Test Friday

M, d , $B(z,R)$, $\bar{B}(z,R)$, $\mathring{A}$ interior , A open every point is interior

HW 3 p108 $A \subset \mathbb{R}$, $B \subset \mathbb{R}^2$, $B = \{(x,y) \in \mathbb{R}^2 \mid x \in A\}$ show B is open
$d((x_1, x_2), (y_1, y_2))$

A open $\Rightarrow \forall a \in A \; \exists \varepsilon_a > 0 \ni |x-a| < \varepsilon_a \Rightarrow x \in A$
$d(x,a) < \varepsilon_a$

w.t.s. B is open so let $(a,x) \in B$, and $a \in A$ so $\exists \varepsilon_a > 0 \ni$
$|y-a| < \varepsilon_a \Rightarrow y \in A$, need to find $\varepsilon > 0 \ni$
$(z,w) \in B((a,x),\varepsilon) \subset \mathbb{R}^2 \Rightarrow (z,w) \in B$
$d((z,w),(a,x)) < \varepsilon \Rightarrow |z-a|^2 + |w-x|^2 < \varepsilon^2$
wts that if $\varepsilon > 0$ is small enough then $(z,w) \in B$
$\therefore$ need to show $z \in A$ w always $\in \mathbb{R}$
to show $z \in A$ we need to show $|z-a| < \varepsilon_a$
But $|z-a|^2 + |w-x|^2 < \varepsilon^2 \Rightarrow |z-a| < \varepsilon$ so take $\varepsilon = \varepsilon_a$

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prop $\mathring{A}$ is the largest open subset of A

prop Intersection of a finite number of open sets is open and the union of any family of open sets is open

pf assume $A_1, \ldots A_n$ open w.t.s. $\bigcap_{i=1}^{n} A_i$ is open
Let $z \in \bigcap_{i=1}^{n} A_i \Rightarrow z \in A_i \; \forall i \in 1, \ldots n \Rightarrow \forall i \; \exists \varepsilon_i > 0 \ni$
$B(z, \varepsilon_i) \subset A_i$, $i \in 1, \ldots, n$. Let $\varepsilon = \min(\varepsilon_i) \Rightarrow B(z,\varepsilon) \subset A_i$
$\Rightarrow B(z,\varepsilon) \subset \bigcap_{i=1}^{n} A_i$

Ex $A_n = (-\frac{1}{n}, \frac{1}{n}) \Rightarrow \bigcap_{n=1}^{\infty} A_n = \{0\}$ which is not open

$\mathring{A}$ is the largest subset of A

In particular $\mathring{A} = \bigcup_{\substack{U \subset A \\ U \text{ open}}} U$ always $\emptyset$

pf Suppose $U \subset A$ is open. Let

wts. $U \subset \mathring{A}$. Let $u \in U$. U is open so $\exists\ \varepsilon > 0 \ni B(u, \varepsilon) \subset \underset{\subset A}{U}$

By defn $u \in \mathring{A} \Rightarrow U \subset \mathring{A}$

finally we need to show $\mathring{A}$ is open.

Let $z \in \mathring{A} \Rightarrow B(z, \varepsilon) \subset A$. But $B(z, \varepsilon)$ is open so

$B(z, \varepsilon) \subset \mathring{A}$ by the first part of proof.