OPINION DYNAMICS WITH RELUCTANT AGENTS

HOI-TO WAI, CHRISTOPHER PATTON

AGENDA

- Opinion Dynamics Model
- Reluctant agents
- Convergence properties
- Numerical simulation
- Extensions

MOTIVATIONS

- **Opinion dynamics studies how agents in a social network interact.**
- **In a social network, agents (people) talks to the others and exchange ideas**
- **Impact: to understand human behavior, etc…**
- **Relevant questions:**
  - do the opinions of agents converge?
  - convergence rate?
  - What’s the effect of the network topology? etc…

OPINION DYNAMICS MODEL

- **In a social network, opinions held by agents *may* be modeled as a *probability distribution*.**
- **As suggested in DeGroot [1], at time k:**

$$\mathbf{w}^{(k)} = \mathbf{A}^{(k)}\mathbf{w}^{(k-1)}$$

$\mathbf{w}^{(k)} \in \mathbb{R}^N$

is the vector of opinion held by the *N* agents in the network

$\mathbf{A}^{(k)} \in \mathbb{R}^{N \times N}$

is the `mixing’ matrix at time *k*.

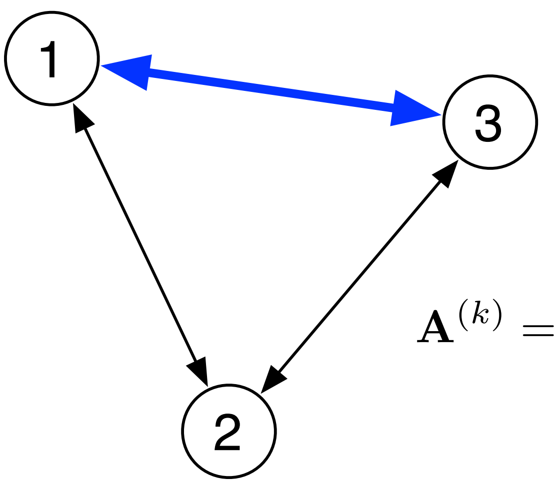
OPINION DYNAMICS MODEL

- **Example: Gossiping model**

$$\mathbf{A}^{(k)} = \mathbf{I} - \frac{1}{2}(\mathbf{e}_i + \mathbf{e}_j)(\mathbf{e}_i + \mathbf{e}_j)^T$$

→ **At time *k*, an edge is selected and the two agents shares their beliefs**

$$w_1^{(k-1)} = 1, w_1^{(k)} = 2$$



$$w_3^{(k-1)} = 3, w_3^{(k)} = 2$$

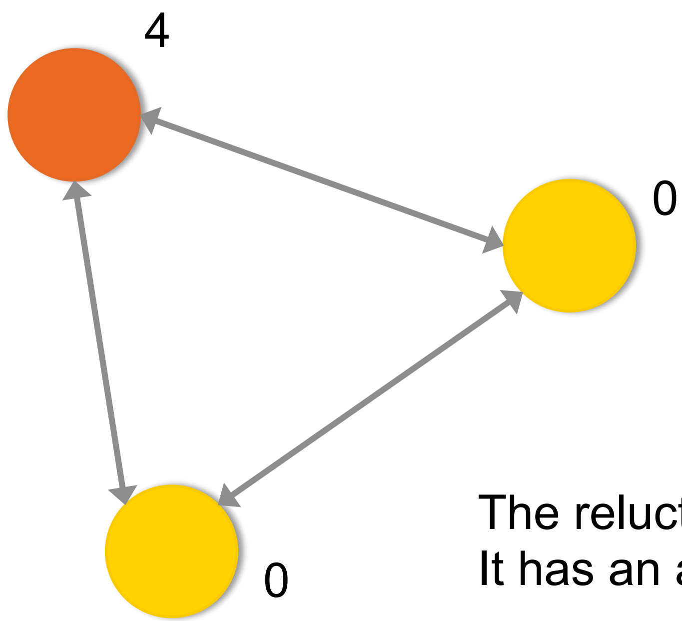
$$\mathbf{A}^{(k)} = \mathbf{I} - \frac{1}{2}(\mathbf{e}_1 + \mathbf{e}_3)(\mathbf{e}_1 + \mathbf{e}_3)^T$$

OPINION DYNAMICS MODEL

- **In Gossiping model, the *true average* can always be found.**
- **Applications & extensions:**
  - Social network
    - The Hegselmann and Krause (HK) model [2] considers when the opinion exchange is bounded by the confidence level → lead to a phase transition for polarization.
  - Wireless sensor network
    - The gossiping model is incorporated in wireless sensor network algorithms for distributed average computation.
- **Assumptions:**
  - The previous models assume that the \`opinion update' happens ***immediately***!!

