HW 1 Review

(2) $x_n \to \frac{1}{4}$ prove $\frac{1}{x_n} \to 4$

Let $\varepsilon > 0$ $\left|\frac{1}{x_n} - 4\right| = \left|\frac{1 - 4x_n}{x_n}\right| = \frac{4\left|x_n - \frac{1}{4}\right|}{|x_n|} = \frac{4}{|x_n|}\left|x_n - \frac{1}{4}\right|$

bound x_n to keep away from 0

$\left|x_n - \frac{1}{4}\right| < \frac{1}{8} \Rightarrow \frac{1}{8} < x_n < \frac{3}{8}$

$\Rightarrow |x_n| > \frac{1}{8}$ if $n > K\left(\frac{1}{8}\right)$

$< \frac{4}{\frac{1}{8}}\left|x_n - \frac{1}{4}\right| = 32\left|x_n - \frac{1}{4}\right|$

$< 32\left(\frac{\varepsilon}{32}\right) = \varepsilon$

$L(\varepsilon) = \max\left\{K\left(\frac{1}{8}\right), \frac{\varepsilon}{32}\right\}$

Let $L(\varepsilon) = \max\left\{K\left(\frac{1}{8}\right), \frac{\varepsilon}{32}\right\}$

(3) pf by contradiction

suppose $x < 0$ $|x_n - x| = x_n - x \not> |x|$

Functions Monday

Metric

$d: M \times M \to [0, \infty)$

① positive definite

② symmetric

③ $\Delta \leq$ (Triangle Inequality)

$d(x,y) \leq d(x,z) + d(z,y)$

Triangle Inequality backwards BW $\Delta \leq$ Test Question

$$|d(x,z) - d(y,z)| \leq d(x,y)$$

$\Delta \leq$

pf $d(x,z) \leq d(x,y) + d(y,z)$ $\quad$ $d(y,z) \leq d(y,x) + d(x,z)$

$d(x,z) - d(y,z) \leq d(x,y)$ $\quad$ $d(y,z) - d(x,z) \leq d(y,x)$

$d(x,y) = d(y,x)$

$-(d(y,z) - d(x,z)) = d(x,z) - d(y,z) \geq -d(x,y)$

$-d(x,y) \leq d(x,z) - d(y,z) \leq d(x,y)$

$\Rightarrow |d(x,z) - d(y,z)| \leq d(x,y)$

$M, d \to$ metric space

$z \in M$, $R > 0$

Open ball centered at O, radius R $\quad$ open $B(z, R) =$

closed $\{x \in M : d(x,z) < R\}$

$\overline{B}(z,R) = \{ \cdots \leq R\}$

Ex $M \subset \mathbb{R}$ $\quad d(x,y) = |y - x|$

① $d(x,y) \geq 0$ $\quad = 0$ iff $x = y$ $\quad$ p.d

② $d(x,y) = d(y,x)$ $\quad$ Symmetric

③ $d(x,y) \leq d(x,z) + d(z,y)$ $\quad \Delta \leq$

satisfied = metric

$|y - x| \leq |z - x| + |y - z|$

$|y - x| = |y - z + z - x| \leq |y - z| + |z - x|$

$\leq d(z,y) + d(x,z)$

$M = \mathbb{R} \quad B(z, R)$

$B(z,R) = \{x : |x-z| < R\}$
$= \{x : -R < |x-z| < R\} = \{x : z-R < x < z+R\}$
$=$ interval $(z-R, z+R)$

$\overline{B}(z,R) = [z-R, z+R]$

$M = \mathbb{Q}$

① $B(z,R) = \overline{B}(z,R)$?
② is $d(x,y) = |x-y|^2$ a metric in $\mathbb{R}$?
③ is $d(x,y) = |x-y|^{\frac{1}{2}}$ . . . . . ?

counter example

① $R \notin \mathbb{Q} \Rightarrow B(z,R) = \overline{B}(z,R)$

②