ECS289F Progress report
— Opinion Dynamics with reluctant agents —

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This document reports on the recent progress we have made for the course project in the two weeks after the project proposal's submission. The goal of this project is to consider a new aspect for the DeGroot's model by considering an opinion dynamic model where a subset of agents are *reluctant* to update their opinion.

Let us first recap on the system model. We consider an undirected graph $G = (V, E)$. Each agent $i \in V$ holds an initial opinion $\boldsymbol{w}_i^{(0)} \in \mathbb{R}^L$. At time k, the agents exchange their beliefs with the others to compute:

$$\hat{\boldsymbol{w}}_i^{(k)} = \sum_{j \in \mathcal{N}_i} P_{ij}^{(k)} \boldsymbol{w}_j^{(k-1)}, \tag{1}$$

where $0 \leq P_{ij}^{(k)} \leq 1$ models the trust agent i has on agent j at time k. Importantly, we assume $\sum_{j=1}^{|V|} P_{ij}^{(k)} = 1$ and $\sum_{i=1}^{|V|} P_{ij}^{(k)} = 1$. Notice that the matrix $\boldsymbol{P}^{(k)}$ is time-variant.

The vector $\hat{\boldsymbol{w}}_i^{(k)}$ is the opinion that agent i is supposed to hold at time k. In DeGroot's model, the agents are updating instantly such that $\boldsymbol{w}_i^{(k)} = \hat{\boldsymbol{w}}_i^{(k)}$. In this case, it is known that $\boldsymbol{w}_i^{(k)}$ converges to the average of $\{\boldsymbol{w}_i^{(0)}\}$ asymptotically, i.e., achieving the 'wisdom of the crowd', under some mild assumptions. However, in our model, some agents are reluctant such that they don't update *immediately*. Instead, $\boldsymbol{w}_i^{(k)}$ is updated by:

$$\boldsymbol{w}_i^{(k)} = \frac{c_i^{(k)}}{\tau_i} \hat{\boldsymbol{w}}_i^{(k)} + \frac{\tau_i - c_i^{(k)}}{\tau_i} \bar{\boldsymbol{w}}_i^{(k - c_i^{(k)})},\ i \in V_r,$$

where $V_r \subseteq V$ is the set of reluctant agents and

$$c_i^{(k)} = \begin{cases} 1 & ,\ \text{if } P_{ij}^{(k)} \neq 0,\ \text{for some } j \in V \ (\text{agent } i \text{ talked at time } k). \\ \min\{c_i^{(k-1)} + 1, \tau_i\} & ,\ \text{otherwise.} \end{cases}$$

is a counter variable and $\tau_i \in \mathbb{Z}$ is the adaptation rate of i. In other words, the reluctant agent will slowly adapt to the new opinion in τ_i time steps. Notice that the 'normal' agents are special case of this with $\tau_i = 1$.

1 Convergence Analysis