta X)^2$, and then use the continuous case of iterated expectations along X (eq 5.33) to find EQ

$$EQ = \int_{-\infty}^{\infty} f_X(t)\ E(Q|X = t)\ dt \tag{3}$$

$$E(Q|X = t) = E[(t^{0.75} - \beta t)^2] = (t^{0.75} - \beta t)^2 \tag{4}$$

X is $U(0, 1)$, so $f_X(t)$ is 1 between the values of 0 and 1, and is zero elsewhere. With this and the value of $E(Q|X = t)$, we can rewrite EQ as the following.

$$EQ = \int_0^1 (t^{0.75} - \beta t)^2\ dt \tag{5}$$

From here it's a simple matter to minimize with respect to β. First, finish computing the integral to get the final function of β to minimize.

$$EQ = \frac{1}{3}(\beta^2 - \frac{24}{11}\beta + \frac{6}{5}) \tag{6}$$

And then take the derivative of the function with respect to β.

$$\frac{d\,EQ}{d\beta} = \frac{2}{3}\beta - \frac{8}{11} \tag{7}$$

And finally set that equal to zero and solve for β, which yields $\beta = \frac{12}{11}$ as the value that minimizes our function.

Now we can move on and compute the bias. The bias is simply $\hat{m}_{Y;X}(t) - m_{Y;X}(t)$, which for the selected β and $t = 0.5$ is

$$Bias(0.5) = \beta 0.5 - 0.5^{0.75} \approx \boxed{\text{-0.0491}} \tag{8}$$

Problem 2

Problem 2.c

Our next data set explores house value throughout California derived from 1990 census data. The state is divided up into 20640 geographic blocks with average population 1425.

We are interested in how the following variables affect median house value: median income, housing median age, total rooms, total bedrooms, population, households, latitude, and longitude.

Here is R's summary output of the data:

```
Median.House.Value Median.Income       Median.Age
 Min.   : 14999     Min.   : 0.4999   Min.   : 1.00
 1st Qu.:119600     1st Qu.: 2.5634   1st Qu.:18.00
 Median :179700     Median : 3.5348   Median :29.00
 Mean   :206856     Mean   : 3.8707   Mean   :28.64
 3rd Qu.:264725     3rd Qu.: 4.7432   3rd Qu.:37.00
 Max.   :500001     Max.   :15.0001   Max.   :52.00
Total.Rooms     Total.Bedrooms     Population
 Min.   :    2   Min.   :   1.0   Min.   :    3
 1st Qu.: 1448   1st Qu.: 295.0   1st Qu.:  787
 Median : 2127   Median : 435.0   Median : 1166
 Mean   : 2636   Mean   : 537.9   Mean   : 1425
 3rd Qu.: 3148   3rd Qu.: 647.0   3rd Qu.: 1725
 Max.   :39320   Max.   :6445.0   Max.   :35682
 Households        Latitude       Longitude
 Min.   :   1.0   Min.   :32.54   Min.   :-124.3
 1st Qu.: 280.0   1st Qu.:33.93   1st Qu.:-121.8
 Median : 409.0   Median :34.26   Median :-118.5
 Mean   : 499.5   Mean   :35.63   Mean   :-119.6
 3rd Qu.: 605.0   3rd Qu.:37.71   3rd Qu.:-118.0
 Max.   :6082.0   Max.   :41.95   Max.   :-114.3
```

Next we use our Parsimony package on the data at various k levels. Here are the calls and output:

```
 > prsm(df[,1],df[,2:9],k=0.01, predacc=ar2, crit='max', printdel=T)
full outcome =  0.6369649
deleted         Total.Rooms
new outcome  =  0.6350863
deleted         Total.Bedrooms
new outcome  =  0.6321316
[1] "Median.Income" "Median.Age"    "Population"    "Households"    "Latitude"      "Longitude"
```

```
> prsm(df[,1],df[,2:9],k=0.05, predacc=ar2, crit='max', printdel=T)
full outcome =  0.6369649
deleted         Median.Age
new outcome  =  0.6243571
deleted         Total.Rooms
new outcome  =  0.6218261
deleted         Total.Bedrooms
new outcome  =  0.6198323
[1] "Median.Income" "Population"    "Households"    "Latitude"      "Longitude"
```

By loosening our k value, the model selector decides to remove Median Age in the latter run. It is interesting to note that longitude and latitude remain in the model, which we will address later.

