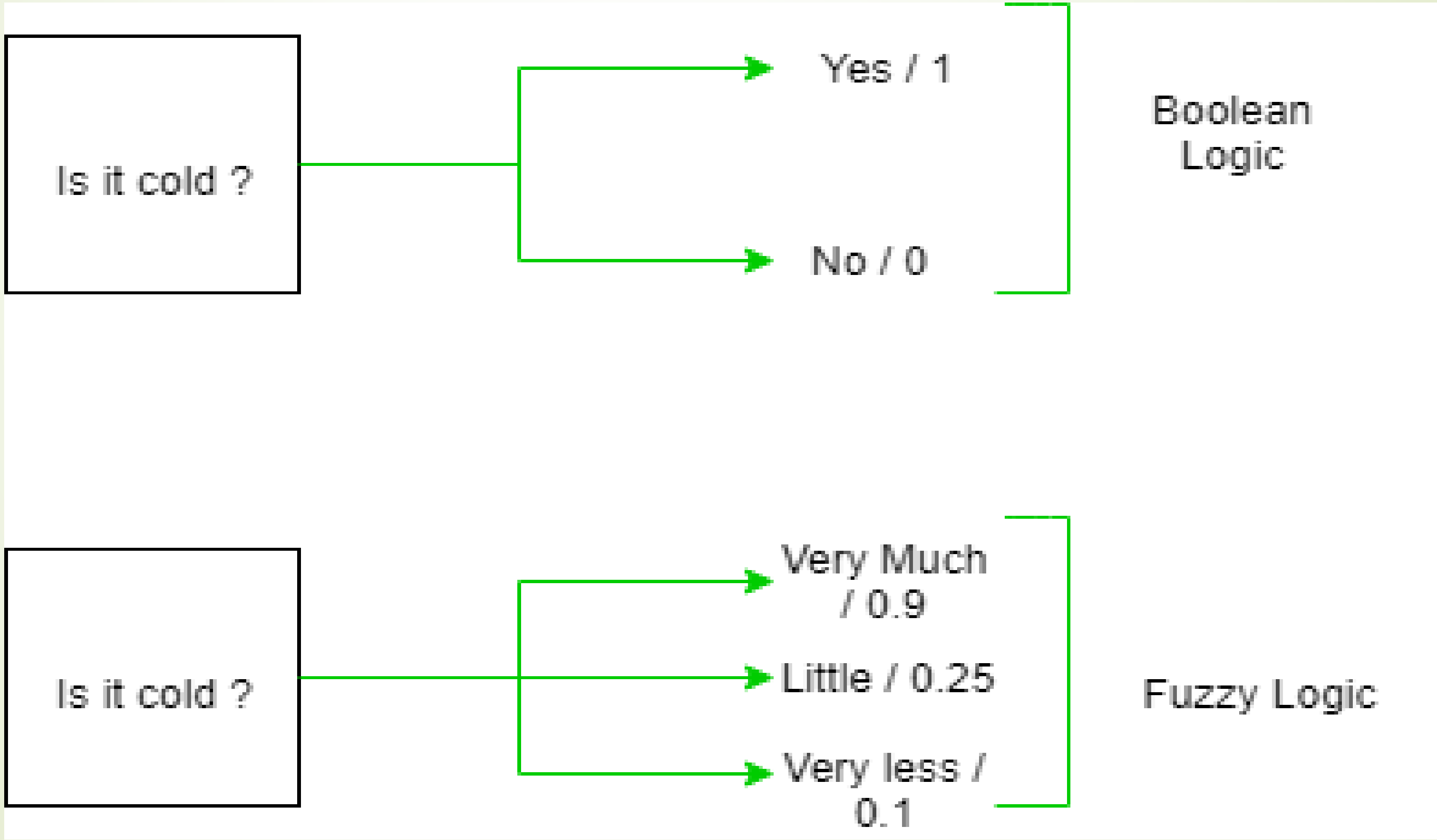
UNIT 2

- The word **fuzzy** refers to things which are not clear or are vague. Any event, process, or function that is changing continuously cannot always be defined as either true or false, which means that we need to define such activities in a Fuzzy manner.
- fuzzy logic provides very valuable flexibility for reasoning. In this way, we can consider the inaccuracies and uncertainties of any situation.

