Bias, Variance and Parsimony in Regression Analysis
ECS 256 Winter 2014

Christopher Patton, cjpatton@ucdavis.edu
Alex Rumbaugh, aprumbaugh@ucdavis.edu
Thomas Provan,tcprovan@ucdavis.edu
Olga Prilepova, prilepova@gmail.com
John Chen, jhochen@ucdavis.edu

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Introduction

Testing Parsimony on Simulated Data

Predictors: $X = X_1, ..., X_{10}$

Response: Y drawn from $U(m_{Y;X}(t) - 1, m_{Y;X}(t) + 1)$

where $m_{Y,X}(t) = t_1 + t_2 + t_3 + 0.1t_4 + 0.01t_5$

Testing Parsimony on Simulated Data

		prsm(k=0.01)	prsm(k=0.05)	sig test
n=100	Run 1	X_1, X_2, X_3, X_9	X_1, X_2, X_3	X_1, X_2, X_3
	Run 2	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3
	Run 3	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3
n=1000	Run 1	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3, X_4
	Run 2	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3
	Run 3	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3
n=10K	Run 1	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3, X_4
	Run 2	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3, X_4
	Run 3	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3, X_4, X_9
n=100K	Run 1	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3, X_4
	Run 2	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3, X_4, X_9
	Run 3	X_1, X_2, X_3	X_1, X_2, X_3	X_1, X_2, X_3, X_4, X_9

Testing Parsimony on Simulated Data

k=0.01	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
N = 100	1	1	1	0.24	0.11	0.14	0.21	0.22	0.26	0.28
N = 1000	1	1	1	0.08	0	0	0	0	0	0
N = 10K	1	1	1	0	0	0	0	0	0	0
N = 100K	1	1	1	0	0	0	0	0	0	0
N = 1M	1	1	1	0	0	0	0	0	0	0
k=0.05	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
N = 100	1	1	0.99	0.1	0.02	0.05	0.04	0.03	0.07	0.02
N = 1000	1	1	1	0	0	0	0	0	0	0
N = 10K	1	1	1	0	0	0	0	0	0	0
N = 100K	1	1	1	0	0	0	0	0	0	0
N = 1M	1	1	1	0	0	0	0	0	0	0

Testing Parsimony on Simulated Data

Sig Test	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
N = 100	1	1	1	0.14	0.03	0.05	0.05	0.03	0.09	0.04
N = 1000	1	1	1	0.31	0.02	0.05	0.05	0.05	0.02	0.04
N = 10K	1	1	1	1	0.04	0.01	0.07	0.07	0.03	0.06
N = 100K	1	1	1	1	0.35	0.06	0.09	0.03	0.05	0.03
N = 1M	1	1	1	1	1	0.05	0.03	0.08	0.02	0.03

California Housing Data

- Derived from 1990 Census
- Response Variable: median house value
- Predictor Variables: median income, housing median age, total rooms, total bedrooms, population, households, latitude, and longitude

Parsimony

Method	Parsimony (k=0.01)	Parsimony (k=0.05)	Sig Test
Columns Deleted	Total Rooms Total Bedrooms	Total Rooms Total Bedrooms Median Age	None
Adjusted R^2	0.6321316	0.6218261	0.6369649

Regression Coefficients

```
Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)    -3.594e+06  6.254e+04 -57.468  < 2e-16 ***
Median.Income   4.025e+04  3.351e+02 120.123  < 2e-16 ***
Median.Age      1.156e+03  4.317e+01  26.787  < 2e-16 ***
Total.Rooms    -8.182e+00  7.881e-01 -10.381  < 2e-16 ***
Total.Bedrooms  1.134e+02  6.902e+00  16.432  < 2e-16 ***
Population     -3.854e+01  1.079e+00 -35.716  < 2e-16 ***
Households      4.831e+01  7.515e+00   6.429 1.32e-10 ***
Latitude       -4.258e+04  6.733e+02 -63.240  < 2e-16 ***
Longitude      -4.282e+04  7.130e+02 -60.061  < 2e-16 ***
```