Next we want to compare our Parsimony package to the significance testing approach.

Here is the summary of the coefficients of our lm() call in R:

```
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)    -3.594e+06  6.254e+04 -57.468  < 2e-16 ***
Median.Income   4.025e+04  3.351e+02 120.123  < 2e-16 ***
Median.Age      1.156e+03  4.317e+01  26.787  < 2e-16 ***
Total.Rooms    -8.182e+00  7.881e-01 -10.381  < 2e-16 ***
Total.Bedrooms  1.134e+02  6.902e+00  16.432  < 2e-16 ***
Population     -3.854e+01  1.079e+00 -35.716  < 2e-16 ***
Households      4.831e+01  7.515e+00   6.429 1.32e-10 ***
Latitude       -4.258e+04  6.733e+02 -63.240  < 2e-16 ***
Longitude      -4.282e+04  7.130e+02 -60.061  < 2e-16 ***
---
```

The significance testing approach deletes no predictors. This data set has a large size (20640) so it makes sense that the predictors are all deemed significant.

Here is a summary of the comparison of methods:

Method	Parsimony (k=0.01)	Parsimony (k=0.05)	Significance Testing
Columns Deleted	Total Rooms Total Bedrooms	Total Rooms Total Bedrooms Median Age	None
Adjusted RSquared	0.6321316	0.6218261	0.6369649

Before discussing conclusions about the data, we address latitude and longitude. Looking at the full summary above, latitude and longitude both have negative coefficient values. This says if we travel east or north, on average house value decreases. However a real estate expert would think you are crazy hearing that statement.

The problem arises with using coordinate data in a linear regression model. Without any interaction terms we are assuming the statement above. We would risk overfitting if we attempted to make a complex formula involving coordinates.

A block's location is clearly important. We can see what blocks are close to one another, and proximity to oceans or mountains. Using location data effectively can lead to powerful models, however our model is not set up to do this. But we can plot our data atop a map which we will show shortly.

We remove latitude and longitude and use the predictors kept by our Parsimony package. Here is the summary of the regression:

```
Coefficients:
```

```
                Estimate Std. Error t value Pr(>|t|)
(Intercept)   -32165.268   2167.358  -14.84   <2e-16 ***
Median.Income  43094.918    284.263  151.60   <2e-16 ***
Median.Age      2000.544     45.080   44.38   <2e-16 ***
Population       -43.045      1.127  -38.20   <2e-16 ***
Households       152.700      3.344   45.66   <2e-16 ***
```

First we see that income positively affects house value which is unsurprising. If people have more income, they can afford more expensive houses.

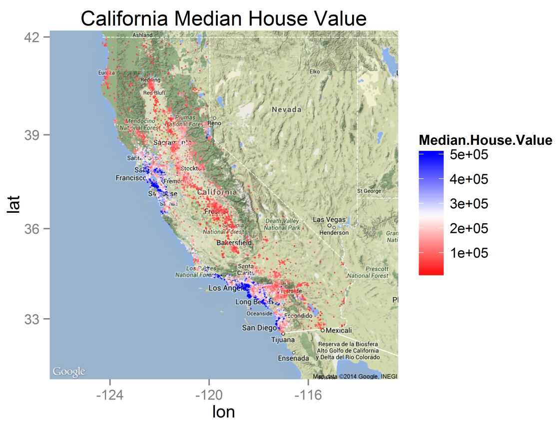
Next looking at age, we see it also positively affects house value. One would expect older people to have more money and this reflected in the high co-variance between these predictors.