Introduction to Fuzzy logic

- **What is Fuzzy Logic?**
- Fuzzy logic is a form of many-valued logic.
- Unlike binary (classical) logic that operates with **true (1)** or **false (0)**, fuzzy logic allows for **degrees of truth** (values between 0 and 1).
- It models **uncertainty**, **imprecision**, and **vagueness**, just like human reasoning.
- A type of logic dealing with **degrees of truth** rather than the usual "true or false".
- Helps model **uncertainty**, **imprecision**, and **human-like decision-making**.

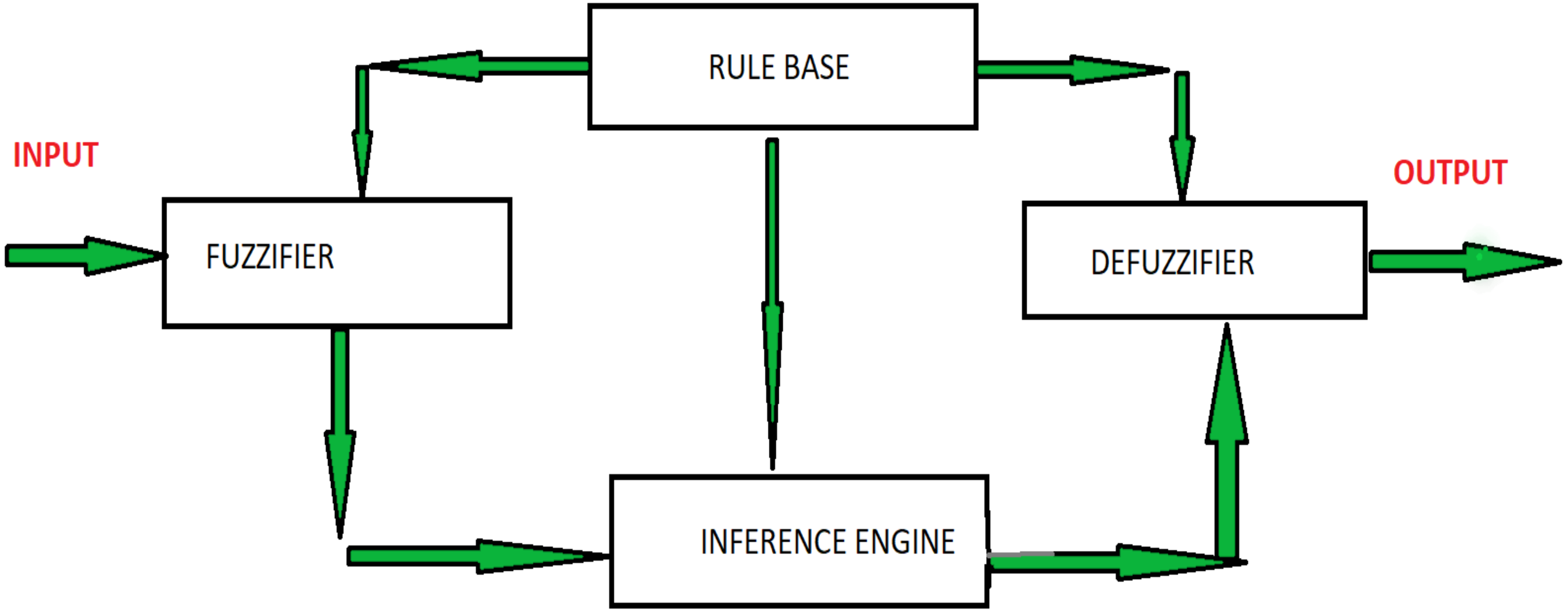
- **Why Fuzzy Logic?**
  - Real-world problems are **not always binary**.
  - Useful in **control systems**, **AI**, **pattern recognition**, **decision-making**, and **automation**.
  - Mimics **human-like reasoning**.
- A type of logic dealing with **degrees of truth** rather than the usual "true or false".
- Helps model **uncertainty**, **imprecision**, and **human-like decision-making**.


