Assignment 3 (ML for TS) - MVA 2022/2023

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1 Introduction

Objective. The goal is to implement (i) a signal processing pipeline with a change-point detection method and (ii) wavelets for graph signals.

Warning and advice.

- Use code from the tutorials as well as from other sources. Do not code yourself well-known procedures (e.g. cross validation or k-means), use an existing implementation.
- The associated notebook contains some hints and several helper functions.
- Be concise. Answers are not expected to be longer than a few sentences (omitting calculations).

Instructions.

- Fill in your names and emails at the top of the document.
- Hand in your report (one per pair of students) by Friday 7^{th} April 11:59 PM.
- Rename your report and notebook as follows:
`FirstnameLastname1_FirstnameLastname1.pdf` and
`FirstnameLastname2_FirstnameLastname2.ipynb`.
For instance, `LaurentOudre_CharlesTruong.pdf`.
- Upload your report (PDF file) and notebook (IPYNB file) using this link: https://www.dropbox.com/request/rmETjrLAH9Li3pf8JvOt.

2 Dual-tone multi-frequency signaling (DTMF)

In the last tutorial, we started designing an algorithm to infer from a sound signal the sequence of symbols encoded with DTMF.

Question 1

Finalize this procedure–in particular, find the best hyperparameters. Describe in 5 to 10 lines your methodology and the calibration procedure (give the hyperparameter values).

Answer 1

To solve the problem at hand, we have implemented a function called *decode_signal*. This function uses the algorithm provided in the lab session to detect the breaking points in the signal. For each window detected (corresponding to two consecutive breaking points), we compute the mean values over the window of the particular 8 frequencies of interest. Since the frequencies are discrete, we take the 8 closest frequencies. We then select the two maximum mean coefficients and check if their values are large enough. If they are, we consult a table to determine which symbol they correspond to. If they are not large enough, we consider the window to be a silence.

To determine the threshold for what constitutes a "large enough" mean value, we conducted experiments on different types of signals. We found that a maximum mean value of over 0.3 generally corresponds to a non-silent window. After obtaining the decoded symbols, we postprocess the results to remove consecutive identical symbols as well as silence windows.

Question 2

What are the two symbolic sequences encoded in the provided signals?

Answer 2

- Sequence 1: [B, 9, 4, B, 3, 8, B, #, 1]
- Sequence 2: [C, D, 1, 1, 2, 6, 3, 9]

3 Wavelet transform for graph signals

Let G be a graph defined a set of n nodes V and a set of edges E. A specific node is denoted by v and a specific edge, by e. The eigenvalues and eigenvectors of the graph Laplacian L are $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ and $u_1, u_2, \ldots, u_n$ respectively.

For a signal $f \in \mathbb{R}^n$, the Graph Wavelet Transform (GWT) of f is $W_f : \{1, \ldots, M\} \times V \longrightarrow \mathbb{R}$:

$$W_f(m, v) := \sum_{l=1}^{n} \hat{g}_m(\lambda_l) \hat{f}_l u_l(v) \tag{1}$$

where $\hat{f} = [\hat{f}_1, \ldots, \hat{f}_n]$ is the Fourier transform of f and $\hat{g}_m$ are M kernel functions. The number M of scales is a user-defined parameter and is set to $M := 9$ in the following. Several designs are available for the $\hat{g}_m$; here, we use the Spectrum Adapted Graph Wavelets (SAGW). Formally, each kernel $\hat{g}_m$ is such that

$$\hat{g}_m(\lambda) := \hat{g}^U(\lambda - am) \quad (0 \leq \lambda \leq \lambda_n) \tag{2}$$

where $a := \lambda_n / (M + 1 - R)$,

$$\hat{g}^U(\lambda) := \frac{1}{2}\left[1 + \cos\left(2\pi\left(\frac{\lambda}{aR} + \frac{1}{2}\right)\right)\right] \mathbb{1}(-Ra \leq \lambda < 0) \tag{3}$$