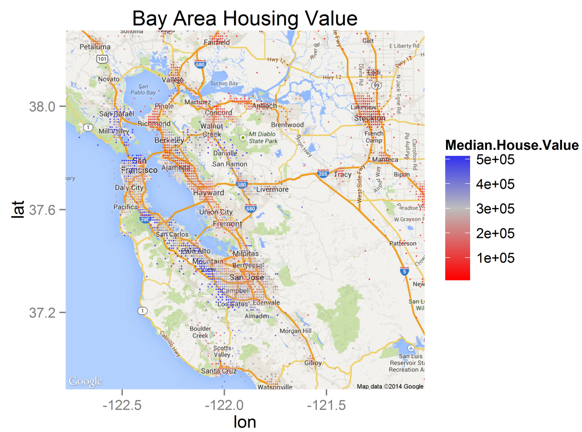
Moving onto population, we see it negatively affects house value. With this data set each block has approximately similar geographic size, so blocks with higher population can be assumed to be denser. From our model we can conclude less dense areas have higher housing value.

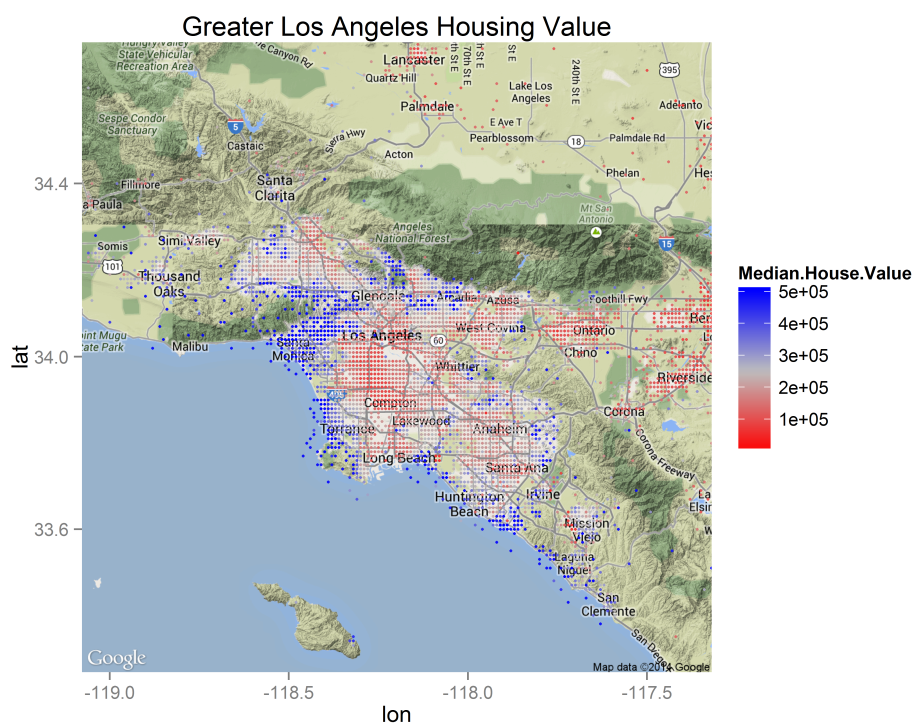
Finally we look at number of households which positively affects house value. This is a bit surprising since population had a negative coefficient. One possible explanation is that our model is detecting large suburban areas which are all houses. Areas with more apartment complexes would fewer number of households and also lower housing value.

Parsimony simplified our model down to four predictors. While we lost the data about rooms, we still have a model we can clearly understand.

Now we make use of our coordinate data using R's ggmap library. First we looked at a map of California as a whole, then zoomed into the Bay Area and the Greater Los Angeles area.