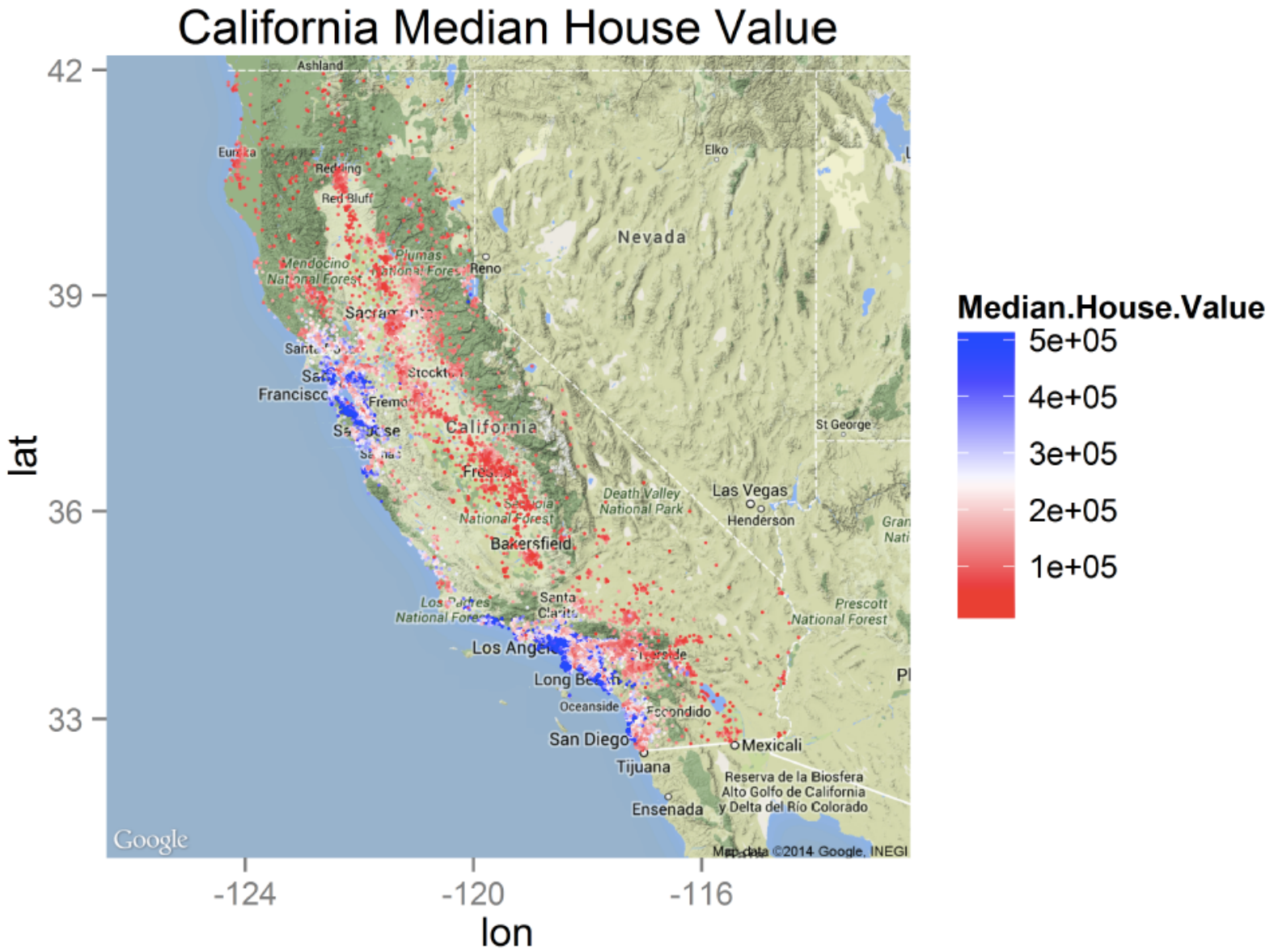
Latitude & Longitude

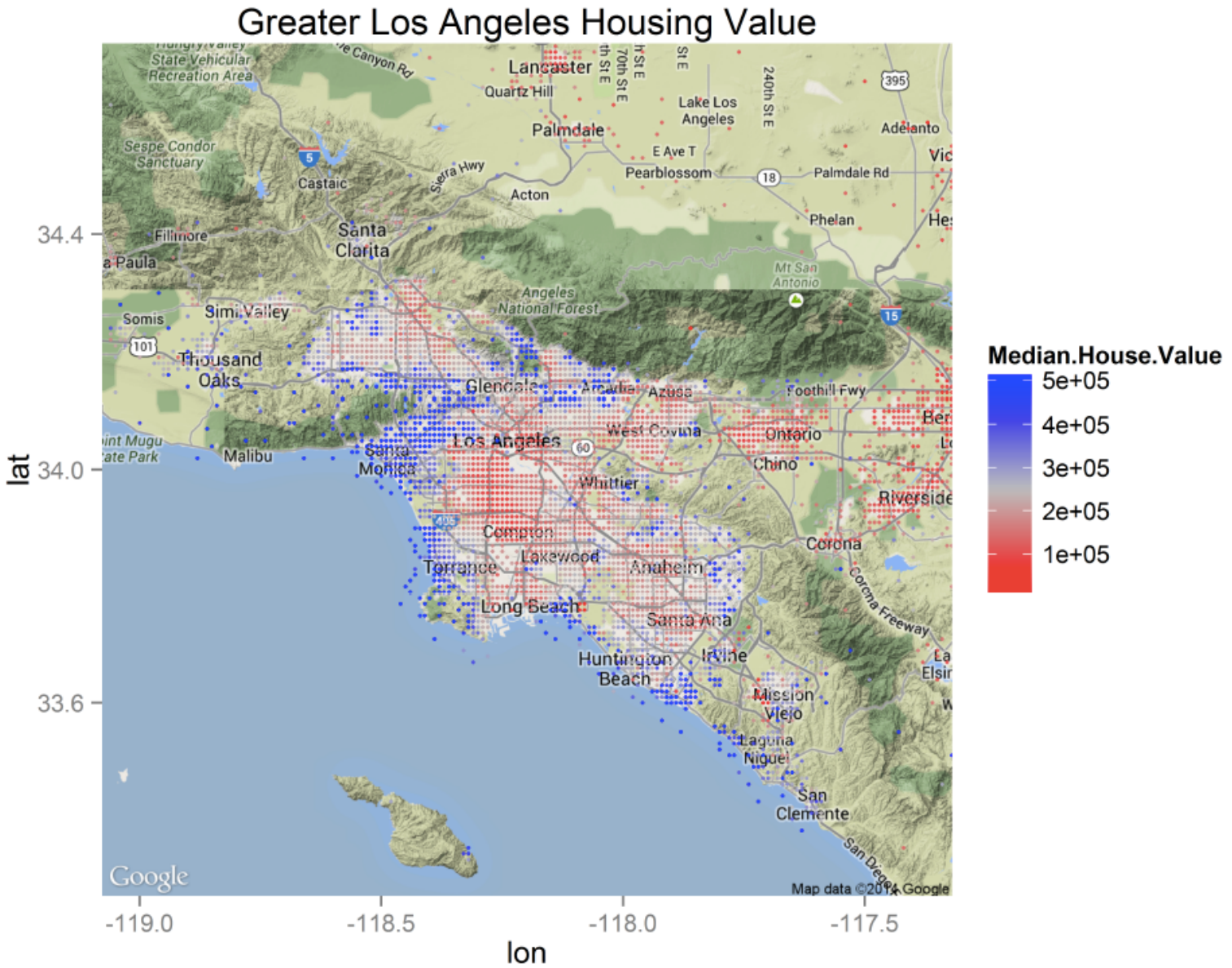
```
Latitude      -4.258e+04  6.733e+02 -63.240  < 2e-16 ***
Longitude     -4.282e+04  7.130e+02 -60.061  < 2e-16 ***
```