Is it cold ?
Yes / 1
No / 0
Boolean Logic
Is it cold ?
Very Much / 0.9
Little / 0.25
Very less / 0.1
Fuzzy Logic

ARCHITECTURE

- **Its Architecture contains four parts :**
- RULE BASE: It contains the set of rules and the IF-THEN conditions provided by the experts to govern the decision-making system, on the basis of linguistic information. Recent developments in fuzzy theory offer several effective methods for the design and tuning of fuzzy controllers. Most of these developments reduce the number of fuzzy rules.
- FUZZIFICATION: It is used to convert inputs i.e. crisp numbers into fuzzy sets. Crisp inputs are basically the exact inputs measured by sensors and passed into the control system for processing, such as temperature, pressure, rpm's, etc.

- INFERENCE ENGINE: It determines the matching degree of the current fuzzy input with respect to each rule and decides which rules are to be fired according to the input field. Next, the fired rules are combined to form the control actions.
- DEFUZZIFICATION: It is used to convert the fuzzy sets obtained by the inference engine into a crisp value. There are several defuzzification methods available and the best-suited one is used with a specific expert system to reduce the error.



FUZZY LOGIC ARCHITECTURE

- A **fuzzy set** is a collection of elements with **degrees of membership** ranging between **0 and 1**.
- Unlike classical (crisp) sets where elements either belong (1) or don’t (0), fuzzy sets allow **partial membership**.
- A fuzzy set **A** in a universe of discourse **X** is defined as:
- A={(x,μA(x)) ∣ x∈X} Where:

• x = element in the universe

• μA(x= **membership function** of x in set A, μA(x)∈[0,1

- Let **X = {10, 20, 30, 40, 50}** be the universe of temperatures (in °C).
- Define fuzzy set **Hot** as:
- Hot={(10,0), (20,0.2), (30,0.5), (40,0.8), (50,1.0}
- At 10°C → membership = 0 → Not hot
- At 30°C → membership = 0.5 → Moderately hot
- At 50°C → membership = 1 → Definitely hot

Membership functions

- fuzziness is best characterized by its membership function. In other words, we can say that membership function represents the degree of truth in fuzzy logic.
- Membership functions characterize fuzziness (i.e., all the information in fuzzy set), whether the elements in fuzzy sets are discrete or continuous.
- Membership functions can be defined as a technique to solve practical problems by experience rather than knowledge.
- Membership functions are represented by graphical forms.
- Rules for defining fuzziness are fuzzy too.

Membership function

- Definition: A graph that defines how each point in the input space is mapped to membership value between 0 and 1. Input space is often referred to as the universe of discourse or universal set (u), which contains all the possible elements of concern in each particular application.

Type	Parameters	Shape	Smooth	Example Use
Triangular	a, b, c	Triangle	No	Speed classification
Trapezoidal	a, b, c, d	Trapezoid	No	Age range, BP range
Gaussian	c, σ	Bell	Yes	Temperature, noise modeling
Bell-Shaped	a, b, c	Bell	Yes	Control systems, human behavior
Sigmoidal	a, c	S-curve	Yes	Threshold logic
Custom	Variable	Flexible	Varies	User-defined logic and modeling

Operations on Fuzzy sets

Fuzzy Set

Fuzzy Set is a Set where every key is associated with value, which is between 0 to 1 based on the certainty .This value is often called as degree of membership. Fuzzy Set is denoted with a Tilde Sign on top of the normal Set notation.

Operations on Fuzzy Set

1. **Union** /Fuzzy OR:

- Consider 2 Fuzzy Sets denoted by A and B, then let's consider Y be the Union of them, then for every member of A and B, Y will be:
- degree_of_membership(Y)= max(degree_of_membership(A), degree_of_membership(B))

2. Intersection/ Fuzzy AND:

- Consider 2 Fuzzy Sets denoted by A and B, then let's consider Y be the Intersection of them, then for every member of A and B, Y will be:
- degree_of_membership(Y)= min(degree_of_membership(A), degree_of_membership(B))