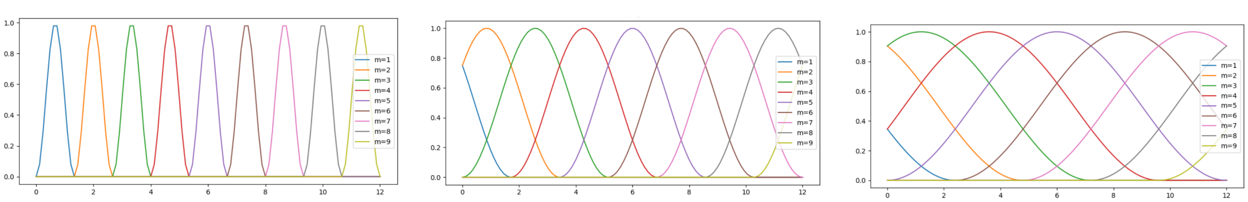
and $R > 0$ is defined by the user.

Question 3

Plot the kernel functions $\hat{g}_m$ for $R = 1$, $R = 3$ and $R = 5$ (take $\lambda_n = 12$) on Figure 1. What is the influence of R?

Answer 3

The parameter R influences the size of the range of values for which the kernels are not null, i.e. the support of the kernel. As R increases, the size of the support also increases. Consequently, as the size of the supports increases, the overlap between the kernels increases as well. It is worth noting that when R is equal to 1, the supports of the kernels are disjoint, whereas they overlap more and more as R increases, for example, for R=3 and R=5.



(a) $R = 1$ (b) $R = 3$ (c) $R = 5$

Figure 1: The SAGW kernels functions

We will study the Molene data set (the one we used in the last tutorial). The signal is the temperature.

Question 4

Construct the graph using the distance matrix and exponential smoothing (use the median heuristics for the bandwidth parameter).

- Remove all stations with missing values in the temperature.
- Choose the minimum threshold so that the network is connected and the average degree is at least 3.
- What is the time where the signal is the least smooth?
- What is the time where the signal is the smoothest?

Answer 4

The stations with missing values are ['ARZAL', 'BATZ', 'BEG_MEIL', 'BREST-GUIPAVAS', 'BRIGNOGAN', 'CAMARET', 'LANDIVISIAU', 'LANNAERO', 'LANVEOC', 'OUESSANT-STIFF', 'PLOUAY-SA', 'PLOUDALMEZEAU', 'PLOUGONVELIN', 'QUIMPER', 'RIEC SUR BELON', 'SIZUN', 'ST NAZAIRE-MONTOIR', 'VANNES-MEUCON'] (18 stations).

The threshold is equal to 0.8315 (mean degree 3.6645).

The signal is the least smooth at 2014-01-10 09:00:00.

The signal is the smoothest at 2014-01-24 19:00:00.

Question 5

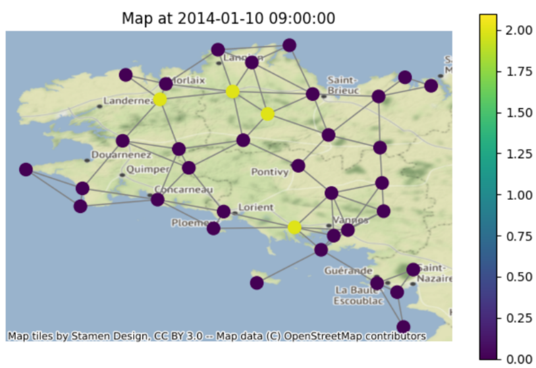
(For the remainder, set $R = 3$ for all wavelet transforms.)

For each node v, the vector $[W_f(1, v), W_f(2, v), \ldots, W_f(M, v)]$ can be used as a vector of features. We can for instance classify nodes into low/medium/high frequency:

- a node is considered low frequency if the scales $m \in \{1, 2, 3\}$ contain most of the energy,
- a node is considered medium frequency if the scales $m \in \{4, 5, 6\}$ contain most of the energy,
- a node is considered high frequency if the scales $m \in \{6, 7, 9\}$ contain most of the energy.

For both signals from the previous question (smoothest and least smooth) as well as the first available timestamp, apply this procedure and display on the map the result (one colour per class).

Answer 5



(a) Least smooth signal

(b) Smoothest signal

(c) First available timestamp

Figure 2: Classification of nodes into low/medium/high frequency

Question 6

Display the average temperature and for each timestamp, adapt the marker colour to the majority class present in the graph (see notebook for more details).