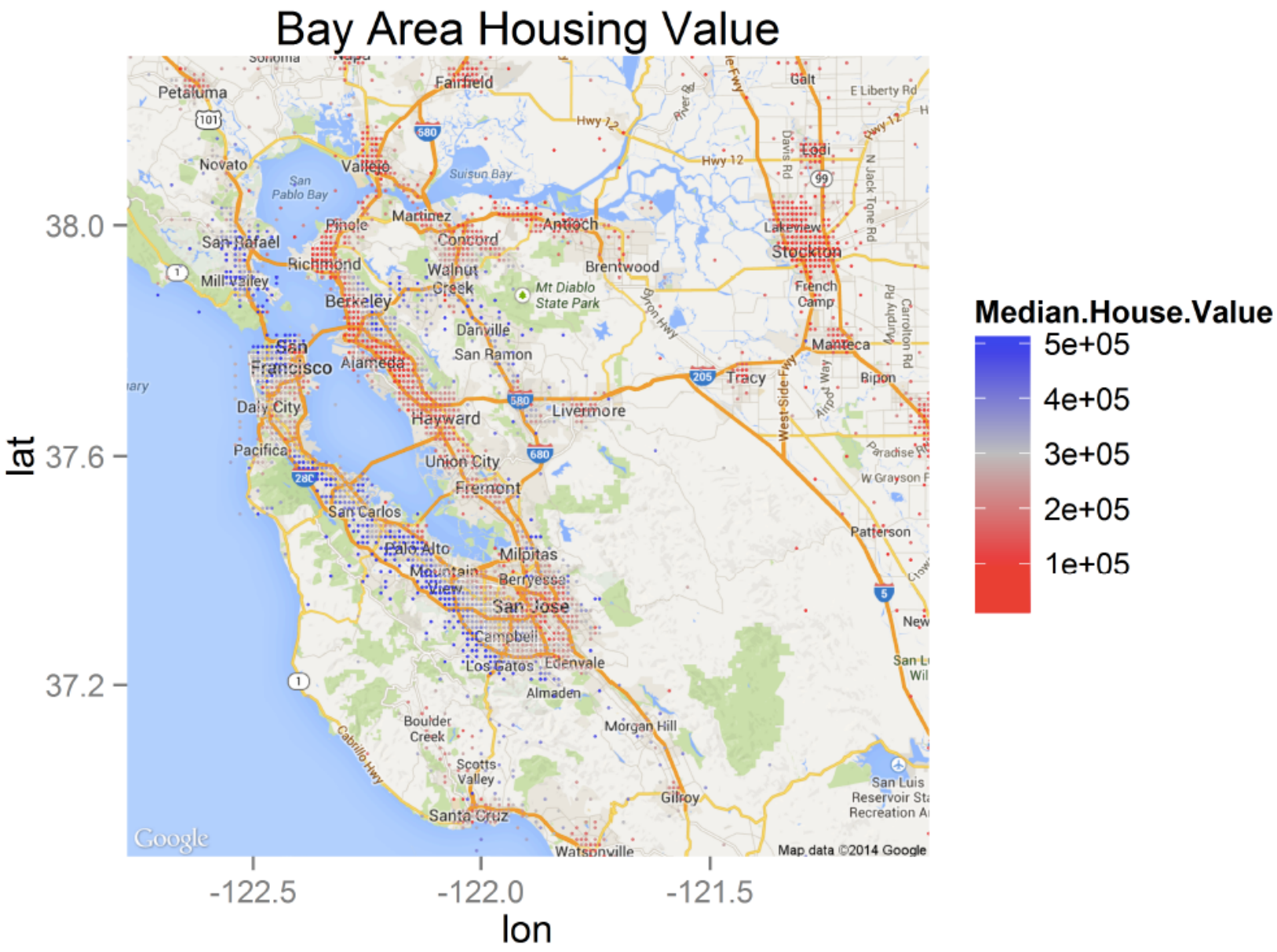
- "Center of Gravity"
- Avoid Overfitting

Understanding

```
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)   -32165.268   2167.358  -14.84   <2e-16 ***
Median.Income  43094.918    284.263  151.60   <2e-16 ***
Median.Age      2000.544     45.080   44.38   <2e-16 ***
Population       -43.045      1.127  -38.20   <2e-16 ***
Households       152.700      3.344   45.66   <2e-16 ***
```