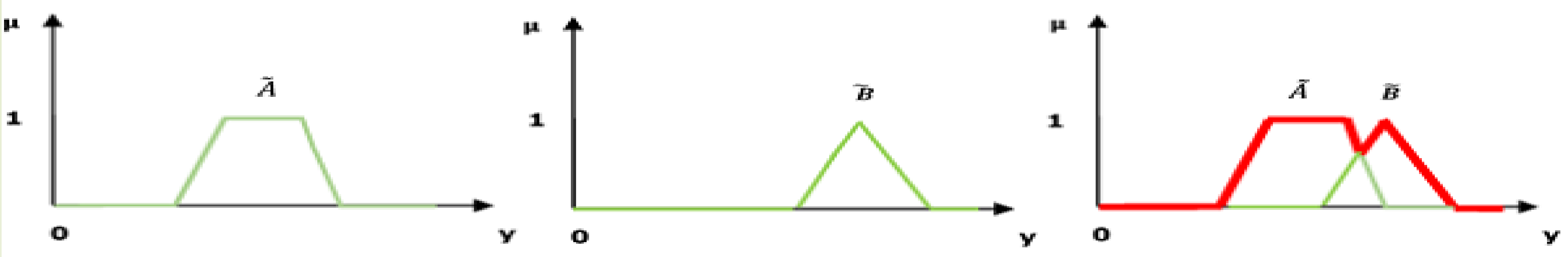
3. Complement/Fuzzy NOT **:**

- Consider a Fuzzy Sets denoted by A , then let's consider Y be the Complement of it, then for every member of A , Y will be:
- degree_of_membership(Y)= 1 - degree_of_membership(A)

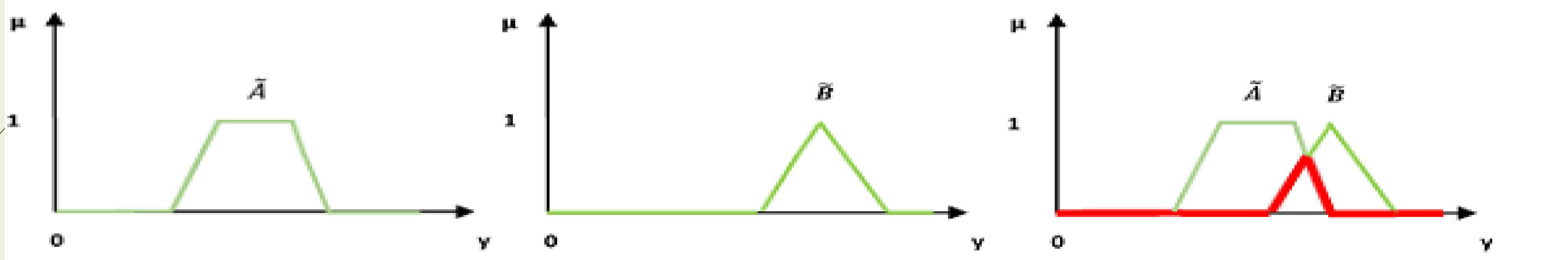
4. Difference :

Consider 2 Fuzzy Sets denoted by A and B, then let's consider Y be the Intersection of them, then for every member of A and B, Y will be:

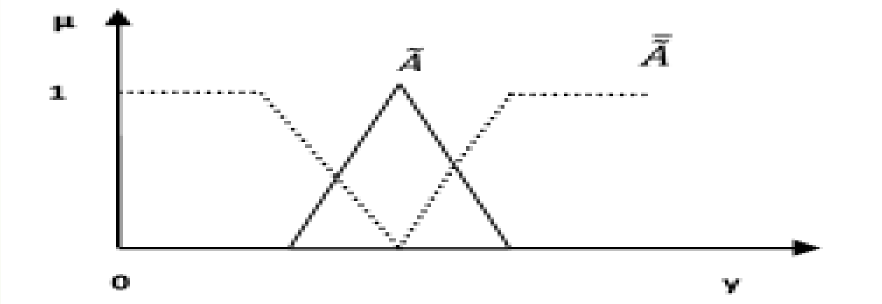
- degree_of_membership(Y)= min(degree_of_membership(A), 1-degree_of_membership(B))