California Median House Value
42
39
36
33
lat
-124
-120
-116
lon
Median.House.Value
5e+05
4e+05
3e+05
2e+05
1e+05


Bay Area Housing Value
38.0
37.6
37.2
lat
-122.5
-122.0
-121.5
lon
Median.House.Value
5e+05
4e+05
3e+05
2e+05
1e+05


Greater Los Angeles Housing Value
34.4
34.0
33.6
lat
-119.0
-118.5
-118.0
-117.5
lon
Median.House.Value
5e+05
4e+05
3e+05
2e+05
1e+05

The visualization helps us see how housing value is clustered in the state. As one might expect, rural areas generally have lower housing value than metropolitan areas. Within the metropolitan areas there are visible divisions as well.

Next we looked at space shuttle data taken from the UCI Machine Learning Repository. The goal was to predict the class given 8 predictors, with 43500 data points. Our logistic regression predicted whether or not the class was of type 1. The description of the data set was very poor, however it matched our criterion for dimension, data points, and regression type. Because of the lack of sufficient description, few conclusions can be drawn from the data, but we can still test our Parsimony package.

Here is the summary of results using Parsimony:

Method	Parsimony (k=0.01)	Parsimony (k=0.05)	Significance Testing
Columns Deleted	V1,V3,V4,V6,V9	V1,V2,V3,V4,V6,V9	V4
AIC	8575.803	8728.906	8475.2

We also tested the various methods against a validation training set. We looked at the probability of correctly identifying a class 1 or not class 1. Also if we identified something as class 1, what was the probability it was actually class 1 and the same for not class 1.

Here is a summary of the results:

Method	Parsimony (k=0.01)	Parsimony (k=0.05)	Significance Testing
P(Correct ID | class 1)	0.9862345	0.9861474	0.9864959
P(Correct ID | not class 1)	0.9285242	0.9285242	0.9169424
P(Correct ID | guess class 1)	0.981276	0.9812744	0.9783135
P(Correct ID | guess not class 1)	0.9466937	0.9463744	0.9470267

The results for the different methods are very close and it is hard to detect much difference in the accuracy levels. There were slightly fewer false positives and there was a slightly better detection of negatives for the Parsimony models. By simplifying the model, there is a possibility of increased accuracy during validation runs.

For large p - large n data set we used the census dataset (https://archive.ics.uci.edu/ml/datasets/Adult), which contains 32561 observations, 13 variable vectors and is based on 1994 Census. There are several categorical variables in this dataset. In order to accommodate regression modeling each categorical vector was split into 0-1 values vectors for each category. According to the website from which the dataset was taken was to predict whether a given person would be making over 50K a year. Our parsimony function at k=0.1 eliminated all but one explanatory variable, which turned out to be "capital gain". This makes sense. If we were to look at only one column in order to predict in a person is making over 50K a year, the capital gain variable would be most informative. At k=0.01 there are many more explanatory variables that enhance the predictions: "age", "education num", "Exec-managerial", "Not-in-family", "Own-child", "Unmarried", "Wife", "capital gain", "capital loss", "hours per week".

Another interesting variable that can be of interest for predictions is "sex", which can be only "Male" or "Female" in the scope of this census. With k=0.01 the following explanatory variables aren't eliminated: "Widowed", "Craft-repair", "Farming-fishing", "Handlers-cleaners", "Transport-moving", "Not-in-family", "Other-relative", "Own-child", "Unmarried", "Wife". Some of them make perfect sense, such as "Wife" variable. If somebody identified as a wife in 1994 Census it is very likely that the person was female. "Widowed" variable makes sense too, and we expect to see a strong negative correlation between identifying as widowed and being female.

With the continuously defined variable an interesting one to predict was "Age". After running our parsimony function with k=0.01, the variables that explain the variance in "Age" are "Self-emp-not-inc", "Assoc-acdm", "Never-married", "Widowed", "Own-child", "hours per week", "salary".

In all of the performed analyses the countries of origin don't seem to play much role, or are too sparse to be good predictors of the outcome. Also it is interesting to point out that the education wasn't highly correlated with the variables of interest. It would be very interesting to compare these results with a more modern census information.