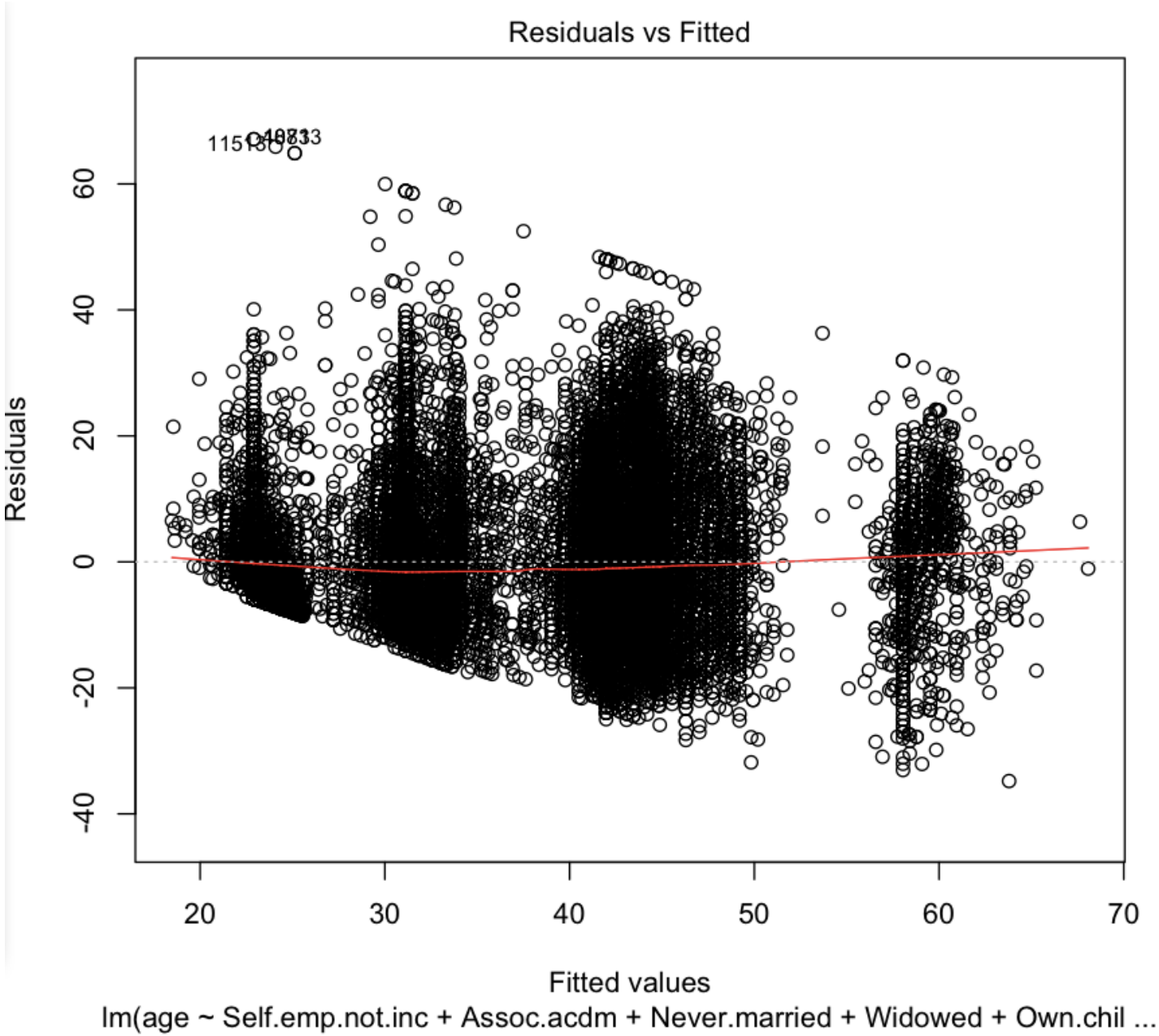

California Median House Value
lat
lon
42
39
36
33
-124
-120
-116
Median.House.Value
5e+05
4e+05
3e+05
2e+05
1e+05
Ashland
Eureka
Redding
Red Bluff
Elko
Nevada
Reno
Mendocino National Forest
Plumas National Forest
Sacramento
Stockton
Fremont
San Jose
California
Fresno
Sequoia National Forest
Death Valley National Park
Las Vegas
Henderson
St George
Bakersfield
Santa Clarita
Los Angeles
Long Beach
Riverside
Oceanside
Escondido
San Diego
Tijuana
Mexicali
Ensenada
Reserva de la Biosfera Alto Golfo de California y Delta del Río Colorado
Prescott National Forest
Google
Map data ©2014 Google, INEGI


Greater Los Angeles Housing Value
lat
lon
34.4
34.0
33.6
-119.0
-118.5
-118.0
-117.5
Median.House.Value
5e+05
4e+05
3e+05
2e+05
1e+05
Lancaster
Quartz Hill
Palmdale
Lake Los Angeles
Adelanto
Pearblossom
Palmdale Rd
Acton
Castaic
Santa Clarita
Fillmore
Phelan
Mt San Antonio
Angeles National Forest
Somis
Simi Valley
Thousand Oaks
Glendale
Azusa
Foothill Fwy
Los Angeles
West Covina
Ontario
Chino
Malibu
Santa Monica
Whittier
Riverside
Compton
Corona
Lakewood
Torrance
Anaheim
Long Beach
Santa Ana
Irvine
Huntington Beach
Mission Viejo
Laguna Niguel
San Clemente
Google
Map data ©2014 Google


Bay Area Housing Value
Median.House.Value
5e+05
4e+05
3e+05
2e+05
1e+05
lat
38.0
37.6
37.2
lon
-122.5
-122.0
-121.5
Petaluma
Fairfield
Novato
Vallejo
Suisun Bay
San Pablo Bay
Martinez
Pinole
Concord
Antioch
San Rafael
Richmond
Walnut Creek
Brentwood
Mt Diablo State Park
Mill Valley
Berkeley
Danville
San Ramon
San Francisco
Alameda
Tracy
Daly City
Hayward
Livermore
Pacifica
Union City
Fremont
San Carlos
Palo Alto
Milpitas
Mountain View
Berryessa
San Jose
Campbell
Los Gatos
Edenvale
Almaden
Boulder Creek
Morgan Hill
Scotts Valley
Gilroy
Santa Cruz
Watsonville
Galt
Lodi
Lakeview
Stockton
French Camp
Manteca
Ripon
Patterson
Google
Map data ©2014 Google