Fuzzy set $\tilde{A}$ Fuzzy set $\tilde{B}$ Union of two Fuzzy sets



Fuzzy set $\tilde{A}$ Fuzzy set $\tilde{B}$ Intersection of two Fuzzy sets



Complement of a fuzzy set

Fuzzy relations

- Fuzzy relation defines the mapping of variables from one fuzzy set to another. Like crisp relation, we can also define the relation over fuzzy sets.
- Let A be a fuzzy set on universe X and B be a fuzzy set on universe Y, then the Cartesian product between fuzzy sets A and B will result in a fuzzy relation R which is contained with the full Cartesian product space

Fuzzy Rules

- **Fuzzy rules** are conditional "IF-THEN" statements used in fuzzy logic systems to represent expert knowledge or decision-making in uncertain environments. A typical fuzzy rule is written as:
  IF x is *A* **THEN** *y* is *B*,
  where x and *y* are variables, and *A* and *B* are fuzzy sets with linguistic labels like "high", "low", or "medium". These rules allow reasoning with vague or imprecise information by combining fuzzy logic operations. Fuzzy rules form the core of fuzzy inference systems, enabling applications in control systems, decision-making, and pattern recognition.
- **Example of Fuzzy Rules**

1. **IF** temperature is *low* **THEN** fan speed is *slow*
2. **IF** temperature is *medium* **THEN** fan speed is *moderate*
3. **IF** temperature is *high* **THEN** fan speed is *fast*

- These rules allow a system (like a smart fan) to adjust speed smoothly instead of switching it on/off abruptly.

Fuzzy Proposition

- Main difference between classical proposition and fuzzy proposition is in the range of their truth values. The proposition value for classical proposition is either true or false but in case of fuzzy proposition the range is not confined to only two values it varies from 2 to n. For example speed may be fast, very fast, medium, slow, and very slow. In fuzzy logic the truth value of fuzzy proposition is also depend on an additional factor known as degree of truth whose value is varies between 0 and 1. For example p: Speed is Slow T(p) = 0.8, if p is partly true T(p) = 1, if p is absolutely true T(p) = 0, if p is totally false So, we can say that fuzzy proposition is a statement p which acquires a fuzzy truth value T(p) ranges from(0 to1)

Different types of Fuzzy Propositions

1. Unconditional and unqualified propositions

The canonical form of this type of fuzzy proposition is p:V is F Where, V is a variable which takes value v from a universal set U. F is a fuzzy set on U that represents a given inaccurate predicate such as fast, low, tall etc.

2. Unconditional and qualified propositions

The canonical form of this type of fuzzy proposition is p:V is F is S Where, V and F have the same meaning and S is a fuzzy truth qualifier Example: Speed is high is very true

3. Conditional and unqualified propositions

The canonical form of this type of fuzzy proposition is p: if X is A, then Y is B

4. Conditional and Qualified Propositions

The canonical form of this type of fuzzy proposition is p: (if X is A, then Y is B) is S Where, all variables have same meaning as previous declare

Fuzzy Propositions

- A fuzzy proposition, much like a classical proposition is a statement that has a truth value. The difference between classical propositions and fuzzy propositions is that classical propositions can have a truth value of either 0 or 1, that is, they can either be entirely false or entirely true. Fuzzy propositions, on the other hand, can have truth values in the range [0, 1].
- For example, if we we have a proposition P, defined as:
  P - The weather is pleasant.
  T(P) = 0, if absolutely false.
  T(P) = 0.2, if mostly false.
  T(P) = 0.4, if partially false.
  T(P) = 0.6, if partially true.
  T(P) = 0.8, if mostly true.
  T(P) = 1, if absolutely true.
  Where T(P) represents the truth value of the proposition.

