

check grade dist

MA 426

2nd Ed

pop tests not graded just for checking

HW not graded should do anyway

$x, y \in \mathbb{R}$ distance between $x, y = |x - y| = d(x, y)$

Ch 2 Set M d = distance or metric on M
 $d: M \times M \rightarrow [0, \infty)$

3 properties of a metric

① $d(x, y) \geq 0 \quad \forall x, y \in M$ and $d(x, y) = 0 \Leftrightarrow x = y$
positive definite \rightarrow

② $d(x, y) = d(y, x)$
symmetric

③ $d(x, y) \leq d(x, z) + d(z, y)$
Triangle inequality

Ex:

$$M \subset \mathbb{R}$$

$$d(x, y) = |x - y|$$

$$x_n \in M \quad (x_n)_{n=1}^{\infty} \quad x \in M$$

$$x_n \rightarrow x$$

We say $x_n \rightarrow x$ if $\forall \varepsilon > 0 \exists k(\varepsilon) > 0 \exists d(x_n, x) < \varepsilon$ if $n > k(\varepsilon)$

$$x_n = \frac{n}{n+1} \quad \text{for } n \in \mathbb{N}$$

$$x_n \rightarrow 1$$

Let $\varepsilon > 0$ then wts $\exists K(\varepsilon) > 0 \ni d(x_n, x) < \varepsilon$ is $n > K(\varepsilon)$

$$d(x_n, x) = d(x_n, 1) = \left| \frac{n}{n+1} - 1 \right| = \left| \frac{-1}{n+1} \right| = \frac{1}{n+1} < \varepsilon$$

Let $K(\varepsilon) = \frac{1}{\varepsilon} - 1$ or $\frac{1}{\varepsilon}$ reasonable ε