SOME AGENTS ARE RELUCTANT!

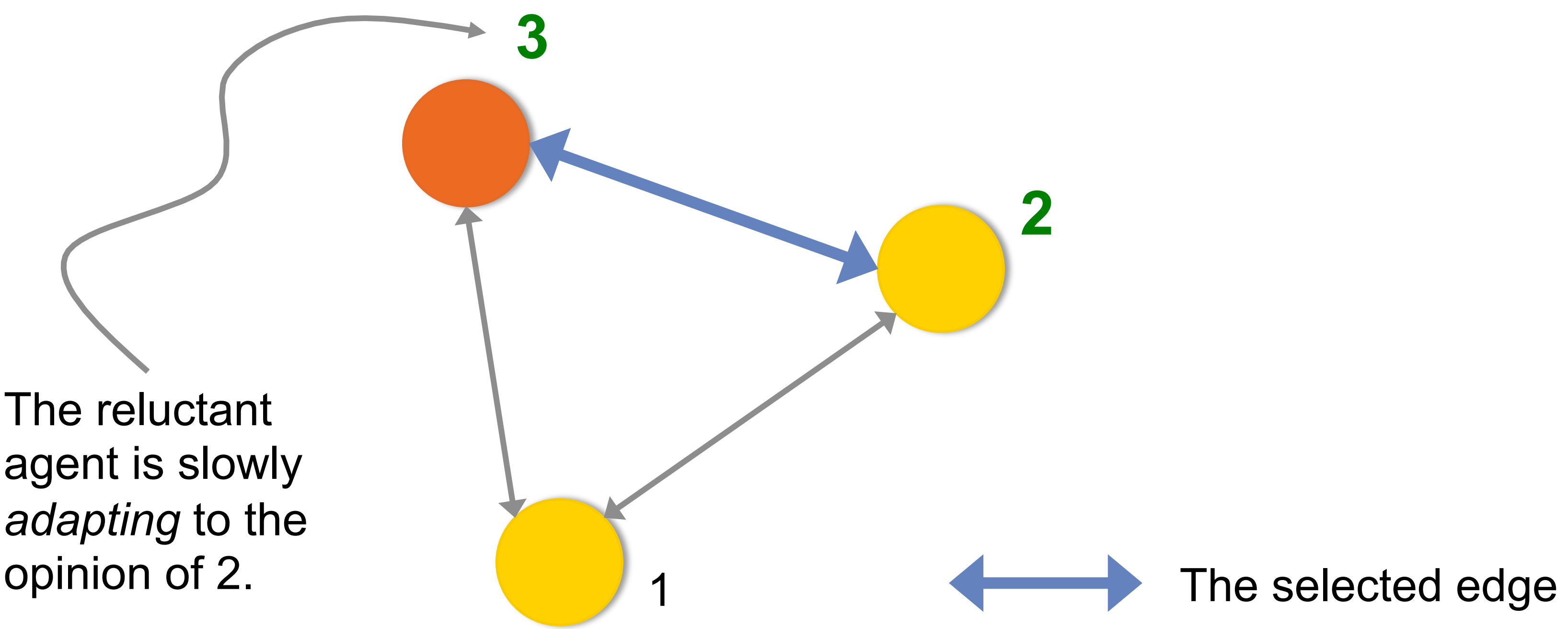
- **Intuition: some agents may be *reluctant* → needs a few more time steps to update their beliefs**
- **Property: i) the reluctant agents take *more time* to update!**
- **E.g.: *(initial states)***



The reluctant agent is colored in orange.
It has an adaptation rate of 2.