- Traditionally, most of us have extensively dealt with two-valued logic. For example:

  0 0
  0 1
  1 0
  1 1

- However, when we deal with fuzzy propositions, we will see ourselves often dealing with multi-valued logic. For example, if we extend our logic to include 0, 1/2 and 1, we will have:

0	0
0	1/2
1/2	0
1/2	1/2
0	1
1	0
1	1
1/2	1
1	1/2

Fuzzy Implications

- A fuzzy implication, also known as a fuzzy if-then rule or a fuzzy conditional statement, takes the form:
  If x is A then y is B.
  Here, A and B are linguistic variables (defined by the two fuzzy sets A and B) on universes of discourses X and Y respectively. 'x is A' is often called the antecedent and 'y is B' is often called the consequence. Here are a few examples of fuzzy implications:

- If the *temperature is high*, then the *pressure is high*.
- If the *number is less than or equal to zero*, then the *number is not a natural number*.
- If the *fruit is ripe* then the *fruit is sweet*, else the *fruit is sour*.

Fuzzy Inference

- **Fuzzy inference** is the process of mapping **input values** to **output values** using **fuzzy logic**. It involves applying a set of **fuzzy IF-THEN rules** to evaluate and derive meaningful outputs from vague or imprecise inputs. Fuzzy inference is the core mechanism in fuzzy logic controllers and expert systems.
- **Steps in Fuzzy Inference Process:**

1. **Fuzzification**
   Converts crisp input values into fuzzy values using **membership functions**.
2. **Rule Evaluation**
   Applies fuzzy rules (e.g., **IF temperature is high THEN fan speed is fast**) and computes the **degree of truth** for each rule.
3. **Aggregation of Rule Outputs**
   Combines the outputs of all rules into a single fuzzy set.
4. **Defuzzification**
   Converts the aggregated fuzzy output into a **crisp (numerical) output**, typically using methods like **centroid**, **bisector**, or **mean of maxima**.

- **Example:**

For a fan control system:

- Input: Temperature = 30°C
- Fuzzy Rule:
  IF temperature is *high* **THEN** fan speed is *fast*

- If 30°C has a **0.8 membership** in the *high* fuzzy set, the output fuzzy set for "fan speed is fast" will also be activated to degree 0.8. After processing all rules, the result is defuzzified to a final fan speed value (e.g., 75%).

Defuzzification

- **Defuzzification** is the final step in a fuzzy logic system where a fuzzy output (which contains degrees of truth across multiple output values) is converted into a single crisp value. This process is necessary because, while fuzzy inference works with fuzzy sets to represent imprecise data, most real-world systems require precise, numerical outputs. Defuzzification translates the fuzzy result of inference into a specific action or decision. Common methods include **Centroid of Area (COA)** or **Center of Gravity (COG)**, **Bisector of Area (BOA)**, **Mean of Maximum (MOM)**, **Smallest of Maximum (SOM)**, and **Largest of Maximum (LOM)**. Among these, the centroid method is widely used because it considers all possible output values and their associated membership values, resulting in a balanced and accurate crisp output.

Fuzzy rule based systems evaluate linguistic if-then rules using fuzzification, inference and composition procedures. They produce fuzzy results which usually have to be converted into crisp output. To transform the fuzzy results in to crisp, defuzzification is performed.

Defuzzification is the process of converting a fuzzified output into a single crisp value with respect to a fuzzy set. The defuzzified value in FLC (Fuzzy Logic Controller) represents the action to be taken in controlling the process.

Different Defuzzification Methods The following are the known methods of defuzzification.

- Center of gravity (COG) / Centroid of Area (COA) Method
- Center of Area / Bisector of Area Method (BOA)
- Weighted Average Method
- Maxima Methods
  1. First of Maxima Method (FOM)
  2. Last of Maxima Method (LOM)
  3. Mean of Maxima Method (MOM)

1. Centroid of Area (COA) / Center of Gravity (COG)

- **Definition**: This method calculates the center of mass (or gravity) of the fuzzy set.
- **Formula**:
- z∗=∫z⋅μ(z) dz/ ∫μ(z)dz
- where μ(z)\mu(z)μ(z) is the membership function.
- **Interpretation**: It finds the balance point of the shape formed by the fuzzy set.
- **Advantages**: Most commonly used due to its accuracy and representation of the entire fuzzy set.
- **Example**: If the output fuzzy set is shaped like a triangle or trapezoid, the centroid will fall at the average point of that shape.