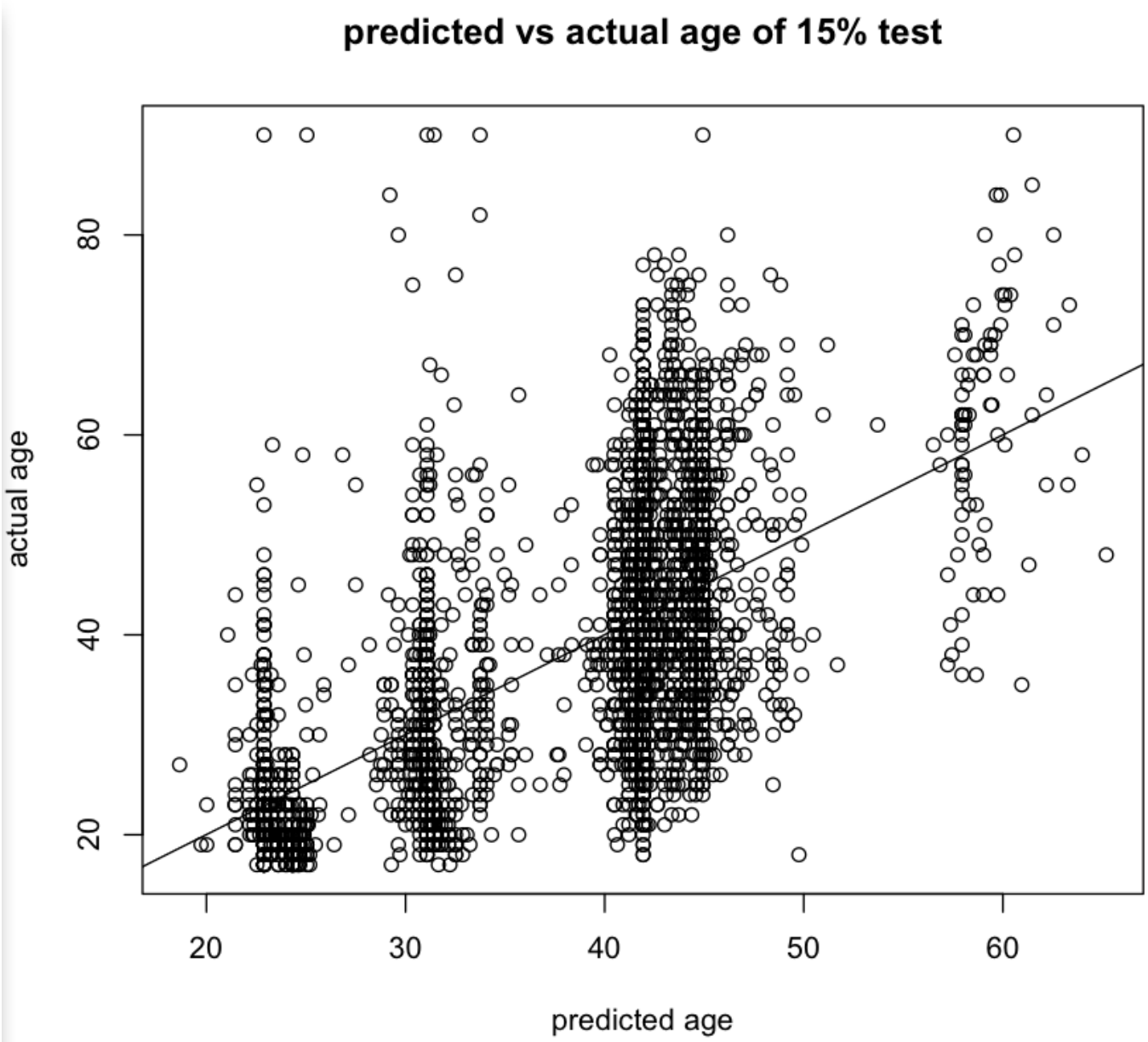
Census Based on 1994

Age


Residuals vs Fitted
Residuals
60
40
20
0
-20
-40
20
30
40
50
60
70
Fitted values
lm(age ~ Self.emp.not.inc + Assoc.acdm + Never.married + Widowed + Own.chil ...