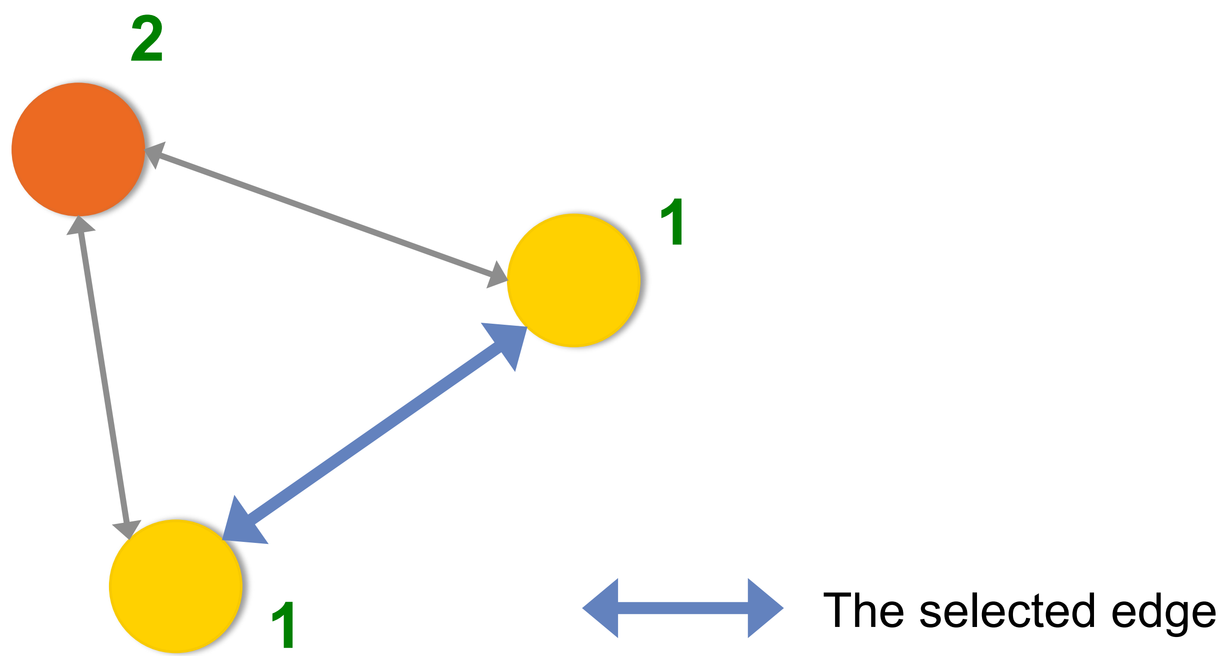
SOME AGENTS ARE RELUCTANT!

- **Intuition: some agents may be *reluctant* → needs a few more time steps to update their beliefs**
- **Property: i) the reluctant agents take *more time* to update!**
- **E.g.:**



SOME AGENTS ARE RELUCTANT!

- **Intuition: some agents may be *reluctant* → needs a few more time steps to update their beliefs**
- **Property: i) the reluctant agents take *more time* to update!**
- **E.g.**

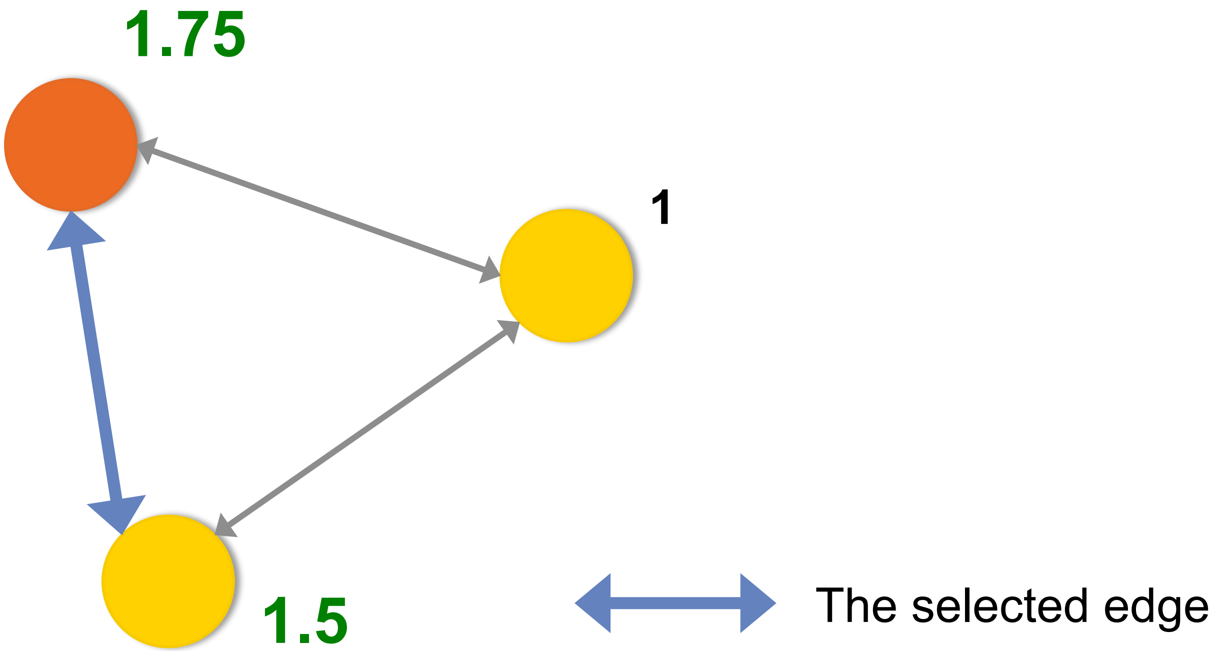


SOME AGENTS ARE RELUCTANT!