2. Bisector of Area (BOA)

- **Definition**: This method finds the vertical line that divides the fuzzy set into two regions with equal area.
- **Interpretation**: It looks for a crisp value where the left area is equal to the right area of the fuzzy region.
- **Use case**: Preferred when symmetry of the result is important, but less commonly used than COA.
- **Drawback**: May give misleading results if the fuzzy set is skewed or irregularly shaped.

Weighted Average Method (also known as the Weighted Mean or Discrete Centroid Method)

- **Definition**: It calculates a weighted average of all output values, where each value is weighted by its membership grade.

Formula:

- $WA = \sum z_i \cdot \mu(z_i)) / \sum \mu(z_i)$
- **Focus**: Uses **all** values and their membership levels.
- **More Accurate**: Because it captures the full shape and distribution of the fuzzy output.

- ## Smallest of Maximum (SOM) Method

Definition:

The **Smallest of Maximum (SOM)** method selects the **smallest value** (i.e., the leftmost point on the x-axis) among all values where the fuzzy output function reaches its **maximum membership grade**.

How It Works:

1. After the **fuzzy inference step**, you obtain a fuzzy output set (a fuzzy region).
2. The membership function μ(z)\mu(z)μ(z) for this output has one or more points where it reaches its **maximum value**
3. The SOM method searches for **all** values z where μ(z)=μmax
4. Among those values, **choose the smallest (leftmost)** value as the final crisp output.

- **Mathematical Expression:**
- If Zmax={z1,z2,...,zn}Z_{ max} = { z_1, z_2, ..., z_n \}Zmax={z1,z2,...,zn} such that
- μ(zi)=μmax
- Then the output is:
- z∗=min(Zmax)
- **Example:**
- Let's say the fuzzy output membership function has the **maximum membership value of 0.8** at three points:
- μ(3)=0.8,μ(5)=0.8,μ(7)=0.8\mu(3)
- Then, using **SOM**:
- z∗=min(3,5,7)=3

- The **Smallest of Maximum (SOM)** method is a fast and straightforward defuzzification technique that selects the **lowest value** among those with the **maximum degree of membership**. While it is not as comprehensive as centroid methods, it is useful in systems where **speed and conservative decisions** are preferred over precision.

- **Last of Maxima Method (LOM)**

Definition:

- The **Last of Maxima (LOM)** method is a defuzzification technique that selects the **largest value (right-most point)** among all those values at which the fuzzy output set reaches its **maximum membership degree**.

How It Works:

1. After fuzzy inference, you have a fuzzy output with a membership function μ(z)\mu(z)μ(z).
2. Identify all values z where the membership function reaches its **maximum value**, say μmax
3. From these values, select the **last** (i.e., **rightmost or largest**) one.

- **Mathematical Expression:**
- If the set of points with maximum membership is:
- Z_{max}={z1,z2,…,zn}. such that. μ(zi)=μ $_{max}$ Then,
- $z^* = max(Z_{max})$
- **Example:**
- Let the fuzzy output membership function have:
- μ(2)=0.6,μ(4)=0.9,μ(6)=0.9,μ(8)=0.9,μ(10)=0.5
- Here, the **maximum membership** is μmax=0.9. at z=4,6,8.
- Using **LOM**, the output is:
- z^*= max(4,6,8)= 8

- The **Mean of Maxima (MOM)** method selects the **average** (or mean) of all values at which the **membership function reaches its maximum value**.
- It is a **balanced** approach among the maxima-based defuzzification methods.
- **How It Works:**

1. Identify the **maximum membership value** μmax\mu_{\text{max}}μmax of the fuzzy output.
2. Find all values of zzz where μ(z)=μmax\mu(z) = \mu_{\text{max}}μ(z)=μmax.
3. Take the **average** (mean) of these values.
4. Return that mean as the crisp output.

Assignment 2

1. Define a **fuzzy set**. Illustrate with an example how fuzzy membership functions differ from crisp sets.
2. Explain different **types of membership functions** with diagrams.
3. Write short notes on:
   a) Fuzzy propositions
   b) Fuzzy implications
   c) Fuzzy inference
4. Describe in detail any **two defuzzification techniques** with examples.
5. Discuss at least **three real-life applications of fuzzy logic** in engineering or daily life.