Answer 6

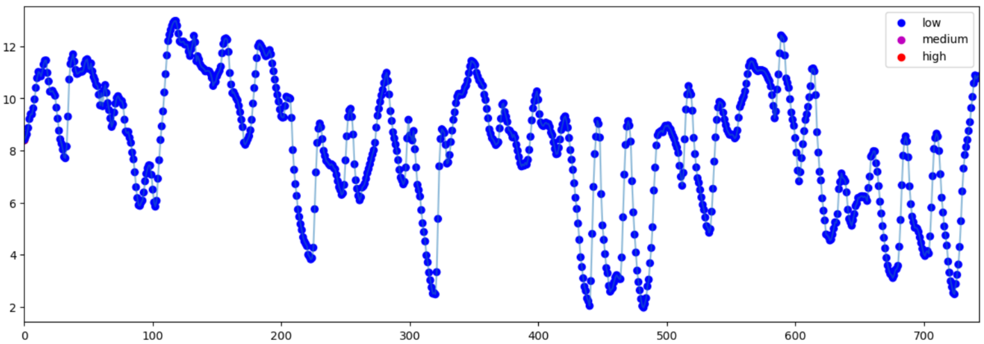


Figure 3: Average temperature. Markers' colours depend on the majority class.

Question 7

The previous graph G only uses spatial information. To take into account the temporal dynamic, we construct a larger graph H as follows: a node is now *a station at a particular time* and is connected to neighbouring stations (with respect to G) and to itself at the previous timestamp and the following timestamp. Notice that the new spatio-temporal graph H is the Cartesian product of the spatial graph G and the temporal graph G' (which is simply a line graph, without loop).

- Express the Laplacian of H using the Laplacian of G and G' (use Kronecker products).
- Express the eigenvalues and eigenvectors of the Laplacian of H using the eigenvalues and eigenvectors of the Laplacian of G and G'.
- Compute the wavelet transform of the temperature signal.
- Classify nodes into low/medium/high frequency and display the same figure as in the previous question.

Answer 7

- We have by definition $H = G \times G'$ thus:

  $$L(H) = L(G) \oplus L(G') = L(G) \otimes I_n + L(G') \otimes I_m$$

  where n is the number of nodes in the spatial graph G and m the number of nodes in the time graph G' and $\otimes$ the Kronecker product.

- - Let $\{\gamma_i\}_{n \times m}$ be the eigenvalues of H and $\{h_i\}_{n \times m}$ the associated eigenvectors.
  - Let $\{\lambda_i\}_n$ be the eigenvalues of G and $\{u_i\}_n$ the associated eigenvectors.
  - Let $\{\mu_i\}_m$ be the eigenvalues of G' and $\{v_i\}_m$ the associated eigenvectors.

  We have the following relations $\forall i \in [0, n-1], \forall j \in [0, m-1]$:

  $$\textbf{Eigenvalues:} \quad \gamma_{m \times i + j} = \lambda_i + \mu_j$$

  $$\textbf{Eigenvectors:} \quad h_{m \times i + j} = u_i \otimes v_j$$

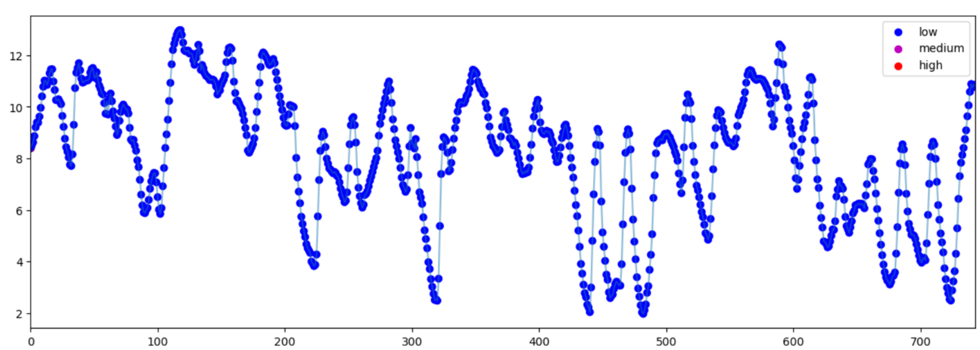


Figure 4: Average temperature. Markers' colours depend on the majority class.