- **Intuition: some agents may be *reluctant* → needs a few more time steps to update their beliefs**
- **Properties:**
  - i) The reluctant agents take *more time* to update!
  - *ii) Moreover, the reluctant agents are eager to switch their opinion while he/she is still adapting (e.g., due to greediness).*

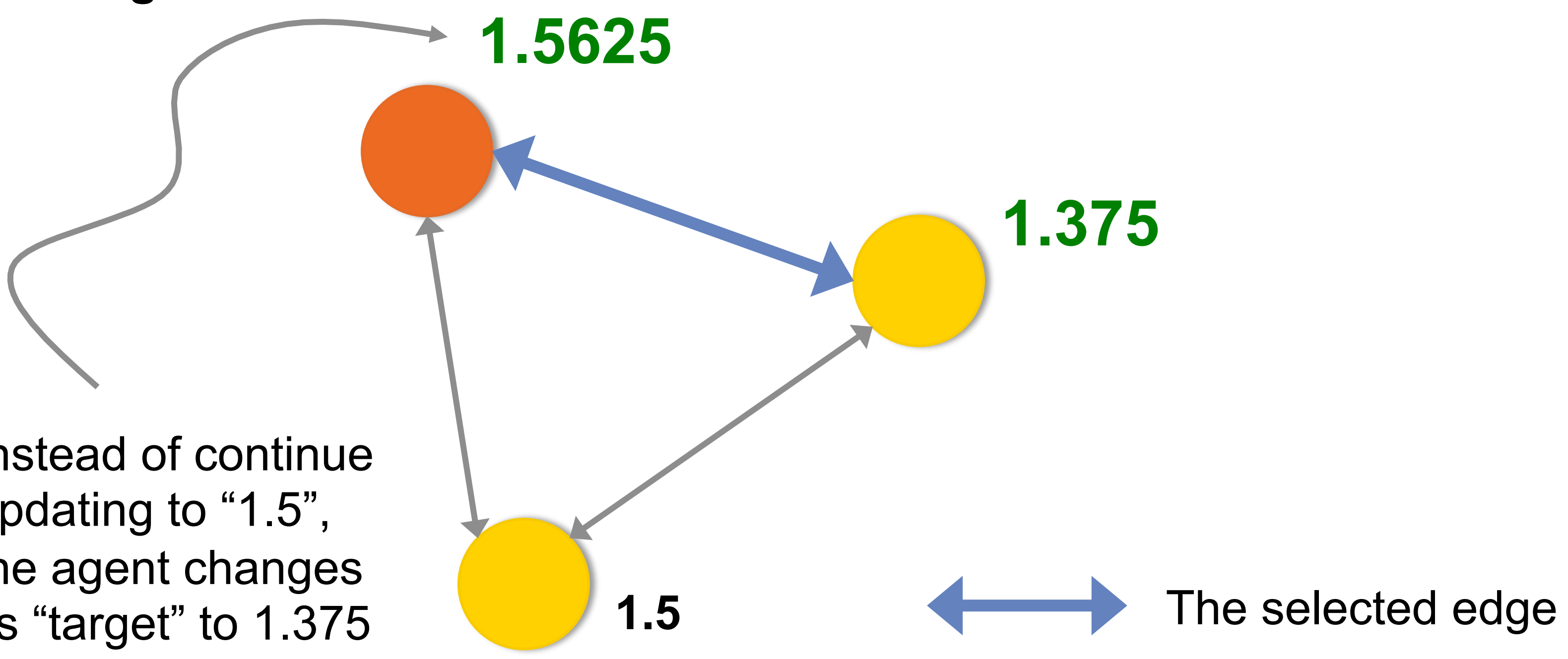
SOME AGENTS ARE RELUCTANT!

- **Intuition: some agents may be *reluctant* → needs a few more time steps to update their beliefs**
- **Property: ii) the reluctant agents are also *eager* to switch their opinion while he/she is still adapting.**
- **E.g.:**



SOME AGENTS ARE RELUCTANT!