Census Based on 1994

Census Based on 1994


predicted vs actual age of 15% test
actual age
predicted age
20
40
60
80
20
30
40
50
60

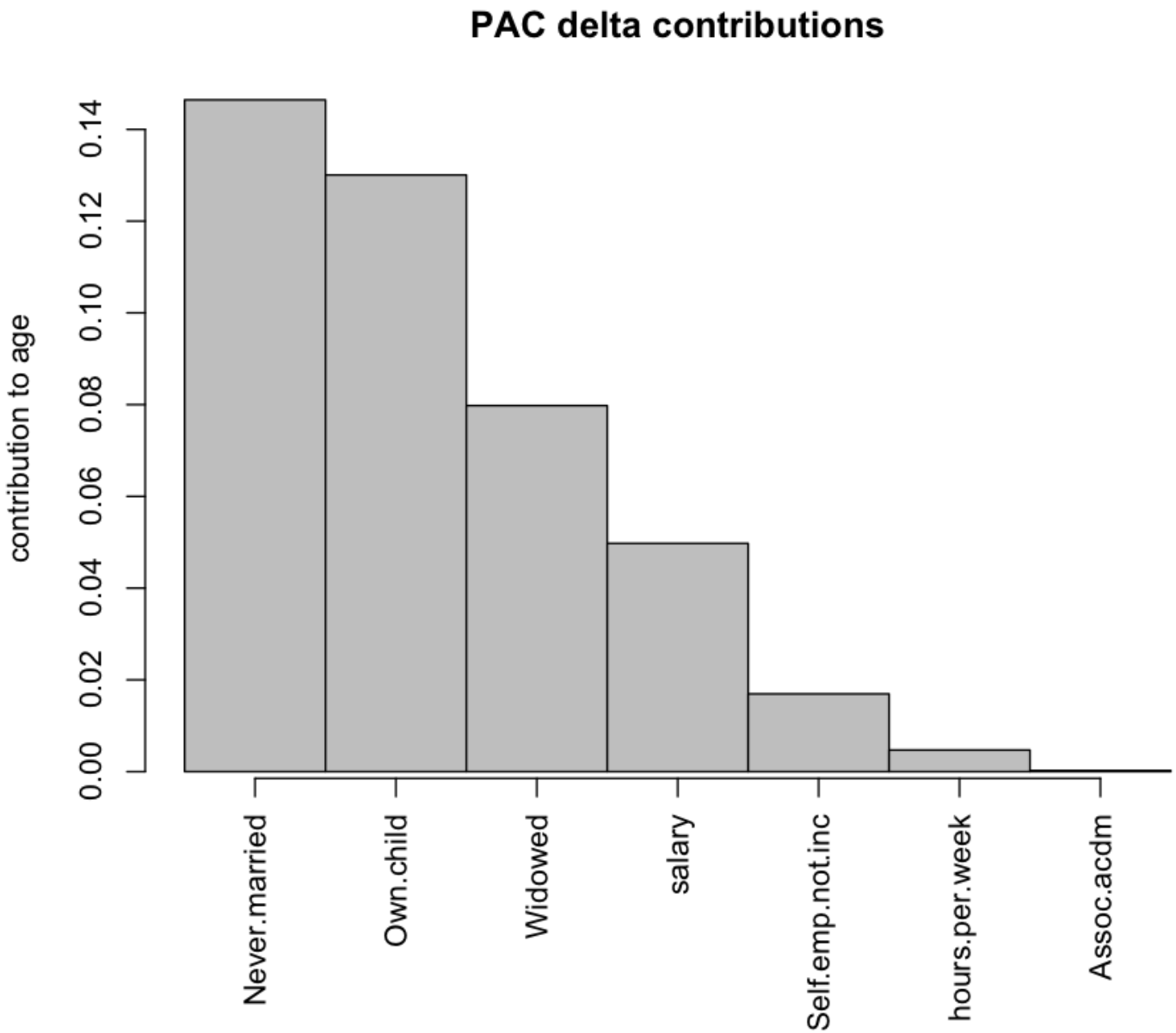

PAC delta contributions
contribution to age
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
Never.married
Own.child
Widowed
salary
Self.emp.not.inc
hours.per.week
Assoc.acdm

Figure:

Small N, Large P

Automobile Data Set:

- UCI Machine Learning Repository
- 195 automobiles,
- 25 attributes per entry.

Goals:

- Determine accurate predictors of vehicle price.
- Gauge characteristics of safe automobiles.

Parsimony: Automobile Prices

- What factors best predict a vehicle's price?
- What are traits that increase price?
- What are the ones that decrease it?

