$d(x,y) = |x-y|^{\frac{1}{2}}$ is a Metric $\Delta \leq$

w.t.s. $|x-y|^{\frac{1}{2}} \leq |x-z|^{\frac{1}{2}} + |z-y|^{\frac{1}{2}}$

$|x-y| \leq |x-z| + 2|x-z|^{\frac{1}{2}}|zy|^{\frac{1}{2}} + |z-y|$

$|x-y| \leq |x-z| + |z-y| \leq |x-z| + 2|x-z|^{\frac{1}{2}}|z-y|^{\frac{1}{2}} + |z-y|$

$(d(x,y))^{\frac{1}{2}} = \hat{d}(x,y)$ $\quad d: M \times M \Rightarrow \mathbb{R}$

$\boxed{\text{Review functions}}$

(M,d) $\quad A \subset M$ $\quad B(z,R) = \{x \in M : d(x,z) < R\}$

$\overline{B}(z,R) = \{x \in M : d(x,z) \leq R\}$

$|x_n - x| < \varepsilon$

$x_n \in B(x,\varepsilon) = (x-\varepsilon, x+\varepsilon)$ $\quad |x-z| < \delta(z,\varepsilon)$

$x \in B(z, \delta(z,\varepsilon))$

<u>Def</u> $A \subset M$ is said to be $\boxed{\text{open}}$ if $\forall\, a \in A\ \exists\ R_a > 0 \ni B(a, R_a) \subset A$

<u>Def</u> z is said to be an <u>interior</u> point of A if $\exists\ \varepsilon > 0 \ni B(z,\varepsilon) \subset A$

$\therefore$ A is open $\Leftrightarrow$ every $z \in A$ is an interior point of A

<u>Ex</u> $B(z,R)$ is an open set

<u>pf</u> Let $x \in B(z,R)$. Then $d(x,z) < R \Rightarrow d(x,z) = R - s$ $\quad s > 0$

wts $\Rightarrow B(x,s) \subset B(z,R)$

Let $y \in B(x,s)$ $\quad$ wts $y \in B(z,R)$

$d(y,z) \leq d(y,x) + d(x,z) < s + (R-s) = R$

$\Rightarrow y \in B(z,R)$ so $B(x,s) \subset B(z,R)$

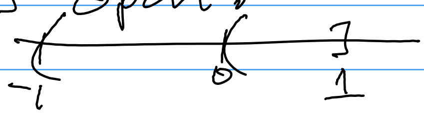
Ex $M = (-1, 1]$ is $[0, 1]$ open

no, let $z = 0$ then $B(z, \varepsilon) = [-\varepsilon, \varepsilon]$ is $\varepsilon < \frac{1}{2}$

$\therefore -\frac{\varepsilon}{2} \in B(z, \varepsilon)\ \forall \varepsilon > 0$ and $-\frac{\varepsilon}{2} \notin [0, 1]$

is $(0, 1]$ open?

yes



$B(1, \varepsilon] = (1-\varepsilon, 1]$

Dfn $\mathring{A}$ = set of all interior points of A (the interior of A)

$\text{Int}(A) = \mathring{A}$

Theorem $A = \mathring{A}$ iff A is open

Furthermore $\mathring{A} = \cup\{U : U \subset A \text{ and } U \text{ is open}\}$

and $\mathring{A}$ is the largest open set contained in A

can assume

Ex $M = \mathbb{R}$ $A = [0, 1] \Rightarrow \mathring{A} = (0, 1)$

if $M = (-1, 1] \Rightarrow \mathring{A} = (0, 1]$

if $M = \mathbb{Q} \Rightarrow \mathring{A} = (0, 1) \cap \mathbb{Q}$

if $M = \mathbb{R}$ and $A = \mathbb{Q}$ then $\mathring{A} = \emptyset$

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(2) Let $S = \{(x,y) \in \mathbb{R}^2 \mid xy > 1\}$ show that S is open

pf Let $(x,y) \in S$ then $xy > 1$ Then $\exists\ \varepsilon > 0 \ni d((x,y), \varepsilon) \subset S$
Let $(x_1,y_1) \in d((x,y),\varepsilon)$. Then $|x-x_1|^2 + |y-y_1|^2 < \varepsilon^2$
and since

(3) Let $A \subset \mathbb{R}$ be open and $B \subset \mathbb{R}^2$ be defined by $B = \{(x,y) \in \mathbb{R}^2 \mid x \in A\}$
show B is open

pf Since A is open $\exists\ \varepsilon_a > 0 \ni \forall a \in A\ |x-a| < \varepsilon \Rightarrow x \in A$.
Then let $(a,y) \in B$ where $a \in A$ then we need to find an $\varepsilon > 0$
$\ni (x,y) \in B((a,y),\varepsilon) \subset \mathbb{R}^2 \Rightarrow (a,y) \in B$. Then $d((a,y),(x_1,y_1))$
$< \varepsilon \Rightarrow |x_1 - a|^2 + |y_1 - y|^2 < \varepsilon^2 \Rightarrow |x_1 - a| < \varepsilon$ so let $\varepsilon = \varepsilon_a$
$\Rightarrow x_1 \in A \Rightarrow (x_1, y_1) \in B$ qed

(5) Let $A \subset \mathbb{R}$ be open and $B \subset \mathbb{R}$ Define $AB = \{xy \in \mathbb{R} \mid x \in A \text{ and } y \in B\}$
is AB nessisarily open
No. If B is closed then AB is not nessesarily open.
If B is open then AB is open

109 (1) Let $S = \{(x,y) \in \mathbb{R}^2 \mid xy \geq 1\}$. Find $\text{in}(S)$ or $\mathring{S}$
$\mathring{S} = \{(x,y) \in \mathbb{R}^2 \mid xy > 1\}$

pf Let $(x,y) \in \mathring{S}$ then we must find $\varepsilon > 0 \ni B((x,y),\varepsilon) \subset \mathring{S}$.
Let $(x_1,y_1) \in \mathring{S}$ then $d((x,y),(x_1,y_1)) = |x_1 - x|^2 + |y_1 - y|^2 < \varepsilon^2$
then $|x_1 - x| < \varepsilon$ and $|y_1 - y| < \varepsilon$