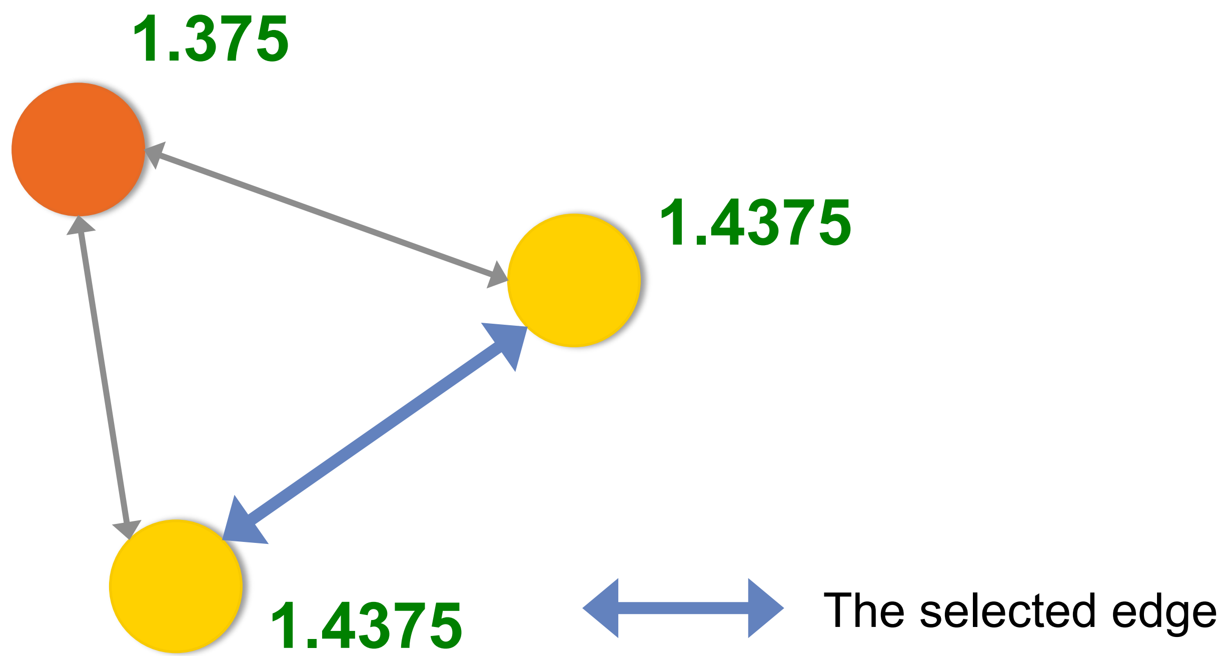
- **Intuition: some agents may be *reluctant* → needs a few more time steps to update their beliefs**
- **Property: ii) the reluctant agents are also *eager* to switch their opinion while he/she is still adapting.**
- **E.g.:**



Instead of continue updating to “1.5”, the agent changes its “target” to 1.375

SOME AGENTS ARE RELUCTANT!

- Intuition: some agents may be *reluctant* → needs a few more time steps to update their beliefs
- Property: ii) the reluctant agents are also *eager* to switch their opinion while he/she is still adapting.
- E.g.:



SOME AGENTS ARE RELUCTANT!

- **Mathematically, we can describe the opinion dynamics model with reluctant agents as:**

$$\hat{w}_i^{(k)} = \sum_j A_{ij}^{(k)} w_j^{(k-1)}$$

$$w_i^{(k)} = \frac{\min\{c_i^{(k)}, \tau_i\}}{\tau_i} \hat{w}_i^{(k-c_i^{(k)}+1)} + \frac{\tau_i - \min\{c_i^{(k)}, \tau_i\}}{\tau_i} w_i^{(k-c_i^{(k)})}$$

where *c* is a counter variable:

$$c_i^{(k)} = \begin{cases} 1 & \text{, if agent } i \text{ talked at time } k. \\ c_i^{(k-1)} + 1 & \text{, otherwise} \end{cases}$$

RESEARCH QUESTION?

- **Our aim is to study the following questions:**
  - Given the reluctant agent model, does the opinions beliefs converge at all?
    - **Yes!** The proof involves developing an equivalent, augmented model.
  - Can the “true average” still be found?
    - **No!** In particular, it is believed that the “more central” and “more reluctant” the reluctant agents are, the more bias they will introduce.
  - What about the convergence rate?
- **These questions will be addressed in the full report.**

SIMULATION