Method	Parsimony (k = 0.01)	Parsimony (k = 0.05)	Significance Testing
Columns Retained	ohcv, twelve-cylinders, engine.size, stroke, compression.ratio, peak.rpm	engine.size	bmw, dodge, 'mercedes-benz', mitsubishi, plymouth, porsche, saab, std, front, wheel.base, length, width, height, curb.weight, dohc, ohc, engine.size, peak.rpm
AIC	0.8676842	0.7888274	0.9308

Significance Testing: Auto Prices

Results of Significance Testing (Auto Price):

```
(Intercept)      -4.234e+04  1.125e+04  -3.764 0.000229 ***
bmw               9.290e+03  8.611e+02  10.788  < 2e-16 ***
dodge            -1.504e+03  8.532e+02  -1.762 0.079785 .
‘mercedes-benz‘   6.644e+03  1.003e+03   6.625 4.17e-10 ***
mitsubishi       -2.628e+03  7.331e+02  -3.585 0.000438 ***
plymouth         -1.628e+03  8.881e+02  -1.833 0.068485 .
porsche           4.053e+03  2.238e+03   1.811 0.071936 .
saab              2.413e+03  1.028e+03   2.347 0.020043 *
std              -1.109e+03  5.129e+02  -2.162 0.031973 *
front            -1.275e+04  2.663e+03  -4.785 3.63e-06 ***
wheel.base        1.141e+02  7.390e+01   1.544 0.124355
length           -7.918e+01  4.225e+01  -1.874 0.062586 .
width             7.652e+02  2.029e+02   3.772 0.000222 ***
height           -1.377e+02  1.164e+02  -1.183 0.238332
curb.weight       3.781e+00  1.118e+00   3.381 0.000890 ***
dohc              1.569e+03  8.067e+02   1.944 0.053451 .
ohc               8.531e+02  4.575e+02   1.865 0.063911 .
engine.size       7.733e+01  1.035e+01   7.470 3.74e-12 ***
peak.rpm          1.522e+00  3.938e-01   3.864 0.000157 ***
---
Multiple R-squared:  0.9373, Adjusted R-squared:  0.9308
F-statistic: 144.5 on 18 and 174 DF,  p-value: < 2.2e-16
```

Top Predictors - Price

- Engine specifications, machinery
- Adds Value: Luxury Brands (BMW, Porsche)
- Reduces Value: Front-based Engine (Found in lower-end vehicles), economy brands (Mitsubishi, Plymouth)

Parsimony: Auto Safety

- Each auto is rated from -3 to 3 by insurers. -3 is safest, 3 is least safe.
- Use logistic regression to determine attributes of safe vehicles

Method	Parsimony (k = 0.01)	Parsimony (k = 0.05)	Significance Testing
Columns Retained	saab, toyota, volkswagen, turbo, two-doors, hatchback, sedan, 4wd, rwd, rear, wheel.base, length, width, height, curb.weight, l, ohc, ohcf ,ohcv, five-cylinders, four-cylinders, three-cylinders, twelve-cylinders, engine.size, 2bbl, idi, mfi, mpfi, spdi, bore, stroke, compression.ratio, horsepower, peak.rpm, city.mpg, highway.mpg	saab, toyota, volkswagen, turbo, two-doors, hatchback, sedan, 4wd, rwd, rear, wheel.base, length, width, height, curb.weight, l, ohc, ohcf ,ohcv, five-cylinders, four-cylinders, three-cylinders, twelve-cylinders, engine.size, 2bbl, idi, mfi, mpfi, spdi, bore, stroke, compression.ratio, horsepower, peak.rpm, city.mpg, highway.mpg	audi, saab, volkswagen, diesel, std, four-doors, 4wd, fwd, 1bbl
AIC	74	74	130.24

Significance Testing: Auto Safety

Results of Significance Testing (Auto Safety):

```
Coefficients:
              stimate Std. Error z value Pr(>|z|)
(Intercept)    E 2.5122     1.1216   2.240  0.02510 *
audi           20.3574  2027.3521   0.010  0.99199
saab           17.7446  1985.9220   0.009  0.99287
volkswagen      1.8112     0.9634   1.880  0.06011 .
diesel         -2.0155     1.2716  -1.585  0.11297
std            -0.4196     1.0765  -0.390  0.69668
‘four-doors‘   -5.9725     1.1293  -5.288 1.23e-07 ***
‘4wd‘          -0.1377     2.1849  -0.063  0.94976
fwd             3.3028     1.1093   2.977  0.00291 **
‘1bbl‘         -4.4965     1.4035  -3.204  0.00136 **
---
Null deviance: 266.06  on 192  degrees of freedom
Residual deviance: 110.24  on 183  degrees of freedom
AIC: 130.24
```

Top Predictors - Safety

- A negative z is a safer vehicle.
- The larger four-doored vehicles tend to be safer than two-doored ones.
- Sporty, rear-wheel drive vehicles tend to be more risky.
- prsm() unsuited for dimension reduction in this case - not enough data points. Plymouth)

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